PSet 2

1F-6

a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $A^3 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ by induction

Let T be set of $n \in \mathbb{N}$ s.t. $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$A^1 = A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow 1$ in A

Assume k in $A \Rightarrow A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

$A^{k+1} = A^k \cdot A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

1F-8

a) $A\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \\ 1 & 4 & -1 \end{bmatrix} = A$

b) $A\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 7 \\ 0 & 0 & 1 \\ 4 & 3 & 1 \end{bmatrix}$

$A \cdot M = N \Rightarrow A = M^{-1}N$

$\det(M) = 2(1-2) - 1(0) + 0(0) = -2 \neq 0 \Rightarrow$ row vectors form parallelepiped with nonzero volume, ie the vectors are not coplanar

$M^{-1} = \frac{1}{-2}\operatorname{adj}(M)$

$\operatorname{adj}(M) = \begin{bmatrix} 1-2 & -(0) & +(0) \\ -(1) & +(2) & -(2) \\ +(2) & -(4) & +(2) \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 2 & -2 \\ 2 & -4 & 2 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & 2 \\ 0 & 2 & -4 \\ 0 & -2 & 2 \end{bmatrix}$

$M^{-1} = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 2 \\ 0 & 2 & -4 \\ 0 & -2 & 2 \end{bmatrix} \Rightarrow A = M^{-1}N = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 2 \\ 0 & 2 & -4 \\ 0 & -2 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 & 7 \\ 0 & 0 & 1 \\ 4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2+8 & -3+6 & -7-1+2 \\ -16 & -12 & 2-4 \\ 8 & 6 & -2+2 \end{bmatrix} \cdot -\frac{1}{2}$

$= -\frac{1}{2}\begin{bmatrix} 10 & 3 & -6 \\ -16 & -12 & -2 \\ 8 & 6 & 0 \end{bmatrix} = \begin{bmatrix} -5 & -3/2 & 3 \\ 8 & 6 & 1 \\ -4 & -3 & 0 \end{bmatrix} = A$

1F-9

$A \cdot A^T = I \Rightarrow$ orthogonal matrix

a) let $r_i = [r_{i_1} \ \ r_{i_2} \ \ r_{i_3}]$ be i^{th} row of A

$$A \cdot A^T = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \cdot \begin{bmatrix} r_1^T & r_2^T & r_3^T \end{bmatrix} = I_{3\times3}$$

$$\Rightarrow \begin{aligned} r_1 r_1^T &= 1 \\ r_2 r_2^T &= 1 \\ r_3 r_3^T &= 1 \end{aligned}$$

But $r_i r_i^T = r_{i_1}^2 + r_{i_2}^2 + r_{i_3}^2 = |r_i|^2$

$\Rightarrow |r_1|^2 = |r_2|^2 = |r_3|^2 = 1 \Rightarrow |r_1| = |r_2| = |r_3| = 1$

b) From

$$A \cdot A^T = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \cdot \begin{bmatrix} r_1^T & r_2^T & r_3^T \end{bmatrix} = I_{3\times3}$$

we have $r_i \cdot r_j^T = 0 \quad i \neq j$

but $r_i \cdot r_j^T = r_{i_1} r_{j_1} + r_{i_2} r_{j_2} + r_{i_3} r_{j_3} = \vec{r_i} \cdot \vec{r_j}$

$\Rightarrow \vec{r_i} \cdot \vec{r_j} = 0 \quad i \neq j$ so each vector represented by a row is orthogonal to each of the vectors repres. by the other rows.

1G-3

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$ if $\det A \neq 0$

$\det(A) = 1(2-(-1)) - 1(-1-1) = 3 - 1(-2) = 5$

$$\text{minors} = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & -2 \\ -2 & 1 & 1 \end{bmatrix} \quad \text{cofactors} = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & 2 \\ -2 & -1 & 1 \end{bmatrix} \quad \text{adj}(A) = \text{cofactors}^T = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5}\text{adj}(A) \Rightarrow X = \frac{1}{5}\begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 6-6 \\ -2-3 \\ 2+3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 0 \\ -5 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

16-4

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$

From 16-3, $A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

$$X = \frac{1}{5}\begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$\Rightarrow \quad 5x_1 = 3y_1 + y_2 - 2y_3$

$5x_2 = -y_1 + 3y_2 - y_3$

$5x_3 = y_1 + 2y_2 + y_3$

16-5 Show $(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} \cdot AB = I = B^{-1}A^{-1}AB = B^{-1}B = I$

1H-3

a) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & c & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$AX = B$

$\det A = 1(2-c) - (-1)(4+1) + 1(2c+1)$

$= 2 - c + 5 + 2c + 1 = c + 8$

$\det A \neq 0 \Rightarrow c + 8 \neq 0 \Rightarrow c \neq -8 \Rightarrow$ only trivial solution $\vec{x} = \vec{0}$

$c = -8 \Rightarrow \det A = 0$

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & -8 & 2 \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ $\quad n_1 \cdot (n_2 \times n_3) = 0 = \det A$

note: $\vec{n}_1, \vec{n}_2, \vec{n}_3$ are coplanar. Not lin. indep.

b) $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = c \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} 2-c & 1 \\ 0 & -1-c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

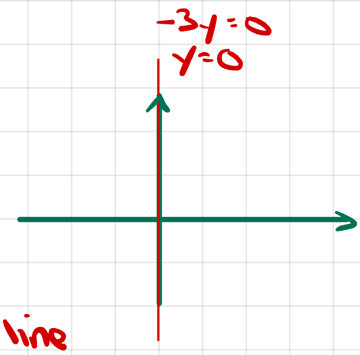
$AX = B$

$\det A = (2-c)(-1-c) = -(2-c)(1+c) = -[2 + 2c - c - c^2] = c^2 - c - 2 = 0$

$\Delta = 1 - 4 \cdot (-2) = 9 \quad c = \frac{1 \pm 3}{2} \begin{matrix} \nearrow 2 \\ \searrow -1 \end{matrix}$

① $c = 2$

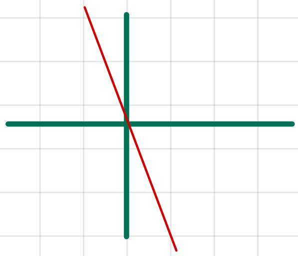
$\begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow y = 0, \; -3y = 0 \Rightarrow y = 0$



Two lin. dependent equations, represent same line

② $c = -1$

$\begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 3x + y = 0 \Rightarrow y = -3x$



There is only one equation.

c) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & -8 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$AX = B$

$\det A = 0$

Solution 1

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & -8 & 2 \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ → vectors normal to planes in the system of eq.

Any solution is on all three planes → position vector parallel to a vector orthogonal to any of the normal vectors

$$n_1 \times n_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = +\hat{\imath}(-1-1) - \hat{\jmath}(1-2) + \hat{k}(1+2)$$

$$= \langle -2, 1, 3 \rangle$$

$\langle x_1, x_2, x_3 \rangle = \langle 0,0,0 \rangle + t\langle -2, 1, 3 \rangle$ = parametric line

$x_1 = -2t$
$x_2 = t$
$x_3 = 3t$

Check solution: $1(-2t) - 1(t) + 1(3t) = -2t - t + 3t = 0$

Solution 2

$k_1[1 \ -1 \ 1] + k_2[2 \ 1 \ 1] = [-1 \ -8 \ 2]$

$k_1 + 2k_2 = -1$ check: $5 + 2(-3) = -1$ ✓
$-k_1 + k_2 = -8$ ⇒ $2k_2 = -6$ ⇒ $k_2 = -3$ ⇒ $k_1 = 2+3 = 5$
$k_1 + k_2 = 2$

$\Rightarrow 5\vec{n}_1 - 3\vec{n}_2 = \vec{n}_3 \Rightarrow \vec{n}_3 \cdot \vec{x} = (5\vec{n}_1 - 3\vec{n}_2)\vec{x} = 5\vec{n}_1 x - 3n_2\vec{x}$

So if we find a solution for the first two equations (i.e. a point on the first two planes) we automatically have a solution for the third plane because the normal vector $\vec{n}_3$ is a linear combin. of $\vec{n}_1$ and $\vec{n}_2$ so it too will necess. be ⊥ to the solution vector.

If $\vec{n}_3$ were l.i. from $\vec{n}_1, \vec{n}_2$ we could not say that a sol. to eq 1) and 2) solved 3).

$x - y + z = 0$
$2x + y + z = 0$

Given a $y = t$, we have: $x - t + z = 0 \Rightarrow x = t - z$

$\Rightarrow 2(t-z) + t + z = 0 \Rightarrow 2t - 2z + t + z = 0$

$\Rightarrow 3t = z$, and $x - t + 3t = 0 \Rightarrow x = -2t$

So, $(-2t, t, 3t)$ is a parametrized solution.

IE-1

a) $P_0\ (2,0,-1) \quad \perp (1,2,-2)$

$$\langle x-2, y, z+1\rangle \langle 1,2,-2\rangle = 0$$

$$x-2+2y-2(z+1)=0$$
$$x+2y-2z=4$$

b) $(0,0,0) \quad (1,1,0) \quad (2,-1,3)$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 2 & -1 & 3 \end{vmatrix} = \hat{\imath}(3)-\hat{\jmath}(3)+\hat{k}(-1-2) = \langle 3,-3,-3\rangle = \vec{n}$$

$$\langle x,y,z\rangle\langle 3,-3,-3\rangle = 0$$

$$3x-3y-3z=0$$

c) $\underset{P_1}{(1,0,1)} \quad \underset{P_2}{(2,-1,2)} \quad \underset{P_3}{(-1,3,2)}$

$\vec{P_1P_2} = \langle 1,-1,1\rangle \qquad \vec{P_1P_3} = \langle -2,3,1\rangle$

$$\vec{P_1P_2} \times \vec{P_1P_3} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ -2 & 3 & 1 \end{vmatrix} = \hat{\imath}(-1-3)-\hat{\jmath}(1+2)+\hat{k}(3-2) = \langle -4,-3,1\rangle$$

$$-4x-3y+z=d \qquad \Rightarrow \quad -4x-3y+z=-3$$
$$-4\cdot 1+1=d=-3$$

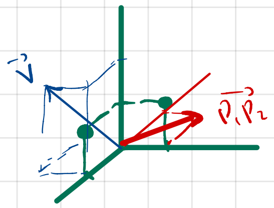
d) $\underset{P_1}{(a,0,0)}, \underset{P_2}{(0,b,0)}, \underset{P_3}{(0,0,c)}$

$\vec{P_1P_2} = \langle -a,b,0\rangle \qquad \vec{P_1P_3} = \langle -a,0,c\rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = \hat{\imath}(bc)-\hat{\jmath}(-ac)+\hat{k}(ab) = \langle bc,ac,ab\rangle$$

$$bcx+acy+abz=d \Rightarrow abc=d \Rightarrow bcx+acy+abz=abc$$
$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

e) $(1,0,1)$, $(0,1,1)$ parallel to $\langle 1,-1,2\rangle = \vec{v}$
P_1 P_2



$\overrightarrow{P_1P_2} = \langle -1,1,0\rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix}$$

$\hat{i}(2) - \hat{j}(-2) + \hat{k}(1-1) = \langle 2,2,0\rangle$

$\langle x-1, y, z-1\rangle \langle 2,2,0\rangle = 0$

$2x-2+2y=0 \Rightarrow x+y=1$

1E-2 $2x-y+z=3$ $\langle 2,-1,1\rangle = \vec{n}_1$
$x+y+2z=1$ $\langle 1,1,2\rangle = \vec{n}_2$

$|\vec{n}_1| = \sqrt{4+1+1} = \sqrt{6}$

$|\vec{n}_2| = \sqrt{1+1+4} = \sqrt{6}$

$\vec{n}_1 \cdot \vec{n}_2 = 2-1+2 = 3 = \sqrt{6}\cdot\sqrt{6}\cdot\cos\alpha$

$\cos\alpha = \frac{3}{6} = \frac{1}{2} \Rightarrow \alpha = \pi/3$

1E-3

a) $P_0 = (1,0,-1)$ $\vec{v} = \langle 2,-1,3\rangle$

$\overrightarrow{P_0P} = \langle x-1, y, z+1\rangle = t\langle 2,-1,3\rangle$

$x-1 = 2t$ $y = -t$ $z+1 = 3$

$x = 1+2t$ $y = -t$ $z = -1+3t$

b) $P_0 = (2,-1,-1)$ $\perp$ to $x-y+2z=3$

$\vec{n} = \langle 1,-1,2\rangle$

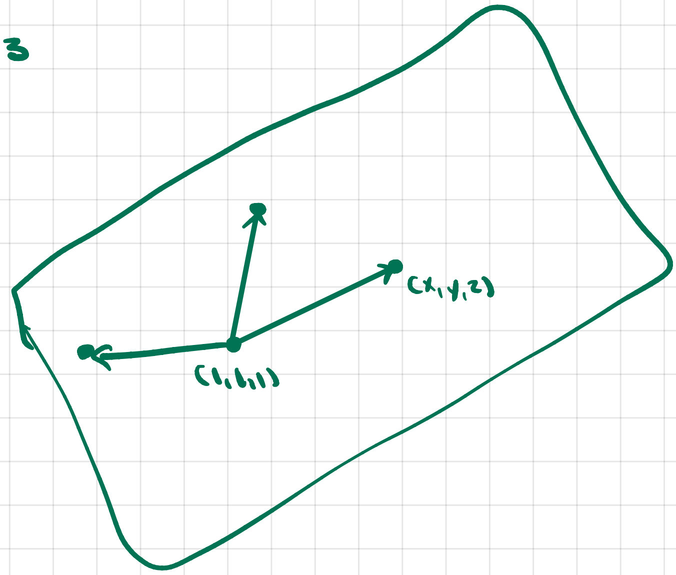
$\overrightarrow{P_0P} = \langle x-2, y+1, z+1\rangle$

our plane should be parallel to $\vec{n}$.

$\overrightarrow{P_0P} = t\vec{n} \Rightarrow x-2 = t \Rightarrow x = 2+t$
$y+1 = -t \Rightarrow y = -1-t$
$z+1 = 2t \Rightarrow z = -1+2t$

c) lines through $P_0 = (1,1,1)$ on plane $x+2y-z=2$

Let $P = (x,y,z)$ be on a line, ie on the plane.



$(x,y,z) = (1,1,1) + t(a,b,c)$, $a+2b-c=2$

$x = 1+at$
$y = 1+bt$
$z = 1+ct$

1E-5 Line through (1,1,-1), perpend. to $x+2y-z=3$, intersection with $2x-y+z=1$

Line: $(x,y,z)=(1,1,-1)+t(1,2,-1)$

$x=1+t$ $\quad 2(1+t)-(1+2t)+(-1-t)=1$

$y=1+2t$ $\quad \cancel{2}+\cancel{2t}-\cancel{1}-\cancel{2t}-\cancel{1}-t=1$

$z=-1-t$ $\quad t=-1$

$\Rightarrow x(-1)=0$

$y(-1)=-1$ $\Rightarrow$ intersection at $(0,-1,0)$

$z(-1)=0$

1E-6 Distance origin to $ax+by+cz=d$

$\langle a,b,c\rangle=\vec{n}$

$t\langle a,b,c\rangle=\langle x,y,z\rangle$ line through origin parallel to $\vec{n}$

$x=at$ $\quad a^2t+b^2t+c^2t=d$

$y=bt$

$z=ct$ $\quad t=\dfrac{d}{a^2+b^2+c^2}$

line crosses plane when $t^*=d/(a^2+b^2+c^2)$

$\langle x,y,z\rangle=\langle at^*,bt^*,ct^*\rangle$

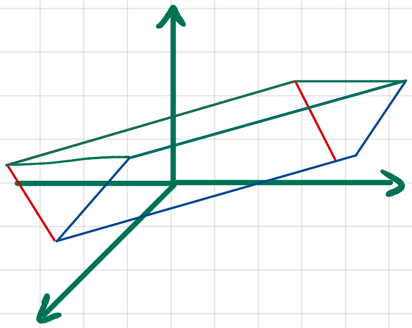
$|\langle x,y,z\rangle|=\sqrt{t^{*2}(a^2+b^2+c^2)}=|t^*|\sqrt{a^2+b^2+c^2}$

$=\dfrac{|d|}{\sqrt{a^2+b^2+c^2}}$

Problem 1

$P_1, P_2, P_3 \in \mathbb{R}^3$

a)



b) Let $\vec{n}_1, \vec{n}_2, \vec{n}_3$ be the normal vectors to the three planes.

$\vec{n}_1$ and $\vec{n}_2$ are always on the same plane. The normal vector $\vec{v}$ to the shaded plane is perpendicular to both original vectors, and is thus on the intersection of the planes defined by using those two original vectors as normal vectors. Now consider a third vector, $\vec{n}_3$ and consider the plane it shares w/ one of the first two vectors, say $\vec{n}_1$. The planes defined by using $\vec{n}_3$ and $\vec{n}_1$ as normal vectors have an intersection, a line parallel to a normal vector of the shaded plane of $\vec{n}_1, \vec{n}_3$. Since this intersection is parallel to the first intersection line, this intersection must be perpendicular to $\vec{n}_2$. Thus it is perpendicular to the plane shared by $\vec{n}_1, \vec{n}_2$.

$\Rightarrow$ The plane shared by $\vec{n}_3, \vec{n}_1$ is the same as the plane shared by $\vec{n}_1, \vec{n}_2$ because they share the same normal vector.

c) Intersection lines all of form

$$\langle x(t), y(t), z(t)\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$$

and they differ with respect to $\langle x_0, y_0, z_0\rangle$.

Any plane perpendicular to these lines obeys $\langle n_1, n_2, n_3\rangle \cdot \langle a, b, c\rangle = 0$

$\langle a, b, c\rangle$ is thus a normal vector for a plane of vectors $\langle n_1, n_2, n_3\rangle$.

But $\langle a, b, c\rangle$ is in each of the original three planes P_1, P_2, and P_3 and is thus perpendicular to their normal vectors. The latter must all be in the plane of vectors $\langle n_1, n_2, n_3\rangle$

Problem 2

Three raw materials: $M_1, M_2, M_3 \rightarrow$

	M_1	M_2	M_3
P_1	1	2	3
P_2	1	3	5
P_3	3	5	8
	137	279	448

The problem description is ambiguous. One interpretation is: To produce 1 unit of P_1 you need $1\ M_1$, $2\ M_2$, $3\ M_3$. Another interpretation is: To produce 1 unit P_1, 1 total unit of raw material is used, so $\frac{3}{6} M_3$, $\frac{2}{6} M_2$, $\frac{1}{6} M_1$.

Interpretation 1

a) $$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 8 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 137 \\ 279 \\ 448 \end{bmatrix}$$

$$AP = M \Rightarrow P = A^{-1}M$$

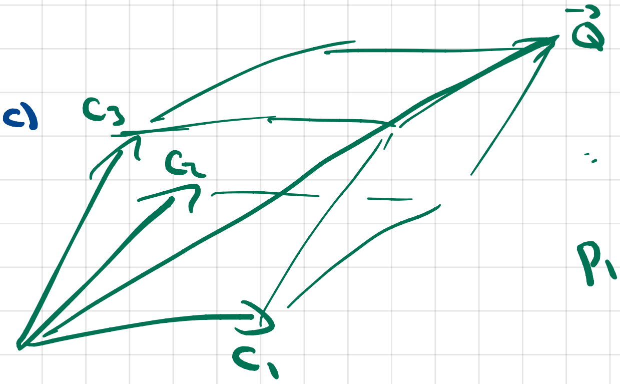
b) $$\det A = +1(24-25) - 1(16-15) + 3(10-9)$$
$$= -1 - 1 + 3 = 1$$

$$A^{-1} = \begin{bmatrix} -1 & -1 & 1 \\ -(-7) & -1 & -2 \\ -4 & -(-1) & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -1 & 1 \\ 7 & -1 & -2 \\ -4 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 7 & -4 \\ -1 & -1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} -1 & 7 & -4 \\ -1 & -1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 137 \\ 279 \\ 448 \end{bmatrix} = \begin{bmatrix} 24 \\ 32 \\ 27 \end{bmatrix}$$

c)



$$\therefore \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix}$$

$$P_1\vec{C_1} + P_2\vec{C_2} + P_3\vec{C_3} = \vec{Q}$$

Interpretation 2

a) $$\begin{bmatrix} 1/6 & 1/9 & 3/16 \\ 2/6 & 3/9 & 5/16 \\ 3/6 & 5/9 & 8/16 \end{bmatrix} = \begin{bmatrix} 1/6 & 1/9 & 3/16 \\ 1/3 & 1/3 & 5/16 \\ 1/2 & 5/9 & 1/2 \end{bmatrix} = A$$

$$A \cdot P = M \Rightarrow \begin{bmatrix} 1/6 & 1/9 & 3/16 \\ 1/3 & 1/3 & 5/16 \\ 1/2 & 5/9 & 1/2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 137 \\ 279 \\ 448 \end{bmatrix}$$

$$\det A = \frac{1}{6}\left[\frac{1}{6} - \frac{25}{16 \cdot 9}\right] - \frac{1}{9}\left[\frac{1}{6} - \frac{5}{32}\right] + \frac{3}{16}\left[\frac{5}{27} - \frac{1}{6}\right] = \frac{1}{864}$$

b) $$A^{-1} = \begin{bmatrix} -6 & 42 & -24 \\ -9 & -9 & 9 \\ 16 & -32 & 16 \end{bmatrix}$$

$$P = A^{-1}M = \begin{bmatrix} 144 \\ 288 \\ 432 \end{bmatrix}$$

c) If $\det(A) = 0$ then $AP = M$ has either 0 or ∞ solutions depending on choice of M. This happens when a column (or row) is a linear combin. of the others.

In this problem we also have the sum of elements of each column equal to 1.

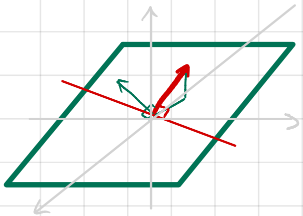
$$\begin{bmatrix} 1/6 & 1/9 & a \\ 2/6 & 3/9 & b \\ 3/6 & 5/9 & c \end{bmatrix} \qquad a+b+c = 1$$

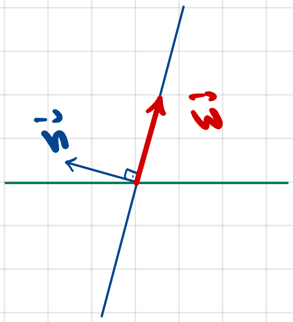
$$a = \frac{3k_1 + 2k_2}{18} \qquad b = \frac{6k_1 + 6k_2}{18} \qquad c = \frac{9k_1 + 10k_2}{18}$$

$$\frac{18k_1 + 18k_2}{18} = 1 \Rightarrow k_1 + k_2 = 1$$

So as long as $k_1 + k_2 = 1$ the proportions in the last row will sum to 1. We are essentially assigning the ratios of P_1 and P_2 for P_3.

Problem 3





$\vec{w}$ is perpendicular to the line of intersection of the plane and the x-y plane.

It is also perpend. to the normal vector $\vec{n}$.

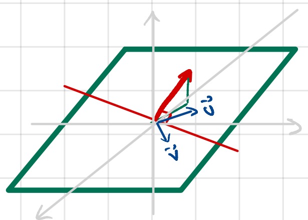
Given a vector on the intersection line, $\vec{v}$, we have $\vec{n} \times \vec{v} = \vec{w}$

$\vec{v}$ can be obtained as the cross product of $\vec{n}$ and a vector normal to the x-y plane, $\hat{h}$

$w = \vec{n} \times (\hat{h} \times \vec{n})$

$\vec{u} \times \vec{v} = \vec{n}$

b)



$\vec{w} = \vec{n} \times (\hat{h} \times \vec{n})$
$= (\vec{u} \times \vec{v}) \times (h \times (\vec{u} \times \vec{v}))$