Operators: are to functions as functions are to numbers

Examples

$Df(t) = f'(t)$

D applied to f

$If = f$ (identity)

I applied to f

operators can be added and multiplied by numbers and/or functions

$tD + 4I : \quad f(t) \to tf'(t) + 4f(t)$

operators can be composed

D^2 (second deriv. operator)

$D^2 = D \cdot D$

$D^2 f = D(Df) = D(f') = f''$

General Linear ODE, order n for $y = y(t)$

$$y^{(n)} + p_1(t)y^{(n-1)} + \dots + p_n(t)y = q(t)$$

with constant coeff:

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = q(t)$$

$p(D)y = q(t)$, $D = \frac{d}{dt}$, $p(D) = D^n + a_1 D^{n-1} + \dots + a_n$ (polynomial differential operator w/ const. coeff.)

Differential Operator Rules

sum rule $\quad [p(D) + q(D)]u = p(D)u + q(D)u$

linearity rule $\quad p(D)(c_1 f + c_2 g) = c_1 p(D)f + c_2 p(D)g$

Also, this linearity remains no matter how many times we apply $p(D)$

multiplication rule $\quad p(D) = g(D)h(D) \Rightarrow p(D)u = g(D)(h(D)u)$

substitution rule $\quad p(D)e^{at} = p(a)e^{at}$

exponential-shift rule $\quad$ let $u = u(t)$. $\quad p(D)e^{at}u = e^{at}p(D+a)u$

Time (Translation) Invariance

$p(D)$ constant coeff. diff. operator

$p(D)x(t) = q(t) \quad \Rightarrow \quad p(D)x(t-c) = q(t-c)$