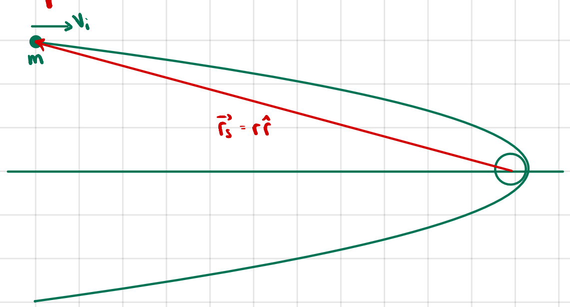
19.4 Conservation of Angular Momentum About a Point

$\Rightarrow$ if $\vec{\tau}_S = 0$ then $\frac{d\vec{L}_S}{dt} = 0 \Rightarrow \vec{L}_{S,f} = \vec{L}_{S,i}$

example 19.4



$$\vec{F}^G_{E,m} = -\frac{G m M_e}{r^2}\hat{r}$$

$\vec{\tau}_S = \vec{r}_S \times \vec{F}^G_{E,m} = \vec{0} \Rightarrow \Delta\vec{L}_S = 0$

$\vec{L}_{S,i} = \vec{r}_{S,i} \times m\vec{v}_i = (x_i\hat{i} + h\hat{j}) \times m v_i \hat{i} = -m v_i h\,\hat{k}$

$\vec{L}_{S,e} = (r_e\hat{i}) \times m v_p(-\hat{j}) = -r_e m v_p \hat{k}$

speed of meteor at nearest approach

$\Delta L_S = 0 \Rightarrow \cancel{m} v_i h = \cancel{m} v_p r_e$ unknowns: v_p, h

$\Delta E = 0 \quad \frac{\cancel{m} v_i^2}{2} = \frac{\cancel{m} v_p^2}{2} - \frac{G\cancel{m} m_e}{r_e}$

$\Rightarrow v_i^2 = v_p^2 - \frac{2Gm_e}{r_e}$

$v_p = \frac{v_i h}{r_e} \Rightarrow v_i^2 = v_i^2 \cdot \frac{h^2}{r_e^2} - \frac{2Gm_e}{r_e}$

$r_e^2 v_i^2 + 2Gm_e r_e = v_i^2 \cdot h^2$

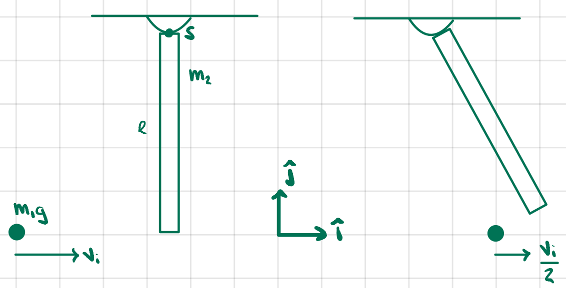
$$h = \sqrt{r_e^2 + \frac{2Gm_e r_e}{v_i^2}}$$

19.5 Angular Impulse

If there is a total applied torque $\vec{\tau}_S$ about a point S over time interval Δt then the torque applies an angular impulse about a point S, given by

$$\vec{J}_S = \int_{t_i}^{t_f} \vec{\tau}_S\, dt = \vec{L}_{S,f} - \vec{L}_{S,i}$$

19.8 Principle of Conservation of Angular Momentum



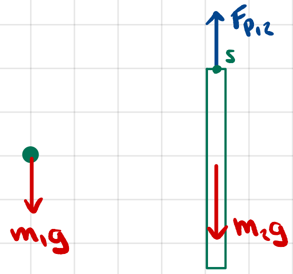
$I_{cm_{rod}} = \frac{m_2 \ell^2}{12}$

system: rod + m_1

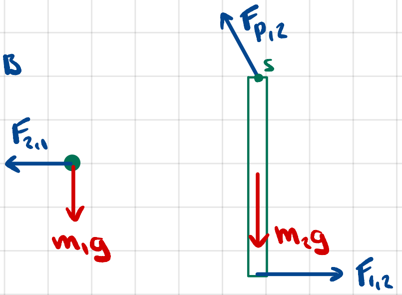
three states: A (immediately before collision), B (immed. after collision), C (rod touches ceiling with zero angular speed

fundamental quantities: momentum, energy, angular momentum

A



B



→ we don't know if the collision is elastic or not, so we don't know if mechanical energy is constant.

$$\vec{\tau}_{S,sys} = \cancelto{0}{\vec{0} \times \vec{F}_{p,12}} + (-\ell \hat{j}) \times (\cancel{\vec{F}_{2,1}} - \cancelto{0}{m_1 g \hat{j}}) + \cancelto{0}{(-\tfrac{\ell}{2}\hat{j}) \times (-m_2 g \hat{j})} + \cancel{(-\ell \hat{j}) \times \vec{F}_{1,2}} = \vec{0}$$

$$\Rightarrow \vec{L}_{S,sys,A} = \vec{L}_{S,sys,B}$$

$$\vec{L}_{S,sys,A} = (-\ell \hat{j}) \times m_1 v_i \hat{i} = m_1 \ell v_i \hat{k}$$

$$\vec{L}_{S,sys,B} = (-\ell \hat{j}) \times m_1 \frac{v_i}{2} \hat{i} + I_S \vec{\omega}_B$$

$$= \frac{\ell m_1 v_i}{2} \hat{k} + I_S \omega_B \hat{k}$$

$$\Rightarrow m_1 \ell v_i = \frac{m_1 \ell v_i}{2} + I_S \omega_B$$

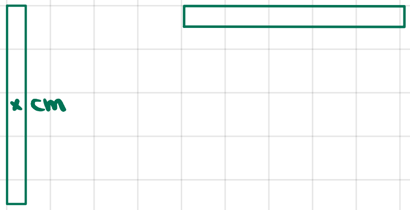
$$\Rightarrow \omega_B = \frac{m_1 \ell v_i}{2 I_S}$$

$$I_S = m_2 d_{cm}^2 + I_{cm} = m_2 \cdot \frac{\ell^2}{4} + \frac{m_2 \ell^2}{12} = \frac{m_2 \ell^2}{3}$$

$$\Rightarrow \omega_B = \frac{3 m_1 \ell v_i}{2 m_2 \ell^2} = \frac{3 m_1 v_i}{2 m_2 \ell}$$

B to C

gravitational force is conservative ⇒ $\Delta E_m = 0$



$$E_B = \frac{I_S \omega_B^2}{2} + \frac{m_1 (v_i/2)^2}{2}$$

$$= \frac{1}{2} \cdot \frac{\cancel{m_2} \cancel{\ell^2}}{\cancel{3}} \cdot \frac{\cancel{3^2} m_1^2 v_i^2}{4 m_2^{\cancel{2}} \cancel{\ell^2}} + \frac{m_1 v_i^2}{8}$$

$$= \frac{3 m_1^2 v_i^2}{8 m_2} + \frac{m_1 v_i^2}{8}$$

$$E_C = m_2 g \frac{\ell}{2} + \frac{m_1 (v_i/2)^2}{2}$$

$$\Delta E_m = 0 \Rightarrow E_B = E_C$$

$$\frac{3 m_1^2 v_i^2}{8 m_2} = m_2 g \frac{\ell}{2}$$

$$\Rightarrow v_i = \frac{\cancelto{4}{8} m_2^2 g \ell}{\cancel{2} \cdot 3 m_1^2}$$

$$\Rightarrow v_i = \frac{4 m_2^2 g \ell}{3 m_1^2}$$

what condition is necessary for the collision to be elastic?

$$E_A = \frac{m_1 v_i^2}{2}$$

$$E_A = E_B \Rightarrow \frac{3 m_1^2 v_i^2}{8 m_2} + \frac{m_1 v_i^2}{8} = \frac{m_1 v_i^2}{2}$$

$$\frac{3 m_1^{\cancel{2}} \cancel{v_i^2}}{\cancel{8} m_2} = \frac{\cancel{3} \cancel{m_1} \cancel{v_i^2}}{\cancel{8}} \Rightarrow \frac{m_1}{m_2} = 1$$