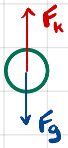
Baseball dropped from helicopter

$$\dot{v} = g - kv$$

$$\dot{v} + kv = g$$

$$ve^{kt} = \int e^{kt} g \, dt + C = \frac{g}{k} e^{kt} + C$$

$$v(t) = Ce^{-kt} + \frac{g}{k}$$



$$F_g + F_k = m\dot{v}(t) \Rightarrow mg + k_1 v = m\dot{v} \Rightarrow \dot{v} = g + \frac{k_1}{m} v$$

$$\dot{v} - \frac{k_1}{m} v = g$$

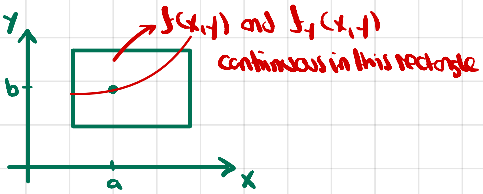
$$ve^{-\frac{k_1 t}{m}} = \int g e^{-\frac{k_1 t}{m}} dt + C$$

$$v = -\frac{gm}{k_1} + Ce^{\frac{k_1}{m} t}$$

$$k = -\frac{k_1}{m} \Rightarrow v(t) = \frac{g}{k} + Ce^{-kt}$$

Existence and Uniqueness

$$\frac{dy}{dx} = f(x,y) \quad y(a) = b$$



$\Rightarrow$ IVP has exactly one sol'n on an open interval I containing a.

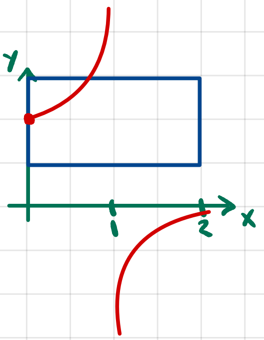
Ex: $\dfrac{dy}{dx} = y^2 = f(x,y)$

$\Rightarrow f$ and f_y continuous on $\mathbb{R}$

$f_y = 2y$

We can choose any interval in $\mathbb{R}$ to find a unique sol'n in for IVP w/ $y(0) = 1$, say $(0, 2)$.
Existence and Uniqueness is guaranteed in $(0,2)$.

The sol'n $y(x) = \dfrac{1}{1-x}$ is discontinuous at $x = 1$.



Example: $xy' = 2y \Rightarrow y' = 2\frac{y}{x} = f(x,y)$

$f_y = \frac{2}{x} \Rightarrow f$ and f_y discont. at $x = 0$

For any $x = a \neq 0$ we can apply Exist./Uniquen. Theorem and conclude there is exactly one sol'n to IVP with initial condition $y(a) = b$, on an open interval around a.

Such sol'ns are: $y^{-1}dy = 2x^{-1} \Rightarrow \ln|y| = 2\ln|x| + C_1 \Rightarrow |y| = e^{2\ln|x|}e^{C_1} = e^{C_1}|x|^2 \Rightarrow y(x) = Cx^2$

All parabolas $y = Cx^2$ pass through $(0,0)$, so IVP w/ $y(0) = 0$ has infinite sol'ns.

On the other hand IVP w/ $y(0) = h \neq 0$ has no solutions.