

# Geometric Methods

→ most 1<sup>st</sup> order equations can't be solved explicitly

→ one tool to solve: **graphical methods** (eg computer visualization)

→ basic concepts

**direction field** for  $y' = f(x, y)$

to each point  $(x, y)$  is associated a slope  $y'(x, y)$

**isoclines** of eq.  $y' = f(x, y)$ : one parameter family of curves  $y' = f(x, y) = m$ ,  $m$  constant.  
"equal-slope"

**integral/solution curves** for a direction field: graphs of solutions to  $y' = f(x, y)$

at each point on one integral curve, the derivative is  $f(x, y)$

→ **Fences**

→ **lower fence** for  $y' = f(x, y)$ : curve that blocks an integral curve from crossing from above. Analogous definition for upper fence.

↳ curve  $y = L(x)$  such that  $L'(x) < f(x, L(x))$

→ **Funnel** for  $y' = f(x, y)$  consists of a pair of fences: an upper fence  $U(x)$  and a lower fence  $L(x)$ , with following properties:

1.  $L(x) < U(x)$  for large enough  $x$

2. The two fences come together asymptotically i.e.  $U(x) - L(x)$  small for large  $x$ .