12.4 Lines and Planes in Space

Straight line in space

determined by two points on it, P_0, P_1

$P_0(x_0, y_0, z_0)$ on line, parallel to $\vec{v} = \langle a, b, c \rangle$

$P(x, y, z)$ on line if $\overrightarrow{P_0P}$ parallel to $\vec{v}$

$\vec{r}_0 = \overrightarrow{OP_0}$ $\quad \vec{r} = \overrightarrow{OP}$

$$\vec{r} - \vec{r}_0 = t\vec{v} \Rightarrow \vec{r} = \vec{r}_0 + t\vec{v} \Rightarrow \begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

parametric eq. of line

$$t = \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

symmetric equations of line

note:

$$\left.\begin{aligned} b(x - x_0) - a(y - y_0) &= 0 \\ c(x - x_0) - a(z - z_0) &= 0 \\ c(y - y_0) - b(z - z_0) &= 0 \end{aligned}\right\}$$

three planes intersecting at a line

normal vectors are $\langle b, -a, 0 \rangle \Rightarrow$ in xy plane $\Rightarrow$ plane $\perp$ xy-plane

$\langle c, 0, -a \rangle \Rightarrow$ in xz plane $\Rightarrow$ $\perp$ xz plane

$\langle 0, c, -b \rangle$