π radians = 180 degrees

$1 \text{ rad} = \frac{180}{\pi}$ degrees

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$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim \frac{1-\cos\theta}{\theta} = 0$

$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - \pi/2) = \cos(\pi/2 - x) = \sin x$

$\sin(x + 2\pi) = \sin x$
$\cos(x + 2\pi) = \cos x$

1 radian = unit of measure of angle based on radius of circle: one radius wrapped around the circumference makes angle of 1 rad.

$2\pi r$ circumference $\to 2\pi$ rad $\to 360°$
$\Rightarrow r$ circum. $\to 1$ rad $\to 180/\pi\,°$

$D_x \sin(x) = \cos(x)$
$D_x \cos x = -\sin x$

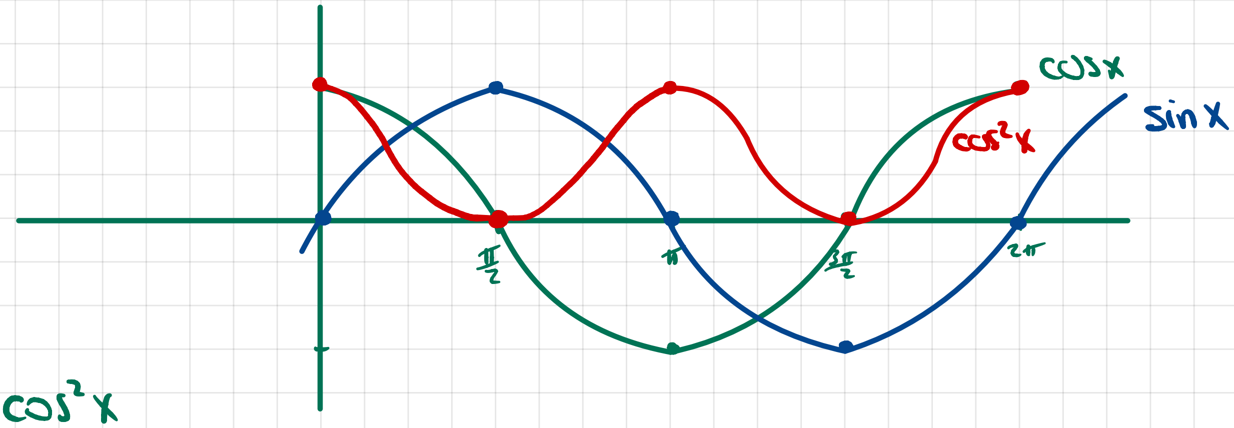
$\tan x = \frac{\sin x}{\cos x} \qquad \tan' x = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\cot x = \frac{\cos x}{\sin x} \qquad \cot' x = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$

$\csc x = \frac{1}{\sin x} \qquad \csc' x = -1 \cdot \sin^{-2} x \cos x = -\frac{\cos x}{\sin^2 x} = -\cos x \cdot \csc^2 x = -\cot x \cdot \csc x$

$\sec x = \frac{1}{\cos x} \qquad \sec' x = -1 \cdot \cos^{-2} x \cdot (-\sin x) = \frac{\sin x}{\cos^2 x} = \sin x \cdot \sec^2 x = \sec x \cdot \tan x$



$y = \cos^2 x$

$y' = 2\cos x(-\sin x) = -2\cos x \sin x$

$y'' = -2(-\sin x)\sin x - 2\cos x \cos x$
$= 2\sin^2 x - 2\cos^2 x$
$= 2(\sin^2 x - \cos^2 x)$

$[0, \frac{\pi}{2}]$

$y'' = 0 \Rightarrow \sin^2 x - \cos^2 x = 0 \Rightarrow \sin^2 x = \cos^2 x$

At $x = 0.5$,

$y(0.5) = \cos^2(1/2)$

$y'(0.5) = -2\underbrace{\cos(1/2)}_{>0}\underbrace{\sin(1/2)}_{>0} < 0$

$\underbrace{y - \cos^2(0.5)}_{\approx 0.7702} = \underbrace{-2\cos(0.5)\sin(0.5)}_{\approx -0.8415}(x - 0.5)$

Ex 8 $D_x \sin^2 3x \cos^4 5x = 2\sin(3x)\cdot\cos(3x)\cdot 3\cdot\cos^4(5x) + \sin^2(3x)\cdot 4\cdot\cos^3(5x)\cdot(-\sin(5x))\cdot 5$
$= 6\sin(3x)\cos(3x)\cos^4(5x) - 20\sin^2(3x)\cos^3(5x)\sin(5x)$

Ex 9 $f(x) = \cos\sqrt{x}$

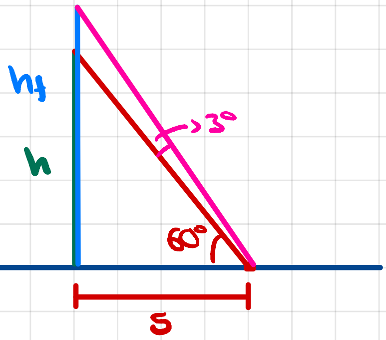
$f'(x) = -\sin(\sqrt{x})\cdot\frac{1}{2}x^{-\frac{1}{2}}$

Ex 10 $y = \sin^2(2x-1)^{3/2} = [\sin(2x-1)^{3/2}]^2$

$y' = 2\sin[(2x-1)^{3/2}]\cdot\cos[(2x-1)^{3/2}]\cdot\frac{3}{2}(2x-1)^{\frac{1}{2}}\cdot 2 =$

Ex 11 $D_x \tan 2x^3 = \frac{1}{\cos^2(2x^3)}\cdot 6x^2$

Ex12



Strategy:

Find initial height ⇒ $\tan\frac{\pi}{3} = \frac{h}{s} \Rightarrow h = s\tan(\frac{\pi}{3}) =$

final height ⇒ $\tan \frac{\pi}{180}\cdot 63 = h_f/s \quad h_f = s\tan\frac{63\pi}{180}$

speed $= \frac{h_f - h}{1s}$

alternatively

h(t) means h changes w/ time.

r $\quad dh/dt = \frac{dh}{d\theta}\cdot\frac{d\theta}{dt}$

we know $h(\theta) = s\tan\theta$ and $\theta'(t)$ at the t we care about, when $\theta = \frac{\pi}{3}$ and $\theta' = \frac{\pi}{60}$.

$\frac{dh}{dt} = s\sec^2(\frac{\pi}{3})\cdot\frac{\pi}{60} = s\cdot 2^2\cdot\frac{\pi}{60} = \frac{\pi}{3}$ m/s

Note the calculation is done at some unspecified t, but there is a specific t; it's just that we know θ' at that t.

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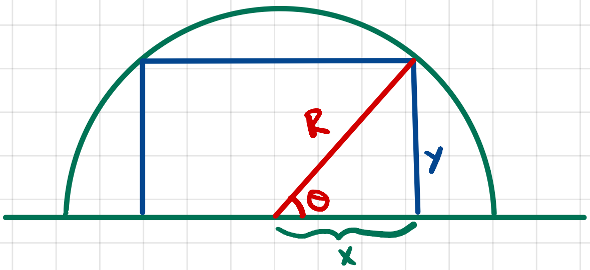

Solution 1

$A(x,y) = 2xy$, $x^2 + y^2 = R^2$ $\Rightarrow A(x) = 2x\sqrt{R^2 - x^2} \Rightarrow A'(x) = 2(R^2 - x^2)^{1/2} + \cancel{2}x\cdot\frac{1}{\cancel{2}}(R^2 - x^2)^{-\frac{1}{2}}(-2x) = 0$

$\Rightarrow 2x^2(R^2 - x^2)^{-1/2} = 2(R^2 - x^2)^{1/2} \Rightarrow x^2 = R^2 - x^2 \Rightarrow x^2 = R^2/2 \Rightarrow x = R/\sqrt{2}$

$\Rightarrow y = R/\sqrt{2} \Rightarrow A(R/\sqrt{2}, R/\sqrt{2}) = R^2$

Solution 2

$A(x,y) = 2xy \Rightarrow A(\theta) = 2R^2\sin\theta\cos\theta \qquad \theta\in[0,\frac{\pi}{2}]$

$x = R\cdot\cos\theta$

$y = R\cdot\sin\theta \qquad A'(\theta) = 2R^2(\cos^2\theta - \sin^2\theta) = 0$

$\theta\in[0,\frac{\pi}{2}] \qquad \Rightarrow \cos^2\theta = \sin^2\theta \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = 1 \Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}$

$f = \sinh(x) = \dfrac{e^x - e^{-x}}{2}$

$f' = \frac{1}{2}(e^x + e^{-x}) = \cosh(x)$

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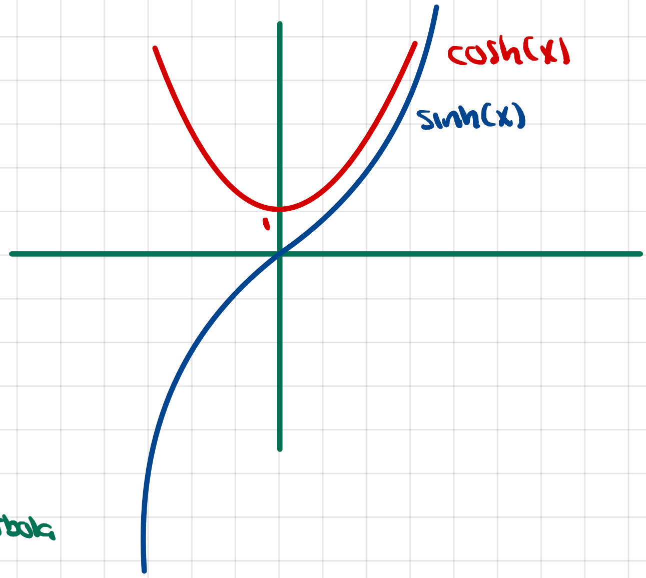
$\cosh^2(x) - \sinh^2(x) = 1$



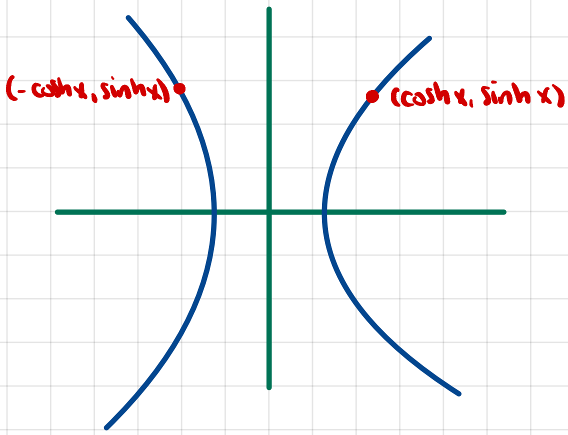
why are they called hyperbolic?

$u = \cosh x$
$v = \sinh x$ $\Rightarrow u^2 - v^2 = 1$, a hyperbola

$u = \cos x$
$v = \sin x$ $\Rightarrow u^2 + v^2 = 1$, a circle

why are they trigonometric?

$x^2 - y^2 = 1$



sinh and cosh have same relation to the (right side) hyperbola above as cos and sin have to the unit circle.

note that cosh > 0 ⇒ for any x, (cosh, sinh) can never have a negative first coordinate.

$\sinh(x+y)$