

Recap of Results

We can always write the eq. for conservation of momentum

$$m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f}$$

or cons with the component in x-direction

$$m_1 v_{1,x,i} + m_2 v_{2,x,i} = m_1 v_{1,x,f} + m_2 v_{2,x,f}$$

In elastic collisions, we write $\Delta K = 0 \Rightarrow K_i = K_f$

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

we can solve for $v_{1,f}$ and $v_{2,f}$ to obtain the 1D energy/momentum principle

$$v_{1,x,i} - v_{2,x,i} = -(v_{1,x,f} - v_{2,x,f}) \Rightarrow v_{1,x,i} = -v_{1,x,f}$$

For totally inelastic collisions, $v_{1,x,f} = 0$ ie $v_{1,x,i} = v_{1,x,f}$

again, conservation of momentum

$$m_1 v_{1,x,i} + m_2 v_{2,x,i} = (m_1 + m_2) v_{x,f}$$

$$\Rightarrow v_{x,f} = \frac{m_1 v_{1,x,i} + m_2 v_{2,x,i}}{m_1 + m_2}$$

We haven't imposed any explicit condition on ΔK but implicitly we have

$$\Delta K = \frac{(m_1 + m_2) v_{x,f}^2}{2} - \frac{m_1 v_{1,x,i}^2}{2} - \frac{m_2 v_{2,x,i}^2}{2}$$

$$\text{simplifying } = \frac{-m_1 m_2 (v_{1,x,i} - v_{2,x,i})^2}{2m_1 + 2m_2} < 0$$

\Rightarrow Kinetic Energy is lost in totally inelastic collision.

Next we defined results concerning the center of mass frame we know \vec{r}_{cm} , so we can obtain my position from CM frame given position in lab frame

$$\vec{r}_1' = \frac{\mu}{m_1} \vec{r}_{12} \quad \vec{v}_1' = \frac{\mu}{m_1} \vec{v}_{12}$$

$$\vec{r}_2' = \frac{\mu}{m_2} \vec{r}_{12} \quad \vec{v}_2' = -\frac{\mu}{m_2} \vec{v}_{12}$$

$$\text{where } \mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$$

But $\vec{v}_{1,2}' = \vec{v}_{1,2}$, ie the relative velocity vector is the same in every reference frame (that is relatively inertial)

This gives us a new result in the 1D elastic collision case.

1D energy/momentum principle $\Rightarrow v_{1,2,i} = -v_{1,2,f}$

$$\vec{v}_1' = \frac{\mu}{m_1} \vec{v}_{1,2} \quad \vec{v}_2' = -\frac{\mu}{m_2} \vec{v}_{1,2}$$

$$\Rightarrow \vec{v}_{1,i}' = \frac{\mu}{m_1} \vec{v}_{1,2,i}$$

$$\vec{v}_{1,f}' = \frac{\mu}{m_1} \vec{v}_{1,2,f} = \frac{\mu}{m_1} (-\vec{v}_{1,2,i})$$

$$\Rightarrow \vec{v}_{1,f}' = -\vec{v}_{1,i}'$$

$$\text{likewise } \vec{v}_{2,f}' = -\vec{v}_{2,i}'$$

In CM frame, in 1D elastic collision, velocities of objects change direction but not magnitude (speed).

Next we look at kinetic energy. Start with the definitions

$$K_{cm} = \frac{m_1 v_{1,i}^2}{2} + \frac{m_2 v_{2,i}^2}{2}$$

$$K_{ground} = \frac{m_1 v_{1,i}^2}{2} + \frac{m_2 v_{2,i}^2}{2}$$

we know the relationship between $v_{1,i}$ and $v_{1,i}'$, and between $v_{2,i}$ and $v_{2,i}'$.

$$\vec{v}_1' = \frac{\mu}{m_1} \vec{v}_{1,2} \quad \vec{v}_2' = -\frac{\mu}{m_2} \vec{v}_{1,2}$$

so we can find K_{cm} in terms of K_{ground} and vice versa
sub v_1' and v_2' into K_{cm} , obtain

$$K_{cm} = K_{ground} + \frac{m_1 + m_2}{2} v_{cm}^2$$

sub v_1' and v_2' into K_{cm} , obtain

$$K_{cm} = \frac{\mu v_{1,2}^2}{2}$$

In an elastic collision, $v_{1,2,i} = -v_{1,2,f}$ so

$$\Delta K_{cm} = 0$$

In a totally inel. collision $v_{1,2,f} = 0$

$$\Delta K_{cm} = -\frac{\mu v_{1,2,i}^2}{2} < 0$$

Finally, note that $K_{ground} = K_{cm} + \text{constant}$

$$\Rightarrow \Delta K_{ground} = \Delta K_{cm}$$