→ there are three facets to the study of diff. eq:

Analytic Methods (aka exact or symbolic methods)

Geometric Methods

Numerical Methods

* most diff. eq. cannot be solved exactly

Errors in Euler's Method

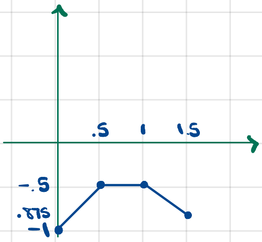
Ex:

$y' = y^2 - x^2$

$y(0) = -1$

$y'' = 2yy' - 2x$

$y''(0) = 2(-1)\cdot 1 - 2\cdot 0 = -2$



Along the first Euler strut, $y = -1 + x$. If we insert this into the diff. eq. we obtain the slopes of solutions along that line.

$y' = (x-1)^2 - x^2 = x^2 - 2x + 1 - x^2 = 1 - 2x$. For x in $[0, 0.5]$, $1 - 2x \leq 1$

ie, starting at any point on the strut except the start point, the slope is too high compared to an actual solution passing through that point.

Note

→ IVP $y' = f(x)$, $y(a) = y_0$ has solution $y(x) = y_0 + \int_a^x f(t)\,dt$

The numerical methods for approximating $y(x)$ correspond to integration approximation techniques.

Euler's Method → Left endpoint Riemann sum

RK2 → trapezoidal rule

RK4 → simpson's rule