simplest multiple integral: $\iint_R f(x,y)\,dA$ of continuous function $f(x,y)$ over the rectangle

$R = [a,b] \times [c,d]$

partition P of R into subrectangles $R_1, \dots, R_k$:

$a = x_0 < x_1 < \dots < x_m = b$

$c = y_0 < y_1 < \dots < y_n = d$

choose arbitrary point (x_i^*, y_i^*) in R_i for each i

$S = \{(x_i^*, y_i^*) \mid 1 \leq i \leq k\}$ - **selection** for partition $P = \{R_i \mid 1 \leq i \leq k\}$

consider $f(x,y)$ above the region R.

ΔA_i is the area of R_i $\Rightarrow$ volume of i^{th} column formed with $f(x_i^*, y_i^*)$ is $f(x_i^*, y_i^*)\Delta A_i$

sum of volumes: $\sum_{i=1}^{k} f(x_i^*, y_i^*)\Delta A_i$

norm $|P|$: max diagonal length among R_i

definition of double integral of f over R: $\iint_R f(x,y)\,dA = \lim_{|P| \to 0} \sum_{i=1}^{k} f(x_i^*, y_i^*)\Delta A_i$

provided this limit exists

How to compute this double integral without actually calculating the limit directly?

theorem $\iint_R f(x,y)\,dA = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx = \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$

iterated single-variable integrations