

PSet 13 - Undeterm. Coeff. - Part I

1. $y'' - y = x^2$ $y(0) = 0$ $y'(0) = -1$

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$2A - C - Ax^2 - Bx = x^2$$

$$-A = 1 \Rightarrow A = -1$$

$$B = 0$$

$$2(-1) - C = 0 \Rightarrow C = -2$$

$$y_p(x) = -x^2 - 2$$

$$y'' - y = 0 \Rightarrow r^2 - 1 = 0 \Rightarrow r = \pm 1$$

$$y_h(x) = c_1 e^x + c_2 e^{-x}$$

$$y(x) = c_1 e^x + c_2 e^{-x} - x^2 - 2 \quad y'(x) = c_1 e^x - c_2 e^{-x} - 2x$$

$$y(0) = c_1 + c_2 - 2 = 0$$

$$y'(0) = c_1 - c_2 = -1$$

$$\begin{aligned} 2c_1 - 2 &= -1 \\ 2c_1 &= 1 \end{aligned} \Rightarrow c_1 = \frac{1}{2}, c_2 = c_1 + 1 = \frac{3}{2}$$

$$y(x) = \frac{e^x}{2} + \frac{3e^{-x}}{2} - x^2 - 2$$

2. $y'' - y' - 2y = 3x + 4$

$$y_p = Ax + B \quad -A - 2Ax - 2B = 3x + 4 \Rightarrow -2A \cdot x - A - 2B = 3x + 4$$

$$y_p' = A$$

$$y_p'' = 0$$

$$\Rightarrow -2A = 3 \Rightarrow A = -\frac{3}{2}$$

$$-A - 2B = 4 \Rightarrow \frac{3}{2} - 2B = 4 \Rightarrow B = \frac{3-8}{4} = -\frac{5}{4}$$

$$y_p(x) = -\frac{3}{2}x - \frac{5}{4}$$

3. $y^{(3)} + 4y' = 3x - 1$

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$y_p^{(3)} = 0$$

$$8Ax + 4B = 3x - 1$$

$$8A = 3$$

$$A = \frac{3}{8}$$

$$4B = -1 \Rightarrow B = -\frac{1}{4}$$

$$y_p = \frac{3}{8}x^2 - \frac{1}{4}x$$

Part II

$$1. x''' - x' = t^2 + 1$$

$$y_p = At^3 + Bt^2 + Ct$$

$$y_p' = 3At^2 + 2Bt + C$$

$$y_p'' = 6At + 2B$$

$$y_p''' = 6A$$

$$\Rightarrow 6A - 3At^2 - 2Bt - C = t^2 + 1$$

$$-3A = 1 \Rightarrow A = -\frac{1}{3}$$

$$-2B = 0 \Rightarrow B = 0$$

$$6A - C = 1 \Rightarrow -2 - C = 1 \Rightarrow C = -2 - 1 = -3$$

$$y_p(t) = -\frac{t^3}{3} - 3t$$

$$r^3 - r = 0 \Rightarrow r(r^2 - 1) = 0, r = 0 \text{ or } r^2 = 1 \Rightarrow r = \pm 1$$

$$y_h(t) = c_1 + c_2 e^t + c_3 e^{-t}$$

$$y(t) = c_1 + c_2 e^t + c_3 e^{-t} - \frac{t^3}{3} - 3t$$