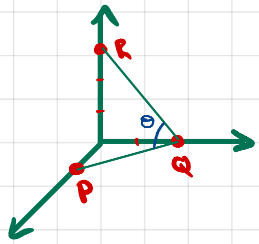
Problem 1



$\vec{OP} = \langle 1,0,0\rangle$
$\vec{OQ} = \langle 0,2,0\rangle$
$\vec{OR} = \langle 0,0,3\rangle$

a) $\vec{QP} = \vec{OP} - \vec{OQ} = \langle 1,-2,0\rangle$

$\vec{QR} = \vec{OR} - \vec{OQ} = \langle 0,-2,3\rangle$

b) $\vec{QP}\cdot\vec{QR} = 4 = \sqrt{5}\sqrt{13}\cos\theta \Rightarrow \cos\theta = \dfrac{4}{\sqrt{65}} \Rightarrow \theta = \cos^{-1}(4/\sqrt{65})$

Problem 2

$P(1,1,1) \qquad Q(0,3,1) \qquad R(0,1,4)$

a) $\vec{PQ} = \langle -1,2,0\rangle \qquad \vec{PR} = \langle 1,0,-3\rangle$

$A = \frac{1}{2}\,|\vec{PQ}\times\vec{PR}|$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ 1 & 0 & -3 \end{vmatrix} = \hat{i}(-6) - \hat{j}(3) + \hat{k}(-2) = \langle -6,-3,-2\rangle$$

$|\vec{PQ}\times\vec{PR}| = \sqrt{36+9+4} = \sqrt{49} = 7$

Area $= \dfrac{7}{2}$

b) $\langle -6,-3,-2\rangle = \vec{n} \perp$ plane containing $\vec{PQ}$ and $\vec{PR}$.

$-6x-3y-2z = -11 \Rightarrow 6x+3y+2z = 11$

c) line through $(1,2,3)$ and $(2,2,0)$ is parallel to $\langle 1,0,-3\rangle$

param. eq: $\langle 1+t, 2, 3-3t\rangle$

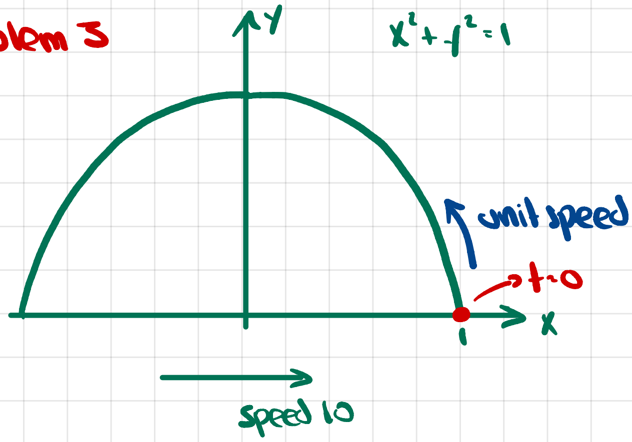
is there intersection w/ plane?

$6(1+t) + 3\cdot 2 + 2(3-3t) = 1$

$6 + \cancel{6t} + 6 + 6 - \cancel{6t} = 1 \Rightarrow 18 = 1$, false $\Rightarrow$ no t gives $\langle x,y,z\rangle$ satisfying the plane eq.

$\Rightarrow$ no intersection $\Rightarrow$ parallel

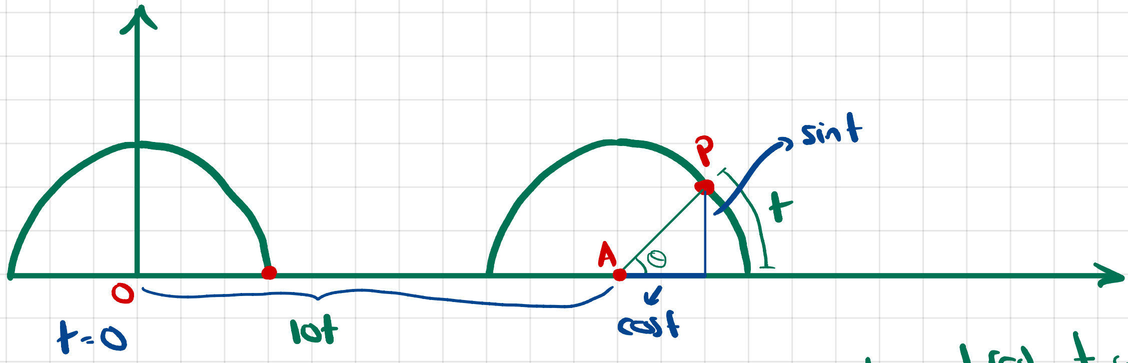
Problem 3



Arc length $= \theta(t)\cdot r = \theta(t) = t$

$\pi = t =$ time to reach back bumper

a)



$$\theta(t) = \frac{1 \text{ rad}}{1 \text{ unit time}} \cdot t \text{ unit time} = t \text{ rad}$$

$\vec{r}(0) = \langle 1, 0 \rangle$

$\vec{OP} = \vec{OA} + \vec{AP}$

$\vec{OA} = \langle 10t, 0 \rangle$

$\vec{AP} = \langle \cos t, \sin t \rangle$

$\vec{OP} = \vec{r}(t) = \langle 10t + \cos t, \sin t \rangle$

$t = 0 \Rightarrow \langle 1, 0 \rangle$

$r(\pi) = \langle 10\pi - 1, 0 \rangle$

b) $\vec{v}(t) = \langle 10 - \sin t, \cos t \rangle$

$|\vec{v}(t)|^2 = V_2(t) = 100 - 20\sin t + \sin^2 t + \cos^2 t = 101 - 20\sin t = \text{speed}^2(t)$

$$\frac{dV_2(t)}{dt} = -20\cos t = 0 \Rightarrow t = \frac{\pi}{2}$$

$$\frac{d^2 v(t)^2}{dt} = 20\sin t$$

At $\pi/2$: $20 > 0 \Rightarrow$ concave up, local min speed at $t = \pi/2$

check boundaries: $t = 0, t = \pi$

$$\left.\begin{aligned} V_2(0) &= 101 - 20\cdot 0 = 101 \\ V_2(\pi) &= 101 - 20\cdot 0 = 101 \\ V_2(\pi/2) &= 101 - 20 = 81 \end{aligned}\right\} \Rightarrow$$

max at $t = 0$ and $t = \pi$, start and end of trajectory.

min at $t = \pi/2$.

Problem 4

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

a) $\det M = 1(-2+1) - 2(-3-2) + 3(-3-4) = -1+10-21 = -12$

b) $$M^{-1} = \frac{1}{-12}\begin{bmatrix} \cdot & a & b \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}^T$$

$$\Rightarrow M^{-1} = \frac{1}{12}\begin{bmatrix} \cdot & \cdot & \cdot \\ -5 & \cdot & \cdot \\ 7 & \cdot & \cdot \end{bmatrix}$$

$a = -[-3-2] = 5$

$b = +[-3-4] = -7$

c) $$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix} \qquad AX = B$$

$$A^{-1} = \frac{1}{12}\begin{bmatrix} 1 & 1 & 4 \\ -5 & 7 & -8 \\ 7 & -5 & 4 \end{bmatrix}$$

$$A^{-1}B = \frac{1}{12}\begin{bmatrix} 1 & 1 & 4 \\ -5 & 7 & -8 \\ 7 & -5 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} t+12 \\ 7t-24 \\ -5t+12 \end{bmatrix} = \vec{r}(t)$$

d) $$\frac{d\vec{r}}{dt} = \langle 1, 7, -5 \rangle / 12$$

Problem 5

a) $P(t)$ with position $\vec{r}(t)$

$P(t)$ lies on plane $4x-3y-2z=6$ $\qquad \vec{n} = \langle 4,-3,-2 \rangle$

$\vec{r}(t)\cdot\vec{n} = \langle x,y,z \rangle \langle 4,-3,-2 \rangle = 6$

b) $$\frac{d\vec{r}}{dt} : \vec{r}'\cdot\vec{n} + \vec{r}\cdot n' = 0$$

$\Rightarrow \vec{v}\cdot\vec{n} = 0$