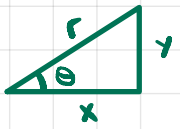
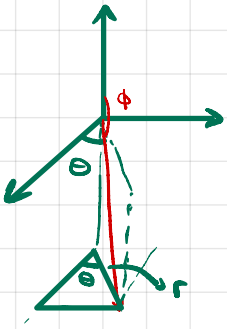
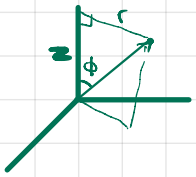
12.8 Cylindrical and Spherical Coordinates

Ex 4: $(8, \frac{5\pi}{6}, \frac{\pi}{3}) = (\rho, \phi, \theta)$







$x = r\cos\theta$

$r = \rho\sin\phi$

$\Rightarrow x = \rho\sin\phi\cos\theta = 8\sin\frac{5\pi}{6}\cos\frac{\pi}{3} = 8\cdot\frac{1}{2}\cdot\frac{1}{2} = 2$

$y = \rho\sin\phi\sin\theta = 8\sin\frac{5\pi}{6}\sin\pi/3 = 8\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$

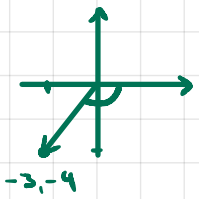
$z = \rho\cos\phi = 8\cdot\cos\frac{5\pi}{6} = 8\cdot\left(-\frac{\sqrt{3}}{2}\right) = -4\sqrt{3}$

$(-3, -4, -12)$ in rect. coord.

$\rho^2 = x^2 + y^2 + z^2 = 9+16+144 = 169 \Rightarrow \rho = 13$

$x^2 + y^2 = r^2 \Rightarrow r = 5$

$\sin\phi = \frac{r}{\rho} = \frac{5}{13}$ $\cos\phi = \frac{z}{\rho} = \frac{-12}{13} \Rightarrow \phi \approx 2.7468$ rad



$\tan(\theta - \pi) = \frac{-4}{-3}$

$\theta = \pi + \tan^{-1} 4/3$

Ex 5 $z = x^2 + y^2 \Rightarrow z = r^2 \Rightarrow \rho\cos\phi = \rho^2\sin^2\phi \Rightarrow \cos\phi = \rho\sin^2\phi \Rightarrow \rho = \frac{1}{\sin\phi}\frac{\cos\phi}{\sin\phi} = \csc\phi\cot\phi$

$\rho = \csc\phi\cot\phi \quad 0 < \phi \leq \pi/2$

Ex 6 $\rho = 2\cos\phi \Rightarrow \rho^2 = 2\rho\cos\phi \Rightarrow x^2 + y^2 + z^2 = 2z \Rightarrow x^2 + y^2 + (z-1)^2 = 1$