13.1 Energy and Conservation of Energy

$\Delta E = \Delta E_{system} + \Delta E_{surroundings} = 0$ (conservation of energy)

closed system: no chg in energy of surroundings → $\Delta E_{system} = 0$

13.2 Kinetic Energy

→ "energy assoc. w/ coherent motion of molecules that constitute a body of mass m"

Def (kinetic energy) Kin. Energy of non-rotating body of mass m moving w/ speed v is $K \equiv \frac{1}{2}mv^2$

SI unit: $\frac{kg\,m^2}{s^2} = J$

Example

From reference frame of the ground or an observer on the ground

$v_{A_1} = 10,\ v_{A_2} = 20$

$v_{B_1} = 50,\ v_{B_2} = 60$

$$\frac{\Delta K_A}{\Delta K_B} = \frac{v_{A_2}^2 - v_{A_1}^2}{v_{B_2}^2 - v_{B_1}^2} = \frac{400-100}{3600-2500} = \frac{300}{1100} = \frac{3}{11}$$

From ref. frame of observer moving at v_{A_1}

$$\frac{\Delta K_A}{\Delta K_B} = \frac{100-0}{2500-1600} = \frac{100}{900} = \frac{1}{9}$$

13.3 Kinematics and Kinetic Energy in one Dimension

13.3.1 Constant Accelerated Motion

rigid body point mass ● at $t=0$, $\vec{v}_{x,i}$

displacement: $\Delta x = v_{x,i}t + \frac{1}{2}a_x t^2$

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{t}$$

$$a_x \Delta x = \frac{v_{x,f} - v_{x,i}}{t}\left(v_{x,i}t + \frac{1}{2}(v_{x,f} - v_{x,i})\cdot t\right)$$

$$= (\dots) = \frac{1}{2}(v_{x,f}^2 - v_{x,i}^2)$$

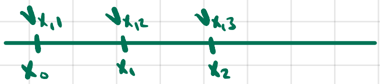
$$\Rightarrow m a_x \Delta x = \frac{1}{2}m(v_{x,f}^2 - v_{x,i}^2) = \Delta K$$

$$\Rightarrow F_x \Delta x = \Delta K$$

13.3.2 Non-constant Accel. Motion

→ Divide displacement into N intervals $j = 1, \dots, N$

$$\sum_{j=1}^{N} (a_{x,j})_{avg}\Delta x_j$$



$$(a_{x,j})_{avg} = \frac{\Delta v_{x,j}}{\Delta t_j} = \frac{v_{x,j+1} - v_{x,j}}{\Delta t_j}$$

$$(a_{x,j})_{avg}\Delta x_j = (v_{x,j+1} - v_{x,j})\frac{\Delta x_j}{\Delta t_j} =$$

$$\frac{v_{x,j}\cancel{\Delta t_j} + \frac{1}{2}(v_{x,j+1} - v_{x,j})\cancel{\Delta t_j}}{\cancel{\Delta t_j}}\cdot(v_{x,j+1} - v_{x,j})$$

$$= v_{x,j}v_{x,j+1} - v_{x,j}^2 + \frac{(v_{x,j+1} - v_{x,j})^2}{2}$$

$$= \frac{\cancel{2v_{x,j}v_{x,j+1}} - 2v_{x,j}^2 + v_{x,j+1}^2 - \cancel{2v_{x,j}v_{x,j+1}} + v_{x,j}^2}{2}$$

$$= \frac{v_{x,j+1}^2 - v_{x,j}^2}{2}$$

$$\sum_{j=1}^{n}(a_{x,j})_{avg}\Delta x_j = \frac{\cancel{v_{x,2}^2} - v_{x,1}^2}{2} + \frac{\cancel{v_{x,3}^2} - \cancel{v_{x,2}^2}}{2} + \dots + \frac{v_{x,n}^2 - \cancel{v_{x,n-1}^2}}{2}$$

$$= \frac{v_{x,n}^2 - v_{x,1}^2}{2}$$

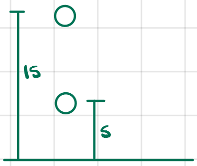
$$\lim_{n\to\infty}\sum_{j=1}^{n}(a_{x,j})_{avg}\Delta x_j = \int_{x_i}^{x_f} a_x(x)\,dx = \frac{1}{2}(v_{x_f}^2 - v_{x_i}^2)$$

$$\int_{x_i}^{x_f} m a_x\,dx = \frac{1}{2}m(v_f^2 - v_i^2) = \Delta K$$

$$\rightarrow \int_{x_i}^{x_f} F_x\,dx = \Delta K$$

Note that this relationship applies for a point mass. For extended bodies it applies to the center of mass.

Ex 13.8 $m = 0.2\,kg$



$$W_g = \int_{y_i}^{y_f} -mg\,dy = -mg(y_f - y_i)$$

$$= \Delta K = \frac{1}{2} m v_f^2$$

$$W_g = \Delta K \Rightarrow -\cancel{m}g(y_f - y_i) = \frac{1}{2}\cancel{m}v_f^2$$

$$\Rightarrow v_f = \pm\sqrt{-2g(y_f - y_i)}$$

$a(t) = -g$

$v(t) = -gt$

$y(t) = y_0 - \frac{gt^2}{2}$

$$y(t_f) = y_f \Rightarrow t_f = \pm\sqrt{\frac{-2(y_f - y_0)}{g}}$$

$$V_{avg} = \frac{y_f - y_i}{\Delta t} = \frac{y_f - y_i}{\sqrt{\frac{-2(y_f - y_0)}{g}}}$$

$$\Rightarrow P_{avg} = F_g \cdot V_{AVG} = -mg \cdot \frac{\sqrt{-2g(y_f - y_i)} - v_i}{\sqrt{\frac{-2(y_f - y_0)}{g}}}$$

$$P = \frac{dW_g}{dt} = \frac{d}{dt} - mg\left(y_0 + \frac{gt^2}{2} - y_0\right) = -mg^2 t$$

$W_g = -0.2 \cdot 9.8(5 - 15) = 19.60\,J$

$v_f = \pm\sqrt{-2 \cdot 9.8 \cdot (-10)} = \pm 14\,m/s$

$$t_f = \pm\sqrt{\frac{-2(-10)}{9.8}} = \pm 1.4285\,s$$

$$V_{avg} = \frac{14 - 0}{1.4285} = -6.99\,m/s$$

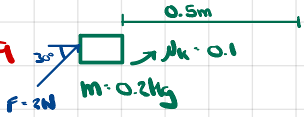
$P_{avg} = -0.2 \cdot 9.8 \cdot (-6.99) = 13.72\,W$

altern. $= \frac{19.60\,J}{1.4285\,s} = 13.72\,W$

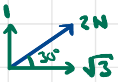
$P = -0.2 \cdot 9.8 \cdot 14 = 27.44\,W$

Note $P(t) = mg^2 t$, power of grav. force linear in time

Ex 13.9



average power of pushing force

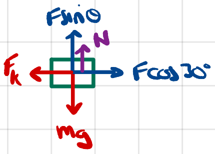


$$V_{Avg} = \frac{0.5 - 0}{\Delta t}$$

$W_F = \sqrt{3} \cdot 0.5$

$$P_{F_{AVG}} = \frac{\Delta W_F}{\Delta t} = \frac{0.5\sqrt{3}}{\Delta t}$$

we need Δt



$2 \cdot \frac{1}{2} + N = 0.2 \cdot 9.8$

$N = 1.96 - 1 = 0.96$

$$2 \cdot \frac{\sqrt{3}}{2} - 0.1 \cdot 0.96 = 0.2 \cdot a \Rightarrow a = 8.18\,m/s^2$$

$a = 8.18$

$v = 8.18t$

$x = 4.09t^2$

$$t_f = \sqrt{\frac{.5}{4.09}} = 0.34\,s = \Delta t$$

$\Rightarrow V_{avg} = 1.43\,m/s$

$P_{avg} = 2.47\,W$

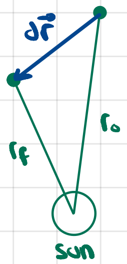
Avg Power Kinetic Friction Force

$F_f = -0.1 \cdot 0.96$

$W_f = -0.1 \cdot 0.96 \cdot 0.5 = -0.048\,J$

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{0.048}{0.34} = -0.1372\,W$$

Ex 13.13



$\vec{r} = r\hat{r}$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

$$\vec{F}_g = F_g\hat{r} = -\frac{Gmm_s}{r^2}\hat{r}$$

$$dW = \vec{F}_g \cdot d\vec{r} = F_g\hat{r}\,(dr\,\hat{r} + r\,d\theta\,\hat{\theta}) = F_g\,dr$$

$$W = \int_{r_i}^{r_f} \vec{F}_g \cdot d\vec{r} = \int_{r_i}^{r_f} F_g\,dr = -Gmm_s\int_{r_i}^{r_f} r^{-2}\,dr = -Gmm_s(-r^{-1})\Big|_{r_i}^{r_f}$$

$$= Gmm_s\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$