14.2 Conservative and Non-conservative Forces

mechanical energy
- ↗ potential energy
- ↘ kinetic energy

conservative force: work done is indep. of path

$$\Rightarrow \oint \vec{F}_c \cdot d\vec{r} = 0$$

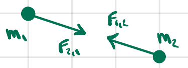
14.3 Changes in Potential Energies of a System

→ When only internal conservative forces act in a closed system, the sum of ΔK and ΔU of the system is zero

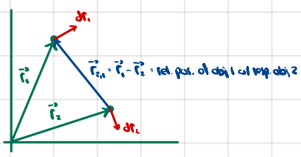
setup

→ closed system, $\Delta E_{sys} = 0$, two masses m_1, m_2

→ there is a single pair of internal forces, and they are conservative.



choose coord. system



relative displacement: $d\vec{r}_{2,1} = d\vec{r}_1 - d\vec{r}_2$

$$\Delta K_{sys} = \Delta K_1 + \Delta K_2 = W_c = \int_A^B \vec{F}_{2,1} \cdot d\vec{r}_1 + \int_A^B \vec{F}_{1,2} \cdot d\vec{r}_2$$

, where A, B are initial and final states

Third Law $\Rightarrow \vec{F}_{1,2} = -\vec{F}_{2,1} \Rightarrow \Delta K_{sys} = \int_A^B \vec{F}_{2,1} \cdot d\vec{r}_1 - \int_A^B \vec{F}_{2,1}\, d\vec{r}_2$

$$= \int_A^B \vec{F}_{2,1}(d\vec{r}_1 - d\vec{r}_2) = \int_A^B \vec{F}_{2,1}\, d\vec{r}_{2,1}$$

$$* \; d\vec{r}_{2,1} = -d\vec{r}_{1,2} \Rightarrow \int_A^B \vec{F}_{2,1}\, d\vec{r}_{2,1} = \int_A^B \vec{F}_{1,2}\, d\vec{r}_{1,2}$$

we now define change in internal potential energy of the system to be neg. of work done by the conservative force when the objects undergo a relative displacement from init. state A to final state B along any displacement that changes the initial state A to the final state B.

$$\Delta U_{sys} = -W_c = -\int_A^B \vec{F}_{2,1}\, d\vec{r}_{2,1} = -\int_A^B \vec{F}_{1,2}\, d\vec{r}_{1,2} = -\Delta K_{sys}$$

this def. is only valid for conservative forces.

14.4.1 chg in Gravit. Pot. En. near Earth Surface

system: Earth, object

gravitational force: internal conservative force acting inside the system

coord. system: origin on Earth surface, $+y$ points away from center of Earth

assumption: displacement of Earth is negligible, only need to consider object displacem.

$$\vec{F}_g = -mg\hat{j}$$
$$d\vec{r} = dy\,\hat{j}$$
$$\vec{F}_g \cdot d\vec{r} = -mg\,dy$$
$$\Delta U_g = -W_g = -(-mg(y_f - y_i)) = mg(y_f - y_i)$$

→ chg in pot. en. is difference in pot. en. fn. we can choose any antideriv. of the work integral.

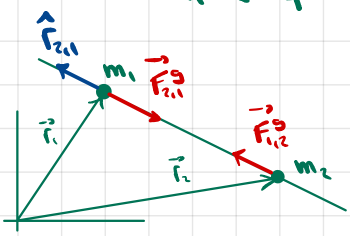
choose a zero reference potential:

$$U_g(0) = 0 \Rightarrow U_g(y) = mgy$$

14.4.3 Inverse Square Gravitation Force

system: two objects of masses m_1, m_2 separated by center-to-center distance $r_{2,1}$

coord. system:



displacement vector: $d\vec{r}_{2,1} = dr_{2,1}\,\hat{r}_{2,1}$

$$\vec{F}^g_{2,1} = -\frac{Gm_1m_2}{r_{2,1}^2}\hat{r}_{2,1}$$

$$\vec{F}^g_{2,1}\, d\vec{r}_{2,1} = -\frac{Gm_1m_2}{r_{2,1}^2}\, dr_{2,1}$$

$$\Delta U_g = -\int_A^B \vec{F}^g_{2,1} \cdot d\vec{r}_{2,1} = -\int_{r_i}^{r_f} -\frac{Gm_1m_2}{r_{2,1}^2}\, dr_{2,1} = -\frac{Gm_1m_2}{r_{2,1}}\Big|_{r_i}^{r_f}$$

$$= -\frac{Gm_1m_2}{r_f} + \frac{Gm_1m_2}{r_i}$$

choose ref. point for zero of potential energy

$$r_i = \infty \quad U(\infty) = 0 \Rightarrow U_g = -\frac{Gm_1m_2}{r}$$