# 23.1 Intro. to Potential Energy

Conservative Force $\vec{F}_c$

$$W = \int_A^B \vec{F}_c \cdot d\vec{s} \quad \text{(path indep.)} = \text{Function}(\vec{r}_B) - \text{Function}(\vec{r}_A)$$

let's call the function $-U(\vec{r})$

$$\Rightarrow W = -U(\vec{r}_B) + U(\vec{r}_A) = -U_B + U_A = -\Delta U$$

$$\Rightarrow \Delta U = -W$$

Work - Kinetic Energy Theorem

$$W = -U_B + U_A = \Delta K = K_B - K_A = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$\Rightarrow \frac{1}{2}mv_A^2 + U_A = \frac{1}{2}mv_B^2 + U_B$$

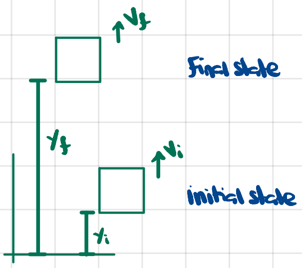
$$\Rightarrow K + U = \underbrace{E_{mech}}_{\text{total mechanical energy}} = \text{constant}$$

$U$ = potential energy

$\Delta K + \Delta U = \Delta E_{mech} = 0$ (for a conservative force)

$$\underbrace{W}_{\text{total work}} = W_c + W_{nc} = \Delta K \Rightarrow W_{nc} = \Delta K + \Delta U$$

## 23.2 Potential Energy for Gravity near Surface of Earth




Def: $U(y_f) - U(y_i) = -\int_{y_i}^{y_f} \vec{F}_g \, d\vec{s}$

negative

$\vec{F}_g = -mg\hat{j}$, conservative force

$$\Rightarrow \Delta U = -(-mg)(y_f - y_i) = mg(y_f - y_i)$$
$$= mg\Delta y$$

$\Rightarrow \Delta y > 0 \Rightarrow \Delta U > 0$

## 23.3 Potential Energy Reference State

$y_p$ : reference point

$U(y_p)$ : reference potential

$U(f) - U(y_p)$

$U(i) - U(y_p)$

$$U(f) - U(y_p) - (U(i) - U(y_p)) = U(f) - U(i) = \Delta U$$

$y_p = 0$, $U(y_p = 0) = 0$

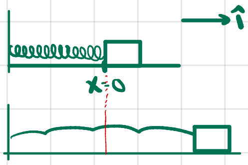
$U(i) - U(y_p) = mg(y_i - y_p) = mgy_i$

$U(f) = mgy_f$

For any height $y$, $U(y) = mgy$ when $U(0) = 0$

potential energy function for gravitational force

## 23.4 Potential Energy of Spring



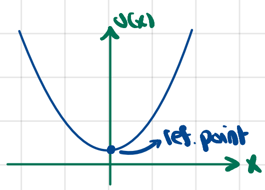

$\vec{F} = -kx\hat{i}$ $\quad d\vec{s} = dx\hat{i}$

$\vec{F} \cdot d\vec{s} = -kx\,dx$

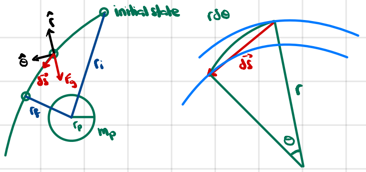
Def: $U(x_f) - U(x_i) = -W_c = -\int_{x_i}^{x_f} (-kx)\,dx = \frac{k(x_f^2 - x_i^2)}{2}$

Def: reference state $x = 0$, $U(x=0) = U_0$

arbitrary state: $x$, $U(x) = U_0 + \frac{kx^2}{2}$



## 23.5 Potential Energy of Gravitation




$\vec{s}(t) = r(t)\hat{r}$

$d\vec{s} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$

$\vec{F}_g = \dfrac{-Gmm_p}{r^2}\hat{r}$     * $\hat{r}\cdot\hat{r} = 1$, $\hat{r}\cdot\hat{\theta} = 0$

$\vec{F}_g \cdot d\vec{s} = \dfrac{-Gmm_p}{r^2}dr$

$$U(r_f) - U(r_i) = -\int_{r_i}^{r_f} \vec{F}_g\, d\vec{s} = -\int_{r_i}^{r_f} \frac{-Gmm_p}{r^2}dr$$

$$= Gmm_p \cdot \left(-\frac{1}{r}\right)\Bigg|_{r_i}^{r_f}$$

$$= -Gmm_p\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

reference state: at $\infty$ $U(\infty) = 0$

arbitrary state: $r \geq r_p$    $U(r) - U(\infty)$

$$= -\frac{Gmm_p}{r}, \quad r \geq r_p$$

## 24.1 Mechanical Energy and Energy Conservation

$E_{mech} = K + U$

$\Delta K + \Delta U = \Delta E_{mech}$

$W_c = -\Delta U$

But, $W = W_c + W_{nc} = \Delta K$

$W_{nc} - \Delta U = \Delta K$

$\Rightarrow W_{nc} = \Delta K + \Delta U = \Delta E_{mech}$

## 24.2 Energy State Diagrams

1. choose initial and final states, and coord. system

initial state:     final state:


2. choose zero potential energy
3. identify K and U for each state

$K_i = 0$     $K_f = \frac{1}{2}mv_f^2$

$U_i = mgR$     $U_f = mgR\cos\theta_f$

4. $W_{nc}$ ?

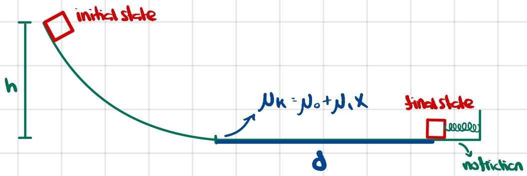
→ assume dome is frictionless $\Rightarrow W_{nc} = 0$

5. apply energy principle

$W_{nc} = (K_f - K_i) + (U_f - U_i)$

$0 = \frac{1}{2}mv_f^2 + mg(R\cos\theta - R)$

## 24.3 Worked Example - Block Sliding Down Circular Slope




$W_{ext} = \Delta_{mech}$

choose zero point for each potential function

$U_g(y=0) = 0$

$U_s(x=0) = 0$

$E_i = U_{gi} = mgy$

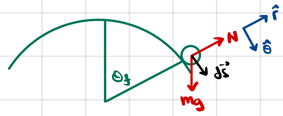
$E_f = \frac{1}{2}kx_f^2$

$$-\int_0^d (\mu_0 + \mu_1 x)mg\,dx = \frac{1}{2}kx_f^2 - mgh$$

$$-\mu_0 mgd - \frac{\mu_1 mgd^2}{2} = \frac{1}{2}kx_f^2 - mgh$$

$$x_f = \sqrt{\frac{1}{k}(2mgh - 2\mu_0 mgd - \mu_1 mgd^2}$$

### 24.4 2nd Law and Energy Conservation




$dW_N = \vec{F}_N d\vec{s} = 0$ because $\vec{F}_N = \vec{N}$, $\vec{N} \perp d\vec{s}$

$W_g = mg\sin\theta\, ds$

$W_g = -\Delta U$

this eq. is not present in the energy condition

$2^{nd}$ law in dir $\perp$ to $d\vec{s}$: $\hat{r}: N - mg\cos\theta = -\dfrac{mv^2}{R}$

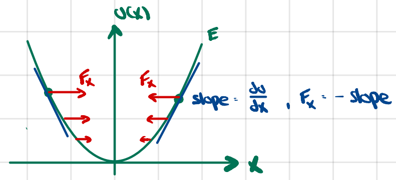
$\hat{\theta}: mg\sin\theta = mR\dfrac{d^2\theta}{dt^2}$, this is integrated w/ respect to ds to obtain the energy principle

## 25.1 Force is the derivative of potential

$$U(x_f) - U(x_i) = -\int_{x_i}^{x_f} F_x^c\,dx = \int_{x_i}^{x_f}\frac{dU}{dx}dx$$

$F_x^c = -\dfrac{dU}{dx}$ (indep. of reference point)

Ex: $U(x) = \frac{1}{2}kx^2$, $U(0) = 0$




$F_x = -kx$

## 25.2 Stable and unstable Equilibrium Points

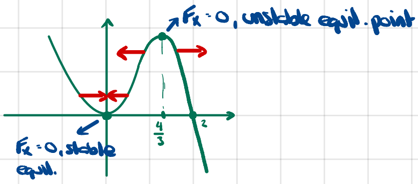
$W_{nc} = 0$

$U(x) = ax^2 - bx^3$

$F_x = -\frac{dU}{dx} = -(2ax - 3bx^2)$

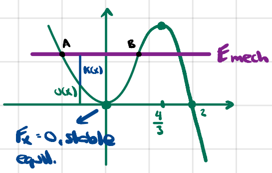
zero points: $x = 0$ and $x = \frac{2a}{3b}$

ex: $a = 4\ J m^{-2}$, $b = 2\ J \cdot m^{-3}$ $\Rightarrow$ $x = \frac{4}{3} m$




## 25.3 Reading Potential Energy Diagrams

$W_{nc} = 0 \Leftrightarrow \Delta E_{mech}$




A, B: turnaround points, $K(x_A) = K(x_B) = 0$