9.2 Polar coordinates

1

a) $(1, \pi/4)$

$x = \cos \pi/4 = \sqrt{2}/2$

$y = \sin \pi/4 = \sqrt{2}/2$

b) $(-2, 2\pi/3)$

$x = -2\cos 2\pi/3 = -2 \cdot (-1/2) = 1$

$y = -2\sin 2\pi/3 = -2(\sqrt{3}/2) = -\sqrt{3}$

c) $(1, -\pi/3)$

$x = 1 \cdot (1/2) = 1/2$

$y = 1 \cdot (-\sqrt{3}/2) = -\sqrt{3}/2$

d) $(3, 3\pi/2)$

$x = 3 \cdot 0 = 0$

$y = 3 \cdot (-1) = -3$

e) $(2, -\pi/4)$

$x = 2 \cdot \sqrt{2}/2 = \sqrt{2}$

$y = 2(-\sqrt{2}/2) = -\sqrt{2}$

f) $(-2, -\frac{7\pi}{6})$

$x = -2\cos(-7\pi/6) = -2(-\sqrt{3}/2) = \sqrt{3}$

$y = -2 \cdot 1/2 = -1$

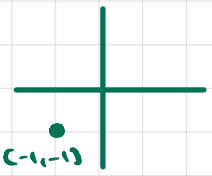
g) $(2, 5\pi/6)$

$x = 2 \cdot (-\sqrt{3}/2) = -\sqrt{3}$

$y = 2 \cdot 1/2 = 1$

2

a) $(-1, -1)$



$1 + 1 = r^2 \Rightarrow r = \sqrt{2}$

because $x < 0$, $\theta = \pi + \tan^{-1}(\frac{-1}{-1})$

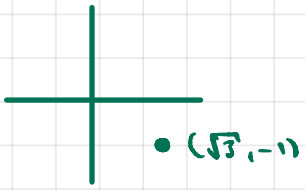
$= \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$(\sqrt{2}, 5\pi/4)$

$(-\sqrt{2}, \pi/4)$

Alternatively, $\theta = \pi/4$, $r = -\sqrt{2}$

b) $(\sqrt{3}, -1)$



$3 + 1 = r^2 \Rightarrow r = 2$

$\theta = \tan^{-1}(-\sqrt{3}/3)$

$\Rightarrow \theta = -\pi/6$

or, $\theta = 5\pi/6$, $r = -2$

$(2, -\pi/6)$

$(-2, 5\pi/6)$

c) $(2, 2)$

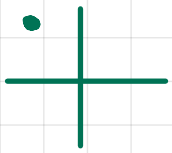
$r = \sqrt{8}$ or $r = -\sqrt{8}$

$\theta = \pi/4$ $\theta = \frac{5\pi}{4}$

$(\sqrt{8}, \pi/4)$

$(-\sqrt{8}, 5\pi/4)$

d) $(-1, \sqrt{3})$



$r = \sqrt{4} = 2$

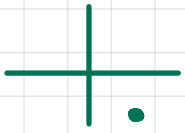
$\theta = \pi + \tan^{-1}(-\sqrt{3}) = \pi - \pi/3 = 2\pi/3$

or $\theta = -\frac{\pi}{3}$ $r = -2$

$(2, 2\pi/3)$

$(-2, -\pi/3)$

e) $(\sqrt{2}, -\sqrt{2})$



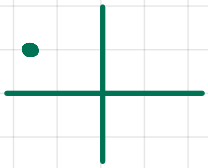
$r = \sqrt{4} = 2$ or $r = -\sqrt{2}$

$\theta = -\pi/4$ $\theta = \frac{3\pi}{4}$

$(2, -\pi/4)$

$(-2, 3\pi/4)$

f) $(-3, \sqrt{3})$



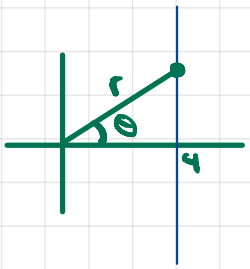
$r = (9+3)^{1/2}$
$= \sqrt{12} = 2\sqrt{3}$

$(2\sqrt{3}, 5\pi/6)$
$(-2\sqrt{3}, -\pi/6)$

$\theta = \pi + \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \pi - \frac{\pi}{6} = 5\pi/6$

or $\theta = -\frac{\pi}{6}$ $r = -2\sqrt{3}$

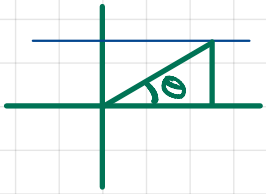
3 $x = 4$



$\cos\theta = \frac{4}{r}$

$r = \frac{4}{\cos\theta}$

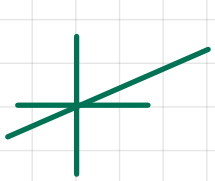
4 $y = 6$



$\sin\theta = \frac{6}{r}$ $\theta \in (0, \pi)$

$r = \frac{6}{\sin\theta}$ $\theta \in (0, \pi)$

5 $x = 3y$

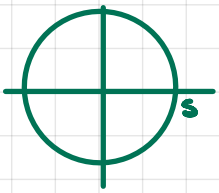


$r\cos\theta = 3r\sin\theta$

$r(3\sin\theta - \cos\theta) = 0 \Rightarrow r = 0$ or $\tan\theta = \frac{1}{3}$ and $r \in \mathbb{R}$

$\tan^{-1}(1/3) = \theta$

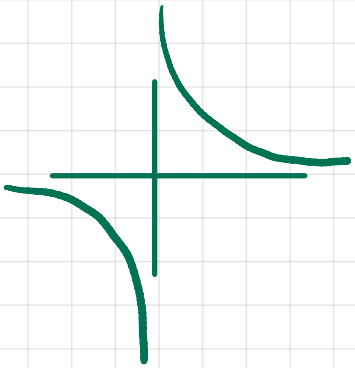
6 $x^2 + y^2 = 25$



$r^2\cos^2\theta + r^2\sin^2\theta = 25$

$r^2 = 25 \Rightarrow r = \pm 5$

7 $xy = 1$



$r\cos\theta \, r\sin\theta = 1$
$r^2 \sin\theta\cos\theta = 1$
$r^2 = \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$
$r^2 = \sec\theta \cdot \csc\theta$

8 $x^2 - y^2 = 1$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta} = \frac{1}{\cos 2\theta} = \sec 2\theta$

9 $y = x^2$

$r\sin\theta = r^2\cos^2\theta$

$r(r\cos^2\theta - \sin\theta) = 0$

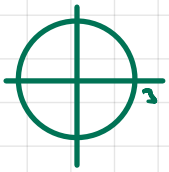
$r = \frac{\sin\theta}{\cos\theta}\frac{1}{\cos\theta} = \tan\theta \cdot \csc\theta$

10 $x + y = 4$

$r(\cos\theta + \sin\theta) = 4$

$r = \frac{4}{\cos\theta + \sin\theta}$

11 $r = 3$

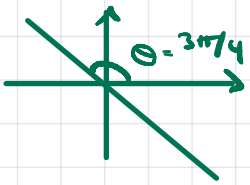


$x = 3\cos\theta$ $\quad x^2 = 9\cos^2\theta = 9(1 - \sin^2\theta) = 9 - y^2 \Rightarrow x^2 + y^2 = 9$

$y = 3\sin\theta$ $\quad y^2 = 9\sin^2\theta$

$9 = x^2 + y^2$

12 $\theta = \frac{3\pi}{4}$



$x = r(-\sqrt{2}/2)$

$y = r(\sqrt{2}/2)$

$-\frac{x \cdot 2}{\sqrt{2}} = \frac{y \cdot 2}{\sqrt{2}} \Rightarrow y = -x$

13 $r = -5\cos\theta$

$x = r\cos\theta$

$x^2 + y^2 = r^2 = r(-5\cos\theta) = -5x \Rightarrow x^2 + y^2 + 5x = 0$

14 $r = \sin(2\theta) = 2\sin\theta\cos\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$r^3 = r \cdot r \cdot 2\sin\theta\cos\theta$

$= 2 \cdot xy$

$x^2 + y^2 = r^2$

$(x^2 + y^2)^{3/2} = r^3 = 2xy$

$(x^2 + y^2)^3 = 4x^2y^2$

15 $r = 1 - \cos 2\theta = 1 - (\cos^2\theta - \sin^2\theta) = 1 - \cos^2\theta + \sin^2\theta = 2\sin^2\theta$

$r^2 = r \cdot 2\sin^2\theta$

$r^3 = 2 \cdot r\sin\theta \cdot r\sin\theta = 2y^2$

$(x^2 + y^2)^{3/2} = r^3 = 2y^2$

$(x^2 + y^2)^3 = 4y^4$

16 $r = 2 + \sin\theta$