63

$P = 2\pi\sqrt{L/g}$ $\quad g = 32\,ft/s$

$P = 2\pi (L/g)^{1/2}$

$\frac{dP}{dL} = 2\pi \cdot \frac{1}{2}(L/g)^{-\frac{1}{2}} \cdot \frac{1}{g} = P'(L)$

$P'(2) = \frac{\pi}{g\sqrt{2/g}}$

64 $V(r) = \frac{4}{3}\pi r^3$

$V(r) = (4\pi r^2) \cdot \frac{r}{3}$

$A(r) = 4\pi r^2 \quad r^2 = \frac{A}{4\pi} \quad r = \sqrt{\frac{A}{4\pi}}$

$V = A \cdot \frac{1}{3}\sqrt{\frac{A}{4\pi}} = \frac{A^{\frac{3}{2}}}{3\sqrt{4\pi}}$

$V'(A) = \frac{\cancel{3}}{2}\frac{A^{\frac{1}{2}}}{\cancel{3}(4\pi)^{\frac{1}{2}}} = \frac{1}{2}\sqrt{\frac{A}{4\pi}} = \frac{1}{4}\sqrt{\frac{A}{\pi}}$

65 $x^2 + y^2 = 1 \Rightarrow y = \pm\sqrt{1-x^2}$

$2x + 2yy' = 0$

$y' = -\frac{x}{y}$

$y' = \frac{-x}{\sqrt{1-x^2}} = -2 \qquad y' = \frac{x}{\sqrt{1-x^2}} = -2$

$-x = -2(1-x^2)^{1/2} \qquad x = -2(1-x^2)^{1/2}$

$x^2 = 4(1-x^2) = 4 - 4x^2 \qquad x^2 = 4(1-x^2)$

$5x^2 = 4$

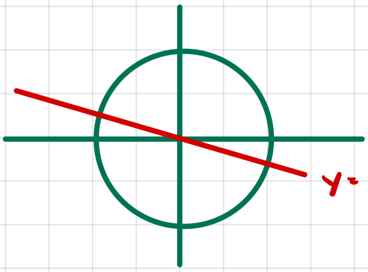
$x^2 = 4/5$

$x = \pm 2/\sqrt{5} = \pm\frac{2\sqrt{5}}{5}$

66 $x^2 + y^2 = 1$, slope tangent $= 3$

$2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$

$y' = 3 \Rightarrow -x = 3y$



$(-3y)^2 + y^2 = 1$

$10y^2 = 1 \Rightarrow y^2 = 1/10 \Rightarrow y = \pm 1/\sqrt{10}$

$y = 1/\sqrt{10} \Rightarrow x = -3/\sqrt{10}$

$y = -1/\sqrt{10} \Rightarrow x = 3/\sqrt{10}$

67 Line through $P(18,0)$, normal to tangent to $y = x^2$ at $Q(a, a^2)$

line through P: $y = m(x-18)$

tangent line at Q: $y - a^2 = 2a(x-a)$

$m = -1/2a \Rightarrow y = \frac{18-x}{2a}$ line through P

But (a, a^2) is on the line through P:

$a^2 = \frac{18-a}{2a} \Rightarrow 2a^3 = 18 - a \Rightarrow 2a^3 + a - 18 = 0$

by inspection, $a = 2$ is a solution $\Rightarrow (a-2)$ is a factor of the cubic

$$\begin{array}{r|l}
 & 2a^2 + 4a + 9 \\
a-2 & 2a^3 + \quad\;\; + a - 18 \\
 & 2a^3 - 4a^2 \\
\hline
 & +4a^2 + a \\
 & 4a^2 - 8a \\
\hline
 & 9a - 18 \\
 & 9a - 18 \\
\hline
 & 0
\end{array}$$

$2a^3 + a - 18 = (a-2)(2a^2 + 4a + 9)$

$\Delta = 16 - 4 \cdot 2 \cdot 9 < 0$

$\Rightarrow$ no further real solutions

$\Rightarrow (2,4)$ is Q, tangent slope 4

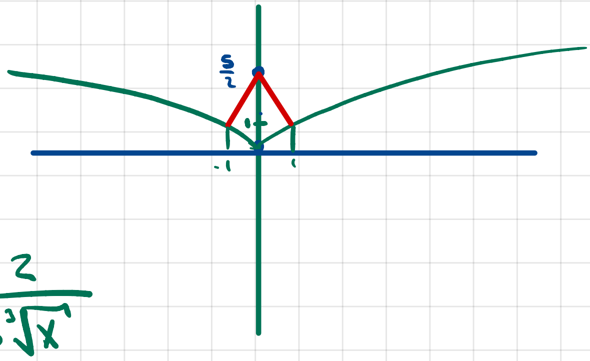
line through P has slope $-1/4$, it has equation:

$y = -\frac{1}{4}(x-18) \Rightarrow x + 4y = 18$

69 $P(0, 5/2)$

$y = x^{2/3}$

$= \sqrt[3]{x^2}$



$y_0 - \frac{5}{2} = -\frac{3\sqrt[3]{x_0}}{2} x_0$

$2x_0^{2/3} - 5 = -3x_0^{4/3}$

$3x_0^{4/3} + 2x_0^{2/3} - 5 = 0$

$3m^2 + 2m - 5 = 0$

$\Delta = 4 + 60 = 64$

$m = \frac{-2 \pm 8}{6} \to 1$, $\to -\frac{5}{3}$

$y' = \frac{2}{3} x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

$y'' = \frac{2}{3}\left(-\frac{1}{3}\right) x^{-\frac{4}{3}} = \frac{-2}{9\sqrt[3]{x^4}} < 0$

slope of normal line $l = -\frac{3\sqrt[3]{x}}{2}$

point of intersection: $(x_0, x_0^{2/3})$

$m = x_0^{2/3} > 0 \Rightarrow x_0^{2/3} = 1 \Rightarrow (x_0^2)^{\frac{1}{3}} = 1$

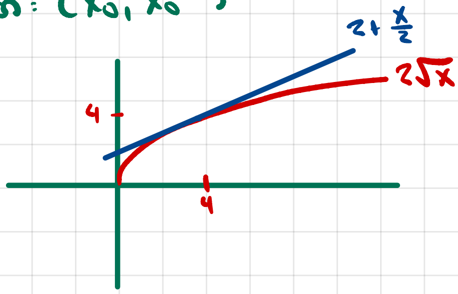
$x_0^2 = 1 \Rightarrow x_0 = \pm 1$

$x_0 = 1 \Rightarrow y_0 = 1^{2/3} = 1$

$x_0 = -1 \Rightarrow y_0 = ((-1)^2)^{1/3} = 1$

51 $y = 2\sqrt{x}$ $x \geq 0$

$x = 4$



$y' = 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} = x^{-\frac{1}{2}}$

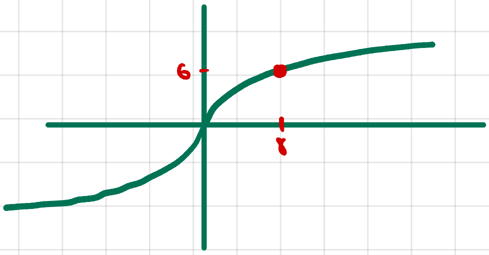
$y'(4) = 1/2$

$y - 4 = \frac{1}{2}(x - 4) \Rightarrow y = 2 + x/2$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2\sqrt{x} = 0 = f(0) \Rightarrow$ continuous on the right of 0

$\lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^+} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h} = \lim \frac{2\sqrt{h}}{h} = \lim_{h \to 0^+} \frac{2}{\sqrt{h}} = +\infty$

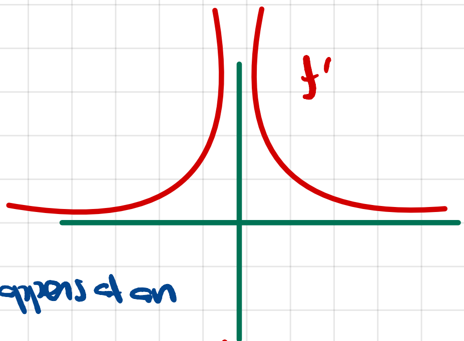
52 $y = 3\sqrt[3]{x}$ $x = 8$ $f(8) = 6$



$f(-x) = 3\sqrt[3]{-x} = -3\sqrt[3]{x} = -f(x) \Rightarrow$ odd

$f' = 3 \cdot \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{x^2}} > 0$

$f'' = -\frac{2}{3} x^{-\frac{5}{3}} = -\frac{2}{3\sqrt[3]{x^5}}$



$f''(-1) = \frac{-2}{3(-1)} = \frac{2}{3}$

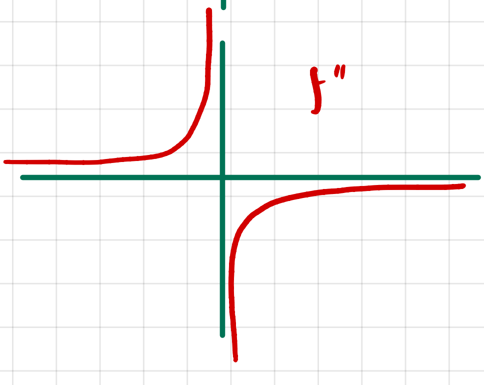
$f''(1) = \frac{-2}{3 \cdot 1} = -\frac{2}{3}$

However f'' is not continuous on $[-1, 1]$ so we can't apply the intermediate value theorem and say $f''(c) = 0$ for some $c \in [-1, 1]$. The change in sign of f'' happens at an infinite discontinuity

$f'(0)$ does not exist, so it cannot be continuous at 0

Furthermore,

$\lim_{x \to 0} f'(x) = +\infty$, so this is an infinite discontinuity



35 $g(t) = \left(1+\frac{1}{t}\right)^2 (3t^2+1)^{1/2}$

$$g'(t) = 2(1+t^{-1})(-1)t^{-2}(3t^2+1)^{1/2} + (1+t^{-1})^2(1/2)(3t^2+1)^{-1/2}\cdot 6t$$

$$= \frac{-2(1+t^{-1})\sqrt{3t^2+1}}{t^2} + \frac{3t(1+t^{-1})^2}{\sqrt{3t^2+1}}$$

$$= \frac{-2(1+t^{-1})(3t^2+1) + 3t^3(1+t^{-1})^2}{t^2\sqrt{3t^2+1}}$$

$$= \frac{\cancel{-6t^2} - 2 - 6t - 2t^{-1} + 3t^3 \cancel{+6t^2} + 3t}{t^2\sqrt{3t^2+1}}$$

$$= \frac{3t^3 - 3t - 2t^{-1} - 2}{t^2\sqrt{3t^2+1}} = \frac{3t^4 - 3t^2 - 2t - 2}{t^3\sqrt{3t^2+1}}$$

37 $f(x) = \frac{2x-1}{(3x+4)^5}$

$$f'(x) = \frac{2(3x+4)^5 - (2x-1)\cdot 5(3x+4)^4\cdot 3}{(3x+4)^{10}}$$

$$= \cancel{(3x+4)^5}\left[2 - \frac{(2x-1)\cdot 15}{3x+4}\right] \Big/ (3x+4)^{\cancel{10}\,5}$$

$$= \left[\frac{6x+8-30x+15}{3x+4}\right] \Big/ (3x+4)^5 = \frac{23-24x}{(3x+4)^6}$$

41 $h(y) = \frac{\sqrt{1+y}+\sqrt{1-y}}{\sqrt[3]{y^5}} = (\sqrt{1+y}+\sqrt{1-y})\cdot y^{\frac{5}{3}}$

$$h'(y) = \frac{\left[\frac{1}{2}(1+y)^{-1/2} + \frac{1}{2}(1-y)^{-1/2}(-1)\right]y^{5/3} - \left[(1+y)^{1/2}+(1-y)^{1/2}\right]\cdot\frac{5}{3}y^{2/3}}{y^{10/3}}$$

$$= \frac{(1+y)^{1/2}(1-y) - (1-y)^{1/2}(1+y)}{2(1+y)(1-y)}\cdot y^{-\frac{5}{3}} - \frac{5}{3}y^{-\frac{8}{3}}\left[\frac{\left[(1+y)^{1/2}+(1-y)^{1/2}\right]\cdot 2(1+y)(1-y)}{2(1+y)(1-y)}\right]$$

$$= y^{-\frac{5}{3}}\left[\frac{\frac{3y(1+y)^{1/2}(1-y) - 3y(1+y)(1-y)^{1/2}}{3y} - \frac{5}{3y}\cdot 2(1+y)(1-y)\left[(1+y)^{1/2}+(1-y)^{1/2}\right]}{2(1+y)(1-y)}\right]$$

$$= y^{-\frac{5}{3}}\left[\frac{3y(1+y)^{1/2}(1-y) - 3y(1+y)(1-y)^{1/2} - 10(1+y)^{1/2} - 10(1-y)^{1/2} + 10y^2(1+y)^{1/2} + 10y^2(1-y)^{1/2}}{6y(1+y)(1-y)}\right]$$

$$= \left[7y^2(1+y)^{1/2} + 7y^2(1-y)^{1/2} + 3y(1+y)^{1/2} - 3y(1-y)^{1/2} - 10(1+y)^{1/2} - 10(1-y)^{1/2}\right] \Big/ 6y^{8/3}(1+y)(1-y)$$

$$= \left[7y^2\left[(1+y)^{1/2}+(1-y)^{1/2}\right] + 3y\left[(1+y)^{1/2}-(1-y)^{1/2}\right] - 10\left[(1+y)^{1/2}+(1-y)^{1/2}\right]\right] \Big/ 6y^{8/3}(1+y)(1-y)$$

$$= \frac{\left[(1+y)^{1/2}+(1-y)^{1/2}\right](7y^2-10) + \left[(1+y)^{1/2}-(1-y)^{1/2}\right]\cdot 3y}{6y^{8/3}(1-y^2)}$$

* differs from solution; may be incorrect or may be correct in different form