31 a) $\frac{dy}{dx} + P(x)y = 0$

$y_c = Ce^{-\int P(x)dx}$

$\frac{dy_c}{dx} = Ce^{-\int P(x)dx} \cdot (-P(x)) = -P(x)y_c \Rightarrow \frac{dy_c}{dx} + P(x)y_c = 0$

b) $\frac{dy}{dx} + P(x)y = Q(x)$

$y_p = e^{-\int P(x)dx}\left[\int Q(x)e^{\int P(x)dx}dx\right]$

$\frac{dy_p}{dx} = -P(x)e^{-\int P(x)dx}\left[\int Q(x)e^{\int P(x)dx}dx\right] + e^{-\int P(x)dx}Q(x)e^{\int P(x)dx}$

$= -P(x)y_p + Q(x) \Rightarrow \frac{dy}{dx} + P(x)y = Q(x)$

c) $\frac{dy_c}{dx} + P(x)y_c = 0$

$\frac{dy_p}{dx} + P(x)y_p = Q(x)$

By superpos. we know $y_p + c_2 y_c$ is sol. to $y'(x) + P(x)y(x) = Q(x)$

$\frac{d}{dx}(y_p + cy_c) = \frac{dy_p}{dx} + c\frac{dy_c}{dx} = (Q(x) - P(x)y_p) + c(-P(x)y_c)$

$= -P(x)(y_p + cy_c) + Q(x)$

$\Rightarrow \frac{d}{dx}(y_p + cy_c) + P(x)(y_p + cy_c) = Q(x)$

$\Rightarrow y_p + y_c$ is a general solution

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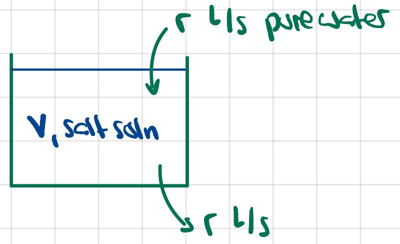

$x(t)$ = Amount salt, grams (g)

$\Delta x = r\frac{L}{s} \cdot 0\frac{g}{L}\Delta t - r\frac{L}{s} \cdot \frac{x(t)}{V}\frac{g}{L}\Delta t \Rightarrow \underbrace{\frac{dx}{dt} = -\frac{rx(t)}{V}}_{\dot{x} + kx = 0 \Rightarrow x = x_0 e^{-kt}} \Rightarrow \frac{1}{x}dx = -\frac{r}{V}dt \Rightarrow \ln x = -\frac{r}{V}t + C \Rightarrow x(t) = Ce^{-\frac{r}{V}t} = x_0 e^{-\frac{r}{V}t}$

$x(t) = Ce^{-\frac{5}{1000}t}$

$\Rightarrow x(t) = 100e^{-t/200}$

$x(0) = 100 = C$

$x(t_{10}) = 10 = 100e^{-t_{10}/200} \Rightarrow e^{-t_{10}/200} = \frac{1}{10} \Rightarrow t_{10} = -200 \cdot \ln(0.1) \approx 460.52$ seconds