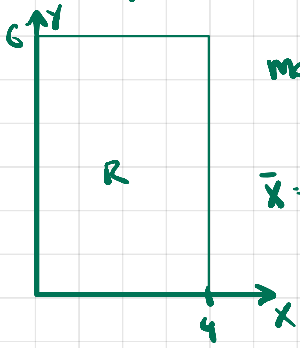
1 $\delta(x,y) = 1$: density in units mass per unit area

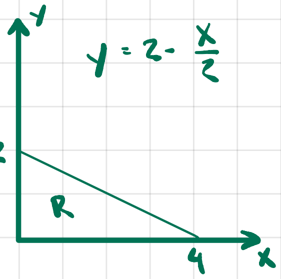


$$\text{mass} = \iint_R dm = \iint_R \delta\, dA = \int_0^6\int_0^4 dx\,dy = 4\cdot 6 = 24$$

$$\bar{x} = \frac{\iint_R x\,dm}{m} = \frac{\int_0^6\int_0^4 x\,dx\,dy}{24} = \frac{\int_0^6 8\,dy}{24} = \frac{48}{24} = 2$$

$$\bar{y} = \frac{\iint_R y\,dm}{m} = \frac{\int_0^6\int_0^4 y\,dx\,dy}{24} = \frac{\frac{1}{2}\cdot 6^2\cdot 4}{24} = 3$$

5



$$\text{mass} = \int_0^2\int_0^{4-2y} dx\,dy = \int_0^2 [4-2y-0]\,dy = [4y - y^2]\Big|_0^2 = 8-4 = 4$$

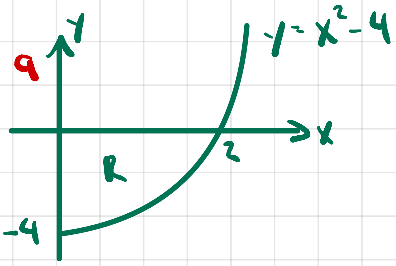
$$\bar{x} = \frac{\int_0^2\int_0^{4-2y} x\,dx\,dy}{m} = \frac{\int_0^2 \frac{1}{2}(4-2y)^2\,dy}{4} = \frac{\int_0^2 (16-16y+4y^2)\,dy}{8}$$

$$= \frac{[16y - 8y^2 + \frac{4}{3}y^3]\Big|_0^2}{8} = \frac{\cancel{32} - \cancel{32} + \frac{32}{3}}{8} = \frac{4}{3}$$

$$\bar{y} = \frac{\int_0^2\int_0^{4-2y} y\,dx\,dy}{m}$$

$$= \frac{\int_0^2 y(4-2y)\,dy}{4} = \frac{\frac{4y^2}{2} - \frac{2y^3}{3}\Big|_0^2}{4} = \frac{8 - \frac{16}{3}}{4} = \frac{8}{3\cdot 4} = \frac{2}{3}$$

9



$$m = \int_0^2\int_0^{x^2-4} dy\,dx = \int_0^2 (x^2-4)\,dx = \left(\frac{x^3}{3} - 4x\right)\Big|_0^2 = \frac{8}{3} - 8 = -\frac{16}{3}$$

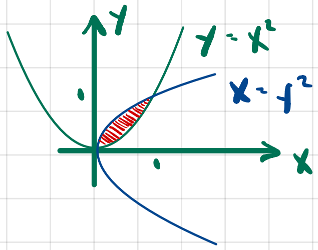
$$\bar{x} = \frac{\int_0^2\int_0^{x^2-4} x\,dy\,dx}{m} = \frac{\int_0^2 (x^3-4x)\,dx}{m} = \frac{\frac{x^4}{4} - 2x^2\Big|_0^2}{m}$$

$$= \frac{4-8}{-\frac{16}{3}} = \frac{4\cdot 3}{16} = \frac{3}{4}$$

$$\bar{y} = \frac{\int_0^2\int_0^{x^2-4} y\,dy\,dx}{m} = \frac{\int_0^2 \frac{1}{2}(x^2-4)^2\,dx}{m} = \frac{\int_0^2 (x^4 - 8x^2 + 16)\,dx}{2m} = \frac{\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]\Big|_0^2}{2m}$$

$$= \frac{\frac{32}{5} - \frac{64}{3} + 32}{-\frac{32}{3}} = -3\left[\frac{1}{5} - \frac{2}{3} + 1\right] = -3\left[\frac{3-10+15}{15}\right] = -3\cdot\frac{8}{15} = -\frac{8}{5}$$

15



$x^2=\sqrt{x} \Rightarrow x^4-x=0 \Rightarrow x(x^3-1)=0 \Rightarrow x^3=1 \Rightarrow x=1$

* R is a symmetric region about $y=x$. Because density is also symmetric about this line, the centroid lies on the line.

$\delta(x,y)=xy$

$$m=\iint_R dm=\iint_R \delta\, dA=\int_0^1\int_{y^2}^{\sqrt{y}} xy\,dx\,dy=\int_0^1 \frac{1}{2}y\,x^2\Big|_{y^2}^{\sqrt{y}}dy=\frac{1}{2}\int_0^1 y(y-y^4)\,dy=\frac{1}{2}\int_0^1(y^2-y^5)\,dy=\frac{1}{2}\left[\frac{y^3}{3}-\frac{y^6}{6}\right]\Big|_0^1$$

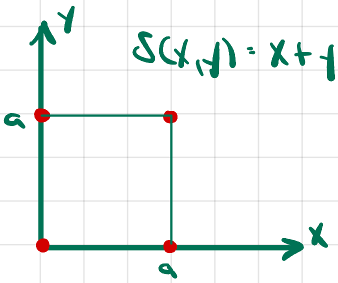
$$=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{6}\right]=\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$$

$$\bar{x}=\frac{\iint_R x\,dm}{m}=\frac{\iint_R x\,\delta\,dA}{m}=\frac{\int_0^1\int_{y^2}^{\sqrt{y}}x^2y\,dx\,dy}{m}=\frac{\int_0^1 y\,\frac{x^3}{3}\Big|_{y^2}^{\sqrt{y}}dy}{m}=\frac{\int_0^1 y(y^{3/2}-y^6)\cdot\frac{1}{3}\,dy}{m}=$$

$$=\frac{\int_0^1\frac{1}{3}(y^{5/2}-y^7)\,dy}{m}=\frac{1}{3m}\cdot\left[\frac{2}{7}y^{7/2}-\frac{y^8}{8}\right]\Big|_0^1=\frac{1}{3\cdot\frac{1}{12}}\left[\frac{2}{7}-\frac{1}{8}\right]=4\left[\frac{16-7}{56}\right]=\frac{9}{14}$$

$$\Rightarrow \bar{y}=\frac{9}{14}$$

21



$\delta(x,y)=x+y$

Symmetric region and density. centroid on $y=x$ line

$\bar{x}=\bar{y}=\frac{7a}{12}$

$$m=\iint_R\delta\,dA=\int_0^a\int_0^a(x+y)\,dx\,dy=\int_0^a\left(\frac{a^2}{2}+y\cdot a\right)dy$$

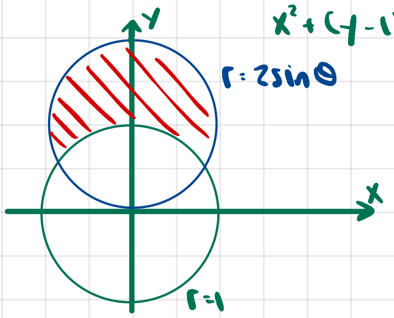
$$=\frac{a^2}{2}a+\frac{1}{2}a\cdot a^2=\frac{a^3}{2}+\frac{a^3}{2}=a^3$$

$$\bar{x}=\frac{\int_0^a\int_0^a x(x+y)\,dx\,dy}{m}=\frac{\int_0^a\left[\frac{1}{3}x^3+y\cdot\frac{1}{2}x^2\right]_0^a dy}{m}$$

$$=\frac{\int_0^a\left[\frac{1}{3}a^3+\frac{y}{2}a^2\right]dy}{m}=\frac{\frac{1}{3}a^4+\frac{1}{2}a^2\cdot\frac{a^2}{2}}{a^3}=\frac{a}{3}+\frac{a}{4}=\frac{7a}{12}$$

29 $r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2+y^2 = 2y \Rightarrow x^2+y^2-2y+1=1$

$x^2+(y-1)^2=1$



intersection

$$\begin{array}{l} r = 2\sin\theta \\ r = 1 \end{array} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6},\ \theta = \frac{5\pi}{6}$$

$\delta(x,y) = y$, Symmetric region and density rel. to y axis $\Rightarrow$ centroid on $x=0$

$$m = \int_{\pi/6}^{5\pi/6}\int_{1}^{2\sin\theta} r^2\sin\theta\, dr\, d\theta = \int_{\pi/6}^{5\pi/6} \frac{1}{3}\sin\theta\left(8\sin^3\theta - 1\right) d\theta = \int_{\pi/6}^{5\pi/6}\left[\frac{8}{3}\sin^4\theta - \frac{1}{3}\sin\theta\right] d\theta$$

$$= \frac{\sqrt{3}}{4} + \frac{2\pi}{3}$$

$$\bar{y} = \frac{\displaystyle\int_{\pi/6}^{5\pi/6}\int_{1}^{2\sin\theta} r^3\sin^2\theta\, dr\, d\theta}{m} = \frac{33\sqrt{3}+36\pi}{12\sqrt{3}+32\pi}$$