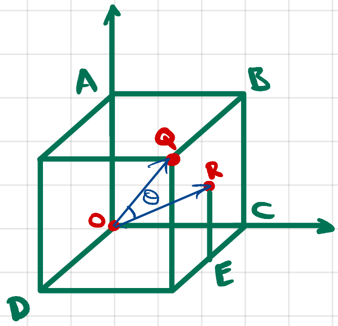
Practice Exam 1



P1

a) $\vec{OA} = \langle 0,0,r\rangle$
$\vec{AB} = \vec{OC} = \langle 0,r,0\rangle$
$\vec{BQ} = \vec{OD} = \langle r,0,0\rangle$

$\vec{OQ} = \vec{OA} + \vec{AB} + \vec{BQ} = \langle r,r,r\rangle$

$\vec{OE} = \langle r/2, r, 0\rangle$

$\vec{ER} = \langle 0,0,r/2\rangle$

$\vec{OE} + \vec{ER} = \langle r/2, r, r/2\rangle = \vec{OR}$

b) $\vec{OQ}\cdot\vec{OR} = \langle r,r,r\rangle\langle r/2, r, r/2\rangle = \frac{r^2}{2} + r^2 + \frac{r^2}{2} = 2r^2 = r\sqrt{3}\cdot\frac{r}{2}\sqrt{6}\cos\theta$

$|\vec{OQ}| = \sqrt{3r^2} = r\sqrt{3}$

$|\vec{OR}| = \sqrt{r^2/4 + \frac{4r^2}{4} + r^2/4} = \sqrt{\frac{6r^2}{4}} = \frac{r}{2}\sqrt{6}$

$\Rightarrow 4r^2 = r^2\sqrt{18}\cos\theta$

$\Rightarrow \cos\theta = \frac{4}{\sqrt{18}} = \frac{4\cdot\sqrt{18}}{18} = \frac{2\sqrt{18}}{9}$

$\theta = \cos^{-1}\left(2\sqrt{18}/9\right) \approx 19.47°$
≈ 0.34 rad

P2

$\vec{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$

$\vec{v}(t) = \langle -3\sin t, 3\cos t, 1\rangle$

speed $= |\vec{v}(t)| = \sqrt{9(\sin^2 t + \cos^2 t) + 1} = \sqrt{10}$

P3

a) $A^{-1} = \frac{1}{2}\begin{bmatrix} \cdot & \cdot & \cdot \\ 2 & \cdot & \cdot \\ -3 & \cdot & \cdot \end{bmatrix}^T \Rightarrow a = 2,\ b = -3$

b) $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$\det A = 2 \neq 0$

$\Rightarrow$ unique solution $A^{-1}B$

$= \frac{1}{2}\begin{bmatrix} 1 & 2 & -3 \\ -1 & -2 & 5 \\ 2 & 2 & -6 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ -4 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 3 & c \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ $\det A = 1(1) - 3(1) + c(2) = 0$
$\Rightarrow 1 - 3 + 2c = 0 \Rightarrow 2c = 2 \Rightarrow c = 1$

$\Rightarrow c = 1 \iff$ no sol. or ∞ sol.

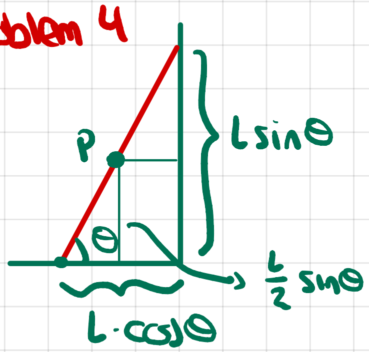
$M = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ $MX = 0$ $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \langle x,y,z\rangle$ such that $\perp$ to the three row vectors

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & 1 \\ 2 & 0 & -1 \end{vmatrix} = \hat{\imath}(-3) - \hat{\jmath}(-1-2) + \hat{k}(-6) = \langle -3, 3, -6\rangle$

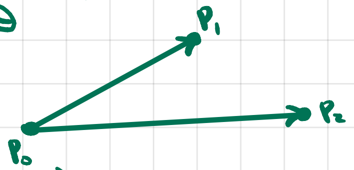
$\langle -3,3,6\rangle\langle 1,1,0\rangle = 0$ as well $\Rightarrow \langle -3,3,6\rangle$ is a solution.

Problem 4



$\vec{OP} = \langle -\frac{L}{2}\cos\theta, \frac{L}{2}\sin\theta \rangle$

Problem 5



a) $P_0 \ (2,1,0)$
$P_1 \ (1,0,1)$
$P_2 \ (2,-1,1)$

$A = \frac{1}{2}\left|\vec{P_0P_1} \times \vec{P_0P_2}\right| = \frac{1}{2}\sqrt{1+1+4} = \frac{\sqrt{6}}{2}$

$\vec{P_0P_1} \times \vec{P_0P_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 0 & -2 & 1 \end{vmatrix} = \hat{i}(-1+2) - \hat{j}(-1) + \hat{k}(2) = \langle 1,1,2 \rangle$

b) $\vec{n} = \langle 1,1,2 \rangle \qquad x + y + 2z = 3$

c) line parallel to $\vec{v} = \langle 1,1,1 \rangle$ passing through $(-1,0,0)$

parametric eq: $\langle -1,0,0 \rangle + t\langle 1,1,1 \rangle = \langle -1+t, t, t \rangle$

intersection with plane from b): $-1+t+t+2t = 3 \Rightarrow 4t = 4 \Rightarrow t = 1$

$\langle 0,1,1 \rangle$ intersection

Problem 6

a) $\frac{d\vec{R}\cdot\vec{R}}{dt} = \frac{d|\vec{R}|^2}{dt} = \frac{d}{dt}(x(t)^2 + y(t)^2 + z(t)^2) = 2xx' + 2yy' + 2zz'$

$= 2\vec{R}\cdot\vec{R}'$

Altern.: $\frac{d\vec{R}\cdot\vec{R}}{dt} = R'\cdot R + R\cdot R' = 2R\cdot R'$

b) $|\vec{R}| = k \Rightarrow \frac{d\vec{R}\cdot\vec{R}}{dt} = \frac{d|\vec{R}|^2}{dt} = \frac{d(k^2)}{dt} = 0 \Rightarrow R\cdot R' = 0 \Rightarrow \vec{R}\cdot\vec{V} = 0$

c) $|\vec{R}| = h, \ \vec{R}\cdot\vec{V} = 0$

$\vec{R}'\cdot\vec{V} + \vec{R}\cdot\vec{A} = 0$

$|\vec{V}|^2 + \vec{R}\cdot\vec{A} = 0 \Rightarrow \vec{R}\cdot\vec{A} = -|\vec{V}|^2$