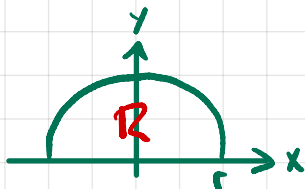**6.6**

Ex 1
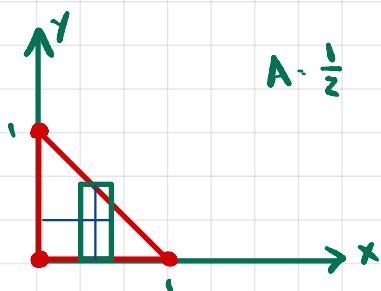

$r(\theta) = r$

$$A = \int_0^{\pi} \frac{1}{2} r^2 \, d\theta = \frac{1}{2} r^2 \theta \Big|_0^{\pi} = \frac{1}{2} \pi r^2$$

$$\bar{x} = \frac{\int_a^b x f(x) \, dx}{A} = \frac{\int_{-r}^{r} x \sqrt{1-x^2} \, dx}{\frac{1}{2} \pi r^2} = \frac{-\frac{(1-x^2)^{3/2}}{3/2} \Big|_{-r}^{r}}{\frac{1}{2} \pi r^2} = \frac{-\frac{2}{3} \left[(1-r^2)^{3/2} - (1-r^2)^{3/2} \right]}{\frac{1}{2} \pi r^2} = 0$$

$$\bar{y} = \frac{\frac{1}{2} \int_b^b f(x)^2 \, dx}{A} = \frac{\frac{1}{2} \int_{-r}^{r} (1-x^2) \, dx}{\frac{1}{2} \pi r^2} = \frac{4r}{3\pi}$$

Ex 2

$A = \frac{1}{2}$

$$\bar{x} = \int_0^1 x f(x) \, dx = \int_0^1 x(1-x) \, dx$$

$$\bar{y} = \int_0^1 \frac{1}{2} f(x) \cdot f(x) \, dx = \frac{1}{2} \int_0^1 (1-x)^2 \, dx$$