

## PSet - Numerical Methods - Part I

### Problem 1

$$y' = x + y^2$$

$$y(0) = 1$$

Estimate  $y(0.1)$ ,  $y(0.2)$ ,  $y(0.3)$  using Euler's Method.

$$h = 0.1$$

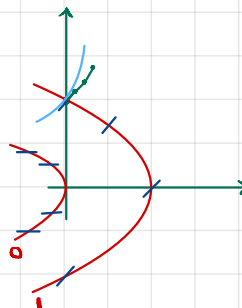
$$y'(0) = 1$$

$$y'' = 1 + 2y'y \Rightarrow y''(0) = 1 + 2 \cdot 1 \cdot 1 = 3 \Rightarrow y \text{ concave at } (0,1)$$

$n$	$x_n$	$y_n$	$m_n$	$h m_n$
0	0	1	1	.1
1	.1	1.1	1.31	0.131
2	.2	1.231	1.715361	0.1715361
3	.3	1.4025361		

↘ likely too low

$$\text{isoclines: } x + y^2 = m \Rightarrow y^2 = m - x \Rightarrow y = \pm \sqrt{m - x}$$
$$x = m - y^2$$



## Part II

### Problem 1

a)  $y' = 2x, y(0) = 0$

$$dy = 2x dx \Rightarrow y = x^2 + C$$

$$y(0) = C = 0 \Rightarrow y(x) = x^2 \Rightarrow y(1) = 1$$

Euler Approx.

$$h = \frac{1}{2}$$

n	$x_n$	$y_n$	$m_n$	$hm_n$
0	0	0	0	0
1	1/2	0	1	1/2
2	1	1/2		

$$h = \frac{1}{3}$$

n	$x_n$	$y_n$	$m_n$	$hm_n$
0	0	0	0	0
1	1/3	0	2/3	2/9
2	2/3	2/9	4/3	4/9
3	1	2/3		

$$h = \frac{1}{n}$$

n	$x_n$	$y_n$	$m_n$	$hm_n$
0	0	0	0	0
1	1/n	0	2/n	2/n <sup>2</sup>
2	2/n	2/n <sup>2</sup>	4/n	4/n <sup>2</sup>
3	3/n	6/n <sup>2</sup>	6/n	6/n <sup>2</sup>
4	4/n	12/n <sup>2</sup>	8/n	8/n <sup>2</sup>
5	5/n	20/n <sup>2</sup>	10/n	10/n <sup>2</sup>

$$(\dots) \quad k \quad k/n \quad \sum_{i=1}^{k-1} 2i/n^2 \quad 2k/n \quad 2k/n^2$$

$$(\dots) \quad n \quad 1 \quad \sum_{i=1}^{n-1} 2i/n^2 \quad 2 \quad 2/n$$

$$\frac{\sum_{i=1}^{n-1} 2i}{n^2} = \frac{\cancel{n(n-1)}}{\cancel{n^2}} = \frac{n(n-1)}{n^2} = 1 - \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{n(n-1)}{n^2} = 1$$

$$b) e(n) = 1 - \frac{n(n-1)}{n^2} = \frac{n^2 - n^2 + n}{n^2} = \frac{1}{n} = h$$

$\Rightarrow$  error  $\propto$  a function of the number of steps is proportional to  $\frac{1}{n} = h$ . ie if we halve  $h$ , we halve the error.