Polynomial Input

polynomial $q(x) = a_n x_n + a_{n-1} x^{n-1} + \cdots + a_0$

note $q(0) = a_0$
$q'(0) = a_1$

Theorem (undetermined coeff.)

$p(0) \neq 0$
$q(x)$ polyn. degree n $\Rightarrow$ $p(D)y = q(x)$ has exactly one sol'n, a polyn. of degree n.

Method of undetermined coeff to compute the polynomial sol'n

linear, time-invariant (LTI) DE $p(D)y = q(x)$

assume a y_p of form $h(x)$, the latter an n^{th} degree polyn. with unknown coeff.

sub y_p into the ODE. Be systematic.

Example $y'' + 3y' + 4y = 4x^2 - 2x$

trial sol'n $y_p = Ax^2 + Bx + C$, $y_p' = 2Ax + B$, $y_p'' = 2A$

$y_p'' + 3y_p' + 4y_p = Ax^2 + Bx + C + 3(Ax + B) + 4 \cdot 2A = (4A)x^2 + (4B + 6A)x + (4C + 3B + 2A)$

compare w/ input signal $(4A)x^2 + (4B + 6A)x + (4C + 3B + 2A) = 4x^2 - 2x$

$\Rightarrow 4A = 4 \Rightarrow A = 1$
$4B + 6A = -2 \Rightarrow 4B + 6 = -2 \Rightarrow B = -2$
$4C + 3B + 2A = 0 \Rightarrow 4C = 6 - 2 \Rightarrow C = 1$

$p(D)y = 0$ has polyn. sol'ns $\Rightarrow$ sol'n of $p(D)y = q$ will be of higher degree than that of $q(x)$

need to bump up degree of trial sol'n by order of lowest derivative.

Ex: $y'' + y' = x + 1$

$y_p = Ax + b$ doesn't work: leads to $A = x + 1$.

Try $y_p = Ax^2 + Bx$, it works: $y_p = \frac{1}{2}x^2$