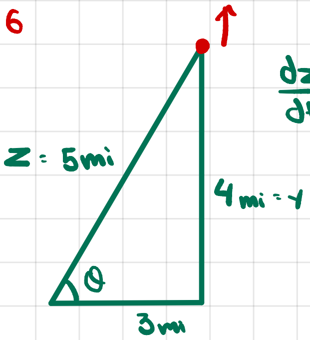
Ex 6



$$\frac{dz}{dt} = 5000 \text{ mi/h at } t=0$$
$$= \frac{5000\text{mi}}{3600\text{s}} = \frac{25}{18} \text{ mi/s}$$

$y = 4 + vt$ $\quad dy/dt = v$

$z = 5 + 5000t$ $\quad dz/dt = 5000$

$\cos\theta = \frac{3}{5}$

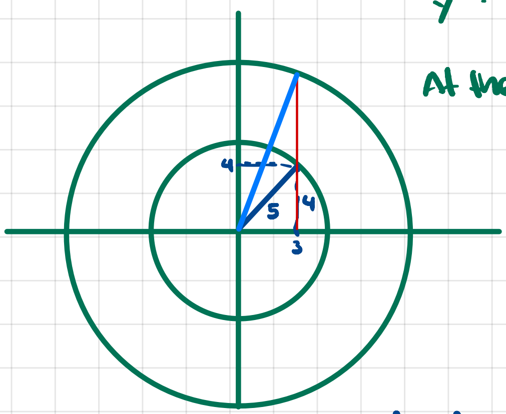
$\sin^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$

$\sin\theta = \frac{4}{5}$

$9 + y^2 = z^2$

$2y \cdot y' = 2zz'$

$y' = \frac{zz'}{y}$

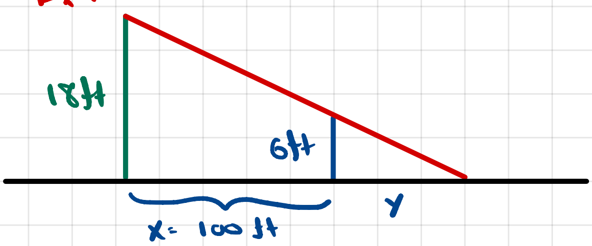


At the time we're interested in, $z = 5, z' = 5000, y = 4$

$y' = \frac{25000}{4} = 6250$ mi/h

y and z are increasing, x is fixed.

Ex 7



$\frac{dx}{dt} = 8$ ft/s

$\frac{x+y}{18} = \frac{y}{6}$

$x = 2y \Rightarrow x = 100 \Rightarrow y = 50$ at t_0

$x' = 2y'$

$y' = \frac{x'}{2}$

$y'(t_0) = \frac{8}{2} = 4$ ft/s

so x grows 8ft/s, y grows 4 ft/s

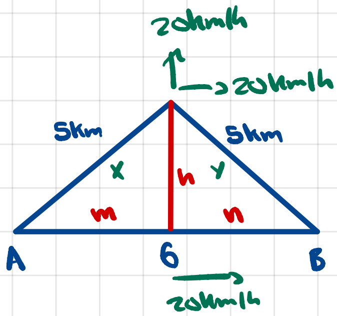
$z = x + y$

$\frac{z}{18} = \frac{z-x}{6} \Rightarrow z = 3z - 3x \Rightarrow 2z = 3x$

$2\frac{dz}{dt} = 3\frac{dx}{dt}$

$\frac{dz}{dt}(t_0) = \frac{3}{2} \cdot 8 = 12$ ft/s

Ex 8



$25 = h^2 + m^2$

$25 = h^2 + n^2$

$m + n = 6$

$25 = 25 - n^2 + m^2$

$0 = -\cancel{n^2} + 36 - 12n + \cancel{n^2}$

$n = 3 \Rightarrow m = 3 \Rightarrow h^2 = 25 - 9 = 16$

$h = 4$

$\frac{dx}{dt} = 28$ km/h $\quad \frac{dy}{dt} = 4$ km/h

$x^2 = h^2 + m^2$

$y^2 = h^2 + n^2$

$m + n = 6$

$2yy' = 2hh' + 2nn'$

$5 \cdot 4 = 4h' + 3n'$

$2xx' = 2hh' + 2mm'$

$xx' = hh' + mm'$

$5 \cdot 28 = 4h' + 3m'$

$20 = 4h' + 3n'$

$140 = 4h' + 3m'$

$m' + n' = 0$

$\Rightarrow 20 = 4h' + 3n'$

$140 = 4h' - 3n'$

$160 = 8h'$

$h' = 20$ km/h

$n' = (20 - 4(20))/3 = -60/3 = -20$ km/h

$m' = (140 - 4 \cdot 20)/3 = 60/3 = 20$ km/h

3.9 Problems

1 $x^2 - y^2 = 1$

$2x - 2y \cdot y' = 0 \Rightarrow y' = \frac{x}{y}$

$y = \pm\sqrt{x^2-1}$

$y' = \frac{1}{2} \cdot \frac{1}{\pm(x^2-1)^{1/2}} \cdot 2x = \frac{x}{\pm(x^2-1)^{1/2}} \Rightarrow y = +\sqrt{x^2-1} \Rightarrow y' = \frac{x}{\sqrt{x^2-1}} = \frac{x}{y}$

$y = -\sqrt{x^2-1} \Rightarrow y' = \frac{x}{-\sqrt{x^2-1}} = \frac{x}{y}$

2 $xy = 1$

$y + xy' = 0 \Rightarrow y' = \frac{-y}{x}$

$y = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2} = \frac{1}{x} \cdot \left(-\frac{1}{x}\right) = \frac{-y}{x}$

3 $16x^2 + 25y^2 = 400$

$32x + 50yy' = 0 \Rightarrow y' = \frac{-32x}{50y} = -\frac{16x}{25y}$

$25y^2 = 400 - 16x^2$

$y^2 = 16 - \frac{16}{25}x^2 \qquad y = \pm\sqrt{16 - \frac{16}{25}x^2}$

$y' = \frac{1}{2(\pm y)} \cdot \left(-\frac{32}{25}x\right) = -\frac{16}{25}\frac{x}{y}$

4 $x^3 + y^3 = 1$

$3x^2 + 3y^2y' = 0 \Rightarrow y' = -\frac{x^2}{y^2}$

$y^3 = 1 - x^3 \Rightarrow y = \sqrt[3]{1-x^3} \Rightarrow y' = \frac{1}{3y^2}(-3x^2) = -\frac{x^2}{y^2}$

7 $x^{2/3} + y^{2/3} = 1$

$y = (1 - x^{2/3})^{3/2} = \sqrt[3]{(1 - \sqrt[3]{x^2})^2}$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0 \Rightarrow y' = \frac{-1}{x^{1/3}} \cdot y^{1/3} = -\sqrt[3]{y/x}$

10 $x^5 + y^5 = 5x^2y^2$

$5x^4 + 5y^4y' = 10xy^2 + 10x^2yy'$

$y'(5y^4 - 10x^2y) = 10xy^2 - 5x^4$

$y' = \frac{2xy^2 - x^4}{y^4 - 2x^2y}$

13 $\cos(x+y) = \sin x \sin y$

$-\sin(x+y)(1+y') = \cos x \sin y + \sin x \cos y$

$= \sin(x+y)$

$1 + y' = -1$

$y' = -2$

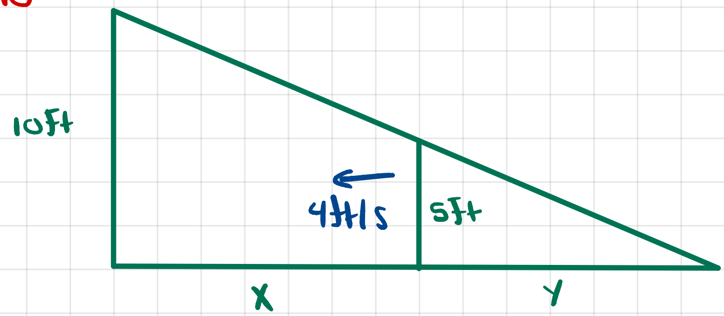
14 $xy = e^{-xy}$

$y + xy' = e^{-xy}(-y - xy') = -ye^{-xy} - xy'e^{-xy}$

$y'x(1 + e^{-xy}) = -y(e^{-xy} - 1)$

$y' = -\frac{y}{x}\frac{(1+e^{-xy})}{(e^{-xy}-1)} = \frac{y(1+e^{-xy})}{x(1-e^{-xy})}$

40



$$\frac{x+y}{10} = \frac{y}{5} \Rightarrow x+y = 2y \Rightarrow x = y$$

$$\frac{dx}{dt} = -4$$

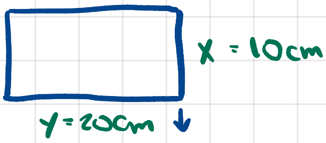
$\Rightarrow$ tip of shadow moving -8 ft/s
shadow size (y) decreasing at -4 ft/s

$$x' = y' \Rightarrow y' = -4$$

$$z = x + y$$

$$z' = x' + y' \Rightarrow z' = -8 \text{ ft/s}$$

41



$$\frac{dx}{dt} = 0.5 \text{ cm/s}$$

$$A'(t) = 4x$$
$$A'(10) = 40$$

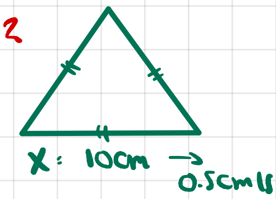
$$y = 2x \text{ cm}$$

$$A = y \cdot x = 2x^2 \text{ cm}^2$$

$$A' = 2 \cdot 2x \cdot x' = 2 \cdot 2 \cdot 10 \text{ cm} \cdot 0.5 \text{ cm/s} = 20 \text{ cm}^2/\text{s}$$

Note that 20cm/s is the rate change assume y is always double x.

42



$$A = \frac{hx}{2}$$

$$h^2 + (x/2)^2 = x^2$$
$$h^2 + x^2/4 = x^2$$
$$2hh' + 2x/4 = 2x$$
$$hh' + x/4 = x$$
$$h' = \frac{3x}{4h}$$

initially $h^2 + 25 = 100$
$$h = \sqrt{75} = 5\sqrt{3}$$

$$h' = \frac{3 \cdot 10}{4 \cdot 5\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

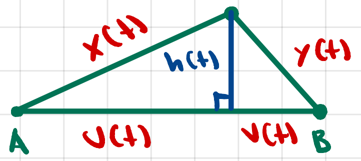
$$\frac{dA}{dt} = \frac{1}{2}\frac{dh}{dx}\frac{dx}{dt}x + \frac{1}{2}h\frac{dx}{dt}$$

$$\frac{1}{2}\frac{dx}{dt}\left[\frac{dh}{dx}x + h\right]$$

with $x = 10$, $h = 5\sqrt{3}$, $dx/dt = 0.5$, $\frac{dh}{dx} = \frac{\sqrt{3}}{2}$

$$\frac{dA}{dt} = \frac{1}{2} \cdot \frac{1}{2}\left[\frac{\sqrt{3}}{2} \cdot 10 + 5\sqrt{3}\right] = \frac{1}{4}\left(\frac{10\sqrt{3} + 10\sqrt{3}}{2}\right) =$$

64



At $t = t_0$, $x(t_0) = 10.4$ $\quad x'(t_0) = 19.2$
$y(t_0) = 5$ $\quad y'(t_0) = -0.6$

$$\begin{aligned} x^2 &= h^2 + u^2 \\ y^2 &= h^2 + v^2 \end{aligned} \Rightarrow \begin{aligned} h^2 &= x^2 - u^2 \\ h^2 &= y^2 - v^2 \end{aligned} \Rightarrow x^2 - u^2 = y^2 - v^2$$

$$v = 12.6 - u \Rightarrow x^2 - u^2 = y^2 - (12.6 - u)^2$$

$$\Rightarrow \text{At } t_0,\ (10.4)^2 - \cancel{u^2} = 5^2 - 12.6^2 + 2 \cdot 12.6u - \cancel{u^2} \Rightarrow u = 9.6 \Rightarrow v = 3$$

$$2xx' - 2uu' = 2yy' - 2(12.6 - u)(-u')$$

$$xx' - \cancel{u}u' = yy' + 12.6u' - \cancel{u}u'$$

$$u' = \frac{xx' - yy'}{12.6}$$

At t_0, $u'(t_0) = \dfrac{10.4 \times 19.2 - 5 \cdot (-0.6)}{12.6} \approx 16.0857$

At this point we know the ship is moving east, towards B horizontally.

At t_0, $h^2 = 10.4^2 - 9.6^2$

Also, $hh' = xx' - uu' \Rightarrow h' = \frac{1}{h}(xx' - uu')$