Planes Tangent to Surfaces

Def:

$f(x,y)$ has continuous partial derivatives on circular disk centered at (a,b) $\Rightarrow$ plane tangent to $z=f(x,y)$ at $P(a,b,f(a,b))$ is plane through P that contains the lines tangent at P to the two curves

$$z=f(x,b),\quad y=b \qquad (x\text{-curves})$$

$$z=f(a,y),\quad x=a \qquad (y\text{-curve})$$

How do we find an eq. for this plane?

Typical non-vertical plane in space: $A(x-a)+B(y-b)+C(z-c)=0,\ C\neq 0$

$\Rightarrow z-c=p(x-a)+q(y-b)$

Note

$y=b \Rightarrow z-c=p(x-a)$, a line

$x=a \Rightarrow z-c=q(y-b)$, a line

These lines are tangent lines if $p=\dfrac{\partial z}{\partial x}$ and $q=\dfrac{\partial z}{\partial y}$

But we know

$$\left.\frac{\partial z}{\partial x}\right|_{(a,b)}=f_x(a,b) \qquad \left.\frac{\partial z}{\partial y}\right|_{(a,b)}=f_y(a,b)$$

$\Rightarrow$ the tangent plane at $P(a,b,f(a,b))$ is

$$z-f(a,b)=f_x(a,b)(x-a)+f_y(a,b)(x-b)$$

or

$$f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)-(z-z_0)=0$$

$\Rightarrow \vec{n}=\langle f_x(x_0,y_0), f_y(x_0,y_0), -1\rangle$