constant coeffs., first order, linear eq. $\dot{y} + ky = q(t)$

solve with integrating factor

$$\dot{y}e^{kt} + ke^{kt}y = e^{kt}q(t) \Rightarrow y(t) = e^{-kt}\left[\int e^{kt}q(t)\,dt + c\right] = e^{-kt}\int e^{kt}q(t)\,dt + ce^{-kt}$$
$$= y_p(t) + c\,y_h(t)$$

Case 1: $k>0$ $\Rightarrow$ exponential decay

input $= 0 \Rightarrow y(t) = ce^{-kt}$, $y \to 0$ as $t \to \infty$

transient

in general $\underbrace{e^{-kt}\int e^{kt}q(t)\,dt}_{\text{steady-state (ie long-term) solution}} + \underbrace{ce^{-kt}}_{\text{transient}}$

* every solution goes asymptotically to the steady state (all solution curves approach the steady-state as $t \to \infty$)

Case 2: $k \leq 0$ $\Rightarrow$ $y_h(t)$ does not go to 0 as $t \to \infty$

no steady-state solution