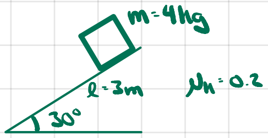
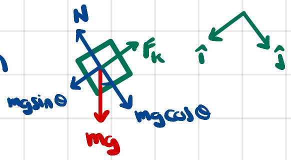
Ex 13.10



$\mu_k = 0.2$

a)



$F_g = mg\sin\theta\,\hat{i} + mg\cos\theta\,\hat{j}$

$F_k = -\mu_k mg\cos\theta\,\hat{i}$

$N = -mg\cos\theta\,\hat{j}$

$W_g = (mg\sin\theta\,\hat{i} + mg\cos\theta\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$

$= mg\sin\theta \cdot dx$

$= 9.8\cdot 4\cdot \frac{1}{2}\cdot 3 = 58.8\text{ J}$

$W_k = -\mu_k mg\cos\theta\,\hat{i}\cdot(dx\,\hat{i} + dy\,\hat{j})$

$= -\mu_k mg\cos\theta\, dx$

$= -0.2\cdot 9.8\cdot 4\cdot \frac{\sqrt{3}}{2}\cdot 3 = -20.36\text{ J}$

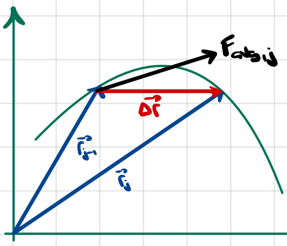
$W_N = 0$

$W_g + W_k + W_N = 38.44 = \Delta K = \quad v = 4.38\text{ m/s}$

13.9 Work Done by Non-constant Force Along Arbitrary Path

position vector $\vec{r}(t)$

non-constant force $\vec{F}$ acts on point mass on 3D curved path



displacement between t_{j-1} and t_j.

avg force in interval $\vec{F}_{j,avg}$

avg work $\Delta W_j = \vec{F}_{j,avg}\cdot\Delta\vec{r}_j$

Total work $= W_N = \sum_{j=1}^{N}\Delta W_j = \sum_{j=1}^{N}\vec{F}_{a,t_j}\cdot\Delta\vec{r}_j$

$W = \lim_{\substack{N\to\infty \\ \Delta r_j\to 0}} \sum \vec{F}_{a,t_j}\cdot\Delta\vec{r} = \int_{r_i}^{r_f}\vec{F}\cdot d\vec{r}$

infinitesimal vector line element

tangent to orbit of the body

limit of $\Delta\vec{r}$

13.9.1 Work in Cartesian coord.

$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$dW = F_x dx + F_y dy + F_z dz$

$W = \int_{\vec{r}_0}^{\vec{r}_f} dW = \int_{\vec{r}_0}^{\vec{r}_f}\vec{F}\cdot d\vec{r} = \int_{\vec{r}_0}^{\vec{r}_f}(F_x dx + F_y dy + F_z dz)$

13.9.2 Work Integral in Cylindrical coord.

$d\vec{r} = dr\,\hat{r} + d\theta\,\hat{\theta} + dz\,\hat{k}$

$\vec{F} = F_r\hat{r} + F_\theta\hat{\theta} + F_z\hat{k}$

$dW = \vec{F}\cdot d\vec{r} = F_r dr + F_\theta d\theta + F_z dz$

$W = \int_{\vec{r}_0}^{\vec{r}_F}(F_r dr + F_\theta d\theta + F_z dz)$

13.11 Work-Kinetic Energy Theorem in 3D

$\vec{F} = m\cdot\vec{a} = m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$

$\int_{\vec{r}_0}^{\vec{r}_i}\vec{F}\cdot d\vec{r} = \int_{\vec{r}_0}^{\vec{r}_i}(F_x dx + F_y dy + F_z dz)$

$= \frac{1}{2}m(v_{x_f}^2 - v_{x_0}^2) + \frac{1}{2}m(v_{y_f}^2 - v_{y_0}^2) + \frac{1}{2}m(v_{z_f}^2 - v_{z_0}^2)$

$\Rightarrow W_F = \Delta K$

13.11.1 Instantaneous Power

$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v} = \frac{dK}{dt} = \frac{1}{2}m\frac{d}{dt}\vec{v}\cdot\vec{v}$

$= \frac{1}{2}m\left(2\frac{d\vec{v}}{dt}\vec{v}\right) = m\vec{a}\vec{v} = \vec{F}\cdot\vec{v}$