

Ex 7.1

 $\uparrow v_0 = 20 \text{ m/s}$
0.145 kg

$$W_g = \int_0^h -mg dx = -mg(h-0) = -(U_h - U_0) = -U_h = -mgh$$

$$-1/mgh = \frac{1}{2}mv(0^2 - v_0^2) \Rightarrow h = \frac{v_0^2}{2g} = \frac{400}{2 \cdot 9.8} = 20.40 \text{ m}$$

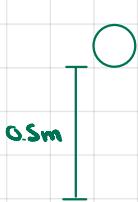
Note that $v=0$ corresponds to when the ball was thrown from. If instead $v_0 = 10$ then

$$W_g = \int_{10}^h -mg dx = -mg(h-10) = \frac{1}{2}mv(-v^2) \Rightarrow h = 10 + \frac{v_0^2}{2g}$$

-DU is still equal to the same quantity, OK

Ex 7.2

a)



$\uparrow v_0 = 20 \text{ m/s}$ The ball starts at rest. $v_0 = K_0 = 0$.

We can define our height to be zero height. Let's use the starting point. $v_0 = 0$, $U_0 = mgv_0 = 0$. Person's hand accelerates the ball to 20 m/s over 0.5m. Gravity is still doing work here, albeit negative work.

$$W_g = -mg(0.5 - 0) = -\frac{mg}{2} = -0.7105 \text{ J}$$

$$W_h = \int_0^{0.5} F_h dx = F_h \cdot \frac{1}{2} \quad \text{Between } 0 \text{ and } 0.5, \text{ total work done equals change in kinetic energy.}$$

$$W_g + W_h = \frac{1}{2}mv^2 \Rightarrow \frac{F_h}{2} = 0.7105 = \frac{1}{2}0.145 \cdot 400 \Rightarrow F_h = 59.42 \text{ N}$$

Note a few ways of looking at the previous calculation.

$$W_g = -\Delta U_g \Rightarrow W_h = \Delta K + \Delta U_g = (K_2 + U_{g2}) - (K_1 + U_{g1}) = (29 + 0.7105) - (0 + 0)$$

ie work done by hand, since >0 , increases E_{mech} of the system ball+earth. Mech. En. started at zero. w/o an external force it wouldn't be possible to increase K_2 w/o also decreasing $U_{g2} = mgh_2$. Because the coord. system is oriented upwards from Earth, h_2 must decrease if the ball is falling.

When the hand releases the ball, E_{mech} is 29.7105 J. Since only gravity is doing work from this point on, mech. energy is conserved. At height h we have

$$E_{\text{mech}} = \frac{1}{2}mv_0^2 + mgh_1 = 29.7105 = \frac{1}{2}mv_h^2 + mgh \Rightarrow v_h = \pm \sqrt{\frac{2}{m}(29.7105 - mgh)}$$

$$v_h (15.5) = 10.29 \text{ m/s}$$

Ex 7.3



There is only the conservative force gravity acting on the ball. Mechanical energy is the same at each point in the trajectory.



$$E_{\text{mech}} = U_0 + K_0 = 0 + \frac{1}{2}mv_0^2 = \frac{mv_0^2}{2}, \text{ where } \vec{v}_0 = \langle v_{0x}, v_{0y} \rangle, v_0 = |\vec{v}_0| = (v_{0x}^2 + v_{0y}^2)^{1/2}$$

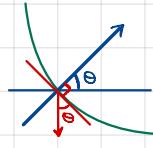
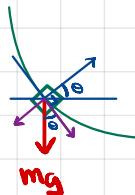
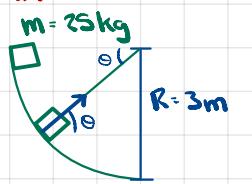
Assump. is that no matter θ , v_0 is constant, i.e. balls launched with same initial speed.

At any other height h from initial point potential energy is higher, speed in y direction is lower, because earth is doing negative work.

$$E_{\text{mech}} = \frac{mv^2}{2} + mgh = \frac{mv_0^2}{2} \Rightarrow v = \pm \sqrt{v_0^2 - 2gh}$$

Note that $v^2 = v_x^2 + v_y^2$ is speed, h is vertical displacement.

Ex 7.4



$$\text{a) } W_g = \int_{\text{top}}^{\text{bottom}} -mg dy = -mg(R - r) = -25 \cdot 9.8(0 - 3) = 735 \text{ J}$$

$$\Rightarrow \Delta U = -735 \text{ J}$$

Intuitively, we know that at the bottom velocity has only an x component, $\vec{v} = \langle v_x, 0 \rangle$. Therefore, $E_{\text{mech}} = \frac{1}{2}mv_{ix}^2$, because $U_i = mg \cdot 0 = 0$. Also, $E_{\text{mech}} = mg \cdot 3$.

Can we equate E_{mech} and E_{kin} ?

There is another force, N , acting on the sphere. The work done by this force is the difference ΔE_{mech} .

$$|N| = |mg \cos \theta|$$

$$N = -mg \sin \theta \cdot \hat{r} = -mg (\cos \theta \hat{i} + \sin \theta \hat{j})$$

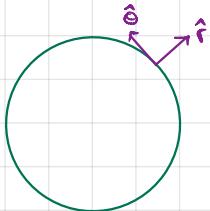
$$\text{Trajectory: } \hat{r} = 3, \theta = 0 \text{ to } \pi/2$$

$$W_N = \int_C -mg \langle \cos \theta, \sin \theta \rangle \langle -\sin \theta, \cos \theta \rangle ds = 0 \text{ because } N \text{ is perpendicular to trajectory.}$$

$$\Rightarrow \Delta E_{\text{mech}} = 0$$

$$\Rightarrow 3mg = \frac{1}{2}mv_{ix}^2 \Rightarrow v_{ix} = \pm 7.66 \text{ m/s. The entire 735 J of pot. energy became kinetic energy.}$$

$$\text{b) } W_g = -\Delta U = mgR \Rightarrow \Delta U = -mgR = \Delta K = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{2gR}$$



$$\hat{r} = \langle \cos\theta, \sin\theta \rangle$$

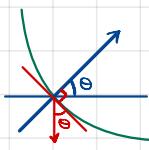
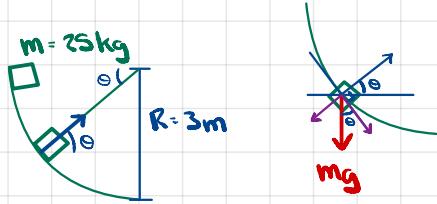
$$\hat{\theta} = \langle -\sin\theta, \cos\theta \rangle$$

$$\frac{d\hat{\theta}}{dt} = \dot{\theta} \langle -\cos\theta, -\sin\theta \rangle = -\dot{\theta}\hat{r}$$

$$\vec{r} = r\hat{r} = r\langle \cos\theta, \sin\theta \rangle$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r\langle -\sin\theta \cdot \dot{\theta}, \cos\theta \cdot \dot{\theta} \rangle = r\dot{\theta}\hat{\theta}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\ddot{\theta}\hat{\theta} + r\dot{\theta}(-\dot{\theta}\hat{r}) = -r\dot{\theta}^2\hat{r} + r\ddot{\theta}\hat{\theta}$$



$$N - mg\sin\theta - m \cdot a_r = m \cdot (-r\dot{\theta}^2) \Rightarrow mr\dot{\theta}^2 = mg\sin\theta(t) - N(t)$$

$$mg\cos\theta = m \cdot a_T = \cancel{r\dot{\theta}^2} \Rightarrow \frac{d^2\theta}{dt^2} = \frac{g}{r} \cos\theta(t)$$

note that $v(t) = \dot{\theta}(t) \cdot R$

$$\text{also } W_g = \int_{f_1}^{f_2} mg dy = -mg(f_2 - f_1) = -\Delta y = \Delta K = \frac{1}{2}mv^2$$

$$\therefore v_f = \sqrt{2g(f_2 - f_1)}$$

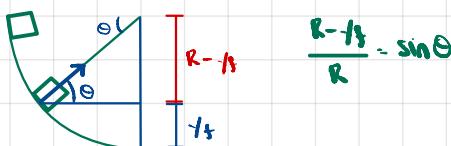
$$a_r = -R\dot{\theta}^2 = -R \left(\frac{v}{R}\right)^2 = -\frac{v^2}{R} = -\frac{2g(R-f_1)}{R}$$

At the bottom, $a_r = -2g$

$$mg - N = m(-2g) \Rightarrow N = 3mg$$

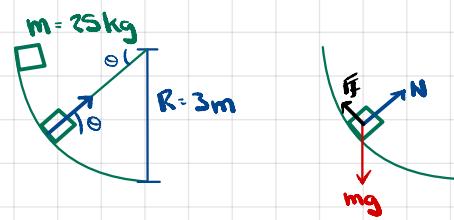
At other points,

$$mg\sin\theta - N = m \frac{(-2g(R-f_1))}{R}$$



$$N = mg \left(\frac{R-f_1}{R} \right) + \frac{2mg}{R} (R-f_1) = 3mg \left(\frac{R-f_1}{R} \right) \text{ this is the normal force at height } f_1.$$

Ex 7.5



$$W_f = \int_C F_f ds = \int_0^{\pi/2} F_f R d\theta = F_f R \cdot \frac{\pi}{2}$$

$$W_f + W_g - \Delta K = \frac{F_f R \pi}{2} + mgR = \frac{m v_f^2}{2}$$

$$F_f = \frac{mv_f^2 - 2mgR}{R\pi} = \frac{m(v_f^2 - 2gR)}{R\pi}$$

$$\Rightarrow F_f = \frac{25(6^2 - 2 \cdot 9.8 \cdot 3)}{3 \cdot \pi} = -60.47 \text{ N}$$

$$\Rightarrow W_f = -60.47 \cdot 3 \cdot \frac{\pi}{2} = -285 \text{ J}$$

$$W_g = 25 \cdot 9.8 \cdot 3 = 735$$

Friction reduced mechanical energy.

$$W_f - \Delta K - W_g = \Delta K + \Delta U = E_{\text{mech}_f} - E_{\text{mech}_i}$$

$$W_f < 0 \Rightarrow \Delta E_{\text{mech}} < 0$$

$$U_f = mg \cdot 0 = 0$$

$$U_i = mg \cdot 3 = 735$$

$$K_f = \frac{m \cdot 6^2}{2} = 450$$

$$K_i = 0$$

Ex 7.6



$$U_i = mg \cdot 0$$

$$K_i = \frac{mv_i^2}{2}$$

$$U_f = mgh$$

$$K_f = 0$$

$$\text{For } m = 12, l = 1.6, \sin \theta = \sqrt{3}/2, v_i = 5, g = 9.8 \Rightarrow h = 1.6 \cdot 0.5 = 0.8$$

$$U_f = 0$$

$$K_i = 12 \cdot 25/2 = 150 \text{ J}$$

$$U_f = 12 \cdot 9.8 \cdot 1.6 \cdot 0.5 = 94.08 \text{ J}$$

$$K_f = 0$$

$$W_f = \Delta E_{\text{mech}} = 94.08 - 150 = -55.92 \text{ J}$$

$$\text{a) } W_f = \int_0^{1.6} F_f dx = F_f \cdot 1.6 = -55.92 \Rightarrow F_f = -34.95 \text{ N}$$

$$\text{b) } U_f = 94.08$$

$$K_i = 0$$

$$U_f = 0$$

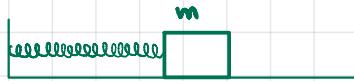
$$K_f = \frac{mv_f^2}{2}$$

$$W_f = \int_{-1.6}^{0} 34.95 dx = 34.95(-1.6) = -55.92 \text{ J}$$

$$-55.92 = \frac{12 \cdot v_f^2}{2} - 94.08$$

$$v_f = 2.52 \text{ m/s}$$

Ex 7.7



$m = 0.2 \text{ kg}$
no friction
massless spring
 $k = 5 \text{ N/m}$

Pull mass, stretch spring to $x = 0.1 \text{ m}$
Before pulling

$$K_0 = U_{el,0} = 0$$

At stretched position

$$K_s = 0$$

$$U_F = \int_0^x F dx$$

$$U_{el} = \int_0^x -kx dx = -k \frac{x^2}{2} = -\Delta U_{el} \Rightarrow U_{el} > 0$$

Potential energy is stored in the spring

$$U_F + U_{el} + \Delta K = 0 \Rightarrow \int_0^x F dx - k \frac{x^2}{2} = U_{el},$$

$$\text{ie } U_F = U_{el},$$

$$\text{Note } \frac{d}{dx} \int_0^x F dx = F(x) = kx,$$

Release the spring

$$U_{el} = \int_{x_1}^{x_2} -kx dx = -\frac{k}{2}(x_2^2 - x_1^2) = -\Delta U_{el} = \Delta K$$

$$-\frac{k(x_2^2 - x_1^2)}{2} = \frac{1}{2}mv_2^2$$

$$\Rightarrow v_2 = \pm \sqrt{-\frac{k}{m}(x_2^2 - x_1^2)}$$

$$v_2 = \pm \sqrt{-\frac{2 \cdot 5}{0.2}(0.08^2 - 0.1^2)} = 0.3 \text{ m/s}$$

Note that this is all just $\Delta E_{\text{mech}} = 0$

$$(U_2 + K_2) - (U_1 + K_1)$$

$$= 5 \cdot \frac{0.08^2}{2} + \frac{1}{2} \cdot 0.2 \cdot v_2^2 - 5 \cdot \frac{0.1^2}{2} - 0 \\ 0.016 \qquad \qquad \qquad 0.025$$

$$K_2 = \frac{1}{2} \cdot 0.2 \cdot v_2^2 = 0.009 \Rightarrow v_2 = \sqrt{\frac{0.009}{0.1}}$$

$$\Rightarrow v_2 = 0.3 \text{ m/s}$$

Ex 7.8 Initial mech. en. is zero. Ext. force does pos. work, increases system energy. U_F is the change in mech. en.

$$U_F + U_{el} = \Delta K$$

$$U_F + \Delta K + \Delta U = \Delta E_{\text{mech}} = (K \frac{x_2^2}{2} + \frac{1}{2}mv_2^2) - 0$$

$$U_F = \int_0^{0.1} 0.61 dx = 0.61 \cdot 0.1 = 0.061 \text{ J}$$

$$0.061 = \frac{5 \cdot 0.1^2}{2} + \frac{0.2v_2^2}{2} \Rightarrow v_2 = \pm 0.6 \text{ m/s}$$

What does the \pm mean?

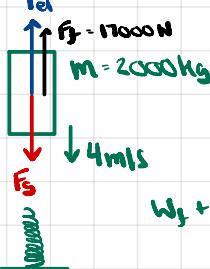
At $x_2 = 0.1$ there is U_2 and K_2 , they sum to 0.061.

If the force instead stops acting, only the conservative elastic force does work. F_{el} does neg. work at first, ie the potential goes up (it goes down), then the opposite occurs as the spring returns towards $x=0$. When it passes $x=0.05$, pot. energy has been converted back to kinetic, and $v = 0.6 \text{ m/s}$.

Ex 7.9

Right before collision $K_1 = \frac{2000 \cdot 16}{2} = 16000 \text{ J}$,

$$U_{el,1} = 0, U_g,1 = 0$$



grav. pot. decreases (g does pos. work), elastic potential increases as spring is compressed

$$W_F + W_g + U_{el} = W_F - \Delta U_g - \Delta U_{el} - \Delta K \Rightarrow \Delta E_{\text{mech}} = W_F$$

$$\text{At } x = -2, \quad W_F = -mg(-2-0) = 2mg = -\Delta U_g$$

$$\Rightarrow \Delta U_g = -2 \cdot 2000 \cdot 9.8 = -39200 \text{ J}$$

$$U_{el} = -\frac{k((-2)^2 - 0^2)}{2} = -2k = -\Delta U_{el} \Rightarrow \Delta U_{el} = 2k$$

$$W_F = \int_{-2}^0 17000 dx = 17000(-2-0) = -34000 \text{ J}$$

$$K_2 = 0 \Rightarrow \Delta K = 0 - 16000 = -16000$$

$$\Rightarrow -34 \cdot 10^3 = -16000 - 39200 + \frac{1}{2}k(-2)^2 \Rightarrow k = 10600 \text{ N}$$

The elevator has kinetic energy, and gravity does work to increase that energy by reducing potential. Were there no friction force, to stop the elevator, the spring would have to do all the negative work to reduce K to zero (so, 16000 J of increased elastic potential) and to absorb gravitational potential that would otherwise become kinetic energy (so, another 39200 J absorbed U_{el}).

With friction, total mech. energy is reduced, and the spring has to do less work (ie absorb less elastic potential).

What happens if we try to define potential energy for friction?



$$x_2 \quad F_f \text{ is constant, but } \ll 0 \text{ if } x_2 > x_1, \gg 0 \text{ if } x_2 < x_1$$

$$W_f = \int_{x_1}^{x_2} -F_f dx = U_f(x_2) - U_f(x_1) = \Delta U_f = \Delta K$$

$$-F_f(x_2 - x_1) = -\Delta U_f \Rightarrow \Delta U_f = F_f x_2 - F_f x_1 = U_f(x) - F_f x$$

$$\text{on the trajectory to the right, } W_f = \Delta K = U_{f_1} - U_{f_2} = K_2 - K_1 \Rightarrow E_{m_1} - E_{m_2}$$

Potential increases.

To the left now

$$W_f = \int_{x_2}^{x_1} F_f dx = F_f(x_1 - x_2) = -\Delta U_f \Rightarrow \Delta U_f = F_f(x_2 - x_1) = U_{f_2} - U_{f_1}$$

Potential increases again. Potential keeps going up, kinetic keeps going down.

With gravity

$$\begin{array}{ll} \square x_2 & \int_{x_1}^{x_2} -mg dx = -mg(x_2 - x_1) < 0 = -\Delta U \Rightarrow U_2 - mgx \\ \square x_1 & \\ \square x_1 & -mg(x_2 - x_1) = \frac{1}{2}mv^2_1 \end{array}$$

$$\text{on the way up } U_1 - U_2 = \Delta K \Rightarrow U_2 + K_2 = U_1 + K_1 \Rightarrow \text{constant v elsewhere.}$$

On the way down

$$\int_{x_2}^{x_1} -mg dx = -mg(x_1 - x_2) > 0 = -\Delta U \Rightarrow U_1 - mgx$$

$$-mg(x_1 - x_2) = \frac{1}{2}m(v_1^2 - v_2^2)$$

Ex 7.10



$$\mu_k = 0.2$$

Straight path

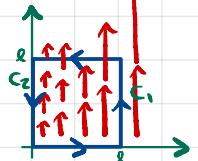
$$W_f = \int_0^{2.5} -(40 \cdot 9.8) \cdot 0.2 dx = -196 J$$

$$W_f + W_g = \Delta K = 0 \Rightarrow W_f = -W_g = 196 J$$

Dogleg Path

$$W_f = \int_0^2 -(40 \cdot 9.8) \cdot 0.2 dx + \int_2^{3.5} -(40 \cdot 9.8) \cdot 0.2 dx = -274 J$$

Ex 7.11 $\vec{F} = Cx\hat{j}$, $C > 0$



$$W_F = \int_{C_1} (Cx\hat{j}) \cdot (dy\hat{j}) + \int_{C_2} 0 \cdot (-dx\hat{j}) \\ = \int_0^L Cx dy = \frac{CL^2}{2}$$

* $\vec{F} = \langle 0, Cx \rangle$ is a vector field.
Is $\vec{F} = \nabla \phi$?

$$\begin{aligned} f_x &= 0 & f_y &= Cx \\ f_{xy} &= 0 & f_{yy} &= C \end{aligned}$$

F is not conservative \Rightarrow non-conservative