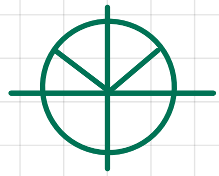
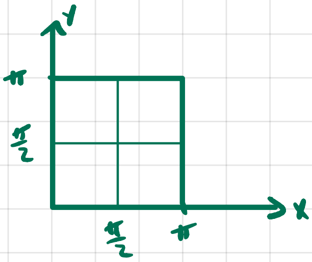
7 $f(x,y) = \sin x \sin y \qquad R: [0,\pi]\times[0,\pi]$

P - four equal squares

$$\iint_R f(x,y)\,dA = \lim_{\|P\|\to 0} \sum f(x_i^*, y_i^*)\Delta A_i \qquad \text{each } \Delta A_i = \frac{\pi^2}{4}$$

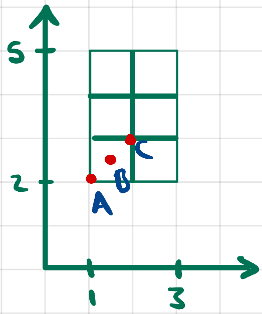




$$\sum_{i=1}^{4} f(x_i^*, y_i^*)\Delta A_i = \frac{\pi^2}{4}\left(f\left(\frac{\pi}{4},\frac{\pi}{4}\right) + f\left(\frac{\pi}{4},\frac{3\pi}{4}\right) + f\left(\frac{3\pi}{4},\frac{\pi}{4}\right) + f\left(\frac{3\pi}{4},\frac{3\pi}{4}\right)\right)$$

$$= \frac{\pi^2}{4}\left[4\cdot\frac{\sqrt2}{2}\cdot\frac{\sqrt2}{2}\right] = \pi^2/2$$

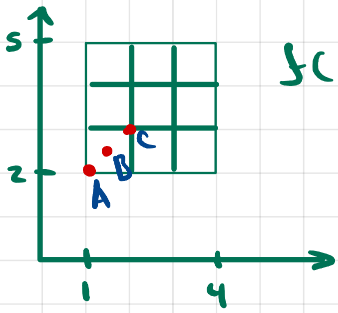
9 $f(x,y) = x^2y^2 \qquad R = [1,3]\times[2,5] \qquad P$ - six unit squares



$$\sum_{i=1}^{6} f(x_i^*, y_i^*)\Delta A_i$$

$f(A)<f(B)<f(C) \Rightarrow L<M<U$

10 $f(x,y) = \sqrt{100-x^2-y^2}$



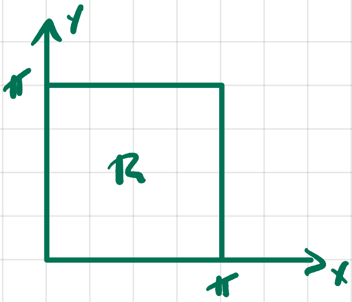
$f(A)>f(B)>f(C) \Rightarrow L>M>U$

13 $$\int_0^3\int_0^3 (xy+7x+y)\,dx\,dy = \int_0^3 \left[\frac{x^2y}{2} + \frac{7x^2}{2} + yx\right]\Big|_0^3 dy = \int_0^3 \left[\frac{9}{2}y + \frac{63}{2} + 3y\right]dy$$

$$= \frac{9y^2}{4} + \frac{63}{2}y + \frac{3y^2}{2}\Big|_0^3 = \frac{9\cdot 9}{4} + \frac{27}{2} + \frac{63\cdot 3}{2} = \frac{513}{4}$$

37 show $0 \leq \int_0^{\pi}\int_0^{\pi} \sin\sqrt{xy}\,dx\,dy \leq \pi^2$

$$\iint_R f(x,y)\,dA = \int_0^{\pi}\int_0^{\pi} \sin\sqrt{xy}\,dx\,dy = \lim_{\|P\|\to 0} \sum_{i=1}^{n} \sin\sqrt{x_i^* y_i^*}\,\Delta A_i$$



$\sin\sqrt{xy} \geq 0$ in R $\Rightarrow$ with any partition,

$f(x_i,y_i)\Delta A_i \geq 0$

$\sin\sqrt{xy} \in [0,1]$ in R

Area $= \pi^2 = \int_0^{\pi}\int_0^{\pi} dx\,dy \geq \iint_R \sin\sqrt{xy}\,dx\,dy$ because $\sin\sqrt{xy} \leq 1$ in R

39 $\iint_R [f(x,y)+g(x,y)]\,dA = \iint_R f(x,y)\,dA + \iint_R g(x,y)\,dA$

$$\iint_R f(x,y)\,dA = \sum f(x_i^*,y_i^*)\Delta A_i$$

$$\iint_R [f(x,y)+g(x,y)]\,dA = \sum (f(x_i^*,y_i^*)+g(x_i^*,y_i^*))\Delta A_i$$

$$= \sum f(x_i^*,y_i^*)\Delta A_i + \sum g(x_i^*,y_i^*)\Delta A_i$$