Pset 10

5E-2 $\displaystyle\int \frac{x\,dx}{(x-2)(x+3)} = \int \left[\frac{A}{x-2} + \frac{B}{x+3}\right] dx$

$$\begin{aligned} x &= A(x+3) + B(x-2) \\ &= Ax + 3A + Bx - 2B \\ &= x(A+B) + (3A-2B) \end{aligned}$$

$\Rightarrow A+B=1 \Rightarrow A=1-B$

$3A-2B=0 \Rightarrow 3(1-B)-2B=0 \Rightarrow 3-3B-2B=0$

$5B=3 \Rightarrow B=\frac{3}{5}$

$\Rightarrow A = \frac{5-3}{5} = \frac{2}{5}$

$$\int \left[\frac{2}{5(x-2)} + \frac{3}{5(x+3)}\right] dx$$

$$= \frac{2}{5}\ln|x-2| + \frac{3}{5}\ln|x+3| + C$$

$$= \ln\left(|x-2|^{2/5}\,|x+3|^{3/5}\right) + C$$

5E-3 $\displaystyle\int \frac{x}{(x^2-4)(x+3)}dx = \int \left[\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x+3}\right] dx$

$A(x-2)(x+3) + B(x+2)(x+3) + C(x+2)(x-2) = x$

$x=2 \Rightarrow B\cdot 4\cdot 5 = 2 \Rightarrow B = \frac{1}{10}$

$x=-2 \Rightarrow A\cdot(-4)(1) = -2 \Rightarrow A = 1/2$

$x=-3 \Rightarrow C(-1)(-5) = -3 \Rightarrow C = 3/5$

$\Rightarrow \displaystyle\int \left[\frac{1}{2(x+2)} + \frac{1}{10(x-2)} + \frac{3}{5(x+3)}\right] dx = \frac{1}{2}\ln|x+2| + \frac{1}{10}\ln|x-2| + \frac{3}{5}\ln|x+3| + C$

5E-5 $\displaystyle\int \frac{3x+2}{x(x+1)^2}dx = \int \left[\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right] dx$

$A(x+1)^2 + Bx(x+1) + Cx = 3x+2$

$x=0 \Rightarrow A\cdot 1^2 = 2 \Rightarrow A=2$

$x=-1 \Rightarrow C(-1) = -3+2 = -1 \Rightarrow C=1$

$x=1 \Rightarrow 2\cdot 2^2 + B\cdot 1\cdot 2 + 1\cdot 1 = 3+2$

$8 + 2B + 1 = 5$

$2B = -4 \Rightarrow B=-2$

$\Rightarrow \displaystyle\int \left[\frac{2}{x} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right] dx = 2\ln|x| - 2\ln|x+1| - (x+1)^{-1} + C$

SE-6 $$\int \frac{2x-9}{(x^2+9)(x+2)}\,dx = \int\left[\frac{A}{x+2}+\frac{Bx+C}{x^2+9}\right]dx$$

$$2x-9 = A(x^2+9)+(Bx+C)(x+2)$$

$$= Ax^2+9A+Bx^2+2Bx+Cx+2C$$

$$= x^2(A+B)+x(2B+C)+(9A+2C)$$

$\Rightarrow A+B=0 \quad \Rightarrow A=-B \Rightarrow B=-A$

$2B+C=2 \quad \Rightarrow C=2-2B=2-2(-A)=2+2A$

$9A+2C=-9$

$\Rightarrow 9A+2(2+2A)=-9$

$9A+4+4A=-9$

$13A=-13 \Rightarrow A=-1$

$B=1$

$C=2-2\cdot 1=0$

$$\int\left[\frac{-1}{x+2}+\frac{x}{x^2+9}\right]dx = -\ln|x+2|+\frac{1}{2}\ln|x^2+9|+C \checkmark$$

$u=x^2+9$

$du=2x\,dx$

$$\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|x^2+9|$$

SE-10

h) $$\int\frac{(x^2+1)\,dx}{x^2+2x+2} = \int\left[1-\underbrace{\frac{(2x+1)}{x^2+2x+2}}\right]dx = \int dx-\int\frac{2u\,du}{u^2}+\underbrace{\int\frac{1}{u^2+1}du}$$

polyn. division

$$\begin{array}{r} 1 \\ x^2+2x+2\,\overline{)\,x^2 \qquad +1} \\ x^2+2x+2 \\ \hline -2x-1 \end{array}$$

$$\frac{x^2+1}{x^2+2x+2}=\frac{-2x-1}{x^2+2x+2}+1$$

$u=x+1$

$\Rightarrow 2x+2=2u,$

$2x+1=2u-1$

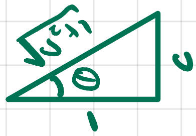
$u^2=x^2+2x+1$

$du=dx$

$= x-\ln|u^2|+\tan^{-1}u$

$= x-\ln|x^2+2x+1|+\tan^{-1}|x+1|+C \checkmark$

$u=\tan\theta$



$du=\sec^2\theta\,d\theta$

$u^2+1=\tan^2\theta+1=\sec^2\theta$

$$=\int\frac{\sec^2\theta\,d\theta}{\sec^2\theta}$$

$$=\int d\theta=\theta+C$$

$=\tan^{-1}u$

$=\tan^{-1}(x+1)$

SF1-A

a) $\int x^a \ln x\,dx \quad (a \neq -1)$

$u = x^a \qquad dv = \ln x\,dx$

$du = a x^{a-1} \qquad v = x\ln x - x$

$$\int x^a \ln x\,dx = x^a(x\ln x - x) - \underbrace{\int (x\ln x - x)a x^{a-1}}$$

$$a\int (x^a \ln x - x^a)\,dx$$

$$= a\int x^a \ln x - \frac{a\,x^{a+1}}{a+1}$$

$(uv)' = u'v + uv'$

$uv' = (uv)' - u'v$

$\int uv'\,dx = uv - \int u'v\,dx$

$$= x^a(x\ln x - x) - a\int x^a \ln x\,dx + \frac{a\,x^{a+1}}{a+1}$$

$$\Rightarrow (1+a)\int x^a \ln x\,dx = x^{a+1}(\ln x - 1) + x^{a+1}\frac{a}{a+1}$$

$$\int x^a \ln x\,dx = \frac{1}{1+a}x^{a+1}\left[\ln x - 1 + \frac{a}{a+1}\right] + C$$

$$= \frac{1}{1+a}x^{a+1}\left[\ln x - \frac{1}{a+1}\right] + C \checkmark$$

SF-2

a) $\int x^n e^{ax}\,dx = \frac{e^{ax}}{a}x^n - \int \frac{e^{ax}}{a} n x^{n-1}\,dx = \frac{e^{ax}x^n}{a} - \frac{n}{a}\int e^{ax}x^{n-1}\,dx \checkmark$

$u = x^n \qquad dv = e^{ax}\,dx$

$du = n x^{n-1} \qquad v = \frac{e^{ax}}{a}$

b) $\int x^2 e^x\,dx = \frac{e^x x^2}{1} - \frac{2}{1}\int e^x x\,dx = e^x x^2 - 2\underbrace{\int e^x x\,dx}_{\substack{n=1\\a=1}}$

$n = 2$

$a = 1$

$$= \frac{e^x x}{1} - \frac{1}{1}\int e^x\,dx$$

$$= e^x x - e^x + C = e^x(x-1) + C$$

$$= e^x x^2 - 2e^x(x-1) + C$$

$$= e^x(x^2 - 2(x-1)) + C \checkmark$$

SF-3 $\int \sin^{-1}(4x)\,dx = \int \csc(4x)\,dx = \frac{1}{4}\int \csc(u)\,du$

$u = 4x \quad du = 4dx$

$= \frac{1}{4}\left(-\ln|\csc u + \cot u| + C\right.$

$= \frac{1}{4}\left[-\ln|\csc 4x + \cot 4x| + C\right.$

Alternatively,

$dv = dx$

$v = x$

$u = \sin^{-1}4x$,

$du = \frac{4dx}{\sqrt{1-16x^2}}$

$$\int \sin^{-1}(4x)\,dx = \sin^{-1}(4x)x - 4\int \frac{x}{\sqrt{1-16x^2}}\,dx$$

$$\int \frac{x\,dx}{\sqrt{1-16x^2}} = \int \frac{\frac{1}{4}\sin\theta \cdot \frac{\cos\theta\, d\theta}{4}}{\cos\theta} = \int \frac{\sin\theta \cancel{\cos\theta}\, d\theta}{16\cancel{\cos\theta}}$$

$x = \frac{1}{4}\sin\theta$

$= \frac{1}{16}\int \sin\theta\, d\theta = -\frac{1}{16}\cos\theta + C$

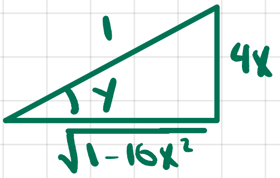
$dx = \frac{\cos\theta}{4}\,d\theta$

$$\sqrt{1-16x^2} = \sqrt{1-16\cdot\frac{1}{16}\sin^2\theta} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$$

$$\Rightarrow \int \sin^{-1}(4x)\,dx = \sin^{-1}(4x)x - 4\left[-\frac{1}{16}\sqrt{1-16x^2}\right] + C$$

$$= \sin^{-1}(4x)x + \frac{1}{4}\sqrt{1-16x^2} + C$$ ✓

what is $\frac{d}{dx}\sin^{-1}4x$?



$\sin y = 4x$

$\sin^{-1}4x = y = f(x)$

$\cos y = \sqrt{1-16x^2}$

$\cos y\, y' = 4$

$y' = \frac{4}{\cos y} = \frac{4}{\sqrt{1-16x^2}}$