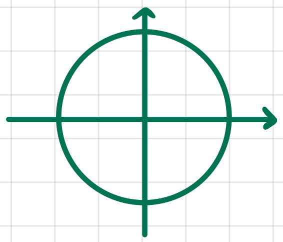
Ex3 $z = 25 - x^2 - y^2$



$$\int_0^{2\pi}\int_0^{5}(25-r^2)\,r\,dr\,d\theta$$

$$z = 25-(x^2+y^2) = 25-r^2$$

Ex4 $z = 8 - r^2$ $\quad z = r^2$

intersection $8-r^2 = r^2 \Rightarrow r^2 = 4 \Rightarrow r = 2$

$$\int_0^{2\pi}\int_0^{2}(8-r^2-r^2)\,r\,dr\,d\theta$$

$$\int \cos^2\theta\,d\theta = \frac{1}{2}\int(1+\cos 2\theta)\,d\theta = \frac{1}{2}\left(\theta+\frac{1}{2}\sin 2\theta\right)$$
$$= \frac{1}{2}(\theta + \sin\theta\cos\theta)$$

3 $r = 1+\cos\theta$

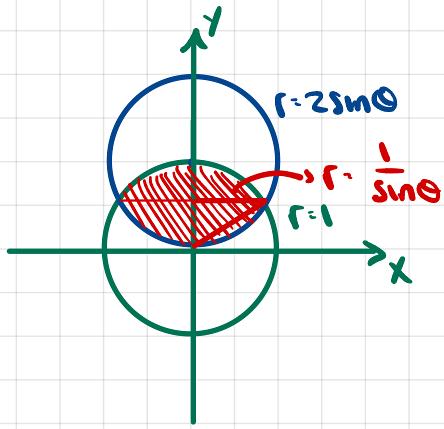
$$2\int_0^{\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta = 2\int_0^{\pi}\frac{(1+\cos\theta)^2}{2}\,d\theta = \int_0^{\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta$$

$$= \theta + 2\sin\theta + \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\,\Big|_0^{\pi}$$

$$= \pi + 0 + \frac{1}{2}\pi + 0 - (0+0+0+0) = \frac{3}{2}\pi$$

θ	r
0	2
$\pi/2$	1
π	0
$3\pi/2$	1
2π	2

5



$r = 2\sin\theta$

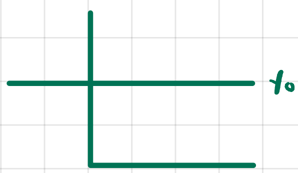
$x = r\cos\theta$
$y = r\sin\theta$

$r^2 = 2r\sin\theta$
$r^2 = 2y$
$\Rightarrow x^2+y^2 = 2y$
$x^2+y^2-2y+1 = 1$
$x^2+(y-1)^2 = 1$

intersection: $1 = 2\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \theta = \frac{5\pi}{6}$; $r=1$

$$\left[\int_{\pi/6}^{5\pi/6}\int_{1/\sin\theta}^{1} r\,dr\,d\theta + \int_{\pi/6}^{\pi/2}\int_0^{1/\sin\theta} r\,dr\,d\theta + \int_0^{\pi/6}\int_0^{2\sin\theta} r\,dr\,d\theta\right]\cdot 2$$

$$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$



$y = y_0 = r\sin\theta$

$r = \frac{y_0}{\sin\theta}$

7 $r = 1 - 2\sin\theta$

$$\int_{\pi/6}^{2\pi/3} \int_0^{1-2\sin\theta} r\,dr\,d\theta$$

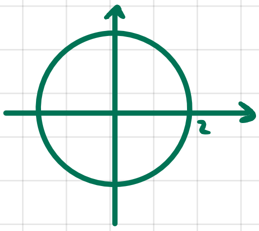
$$\int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$
$$= \frac{1}{2}\left(x - \sin(2x)\cdot\frac{1}{2}\right)$$
$$= \frac{1}{2}x - \frac{1}{2}\sin x\cos x$$

θ	r
0	1
$\pi/6$	0
$\pi/2$	-1
$5\pi/6$	0
π	1
$3\pi/2$	3

$$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1-2\sin\theta)^2\,d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1 - 4\sin\theta + 4\sin^2\theta)\,d\theta$$

$$= \frac{1}{2}\Big[\theta + 4\cos\theta + \theta - 2\sin\theta\cos\theta\Big]\Big|_{\pi/6}^{5\pi/6} = \pi - \frac{3\sqrt{3}}{2}$$

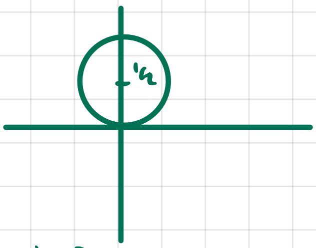
9 $z = \sqrt{x^2+y^2}$ $\quad r = 2$



$$\int_0^{2\pi}\int_0^2 r^2\,dr\,d\theta = \frac{1}{3}\int_0^{2\pi} 8\,d\theta = \frac{8}{3}\cdot 2\pi$$

11 $z = 10 + 2x + 3y$ $\quad r = \sin\theta$

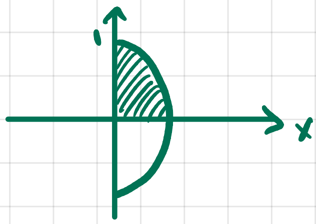
$r^2 = r\sin\theta = y$
$x^2 + y^2 = y$
$x^2 + y^2 - y + \frac{1}{4} = \frac{1}{4}$
$x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$



$$\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta = \frac{\pi}{4}$$

$$\pi\cdot\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$$

13 $$\int_0^1\int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy = \int_0^{\pi/2}\int_0^1 \frac{1}{1+r^2}\cdot r\,dr\,d\theta = \pi\ln 2/4$$



17 $$\int_0^1\int_0^{\sqrt{1-y^2}} \sin(x^2+y^2)\,dx\,dy = \int_0^{\pi/2}\int_0^1 \sin(r)\,r\,dr\,d\theta = (1-\cos(1))\frac{\pi}{4}$$