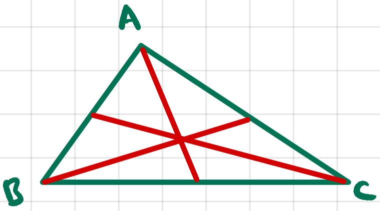
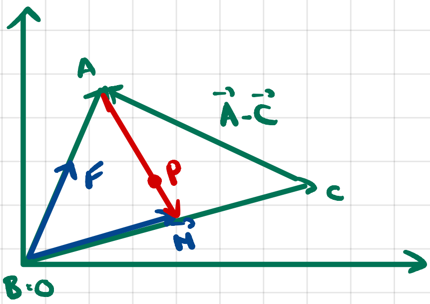
Realization 1

Show that the three medians of a triangle intersect at a point $2/3$ of the way from each vertex.





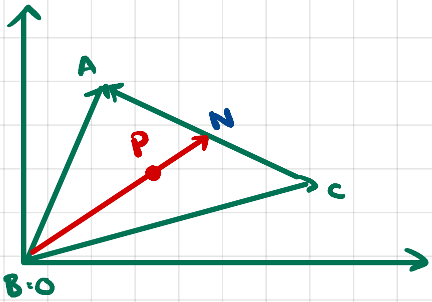
$\vec{ON} = \frac{1}{2}\vec{OC}$

$\vec{OP} = \vec{OA} + \frac{2}{3}\vec{AM}$

$\vec{AM} = \vec{OM} - \vec{OA} = \frac{1}{2}\vec{OC} - \vec{OA}$

$\vec{OP} = \vec{OA} + \frac{2}{3}\left(\frac{1}{2}\vec{OC} - \vec{OA}\right) = \vec{OA} + \frac{1}{3}\vec{OC} - \frac{2}{3}\vec{OA}$

$= \frac{1}{3}\vec{OA} + \frac{1}{3}\vec{OC}$



$\vec{ON} = \vec{OA} + \vec{AN}$

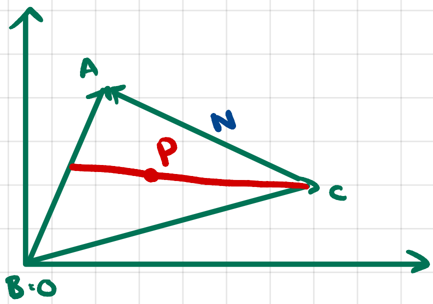
$\vec{OP} = \frac{2}{3}\vec{ON}$

$\vec{AN} = \frac{\vec{AC}}{2} = \frac{1}{2}\cdot(\vec{OC} - \vec{OA})$

$\Rightarrow \vec{ON} = \vec{OA} + \frac{1}{2}\vec{OC} - \frac{1}{2}\vec{OA} = \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OC}$

$\Rightarrow \vec{OP} = \frac{2}{3}\left(\frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OC}\right) = \frac{1}{3}\vec{OA} + \frac{1}{3}\vec{OC}$

We can do the same thing to show
$P = \frac{1}{3}\vec{OA} + \frac{1}{3}\vec{OC}$ also in the last
median.



Note the strategy to solve this problem:

We want position vector $\vec{OP}$ in terms of known position vectors $\vec{OA}$ and $\vec{OC}$.

Each vector should be written in terms of these known vectors.