second-order, constant coeff., linear, homog. eq. (spring + dashpot)

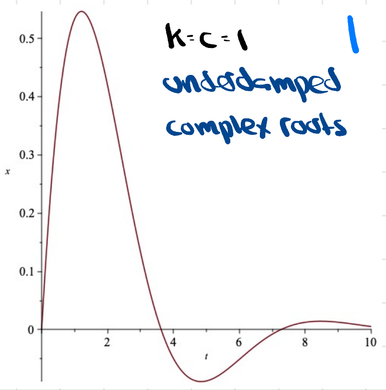
* $mx'' = -kx - cx'$

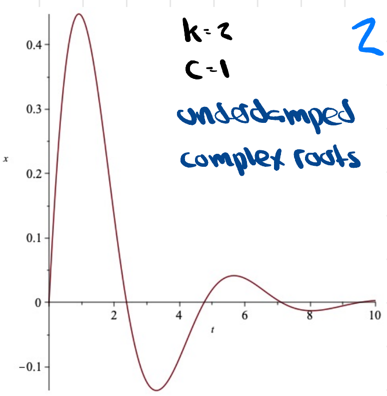
Newton Hooke Dashpot damping

$x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$

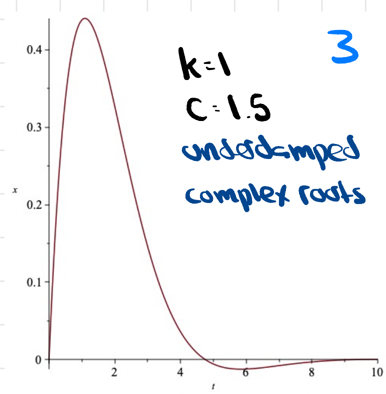
Let's make m=1

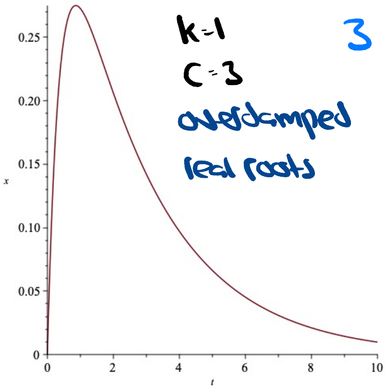
$\Rightarrow x'' + cx' + kx = 0$

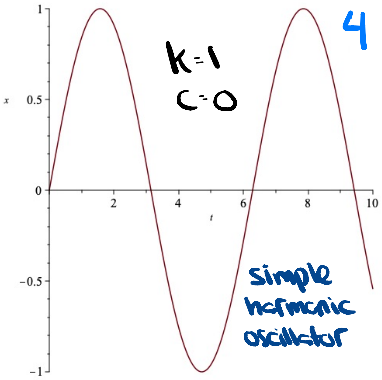


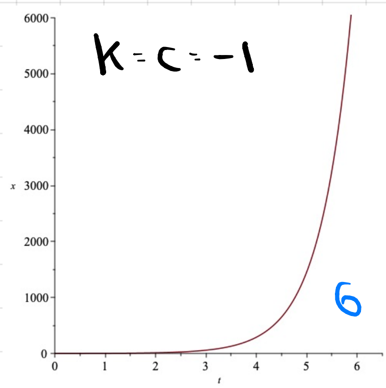


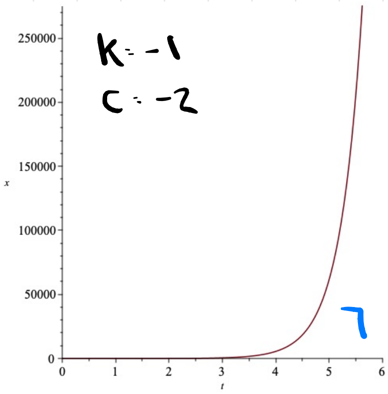
we have a damping effect in a sinusoidal. Roots are complex. Stronger spring vs dashpot means more oscillations

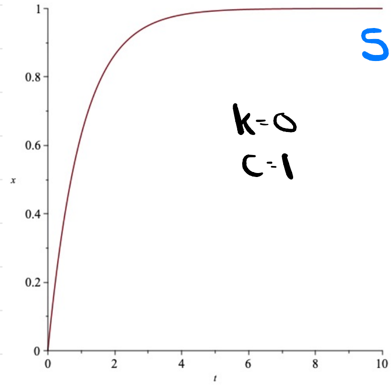


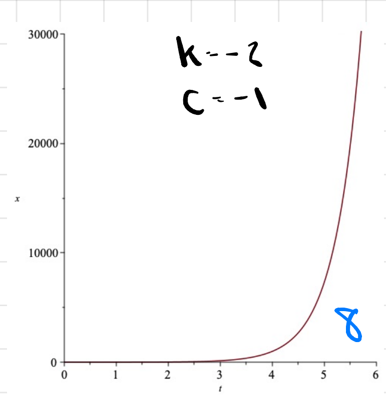


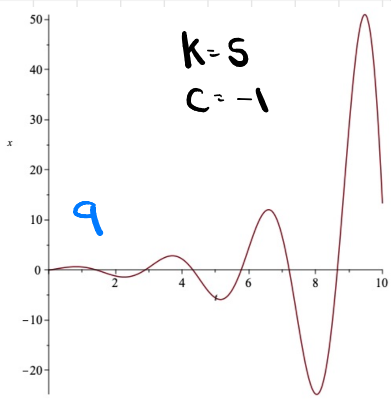


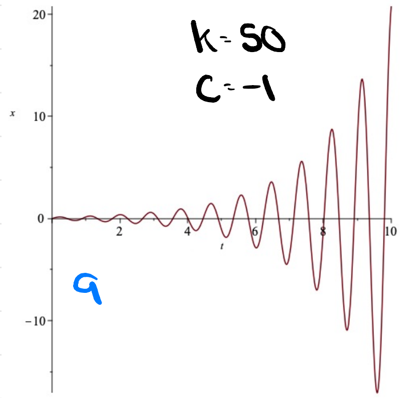


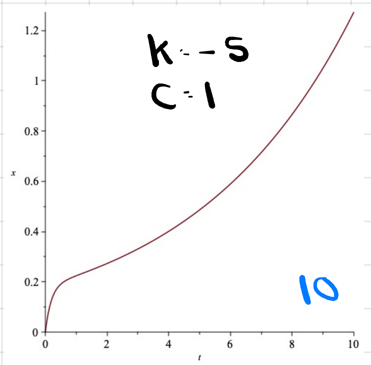


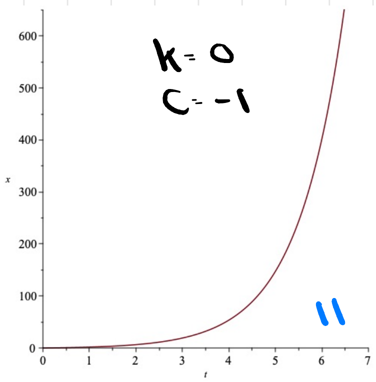


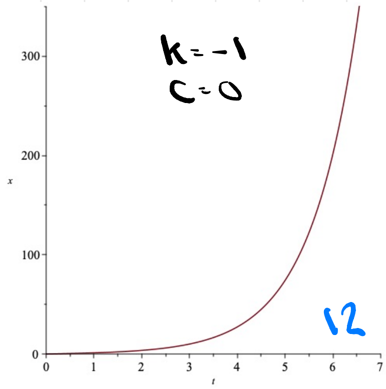












$x'' + Ax' + Bx = 0$

$r^2 + Ar + B = 0$

$\Delta = A^2 - 4B$

$$r = \frac{-A \pm \sqrt{A^2 - 4B}}{2}$$

$$y_1 = e^{t\left(\frac{-A + \sqrt{A^2-4B}}{2}\right)}$$

$$y_2 = e^{t\left(\frac{-A - \sqrt{A^2-4B}}{2}\right)}$$

$$y(t) = C_1 e^{t\left(\frac{-A + \sqrt{A^2-4B}}{2}\right)} + C_2 e^{t\left(\frac{-A - \sqrt{A^2-4B}}{2}\right)}$$

$y(0) = 0, \; y'(0) = 1$

$$y' = \frac{-A + \sqrt{A^2-4B}}{2} C_1 e^{t\left(\frac{-A + \sqrt{A^2-4B}}{2}\right)} + \frac{-A - \sqrt{A^2-4B}}{2} C_2 e^{t\left(\frac{-A - \sqrt{A^2-4B}}{2}\right)}$$

$y(0) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$

$$y'(0) = \frac{-A + \sqrt{A^2-4B}}{2} C_1 + \frac{-A - \sqrt{A^2-4B}}{2} C_2 = 1 \Rightarrow \frac{C_1\left(\cancel{-A} + \sqrt{A^2-4B} \; \cancel{+A} + \sqrt{A^2-4B}\right)}{2} = 1$$

$$\Rightarrow C_1 = \frac{1}{\sqrt{A^2-4B}}, \quad C_2 = -\frac{1}{\sqrt{A^2-4B}}$$

$$\Rightarrow y(t) = \frac{e^{t\left(\frac{-A + \sqrt{A^2-4B}}{2}\right)} - e^{t\left(\frac{-A - \sqrt{A^2-4B}}{2}\right)}}{\sqrt{A^2-4B}}$$

undefined when $A^2 - 4B = 0 \Rightarrow A^2 = 4B$ e.g. $y'' + 2y' + y = 0$

$y'' + 3y' + \frac{9}{4}y = 0$

$y'' + 4y' + 4y = 0$