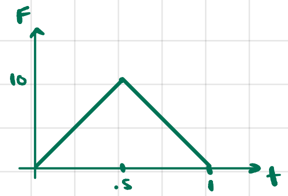
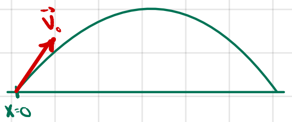
# Ex 10.1 $\Delta t = 1\,s$

$F(t) = \begin{cases} bt\hat{i} & 0 \le t \le 0.5 \\ (d-bt)\hat{i} & 0.5 \le t \le 1 \end{cases}$ $\quad b = 2\cdot 10 \quad d = 2\cdot 10$




$$I = \int_0^{.5} 20t\,dt + \int_{.5}^{1} (20-20t)\,dt = 10\left(\tfrac{1}{4}-0\right) + 20\left(\tfrac{1}{2}\right) - 10\left(1-\tfrac{1}{4}\right)$$
$$= 2.5 + 10 - 10 + 2.5 = 5$$

# Ex 10.5 - Exploding Projectile




$$\vec{F} = m_1\frac{d\vec{v}}{dt} \Rightarrow \int_{t_0}^{t_1}\vec{F}\,dt = \int_{t_0}^{t_1} m_1\frac{d\vec{v}}{dt}\,dt = m_1 v_1 - m_1 v_0$$

$$\int_{t_0}^{t_1}\vec{F}_g\,dt = \int_{t_0}^{t_1} -m_1 g\hat{j}\,dt = -m_1 g(t_1-t_0)\hat{j}$$

$$= (\cancel{m_1 v_y(t_1)}^{\,0} - m_1 v_y(t_0))\hat{j} + (m_1 v_x(t_1) - m_1 v_x(t_0))\hat{i}$$

$\Rightarrow v_y(t_0) = g(t_1 - t_0), \quad v_x(t_1) = v_x(t_0)$

Gravitational force acted during $t_0$ to $t_1$ and reduced momentum in y direction to 0.

Momentum in x dir. stayed the same.

$\vec{p}_0 = m_1(v_x(t_0)\hat{i} + v_y(t_0)\hat{j}) = m_1(v_x(t_0)\hat{i} + g(t_1-t_0)\hat{j})$

$\vec{p}_1 = m_1 v_x(t_1)\hat{i} = m_1 v_x(t_0)\hat{i} = m_1 v_{1,x}$

From kinematics perspective, y direction

$a_y = -g$

$v_y = v_{y_0} - gt$

$y = v_{y_0}t - \frac{gt^2}{2}$

Note that impulse is a definite integral and is the $\Delta$ of momentum.

From $t_0$ to $t_1$, speed went from $v_{y_0}$ to 0. We calculated this change as $-g(t_1 - t_0)$.

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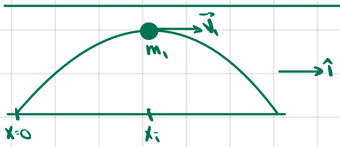



At $t = t_1$, $\vec{p}_1 = m_1 v_x(t_0)\hat{i}$. There is an explosion. Whatever caused the explosion is intact on the two chunks that split off.

$\vec{p}_2 = (-m_2 v_{2x} + m_3 v_{3x})\hat{i}$

we can equate momentum before and after explosion

$m_1 v_{1,x} = -\frac{1}{4}m_1 v_{2,x} + \frac{3}{4}m_1 v_{3,x}$

but $m_2$ falls to the initial position. $m_2$ starts with a zero vertical speed on the descent, therefore it will reach the ground in the same time it took for $m_1$ to reach the explosion point. Since this is also the time it took to cover the horizontal distance, $v_2$ must equal $v_{1,x}$.

$$v_{1,x} = \frac{-v_{1,x}}{4} + \frac{3v_{3,x}}{4} \Rightarrow 5v_{1,x} = 3v_{3,x} \Rightarrow v_{3,x} = \frac{5}{3}v_{1,x}$$

$$\int_0^{t_f} v_{3,x}\,dt = \frac{5}{3}\int_0^{t_f} v_{1,x}\,dt = \frac{5}{3}x_i$$

$\Rightarrow$ The larger piece travelled $x_i + \frac{5}{3}x_i$.

## Solution using center of mass

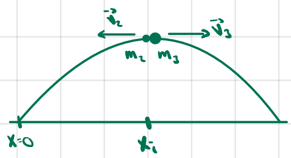
$\vec{r}_{cm} = \dfrac{m_1\vec{r}_1(t)}{m_1} = \vec{r}_1(t)$ so up to the explosion the center of mass is the position of the mass. There is an external force acting on the system: $F_g$. It accelerates the center of mass, but this only actually has an effect in the y direction.

The center of mass has an x-dir velocity that does not change: $a_{cm_x} = 0$. Internal forces (like those involved in the explosion) do not accelerate the center of mass. Therefore, the x-dir center of mass is the same whether or not the explosion occurs.

W/o the explosion, the center of mass is at $2x_i$ when the mass hits the ground. With the explosion:

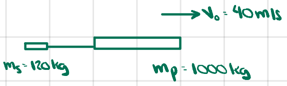
$$x_{cm} = \frac{m_2\cdot 0 + m_3\cdot x_f}{m_2 + m_3} = \frac{\frac{3}{4}m_1 x_f}{m_1} = \frac{3x_f}{4} = 2x_i$$

$\Rightarrow x_f = \frac{8}{3}x_i$

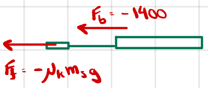



$m_3 = 3m_2$

# Ex 10.6




system: sandbag + rope + plane




$= -0.4 \cdot 120 \cdot 9.8$

When the plane snags the sandbag, it loses some momentum

$m_p \cdot v_{0p} + m_s \cdot v_{0s} = (m_p + m_s) \cdot v_1$

$1000 \cdot 40 + 120 \cdot 0 = 1120 \cdot v_1 \Rightarrow v_1 = \frac{250}{7} \approx 35.71\,m/s$

This calculation assumes the snag collision was instantaneous.

After collision, initial momentum is $p_0 = (1000+120) \cdot 35.71 = 39995.20$

Friction and the brakes provide impulse that reduces momentum to zero.

$\int_0^{t_f} (F_f + F_b)\,dt = -39995.20$

$-1870 t_f = -39995.20 \Rightarrow t_f = 21.38\,s$

here we calc. time to reach momentum of zero under impulse from friction and brakes

$F_R = m_T \cdot a \Rightarrow -1870.40 = 1120 \cdot a \Rightarrow a = -1.67\,m/s^2$

$v(t) = 35.71 - 1.67t$

$v = 0 \Rightarrow t = 21.38\,s$

here we calc. time to reach zero velocity under an acceleration due to friction and braking forces.

$x(t) = 35.71t - \frac{1.67t^2}{2}$

$x(21.38) = 381.79\,m$

Note:

$W = \int_0^{x_f} F_R\,dx = \frac{-1120 \cdot 35.71^2}{2} = -1870\,x_f$

$\Rightarrow x_f = 381.79\,m$

# Ex 10.2

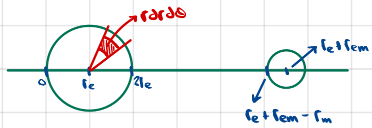
$r_{em} = 3.84 \cdot 10^8\,m$

$m_e = 5.98 \cdot 10^{24}\,kg$

$m_m = 7.34 \cdot 10^{22}\,kg$

$r_e = 6.37 \cdot 10^6\,m$

$r_m = 1.74 \cdot 10^6\,m$

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$\vec{r}_{cm} = \frac{1}{m_e + m_m}\left[m_e \vec{r}_{e_{cm}} + m_m \vec{r}_{m_{cm}}\right]$

$\vec{r}_{e_{cm}} = \frac{\int_{-\pi/2}^{\pi/2}\int_0^{2r_e\cos\theta} \langle r\cos\theta, r\sin\theta\rangle \rho r\,dr\,d\theta}{\int_0^{2\pi}\int_0^{r_e} \rho r\,dr\,d\theta}$

$(x - r_e)^2 + y^2 = r_e^2$

$x^2 + y^2 - 2xr_e + r_e^2 = r_e^2$

$x^2 + y^2 = 2xr_e = 2r_e r\cos\theta = r^2$

$\Rightarrow r = 2r_e\cos\theta$

$\Rightarrow x_{e_{cm}} = \frac{\int_{-\pi/2}^{\pi/2}\int_0^{2r_e\cos\theta} r^2\cos\theta\,\rho\,dr\,d\theta}{\int_{-\pi/2}^{\pi/2}\int_0^{2r_e\cos\theta} \rho r\,dr\,d\theta} = r_e$

$y_{e_{cm}} = 0$

This calculation was just for "fun".

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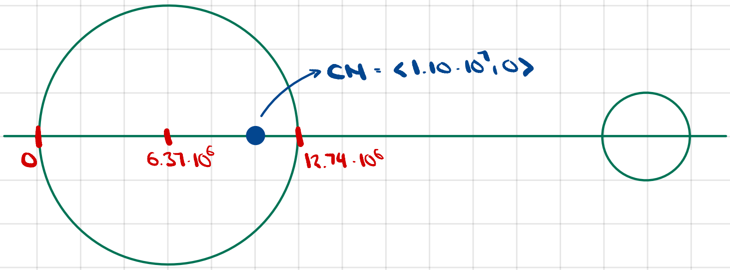
$r_{e_{cm}} = r_e\hat{i}$

$r_{m_{cm}} = (r_e + r_{em})\hat{i}$

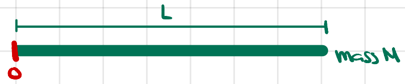
$r_{cm} = \frac{1}{m_e + m_m}\left[m_e r_e \hat{i} + m_m (r_e + r_{em})\hat{i}\right]$

$= \frac{r_e(m_e + m_m) + m_m r_{em}}{m_e + m_m}\hat{i} = 1.10 \cdot 10^7\,m$

Center of mass is within the Earth.



Ex 10.3

a) 


uniform rod: constant density $\lambda = \frac{M}{L}$

$$\vec{r}_{cm} = \frac{\int_0^L x\hat{\imath}\,\cancel{\lambda}\,dx}{\int_0^L \cancel{\lambda}\,dx} = \frac{\frac{1}{2}L^2\hat{\imath}}{L} = \frac{1}{2}L\hat{\imath}$$

b) non-uniform rod $\lambda(x) = \frac{\lambda_0}{L^2}x^2$

$$\vec{r}_{cm} = \frac{\frac{\lambda_0}{L^2}\int_0^L x^3\hat{\imath}\,dx}{\frac{\lambda_0}{L^2}\int_0^L x^2\,dx} = \frac{\frac{1}{4}L^4\hat{\imath}}{\frac{1}{3}L^3} = \frac{3}{4}L\hat{\imath}$$

Ex 10.4