PSet 9

SB-9 $\int e^x(1+e^x)^{-1/3}dx = \int (1+u)^{-1/3}\cdot du = \frac{(1+u)^{2/3}}{2/3} + C = \frac{3}{2}(1+e^x)^{2/3} + C$

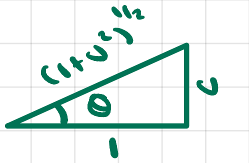
$u = 1+e^x \quad du = e^x dx$

SB-11 $\int \sec^2 9x\, dx = \frac{\tan 9x}{9} + C$

SB-13 $\int \frac{x^2}{1+x^6}dx = \frac{1}{3}\int \frac{du}{1+u^2} = \frac{1}{3}\int \frac{\cancel{\sec^2}\theta\, d\theta}{\cancel{\sec^2}\theta} = \frac{1}{3}\theta + C = \frac{1}{3}\tan^{-1}u + C$

$= \frac{1}{3}\tan^{-1}x^3 + C$

$u = x^3 \quad du = 3x^2dx$



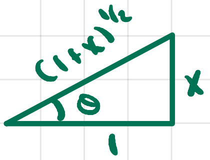
$1+u^2 = 1+\tan^2\theta = \sec^2\theta$

$u = \tan\theta \quad du =$

$\sin\theta = \frac{u}{(1+u^2)^{1/2}}$

$\cos\theta = \frac{1}{(1+u^2)^{1/2}}$

SB-16 $\int_{-1}^{1} \frac{\tan^{-1}x\, dx}{1+x^2} = \int_{-\pi/4}^{\pi/4} \frac{\theta\cdot\sec^2\theta\, d\theta}{\sec^2\theta} = \int_{-\pi/4}^{\pi/4}\theta\, d\theta = \frac{\theta^2}{2}\Big|_{-\pi/4}^{\pi/4} = \frac{\pi^2}{32} - \frac{(-\pi/4)^2}{2} = 0$



$x = \tan\theta \quad dx = \sec^2\theta\, d\theta, \quad -1 = \tan\theta \Rightarrow \theta = -\frac{\pi}{4}, \quad 1 = \tan\theta \Rightarrow \theta = \pi/4$

$1+x^2 = 1+\tan^2\theta = \sec^2\theta$

$\tan^{-1}(\tan\theta) = \theta$

SC-5 $\int \sin^3x\cos^2x\, dx = \int \sin x(1-\cos^2x)\cos^2x\, dx = \int(1-u^2)u^2(-du) = -\int(u^2-u^4)du$

$u = \cos x \quad du = -\sin x\, dx \qquad = -\left[\frac{u^3}{3} - \frac{u^5}{5}\right] + C = \frac{\cos^5x}{5} - \frac{\cos^3x}{3} + C$

SC-7 $\int \sin^2(4x)\cos^2(4x)dx = \frac{1}{4}\int(2\sin 4x\cos 4x)^2dx = \frac{1}{4}\int \sin^2 8x\, dx$

$= \frac{1}{4}\cdot\frac{1}{2}\int(1-\cos 16x)dx = \frac{1}{8}\left(x - \frac{\sin 16x}{16}\right) + C$

5C-9 $\int \sin^3 x \sec^2 x\,dx = \int \sin x(1-\cos^2 x)\cdot\frac{1}{\cos^2 x}\,dx = \underbrace{\int \frac{\sin x}{\cos^2 x}\,dx} - \int \sin x\,dx = \cos^{-1}x + \cos x + C$

$u = \cos x$

$du = -\sin x\,dx$

$= \int \frac{-du}{u^2} = -\frac{u^{-1}}{-1} = u^{-1}$

$= \cos^{-1}x$

5C-11 $\int \sin x \cos 2x\,dx = \int \sin x(\cos^2 x - \sin^2 x)\,dx = \int \cos^2 x \sin x\,dx - \int \sin^3 x\,dx$

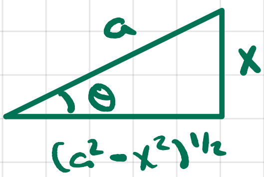
$\int \cos^2 x \sin x\,dx = -\frac{\cos^3 x}{3}$

$\int \sin^3 x\,dx = \int \sin x(1-\cos^2 x)\,dx = \int \sin x\,dx - \int \cos^2 x \sin x\,dx = -\cos x + \frac{\cos^3 x}{3}$

$\int \sin x \cos 2x\,dx = \frac{-\cos^3 x}{3} - \left(-\cos x + \frac{\cos^3 x}{3}\right) + C = \cos x - \frac{2}{3}\cos^3 x + C$

5D-1 $\int \frac{dx}{(a^2-x^2)^{3/2}} = \int \frac{\cancel{a}\cancel{\cos}\theta\,d\theta}{a^{\cancel{3}\,2}\cos^{\cancel{3}\,2}\theta} = \frac{1}{a^2}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{a^2}\int \sec^2\theta\,d\theta$

$= \frac{1}{a^2}\cdot\tan\theta + C$



$\tan\theta = \frac{x}{(a^2-x^2)^{1/2}}$

$= \frac{x}{a^2(a^2-x^2)^{1/2}} + C$

$x = a\sin\theta \qquad dx = a\cos\theta\,d\theta$

$a^2 - x^2 = a^2 - a^2\sin^2\theta = a^2(1-\sin^2\theta)$

$= a^2\cos^2\theta$

5D-2 $\int \frac{x^3}{\sqrt{a^2-x^2}}\,dx = \int \frac{a^3\sin^3\theta\cdot\cancel{a}\cancel{\cos}\theta\,d\theta}{\cancel{a}\cancel{\cos}\theta} = a^3\int \sin^3\theta\,d\theta = a^3\left(-\cos\theta + \frac{\cos^3\theta}{3}\right) + C$

$x = a\sin\theta$

$\cos\theta = \frac{(a^2-x^2)^{1/2}}{a}$

$= a^3\left[-\frac{(a^2-x^2)^{1/2}}{a} + \frac{(a^2-x^2)^{3/2}}{3a}\right] + C$

$= a^2(a^2-x^2)^{1/2}\left[\frac{a^2-x^2}{3} - 1\right] + C$

5D-7 $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = \int \frac{\cancel{a}\tan\theta}{\cancel{a^2}\sec^{\cancel{2}}\theta}\cdot \cancel{a}\tan\theta\cancel{\sec\theta}\,d\theta = \int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \frac{(\sec^2\theta-1)}{\sec\theta}\,d\theta$

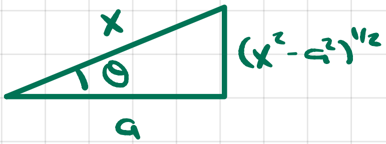
$x = a\sec\theta \Rightarrow \cos\theta = \frac{a}{x}$

$= \int \sec\theta\,d\theta - \int \cos\theta\,d\theta$

$dx = a\tan\theta\sec\theta\,d\theta$

$= \ln|\sec\theta+\tan\theta| - \sin\theta + C$

$= \ln\left|\frac{x}{a} + \frac{(x^2-a^2)^{1/2}}{a}\right| - \frac{(x^2-a^2)^{1/2}}{x} + C$



$x^2-a^2 = a^2(\sec^2\theta-1) = a^2\tan^2\theta$

5D-10 $\displaystyle\int \frac{dx}{(x^2+4x+13)^{3/2}} = \int \frac{dx}{[x^2+4x+4+9]^{3/2}} = \int \frac{dx}{[(x+2)^2+3^2]^{3/2}}$

$x+2 = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$\displaystyle = \int \frac{\cancel{3}\cancel{\sec^2}\theta\,d\theta}{3^{\cancel{3}2}\sec^{\cancel{3}}\theta} = \frac{1}{9}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{9}\int \cos\theta\,d\theta$

$(x+2)^2+3^2 = 3^2\tan^2\theta + 3^2$

$= 3^2(\tan^2\theta+1) = 3^2\sec^2\theta$

$[3^2\sec^2\theta]^{3/2} = (3\sec\theta)^3$

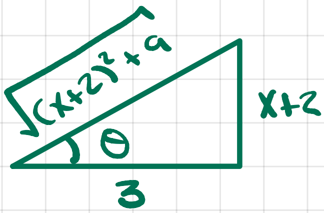
$= \frac{1}{9}\sin\theta + C$

$= \frac{1}{9}\cdot\frac{x+2}{\sqrt{(x+2)^2+9}} + C$

$\int \cos^2\theta = \frac{1}{2}\int(1+\cos 2\theta)\,d\theta$

$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]$

$= \frac{1}{2}[\theta + \sin\theta\cos\theta]$



$\sin\theta = \frac{x+2}{\sqrt{(x+2)^2+9}}$