

volume between $z = x^2 + y^2$ and $z = 2y$

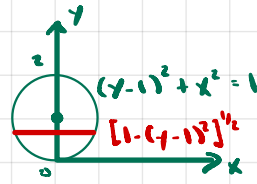
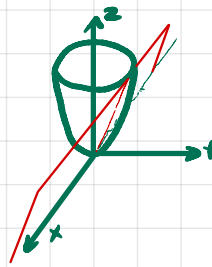
$$2y = x^2 + y^2$$

$$y^2 - 2y + 1 + x^2 = 1$$

$$(y-1)^2 + x^2 = 1$$

$$z = x^2 + \frac{z^2}{2} \Rightarrow 2z = 2x^2 + z^2$$

$$z^2 - 2z + 1 + 2x^2 = 1 \Rightarrow (z-1)^2 + 2x^2 = 1$$



$$y^2 - 2y + x^2 = 0$$

$$r^2 - 2r \sin \theta = 0$$

$$r(r - 2 \sin \theta) = 0$$

$$r = 2 \sin \theta$$

$$\int_0^2 \int_{-[1-(y-1)^2]^{1/2}}^{[1-(y-1)^2]^{1/2}} \int_{x^2+y^2}^{2y} dz dx dy$$

$$= \int_0^\pi \int_0^{2 \sin \theta} \int_{r^2}^{2r \sin \theta} dz r dr d\theta = \frac{\pi}{2}$$