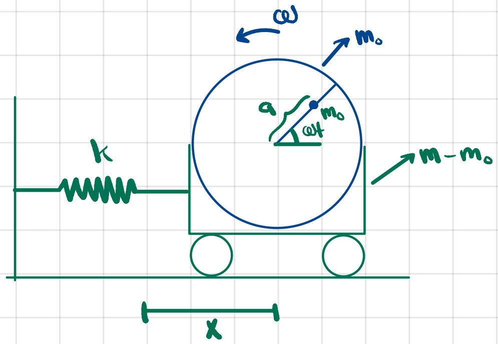
2.6 Forced Oscillations and Resonance

Example of setup that produces mass-spring system with sinusoidal input:



displacement of centroid $= \bar{x} = \dfrac{(m-m_0)x + m_0(x + a\cos\omega t)}{m} = x + \dfrac{m_0 a}{m}\cos\omega t$

2nd law: $m\bar{x}'' = -kx$

sub in $\bar{x}$: $mx'' - m_0 a\omega^2 \cos\omega t = -kx$

$mx'' + kx = m_0 a \omega^2 \cos\omega t$

This is a mass-spring system with external input

call $F_0 = m_0 a \omega^2 \Rightarrow mx'' + kx = F_0 \cos\omega t$

To study such equations, start w/ $mx'' + kx = F_0 \cos\omega t$

$p(r) = mr^2 + k = 0 \Rightarrow r^2 = -\dfrac{k}{m} \Rightarrow r = \pm (k/m)^{1/2} i$

$x_{c_1} = e^{t(\frac{k}{m})^{1/2} i} = \cos t\sqrt{\dfrac{k}{m}} + i \sin t\sqrt{\dfrac{k}{m}}$

$x_c(t) = c_1 \cos t\sqrt{\dfrac{k}{m}} + c_2 \sin t\sqrt{\dfrac{k}{m}} = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$

ω_0 = natural frequency

Try $x_p = A\cos\omega t$, ie a sinusoid with angular freq $\omega \neq \omega_0$

$\Rightarrow x_p(t) = \dfrac{F_0/m}{\omega_0^2 - \omega^2}\cos\omega t$

$\Rightarrow x(t) = c_1 \cos\omega_0 t + c_2 \sin\omega_0 t + \dfrac{F_0/m}{\omega_0^2 - \omega^2}\cos\omega t$

$= C\cos(\omega t - \alpha) + \dfrac{F_0/m}{\omega_0^2 - \omega^2}\cos\omega t$ a superpos. of two oscill.