Gravitational Attraction

Newton's law of Gravitation $\Rightarrow$ $\vec{F} = \dfrac{GMm}{|\vec{R}|^2}\vec{r}$

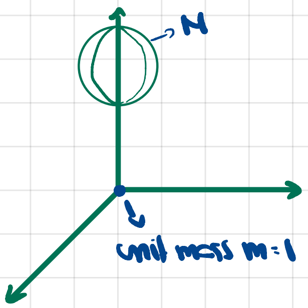
point mass, mass = M

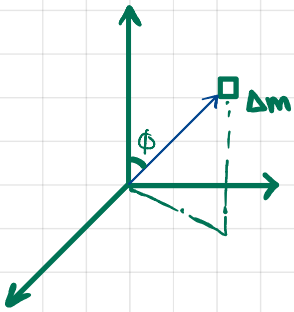
$\vec{r} \to dir(\vec{R})$

position vector from origin

Force exerted by point mass M on point mass m

Now imagine the mass M is a solid body, not a point mass



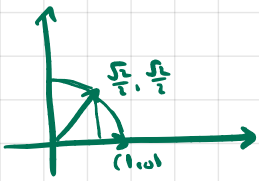


$\Delta m \approx \delta(x,y,z)\cdot\Delta V$
point mass

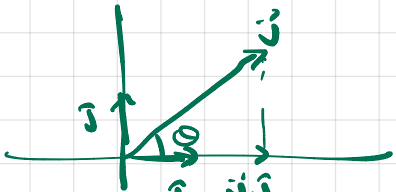
Divide the solid body into small masses, each approx. a point mass Δm.

Calculate gravitational force exerted by each such approximate point mass, sum force of all such point masses.

$\Delta F = \dfrac{G\Delta m \cdot 1}{\rho^2}\vec{r} = \dfrac{G\delta\Delta V}{\rho^2}\vec{r}$



Note the result is a vector, $\vec{r} = \langle \cos\alpha, \cos\beta, \cos\phi \rangle$, ie a vector of direction cosines of $\vec{R}$.



$\hat{i} = \dfrac{\vec{v}\cdot\hat{i}}{|\vec{v}|}$

$\vec{v}\cdot\hat{i} = |\vec{v}|\cos\theta$

$\dfrac{\vec{v}}{|\vec{v}|}\cdot\hat{i}$

$\Delta F_z = \dfrac{G\delta\Delta V}{\rho^2}\cos\phi$

$F_z = \lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\dfrac{G\Delta m_i\cos\phi}{\rho^2} = G\iiint_V \dfrac{\cos\phi}{\rho^2}\delta\, dV$, the z component of the gravitational force exerted by V on point mass at origin.

In spherical coord., $dV = \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

$G\iiint \delta\cos\phi\sin\phi\, d\rho\, d\phi\, d\theta$