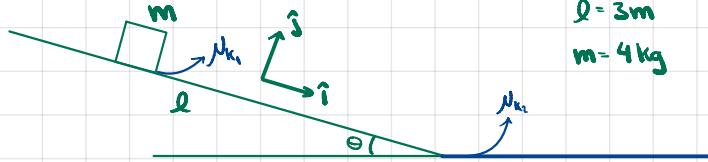


PSet 7

Problem 1



$$\begin{aligned} \theta &= 30^\circ \\ \mu_{k_1} &= 0.2 \\ \mu_{k_2} &= 0.3 \\ l &= 3 \text{ m} \\ m &= 4 \text{ kg} \end{aligned}$$

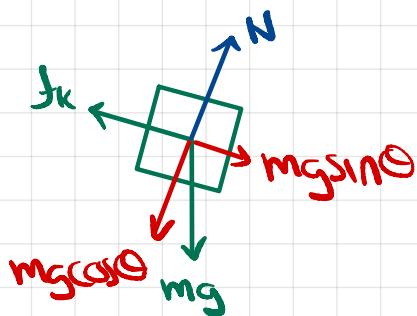
a) while sliding down the plane, friction does negative work.

$$f_k = -\mu_k N \hat{i}$$

$$\text{2nd law, } \hat{j} \text{ dir: } N - mg \cos \theta = 0 \Rightarrow N = mg \cos \theta$$

$$\Rightarrow f_k = -\mu_k mg \cos \theta \hat{i}$$

$$W_k = \int_C f_k \cdot d\vec{s} = -\mu_k mg \cos \theta \int_0^l dx = -l \mu_k mg \cos \theta = -3 \cdot 0.2 \cdot 4 \cdot 9.8 \cdot \frac{\sqrt{3}}{2} = -20.369 \text{ J}$$



$$\text{b) } \vec{F}_g = mg \sin \theta \hat{i} + mg \cos \theta \hat{j}$$

$$W_g = \int_0^l mg \sin \theta dx = l mg \sin \theta = 34.98 \cdot 0.5 = 58.80 \text{ J}$$

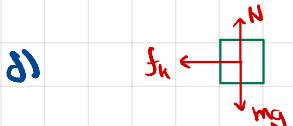
$$\text{c) } \vec{F} = (mg \sin \theta - \mu_k N) \hat{i} + (N - mg \cos \theta) \hat{j}$$

$$\Rightarrow \vec{F} = (mg \sin \theta - \mu_k mg \cos \theta) \hat{i}$$

$$d\vec{s} = dx \hat{i} + dy \hat{j}$$

$$W_i = \int_C \vec{F} \cdot d\vec{s} = mg(\sin \theta - \mu_k \cos \theta) \int_0^l dx = l mg(\sin \theta - \mu_k \cos \theta) = 58.80 - 20.369 = 38.431 \text{ J}$$

$$W_i = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2) \Rightarrow 38.431 = \frac{1}{2} \cdot 4 (v_f^2 - 0) \Rightarrow v_f^2 = (38.431 / 2)^{1/2} = 4.38 \text{ m/s}$$

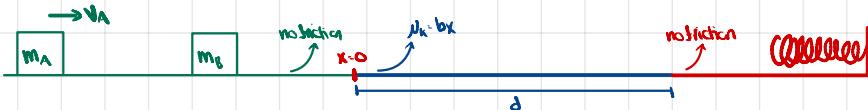


$$W_k = \int_C f_k \cdot d\vec{s} = \int_0^l (-\mu_k mg \hat{i}) (dx \hat{i}) = -\mu_k mg \int_0^l dx = -\mu_k mg l$$

$$\text{e) } W_k = \Delta K = \frac{1}{2} m(0^2 - v_i^2) \Rightarrow -\mu_k mg l = -\frac{1}{2} v_i^2 \Rightarrow l = \frac{v_i^2}{2\mu_k mg} = \frac{(3.7964)^2}{2 \cdot 0.3 \cdot 9.8} = 2.45 \text{ m}$$

$$\begin{aligned} &\rightarrow 4.38 \cos \theta = 4.38 \cdot \frac{\sqrt{3}}{2} \\ &= 3.7964 \text{ m/s} = v_i \\ &\rightarrow 4.38 \text{ m/s} \end{aligned}$$

Problem 2 - Collision and Sliding on a Rough Surface



1. conservation of momentum \Rightarrow determine velocity of $(m_A + m_B)$ at $x=0$

2. work done by friction = Δ kinetic energy \Rightarrow velocity at d found

3. work done by spring = Δ kinetic energy \Rightarrow compression distance found

$$1. m_A v_A = (m_A + m_B) v_0 \Rightarrow v_0 = \frac{m_A}{m_A + m_B} v_A$$

$$2. \int_0^d -bx(m_A + m_B)g dx = \frac{1}{2}(m_A + m_B)(v_d^2 - v_0^2)$$

$$-bg(m_A + m_B) \frac{d^2}{2} = \cancel{\frac{1}{2}}(m_A + m_B)(v_d^2 - v_0^2)$$

$$\therefore v_d^2 = v_0^2 - bgd^2 = \frac{m_A^2}{(m_A + m_B)^2} v_A^2 - bgd^2$$

$$3. \int_0^D -kx dx = \frac{1}{2}(m_A + m_B)(-v_d^2)$$

$$\therefore -\frac{kD^2}{2} = \frac{1}{2}(m_A + m_B)(-v_d^2)$$

$$D = \sqrt{\frac{(m_A + m_B)v_d^2}{k}} = \sqrt{\frac{(m_A + m_B)}{k} \left[\frac{m_A^2}{(m_A + m_B)^2} v_A^2 - bgd^2 \right]}$$

momentum principle applied to 2.

$$\Delta \text{momentum} = \int f_k dt$$

$$(m_A + m_B)(v_d - v_0) = \int_0^{t_d} -bx(t)(m_A + m_B)g dt$$

$$v_d - v_0 = -bg \int_0^{t_d} x(t) dt$$

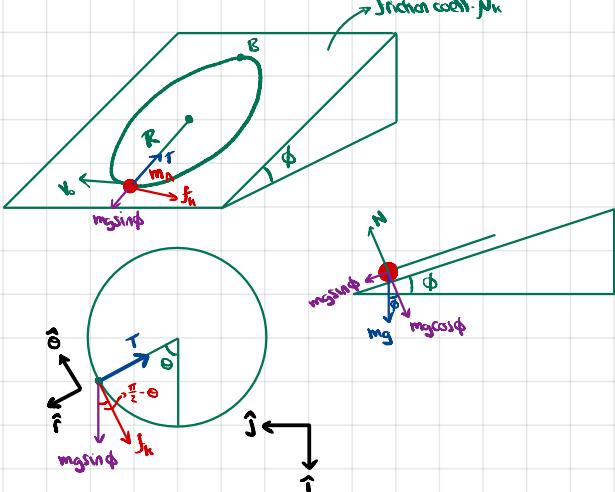
time	momentum
t	$m_T v(t)$

$t + \Delta t$	$m_T v(t + \Delta t)$
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$$\Delta p = m_T \Delta v \Rightarrow \frac{\Delta p}{\Delta t} = \frac{\Delta v}{\Delta t} m_T \Rightarrow \frac{dp}{dt} = (m_A + m_B) \frac{dv}{dt}$$

$$F = \frac{dp}{dt} \Rightarrow -bx(t) = (m_A + m_B) \frac{dv}{dt}$$

Problem 3 - Inclined Plane



a) mass \$A\$ has circular motion \rightarrow acceleration both tangentially and radially
we want to calculate work along a curve, a line integral $\int_C \vec{f}_k \cdot d\vec{s}$.

$$\begin{aligned}\vec{f}_k &= -N_k N \hat{\theta} = -N_k mg \cos \phi (-\sin \theta(t) \hat{i} + \cos \theta(t) \hat{j}) \\ &= N_k mg \cos \phi \sin \theta(t) \hat{i} - N_k mg \cos \phi \cos \theta(t) \hat{j}\end{aligned}$$

$$\begin{aligned}\int_C \vec{f}_k \cdot d\vec{s} &= \int_C \vec{f}_k \cdot \vec{v} dt \\ &\quad \leftarrow \int_C (-N_k N \hat{\theta})(R \omega \hat{\theta}) dt = -N_k NR \int_0^T \theta'(t) dt = -N_k NR \theta(t_f) - N_k NR \frac{\pi}{2} \\ &\quad \leftarrow \int \vec{f}_k \cdot \vec{v} dt - \int \vec{f}_k \cdot \hat{r} ds = \int_0^{T_2} \frac{(-N_k N \hat{\theta})(R \omega \hat{\theta})}{R \omega} \cdot R d\theta = -N_k NR \int_0^{T_2} d\theta = -N_k NR \frac{\pi}{2}\end{aligned}$$

b) At \$B\$ $\vec{v} = \vec{0}$. Motion is no longer circular, and the only force causing acceleration is $mg \cos \phi$.

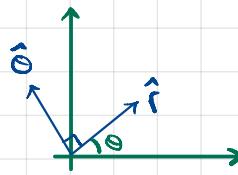
$$+ \vec{f}(t) = R \hat{r} \quad \frac{d\vec{f}(t)}{dt} = R \cdot \frac{d\hat{r}}{dt} = \vec{J}(t)$$

$$\frac{d\hat{r}}{dt} = -\sin \theta(t) \theta'(t) \hat{i} + \cos \theta(t) \theta'(t) \hat{\theta}$$

$$\begin{aligned}\vec{J}(t) &= -R \sin \theta \hat{i} + R \cos \theta \hat{\theta} \\ &= R \theta' (-\sin \theta \hat{i} + \cos \theta \hat{\theta}) = R \theta' \hat{\theta}\end{aligned}$$

$$\begin{aligned}\vec{a}(t) &= (-R \cos \theta \cdot \theta'^2 - R \sin \theta \cdot \theta'') \hat{i} + (-R \sin \theta \cdot \theta'^2 + R \cos \theta \cdot \theta'') \hat{\theta} \\ &= -R \theta'^2 \cos \theta \hat{i} - R \theta'^2 \sin \theta \hat{\theta} - R \sin \theta \hat{i} + R \cos \theta \hat{\theta} \\ &= -R \theta'^2 \hat{i} + R \theta'' \hat{\theta} = a_r \hat{i} + a_\theta \hat{\theta}\end{aligned}$$

$$\vec{r}(t) = R \hat{r} \quad \vec{J}(t) = R \theta' \hat{\theta} \quad \vec{a}(t) = -R \theta'^2 \hat{i} + R \theta'' \hat{\theta}$$



$$\begin{aligned}\hat{r} &= \langle \cos \theta, \sin \theta \rangle \\ \hat{\theta} &= \langle -\sin \theta, \cos \theta \rangle\end{aligned}$$

$$\vec{f}_k + \vec{F}^g + \vec{T} = m \vec{a}$$

$$\vec{f}_k = -N_k mg \cos \phi \hat{\theta}$$

$$\vec{F}^g = mg \sin \phi \cos \theta(t) \hat{r} + mg \sin \phi \sin \theta(t) \hat{\theta}$$

$$\vec{T} = -T \hat{r}$$

$$-N_k mg \cos \phi + mg \sin \phi \sin \theta(t) = m a_r = m R \theta''(t)$$

$$\Rightarrow -T + mg \sin \phi \cos \theta(t) = -m R \theta''(t)^2$$