1.4 Separable Equations

$$\frac{dy}{dx} = H(x,y) = g(x)h(y) = \frac{g(x)}{f(y)}$$

$$\Rightarrow f(y)\frac{dy}{dx} = g(x) \xrightarrow[\text{concise notation}]{} f(y)\,dy = g(x)\,dx$$

$$\int f(y(x))\frac{dy}{dx}\,dx = \int g(x)\,dx + C \qquad F(y) = \int f(y)\,dy = \underbrace{\int g(x)\,dx}_{G(x)} + C$$

Note

$$D_x[F(y(x))] = F'(y(x))\,y'(x) = f(y)\frac{dy}{dx} = g(x) = D_x(G(x))$$

$$\Rightarrow D_x F(y(x)) = D_x G(x))$$

$$\Rightarrow F(y(x)) = G(x) + C$$

ie $\quad f(y)\,dy = g(x)\,dx \xrightarrow{\text{integrate}} F(y) = \int f(y)\,dy = \underbrace{\int g(x)\,dx}_{G(x)} + C$

$F(y) = \int f(y)\,dy = \int g(x)\,dx + C \xrightarrow{\text{differentiate}} f(y)\frac{dy}{dx} = g(x) \xrightarrow{\text{different notation}} f(y)\,dy = g(x)\,dx$

Example $x^2 + y^2 = R$

$2x + 2yy' = 0$

$\Rightarrow K(x,y) = x^2 + y^2 - R = 0$ is an implicit solution of $x + yy' = 0$ because $y = y(x) = (R - x^2)^{1/2}$ satisfies $H(x,y) = 0$ on an interval $[-\sqrt{R}, \sqrt{R}]$, and $y(x)$ is a solution of $x + yy' = 0$