19. $v(0) = v_0 = 0$

$a = 4$ ft/s^2

water resistance $\Rightarrow$ decel. $\frac{v^2}{400}$ ft/s^2

$\Rightarrow \frac{dv}{dt} = 4 - \frac{v^2}{400} = \frac{1600 - v^2}{400}$

$\frac{1}{1600 - v^2} dv = \frac{1}{400} dt$

$\frac{1}{1600 - v^2} = \frac{1}{(40+v)(40-v)} = \frac{A}{40+v} + \frac{B}{40-v}$ $\Rightarrow 1 = A(40-v) + B(40+v) \Rightarrow 1 = 40A - Av + 40B + Bv = v(B-A) + 40(B+A)$

$\Rightarrow B = A,\ 40(B+A) = 1 \Rightarrow 2A = \frac{1}{40} \Rightarrow A = B = \frac{1}{80}$

use partial fractions expansion

$\int \frac{1}{80(40+v)} dv + \int \frac{1}{80(40-v)} dv = \int \frac{1}{400} dt$

$\frac{1}{80} \ln(40+v) - \frac{1}{80} \ln(40-v) = \frac{t}{400}$

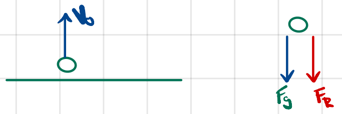
$\frac{1}{\cancel{80}} \ln \frac{40+v}{40-v} = \frac{t}{\cancel{400}_5}$

$\Rightarrow \frac{40+v}{40-v} = e^{\frac{t}{5}} \Rightarrow 40 + v = e^{t/5}(40 - v) \Rightarrow v(1 + e^{t/5}) = 40(e^{t/5} - 1) \Rightarrow v(t) = 40 \frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1}$

$\lim_{t \to \infty} v(t) = 40 \lim \frac{\frac{1}{5} e^{t/5}}{\frac{1}{5} e^{t/5}} = 40$ ft/s limiting velocity

$v(10) = 40 \frac{e^2 - 1}{e^2 + 1} \approx 30.46$ ft/s

21.



$F_R = \pm k v^2 \quad k>0 \quad = -kv|v|$

upward motion

2nd law: $-mg - kv^2 = m\dot{v} \Rightarrow \dot{v} = -g - \frac{k}{m}v^2 = -g\left(1+\frac{k}{mg}v^2\right)$

$$\frac{dv}{1+\frac{k}{mg}v^2} = -g\,dt$$

$$\int \frac{dv}{1+\frac{k}{mg}v^2} = \sqrt{\frac{mg}{k}}\int\frac{du}{1+u^2} = \sqrt{\frac{mg}{k}}\left(\tan^{-1}u + C\right) \Rightarrow \sqrt{\frac{mg}{k}}\left(\tan^{-1}\sqrt{\frac{k}{mg}}\,v + C\right) = -gt$$

$u = \sqrt{\frac{k}{mg}}\,v \Rightarrow du = \sqrt{\frac{k}{mg}}\,dv$ | $\tan^{-1}\sqrt{\frac{k}{mg}}\,v + C = \sqrt{\frac{k}{mg}}(-gt) = -\sqrt{\frac{kg}{m}}\,t$

$$\tan^{-1}\sqrt{\frac{k}{mg}}\,v = -\sqrt{\frac{kg}{m}}\,t + C$$

$$\sqrt{\frac{k}{mg}}\,v = \tan\left[-t\sqrt{\frac{kg}{m}} + c\right] \Rightarrow v(t) = \sqrt{\frac{mg}{k}}\tan\left[-t\sqrt{\frac{kg}{m}} + c\right]$$

$$v(0) = \frac{mg}{\;} \cdot \tan C \Rightarrow \tan C = \frac{k}{\;}v_0 \Rightarrow C = \tan^{-1}\left[v_0\sqrt{\frac{k}{mg}}\right]$$

$$y(t) = y_0 + \sqrt{\frac{mg}{k}} \ln \left| \frac{\cos C_1}{\cos(C_1 - t\sqrt{kg/m}} \right|$$

$$v(t) = \sqrt{\frac{mg}{k}} \cdot \tan\left[-t\sqrt{\frac{kg}{m}} + \tan^{-1}\left[v_0 \sqrt{\frac{k}{mg}} \right] \right] = \sqrt{\frac{mg}{k}} \cdot \tan\left[-t\sqrt{\frac{kg}{m}} + C_1 \right]$$

$$v(t) = 0 \Rightarrow -t\sqrt{\frac{kg}{m}} + C_1 = 0 \qquad t_m = \sqrt{\frac{m}{kg}}\, C_1$$

$$y_m = y_0 + \sqrt{\frac{mg}{k}} \ln \left| \frac{\cos C_1}{\cos(C_1 - C_1)} \right| = y_0 + \sqrt{\frac{mg}{k}} \ln \left| \cos C_1 \right|$$