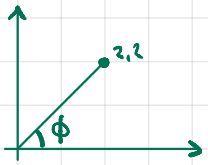
Pset - Sinusoids

Problem 1

a) $2\cos(3t) + 2\sin(3t) = 2\sqrt{2}\cos(3t - \pi/4)$



$\langle \cos 3t, \sin 3t \rangle \cdot \langle 2,2 \rangle = 1 \cdot \sqrt{8}\cos(3t - \pi/4)$

$\phi = \tan^{-1} 1 \Rightarrow \phi = \pi/4$

b) $\sqrt{3}\cos \pi t - \sin \pi t = \sqrt{1+3}\cos(\pi t + \pi/6) = 2\cos(\pi t + \pi/6)$

$\phi = \tan^{-1} -\frac{1}{\sqrt{3}} = -\pi/6$

c) $\cos(t - \pi/8) + \sin(t - \pi/8) = \sqrt{2}\cos(t - \frac{\pi}{8} - \frac{\pi}{4}) = \sqrt{2}\cos(t - 3\pi/8)$

$\phi = \tan^{-1} 1 \Rightarrow \phi = \pi/4$

Problem 2 $\int e^{2x}\sin x\, dx = \text{Im}\left(\int e^{x(2+i)}dx\right) = \frac{2e^{2x}}{5}\sin x - \frac{e^{2x}}{5}\cos x$

$\sin x = \text{Im}(e^{ix})$

$e^{2x}\sin x = \text{Im}(e^{2x+ix}) = \text{Im}(e^{x(2+i)})$

$$\int e^{x(2+i)}dx = \frac{1}{2+i}e^{x(2+i)} = \frac{2-i}{5}e^{2x}(\cos x + i\sin x) = \left[\frac{2e^{2x}}{5} - \frac{e^{2x}}{5}i\right](\cos x + i\sin x)$$

$$= \frac{2}{5}e^{2x}\cos x + \frac{e^{2x}}{5}\sin x + i\left[\frac{2e^{2x}}{5}\sin x - \frac{e^{2x}}{5}\cos x\right]$$