Problem Set 9

4D-1

a) $\vec{F} = \langle 2y, x \rangle \qquad C: x^2+y^2=1 \qquad \Rightarrow \operatorname{curl}\vec{F} = 1-2 = -1$

Green's theorem: $\int_C \vec{F}\cdot d\vec{r} = \iint_R \operatorname{curl}\vec{F}\, dA = -1\ \text{Area}(\text{circle } R) = -\pi$

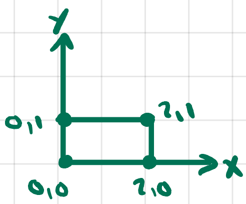
Direct calculation: $\int_C \vec{F}\cdot d\vec{r} = \int_C 2y\,dx + x\,dy = \int_0^{2\pi} 2\sin\theta(-\sin\theta\, d\theta) + \cos\theta \cdot \cos\theta\, d\theta$

$y = \sin\theta \quad dy = \cos\theta\, d\theta \qquad x = \cos\theta \quad dx = -\sin\theta\, d\theta$

$= \int_0^{2\pi} (\cos^2\theta - 2\sin^2\theta)\, d\theta = \int_0^{2\pi} (1 - 3\sin^2\theta)\, d\theta = \int_0^{2\pi} (1 - 3\cdot\frac{1}{2}(1-\cos 2\theta))\, d\theta$

$= \int_0^{2\pi} (-\frac{1}{2} + \frac{3}{2}\cos 2\theta)\, d\theta = \left[-\frac{1}{2}\theta + \frac{3}{2}\cdot\frac{\sin 2\theta}{2}\right]_0^{2\pi} = (-\pi + 0 - (0+0)) = -\pi$

b) $\vec{F} = \langle x^2, x^2 \rangle$



$M = x^2 = N$

Green's: $\oint_C \vec{F}\cdot d\vec{r} = \iint_R (N_x - M_y)\, dA = \iint_R 2x\, dA = \int_0^1\int_0^2 2x\, dx\, dy = 2\cdot m\cdot \bar{x},\ m = \text{area}(R),\ \bar{x} = 1$

$\Rightarrow\ = 2\cdot 2\cdot 1 = 4$

Direct: $\oint_C \vec{F}\cdot d\vec{r} = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$

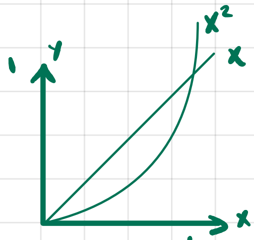
$\int_{C_1} \vec{F}\cdot d\vec{r} = \int_{C_1} x^2 dx + x^2 dy = \int_0^2 x^2\, dx = \frac{8}{3}$

$\int_{C_2} \vec{F}\cdot d\vec{r} = \int_0^1 4\, dy = 4$

$\int_{C_3} \vec{F}\cdot d\vec{r} = \int_2^0 x^2\, dx = -\int_{C_1} \vec{F}\cdot d\vec{r} = -8/3$

$\int_{C_4} \vec{F}\cdot d\vec{r} = \int_{C_4} 0\, dy = 0$

$\oint_C \vec{F}\cdot d\vec{r} = \frac{8}{3} + 4 - \frac{8}{3} + 0 = 4$



c) $\vec{F} = \langle xy, y^2 \rangle \qquad C: y = x^2,\ y = x,\ 0 \le x \le 1$

Green's: $\oint_C \vec{F}\cdot d\vec{r} = \int_0^1\int_{x^2}^{x} -x\, dy\, dx = \int_0^1 -x(x - x^2)\, dy = \int_0^1 (x^3 - x^2)\, dx = \frac{x^4}{4} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}$

Direct: $\int_0^1 x^3 dx + x^4\cdot 2x\, dx + \int_1^0 x^2 dx + x^2 dx = \frac{x^4}{4} + 2\frac{x^6}{6}\Big|_0^1 + \frac{2x^3}{3}\Big|_1^0 = \frac{1}{4} + \frac{1}{3} - \frac{2}{3} = \frac{3+4-8}{12}$

$= -\frac{1}{12}$

4D-2 $\oint_C 4x^3y\,dx + x^4\,dy$

$\vec{F} = \langle P, Q\rangle = \langle 4x^3y, x^4\rangle$

$P_y = 4x^3 \quad Q_x = 4x^3 \Rightarrow \text{curl}\,\vec{F} = 0 \Leftrightarrow \vec{F} \text{ conservative} \Leftrightarrow \vec{F} = \vec{\nabla}f \Leftrightarrow \text{path independent}$

$\Rightarrow \oint_C \vec{F}\cdot d\vec{r} = \oint_C \vec{\nabla}f\cdot d\vec{r} = f(P_0) - f(P_0) = 0$

Fundamental Theorem for Line Integrals

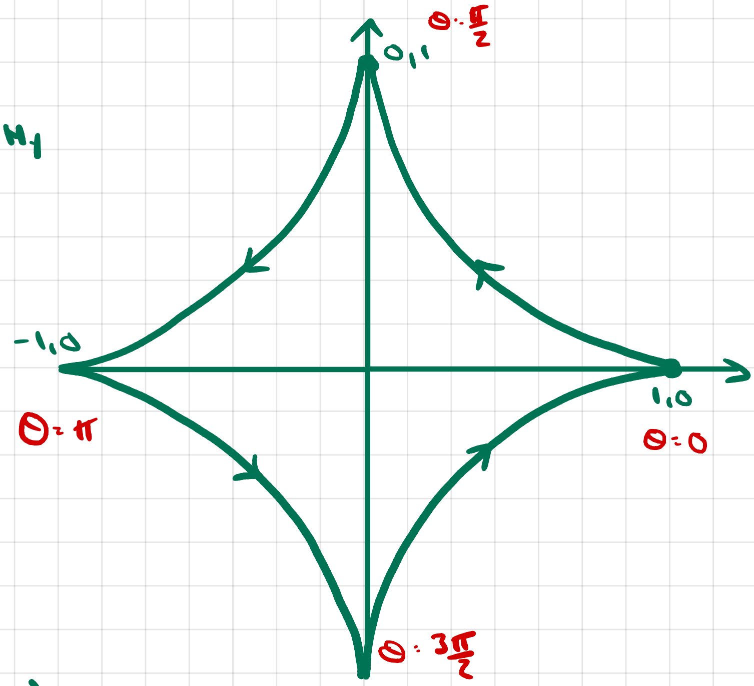
4D-3 $x^{2/3} + y^{2/3} = 1$, area inside?

$\text{Area} = \iint_R dA = \oint_C \vec{F}\cdot d\vec{r}$, $N_x - M_y = 1 \Rightarrow N_x = 1 + M_y$
$\Rightarrow N = x + M_y x$

$x = \cos^3\theta \quad y = \sin^3\theta$

$dx = 3\cos^2\theta(-\sin\theta)\,d\theta$
$dy = 3\sin^2\theta\cos\theta\,d\theta$

$\begin{cases} M_y = 2 \Rightarrow M = 2y \\ N = x + 2x = 3x \end{cases}$



θ	x	y
	$\cos^3\theta$	$\sin^3\theta$
0	1	0
π/2	0	1
π	−1	0
3π/2	0	−1

$\int_C M\,dx + N\,dy$

$= \int_C 2y\,dx + 3x\,dy$

$= \int_0^{2\pi} 2\sin^3\theta \cdot 3\cos^2\theta(-\sin\theta)\,d\theta + 3\cos^3\theta \cdot 3\sin^2\theta\cos\theta\,d\theta$

$= \int_0^{2\pi} -6\sin^4\theta\cos^2\theta\,d\theta + 9\cos^4\theta\sin^2\theta\,d\theta = (\dots \text{Maple}) = \frac{3\pi}{8}$

Calculation is easier choosing a different $\vec{F}$:

$\begin{cases} M_y = -\frac{1}{2} \Rightarrow M = -\frac{y}{2} \\ N = x - \frac{x}{2} = \frac{x}{2} \end{cases} \Rightarrow \vec{F} = \langle -y/2, x/2\rangle$

$\oint_C \vec{F}\cdot d\vec{r} = \frac{1}{2}\int_0^{2\pi} -y\,dx + x\,dy = \frac{1}{2}\int_0^{2\pi} -\sin^3\theta \cdot 3\cos^2\theta(-\sin\theta)\,d\theta + \cos^3\theta \cdot 3\sin^2\theta\cos\theta\,d\theta$

$= \frac{3}{2}\int_0^{2\pi} \sin^4\theta\cos^2\theta + \cos^4\theta\sin^2\theta\,d\theta = \frac{3}{2}\int_0^{2\pi} (\sin^2\theta + \cos^2\theta)\frac{\sin^2 2\theta}{4}\,d\theta$

$= \frac{3}{8}\int_0^{2\pi} \sin^2 2\theta\,d\theta = \frac{3}{16}\int_0^{2\pi}(1 - \cos 4\theta)\,d\theta = \frac{3}{16}\left(\theta - \frac{1}{4}\sin 4\theta\right)\Big|_0^{2\pi}$

$\sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta)$

$= \frac{3}{16}\left[(2\pi - 0) - (0 - 0)\right] = \frac{3\pi}{8}$

4D-4 $\oint_C -y^3dx + x^3dy = \iint_R 3(x^2+y^2)\,dA > 0$ because $\text{curl}\,\vec{F} \geq 0\ \forall x,y$

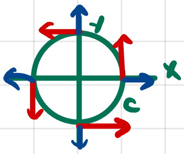
$\vec{F} = \langle -y^3, x^3 \rangle = \langle P, Q \rangle$

$P_y = -3y^2,\ Q_x = 3x^2 \Rightarrow \vec{F} \neq \vec{\nabla} f$

$\text{curl}\,\vec{F} = 3x^2 + 3y^2$

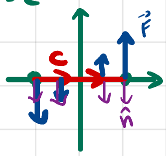
4D-5 $\oint_C xy^2dx + (x^2y + 2x)dy = \iint_R 2\,dA = 2\,\text{area}(R)$ $\quad \vec{F} = \langle xy^2, x^2y + 2x \rangle \quad \text{curl}\,\vec{F} = 2xy + 2 - 2xy = 2$

4E-1 $\vec{F} = \langle -y, x \rangle$



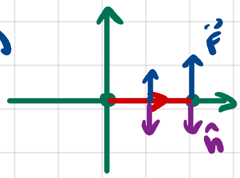
a) Flux = 0
$\vec{F}\cdot\hat{n} = 0\ \forall x,y$ on C

b) $\int_C \vec{F}\cdot\hat{n}\,ds = \int_C -y\,dy - x\,dx = \int_{-1}^{1} -x\,dx = -\frac{1}{2}x^2\Big|_{-1}^{1} = 0$



between $x=-1$ and $x=0$, $\int \vec{F}\cdot\hat{n}\,ds > 0$, between $x=0$ and $x=1$ $\int \vec{F}\cdot\hat{n}\,ds < 0$ in same magnitude, ie they cancel out.

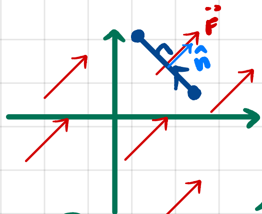
c)



$\int_C \vec{F}\cdot\hat{n}\,ds = \int_C |\vec{F}|(-1)\,ds = -\int_C x\,ds = -\bar{x}\cdot m,\ m = \int ds = 1,\ \bar{x} = \frac{1}{2}$
$= -\frac{1}{2}$

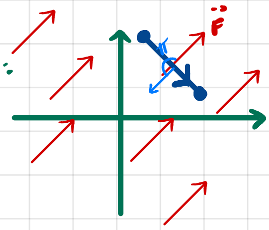
$n = \langle 0, -1 \rangle \Rightarrow \vec{F}\cdot\hat{n} = -x,\ ds = dx \Rightarrow \int_C \vec{F}\cdot\hat{n}\,ds = \int_0^1 -x\,dx = -\frac{1}{2}$

4E-2 $\vec{F} = \langle 1, 1 \rangle$

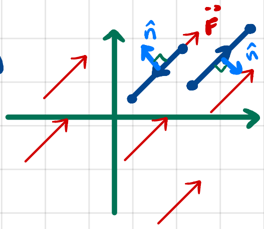


a) $\cos\theta = 1$ for θ angle between $\vec{F}$ and $\hat{n}$. $\theta = 0$, $\vec{F}\cdot\hat{n}$ maximized.

b) $\cos\theta = -1$, $\vec{F}\cdot\hat{n}$ minimized, $\theta = \pi \Rightarrow$ C perpendicular to $\vec{F}$, in direction as follows:



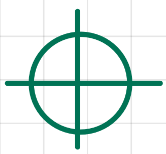
c)



$\cos\theta = 0 \Rightarrow \theta = \pi/2$
C parallel to $\vec{F}$

d) $\int_C \vec{F}\cdot\hat{n}\,ds = -1 = \int_C \nabla(x+y)\cdot\hat{n}\,ds = f(x_1,y_1) - f(x_0,y_0)$, where $f(x,y) = x+y$

$(x_1 + y_1) - (x_0 + y_0) = -1 \Rightarrow (y_1 - y_0) + (x_1 - x_0) = -1 \Rightarrow y_1$



$\vec{F}\cdot\hat{n} = |\vec{F}||\hat{n}|\cos\theta = -1 = \sqrt{2}\cos\theta \Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \Rightarrow \theta =$

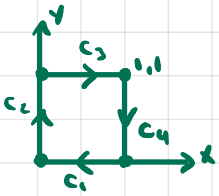
e) max: $\hat{n} = \frac{\langle 1,1\rangle}{\sqrt{2}}$, $\vec{F}\cdot\hat{n} = \frac{2}{\sqrt{2}} \Rightarrow \int_C F\cdot\hat{n}\,ds = \frac{2}{\sqrt{2}}\int_C ds = 2/\sqrt{2} = \sqrt{2}$

min: $\hat{n} = -\frac{\langle 1,1\rangle}{\sqrt{2}}$, $\vec{F}\cdot\hat{n} = -\frac{2}{\sqrt{2}} \Rightarrow \int_C \vec{F}\cdot\hat{n}\,ds = -2/\sqrt{2} = -\sqrt{2}$

4E-3 $\vec{F} = \langle x^2, xy\rangle$ $\quad r(t) = \langle t+1, t^2\rangle \quad 0\le t\le 1 \quad dx = dt \quad dy = 2t\,dt$

$$\int_C \vec{F}\cdot\hat{n}\,ds = \int_C x^2\,dy - xy\,dx = \int_0^1 [(t+1)^2\cdot 2t\,dt - (t+1)t^2\,dt]$$
$$= \int_0^1 [(t^2+2t+1)\cdot 2t - t^3 - t^2]\,dt = \int_0^1 [2t^3+4t^2+2t-t^3-t^2]\,dt$$
$$= \int_0^1 [t^3+3t^2+2t]\,dt = \left[\frac{t^4}{4}+t^3+t^2\right]\Big|_0^1 = \frac{1}{4}+1+1 = \frac{9}{4}$$

4E-4



$\vec{F} = \langle x,y\rangle$

Flux $= \int_C \vec{F}\cdot\hat{n}\,ds = -\oint_C \vec{F}\cdot\hat{n}\,ds = -\iint_R 2\,dA = -2\cdot 1 = -2$

div $\vec{F} = 1+1 = 2$

direct: $\int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4} = -2$

$\int_{C_1} \cancel{x}^{0}\,dy - y\,\cancel{dx}^{0} = 0$

$\int_{C_2} x\,\cancel{dy}^{0} - y\,dx = \int_0^1 -1\,dx = -1$

$\int_{C_3} x\,dy - y\,\cancel{dx}^{0} = \int_1^0 dy = -1$

$\int_{C_4} x\,\cancel{dy}^{0} - \cancel{y}^{0}\,dx = 0$

4E-5 $|\vec{F}| = r^m$

a) $\oint_C \vec{F}\cdot\hat{n}\,ds = \oint_C |\vec{F}|\,ds = \oint_C r^m\,ds = \oint_C a^m\,ds = a^m\cdot 2\pi a = 2\pi a^{m+1}$

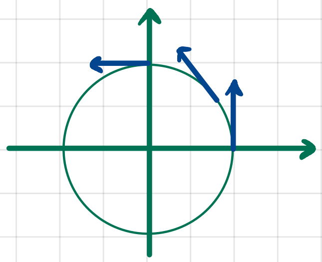
$\vec{F}\cdot\hat{n}$ on C is $|\vec{F}|$ because both $\vec{F}$ and $\hat{n}$ point radially outwards.

If we were to calculate directly, we can't use Green's Theorem directly because $\vec{F}$ not defined at origin.

4F-2 $\vec{F} = \omega\langle -y, x\rangle$ = velocity of rotating fluid

a) div $\vec{F} = 0$ $\quad$ curl $\vec{F} = \omega(1+1) = 2\omega$

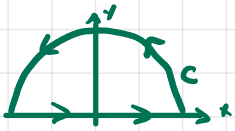
b)



if flux measures how much a flow is "expanding", in this case it is not expanding but rotating.

c)

4F-3 $\vec{F} = \langle x, y \rangle$



G.T.: $\oint_C \vec{F}\cdot\hat{n}\,ds = \iint_R \operatorname{div}\vec{F}\,dA$

$\oint_{C_1} \vec{F}\cdot\hat{n}\,ds = \int_{C_1} x\,dy - y\,dx$, $x = \cos t$, $y = \sin t$, $dx = -\sin t\,dt$, $dy = \cos t\,dt$

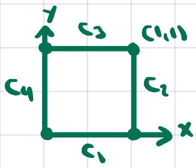
$= \int_0^{\pi} \cos^2 t\,dt + \sin^2 t\,dt = \pi$

$\oint_{C_2} \vec{F}\cdot\hat{n}\,ds = \int x\cdot 0 - 0\cdot dx = 0$

$\Rightarrow \int_C \vec{F}\cdot\hat{n}\,ds = \pi$

$\iint_R 2\,dA = \int_0^{\pi}\int_0^1 2r\,dr\,d\theta = 2\int_0^{\pi} \frac{1}{2}\,d\theta = \pi$

4F-4 $\vec{F} = \langle x^2, xy \rangle$



$\oint_C \vec{F}\cdot\hat{n}\,ds = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$

curl $\vec{F} = y - 0 = y \neq 0$ $\Rightarrow$ not path independent, not conservative

$\int_{C_1} x^2\,dy - xy\,dx = \int_{C_1} x^2\cdot 0 - 0\,dx = 0$

$\int_{C_4} 0\,dy - 0\cdot 0 = 0$

$\int_{C_2} dy = \int_0^1 dy = 1$

$\int_{C_3} -x\,dx = \int_1^0 -x\,dx = -\frac{x^2}{2}\Big|_1^0 = \frac{1}{2}$

$\Rightarrow \oint_C \vec{F}\cdot\hat{n}\,ds = 3/2$

$\iint_R (2x + x)\,dA = \int_0^1\int_0^1 3x\,dx\,dy = \int_0^1 \frac{3}{2}\,dy = \frac{3}{2}$

Problem 1

$\vec{F} = \langle y^3 - 6y, 6x - x^3 \rangle$

a) C w/ largest $\oint_C \vec{F} \cdot d\vec{r}$

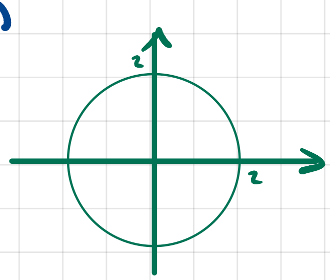
$\oint \vec{F} d\vec{r} = \iint_R \text{curl}\, \vec{F}\, dA$

$\text{curl}\, \vec{F} = 6 - 3x^2 - (3y^2 - 6) = 12 - 3x^2 - 3y^2$

if we view $\iint_R (12 - 3x^2 - 3y^2)\, dA$ as a volume, it is max where $\text{curl}\, \vec{F} > 0$, ie $12 - 3x^2 - 3y^2 > 0$

$\Rightarrow x^2 + y^2 < 4$

b)



$$\int_0^{2\pi}\int_0^2 (12 - 3r^2)\, r\, dr\, d\theta = \int_0^{2\pi} 6r^2 - \frac{3}{4}r^4 \Big|_0^2 d\theta = \int_0^{2\pi} (24 - 12)\, d\theta = 12 \cdot 2\pi = 24\pi$$

Problem 2

Problem setup

Green's theorem in Normal Form + Principle of Conservation of mass

$\Rightarrow$ div $\vec{F}$ of $\vec{F}(x,y)$ is signed rate of mass per unit time per unit area which originates at (x,y), ie source or sink rate

Non-steady flow $\vec{F}(x,y,t)$

Equation of Continuity for fluid flows

ρ: density

div $\vec{F}(x,y,t)$ defined w/ respect to only the space variables

$= M_x + N_y$

For $\vec{F}(x,y,t) = \rho(x,y,t)\ \vec{v}(x,y,t)$

Eq. of continuity: $\frac{\partial\rho}{\partial t} + \operatorname{div}\vec{F} = 0$

* For steady flows $\rho = \rho(x,y)$, $\vec{v} = \vec{v}(x,y)$ $\Rightarrow$ eq. of continuity is div $\vec{F} = 0$

a) $$\iint_R (\partial\rho/\partial t + \operatorname{div}\vec{F})\,dA = 0 \Rightarrow \underbrace{\iint_R \partial\rho/\partial t\,dA}_{\text{rate of change of mass in R}} = -\underbrace{\oint \vec{F}\cdot\hat{n}\,ds}_{\text{source rate}}$$

$$\iint_R \partial\rho/\partial t\,dA = \frac{d}{dt}\iint_R \rho(x,y,t)\,dA = \frac{d}{dt}\,\text{mass}_R(t)$$

$\Rightarrow$ mass is conserved if rate of change of mass equals net outflow of mass.

b) $g(x,y,t)$ $\quad \nabla g = \langle g_x, g_y\rangle$ $\quad \vec{G}(x,y,t) = \langle M(x,y,t), N(x,y,t)\rangle$

$$\operatorname{div}(g\vec{G}) = \operatorname{div}(\langle gM, gN\rangle) = g_xM + gM_x + g_yN + gN_y = g(M_x+N_y) + \nabla g\cdot\vec{G} = g\operatorname{div}\vec{G} + \nabla g\cdot\vec{G}$$

c) $\frac{D}{Dt}$ = convective derivative

"rate of change along a moving path of some physical quantity (scalar or vector) being transported by fluid current"

$f(x,y,t)$ smooth $\quad \nabla f = \langle f_x, f_y \rangle$ gradient in space variables

path of point mass in fluid flow

$\vec{r}(t) = \langle x(t), y(t) \rangle$ smooth curve

$\vec{v} = \vec{r}\,'(t)$

x, y follow a path parametrized by t

$$\frac{Df}{Dt} = \frac{d}{dt} f(\vec{r}(t), t)$$

If we take f to be $\rho(x,y,t)$, $\frac{Df}{Dt}$ tells us how density changes along the path of the particle

If $\frac{D\rho}{Dt} = 0$, the fluid is incompressible.

$\Rightarrow$ mass density constant along the paths of the flow

we have

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f$$

show $\vec{F}(x,y,t) = \rho(x,y,t)\,\vec{v}(x,y,t)$ incompressible

$\iff \operatorname{div}(\vec{v}) = 0$

eq. continuity: $\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{F} = 0$

$\operatorname{div}(g\vec{G}) = g \operatorname{div}(\vec{G}) + \nabla g \cdot \vec{G}$

$\frac{D\rho}{Dt} = \frac{d}{dt}\rho(\vec{r}(t), t) = \nabla \rho \cdot \vec{v} + \frac{\partial \rho}{\partial t} = 0 \Rightarrow$ fluid incompressible

$$\vec{F} = \rho(x,y,t)\,\vec{v}(x,y,t) = \langle \rho(x,y,t) v_1(x,y,t), \rho(x,y,t) v_2(x,y,t) \rangle$$

$\operatorname{div}(\vec{v}) = 0 \Rightarrow \vec{F}(x,y,t)$ incompressible

$$\operatorname{div}\vec{F} = \rho_x v_1 + \rho v_{1_x} + \rho_y v_2 + \rho v_{2_y} = \rho(v_{1_x} + v_{2_y}) + \nabla\rho \cdot \vec{v} = \rho\, \overset{0}{\operatorname{div}\vec{v}} + \nabla \rho \cdot \vec{v}$$

$\Rightarrow \operatorname{div}\vec{F} = \nabla\rho \cdot \vec{v}$

eq. continuity: $\frac{\partial \rho}{\partial t} + \nabla\rho \cdot \vec{v} = 0 = \frac{d}{dt}\rho(\vec{r}(t), t) = \frac{D\rho}{Dt} \Rightarrow$ fluid incompressible

$\vec{F}(x,y,t)$ incompressible $\Rightarrow \operatorname{div}(\vec{v}) = 0$

eq. continuity: $\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho\vec{v}) = \frac{\partial \rho}{\partial t} + \rho \operatorname{div}(\vec{v}) + \nabla\rho \cdot \vec{v} = 0 = \rho \operatorname{div}(\vec{v}) + \cancel{\frac{D\rho}{Dt}}^{\,0} = 0$

$\Rightarrow \operatorname{div}(\vec{v}) = 0$

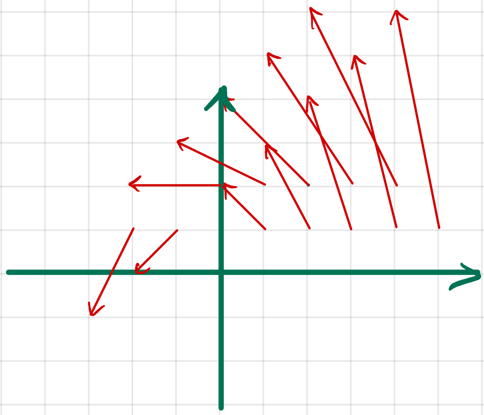
Problem 3

i) $\vec{v}(x,y,t) = t\langle -y, x\rangle$, $\rho(x,y,t) = \sqrt{x^2+y^2}$

$\operatorname{div}\vec{v} = 0 \Rightarrow$ incompressible flow $\Rightarrow \frac{\partial \rho}{\partial t} = 0$

ρ does not depend explicitly on t, and flow is incompressible

$\Rightarrow$ stratified flow



level curves of ρ are concentric circles

$$\frac{D\rho}{Dt} = \rho_x x' + \rho_y y' + \frac{\partial \rho}{\partial t} = 0$$

$$= \frac{2xx' + 2yy'}{2\sqrt{x^2+y^2}} = \frac{\vec{r}(t)\cdot\vec{v}(t)}{\sqrt{x^2+y^2}} \Rightarrow \vec{r}(t)\cdot\vec{v}(t) = 0$$

Eq. continuity: $\cancel{\frac{\partial \rho}{\partial t}}^{0} + \operatorname{div}\vec{F} = 0 \Rightarrow \rho \cancel{\operatorname{div}}^{0}\vec{v} + \vec{v}\nabla\rho = 0$

$$\vec{v}\cdot\nabla\rho = \frac{-tyx + txy}{\sqrt{x^2+y^2}} = 0$$

* $\frac{D\rho}{Dt} = 0 \Rightarrow$ incompressible

$\rho = \rho(x,y,z)$, incompress.

$\Rightarrow$ stratified

$\Rightarrow \vec{v}\cdot\nabla\rho = 0 \Rightarrow$ trajectory along level density curves

$\rho = \rho(x,y,z)$, $\vec{v} = \vec{v}(x,y,z)$

$\Rightarrow$ steady

$\Rightarrow$ trajectories have same shape through time

ii) $\vec{v} = \frac{1}{1+t}\langle x, -y\rangle$ $\quad \rho(x,y,t) = xy$

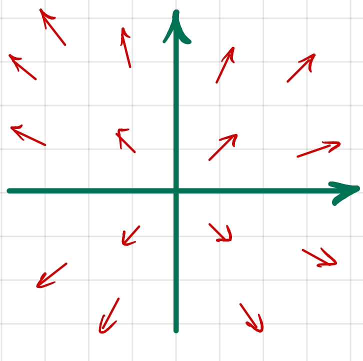
$\operatorname{div}\vec{v} = \frac{1}{1+t} - \frac{1}{1+t} = 0 \Rightarrow$ incompressible. also, stratified.

eq. continuity: $\frac{\partial \rho}{\partial t} + \operatorname{div}\vec{F} = 0 \Rightarrow \operatorname{div}\vec{F} = 0$

$\Rightarrow \rho \cancel{\operatorname{div}}^{0}\vec{v} + \vec{v}\nabla\rho = 0 \Rightarrow \vec{v}\nabla\rho = 0$

$\nabla\rho = \langle y, x\rangle$

$$\vec{v}\cdot\nabla\rho = \frac{yx - xy}{1+t} = 0$$



iii) $v(x,y,t) = t\langle x, y\rangle$ $\quad \rho(x,y,t) = e^{-t^2}$

$\operatorname{div}\vec{v} = t + t = 2t \Rightarrow$ not incompressible

$\partial\rho/\partial t + \operatorname{div}\vec{F} = -2te^{-t^2} + 2te^{-t^2} + 0 = 0$