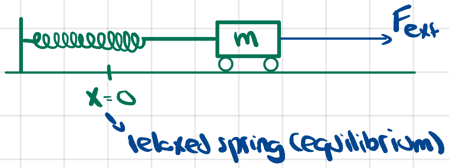
→ second-order equations are the basis of analysis of mechanical and electrical systems

setup



(mechanical system)

F_{ext}: wind blowing on sail, gravity, etc

$m\ddot{x} = F_{spr} + F_{ext}$

$F_{spr}(x) = -kx \quad \Rightarrow \quad m\ddot{x} + kx = F_{ext}$

Friction in the system
↳ depends on motion of the mass

assumption: friction depends only on mass velocity, not position

Friction acts as a damping force. In practice such damping is controlled using a device called a dashpot.

$F_{dash}(\dot{x}) = -b\dot{x}$, $b>0$
linear damping
b: damping constant

$$\boxed{\Rightarrow \quad m\ddot{x} + b\dot{x} + kx = F_{ext} \qquad \text{(DE for displacement } x \text{ of the mass from equilibrium)}}$$

Linear DEs

General Form: $\underbrace{a_n x^{(n)} + a_{n-1} x^{(n-1)} + \dots + a_1\dot{x} + a_0 x}_{\text{system}} = q(t)$
↳ input signal

a_n: coeff., may depend on t not x
represent parameters of the system

a_n's constant ⇒ constant coeff. lin. eq.

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we will focus on $\quad m\ddot{x} + b\dot{x} + kx = F_{ext}$

assumptions
- → $F_{ext} = 0$
- → $m > 0$ (realistic physical system)

special cases
- → $b = 0$ (no damping force, "undamped case", simple harmonic oscillator)
- → $k = 0$ (no spring force)

Simple Harmonic Oscillator

$\ddot{x} + \frac{k}{m}x = 0$. $\quad \omega = \sqrt{k/m} \quad \Rightarrow \quad \ddot{x} + \omega^2 x = 0$

Two solutions: $x_1(t) = \cos\omega t$, $x_2(t) = \sin\omega t \quad \Rightarrow \quad x(t) = a\cos\omega t + b\sin\omega t = A\cos(\omega t - \phi)$

$a_n x^{(n)} + \dots + a_1 \dot{x} + a_0 x = 0$ homog., constant coeff., linear!

guess solution $x(t) = e^{rt}$

$\Rightarrow p(r) = a_n r^n + \dots + a_1 r + a_0 = 0$

$x(t) = e^{rt}$ is a solution if r is root of $p(r)$

Any linear comb. of n independent solutions is also a solution.

A solution $x(t) = ce^{rt}$ is called a modal solution, ce^{rt} is called a mode of the system.