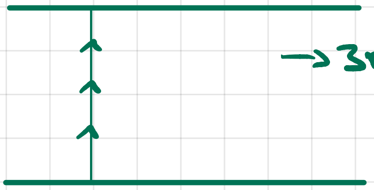
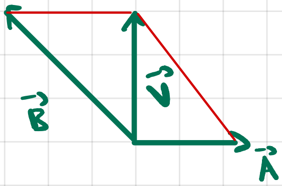
Vectors

1

a)



$\vec{A}\cdot\vec{A} = a_1^2 + b_1^2 = |\vec{A}|^2$



$\vec{V} = \vec{A} + \vec{B}$

$\vec{V}\cdot\vec{A} = 0 = |\vec{A}|^2 + \vec{A}\cdot\vec{B} \rightsquigarrow \vec{A}\cdot\vec{B} = -9 = |\vec{A}||\vec{B}|\cos\theta$

$\Rightarrow -9 = 3\cdot 6\cos\theta \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{4\pi}{3}$

b) $\vec{V}\cdot\vec{A} = 0 = |\vec{A}|^2 + \vec{A}\cdot\vec{B}$

$= 36 + \vec{A}\cdot\vec{B} \Rightarrow \vec{A}\cdot\vec{B} = -36 = 6\cdot 3\cdot\cos\theta \Rightarrow \cos\theta = -2$

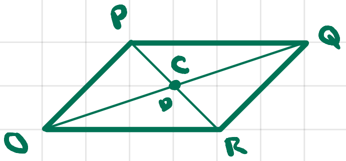
$\Rightarrow$ not possible

2 $\vec{v} = \langle 2,3\rangle$

$|\vec{v}| = \sqrt{4+9} = \sqrt{13}$

$\hat{v} = \langle 2/\sqrt{13}, 3/\sqrt{13}\rangle$

3



Let C be halfway between O and Q $\Rightarrow \vec{OC} = \frac{1}{2}\vec{OQ}$

" D be " " P " R $\Rightarrow \vec{OD} = \frac{1}{2}(\vec{OP} + \vec{OR})$

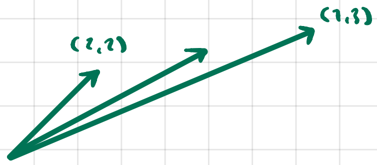
But $\vec{OQ} = \vec{OR} + \vec{OP} \Rightarrow C = \frac{1}{2}(\vec{OR} + \vec{OP}) \Rightarrow$ D and C are same point.

4

$\vec{A} = \langle a_1, a_2\rangle$

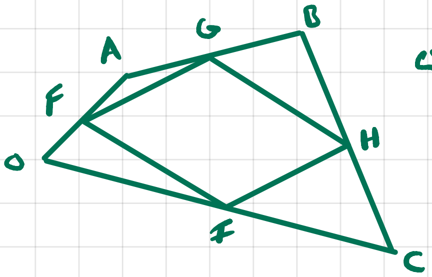
$\vec{B} = \langle b_1, b_2\rangle$

$\frac{1}{2}(\vec{A} + \vec{B}) = \left\langle \frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}\right\rangle$

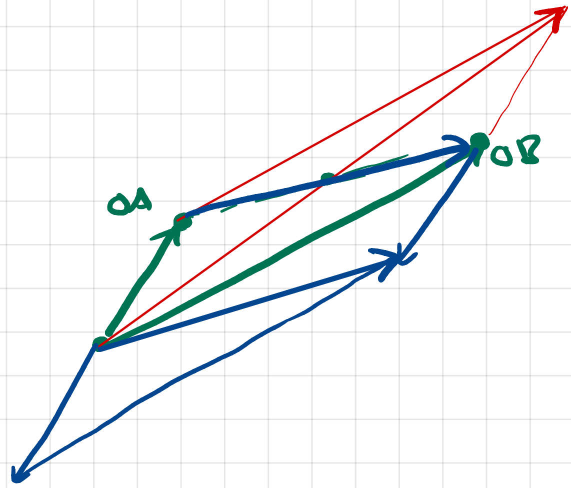


$\langle 4.5, 2.5\rangle$

4



we want to show $\vec{FG} = \vec{IH}$ and $\vec{GH} = \vec{FI}$.



$$\vec{FG} = \vec{OG} - \vec{OF} = \frac{1}{2}\vec{OB} + \cancel{\frac{1}{2}\vec{OA}} - \cancel{\frac{1}{2}\vec{OA}} = \frac{1}{2}\vec{OB}$$

$$\vec{OF} = \frac{1}{2}\vec{OA}$$

$$\vec{OG} = \frac{1}{2}(\vec{OB} + \vec{OA})$$

$$\vec{IH} = \vec{OH} - \vec{OI} = \frac{1}{2}\vec{OB} + \frac{1}{2}\vec{OC} - \frac{1}{2}\vec{OC} = \frac{1}{2}\vec{OB}$$

$$\vec{OH} = \frac{1}{2}(\vec{OB} + \vec{OC})$$

$$OI = \frac{1}{2}\vec{OC}$$

Since two opposite sides are equal and parallel, the midpoints form a parallelogram.