1. a) $x''' + x = 2\cos t$

$z''' + z = 2e^{it}$

$p(r) = r^3 + 1 = 0 \Rightarrow r^3 = -1 \Rightarrow r = (-1)^{1/3}$

$r = \rho e^{i\phi}$

$-1 = \cos(\pi + 2\pi k) + \sin(\pi + 2\pi k) = e^{i(\pi+2\pi k)}$

$\Rightarrow \rho e^{i\phi} = e^{\frac{i(\pi+2\pi k)}{3}} \Rightarrow \rho = 1,\ \phi = \frac{\pi + 2\pi k}{3} \quad k \in \mathbb{Z}$

roots:

$e^{\frac{i\pi}{3}} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

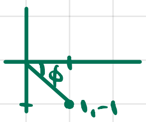
$e^{i\pi} = -1$

$e^{\frac{5\pi}{3}i} = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

$p(i) = i^3 + 1 = -i + 1 \neq 0$

$z_p = \frac{2}{1-i}e^{it} = \frac{\cancel{2}(1+i)}{\cancel{2}}(\cos t + i\sin t)$

$Re(z_p) = \cos(t) - \sin(t) = \sqrt{2}\cos(t + \pi/4) = x_p(t)$



b) gain $= \frac{\sqrt{2}}{2} = \left|\frac{1}{p(i)}\right| = \frac{1}{\sqrt{2}}$

phase lag $= -\pi/4$