2.2 General sol'ns of Linear Eq.

Same steps as in the second order case

n^{th}-order linear DE $\quad P_0(x)y^{(n)} + P_1(x)y^{(n-1)} + \dots + P_{n-1}(x)y' + P_n y = F(x)$

we assume $P_i(x)$ and $F(x)$ contin. on open I

also, $P_0(x) \neq 0$ in $I \Rightarrow y^{(n)} + p_1(x)y^{(n-1)} + \dots + p_{n-1}(x)y' + p_n(x)y = f(x)$

Same theorems as before

Th 1 Principle of Superposition for Homog. Eq.

$y_1, \dots, y_n$ n sol'ns in $I \Rightarrow y = c_1 y_1 + \dots + c_n y_n$ also a sol'n.

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a particular sol'n of an n^{th}-order lin DE is determined by n initial conditions

Th 2 Existence and Uniqueness for Lin. Eq.

$p_1, \dots, p_n, f$ cont on I containing a
$b_0, \dots, b_{n-1}$ numbers

$\Rightarrow$ IVP $\quad y^{(n)} + p_1(x)y^{(n-1)} + \dots + p_{n-1}(x)y' + p_n(x)y = f(x)$

$y(a) = b_0,\ y'(a) = b_1, \dots, y^{(n-1)}(a) = b_{n-1}$

has exactly one sol'n on the whole I

Def Lin. Indep of Functions

$f_1, \dots, f_n$ are l.d on $I \iff \exists\, c_1, \dots, c_n$ not all zero s.t. $c_1 f_1 + \dots + c_n f_n = 0$

How to determine linear dependence/independence?

consider $c_1 f_1 + \dots + c_n f_n = 0$
$\Rightarrow c_1 f_1' + \dots + c_n f_n' = 0$
$(\dots)$
$c_1 f_1^{(n-1)} + \dots + c_n f_n^{(n-1)} = 0$

This is a system of n lin eq in n unknowns $c_1, \dots, c_n$. It has a nontrivial solution if the determ. of the matrix

$$\begin{bmatrix} f_1 & \cdots & f_n \\ \vdots & & \\ f_1^{(n-1)} & & f_n^{(n-1)} \end{bmatrix}$$

is non-zero. This det. is called the Wronskian, $W(f_1, \dots, f_n)$.

Thus if a non-trivial sol'n exists (ie $f_1, \dots, f_n$ are l.d.) then $W(f_1, \dots, f_n) \neq 0$.

Theorem General Sol'ns of Homog. Eq.

$y_1, \dots, y_n$ n l.i. sol'ns $\Rightarrow$ any other sol'n $Y(x)$ can be expressed $\sum_{i=1}^{n} c_i y_i$

Non-homog. Eq.

Assume we know a sol'n y_p to $y^{(n)} + p_1(x)y^{(n-1)} + \dots + p_{n-1}(x)y' + p_n(x)y = f(x)$

Assume Y is yet another sol'n.

Take $y_c = Y - y_p$. Subbing this into the DE $\Rightarrow$ y_c solves the homog. Eq.

Thus we can express y_c as $\sum_{i=1}^{n} c_i y_i$

$\Rightarrow Y = y_c + y_p$ ie a general solution is sum of complementary and particular sol'ns.