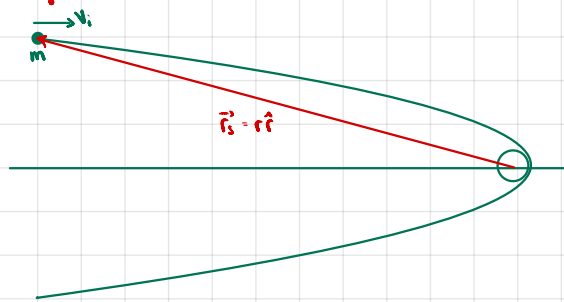


19.4 Conservation of Angular Momentum About a Point

→ if $\vec{\tau}_S = 0$ then $\frac{d\vec{L}_S}{dt} = 0 \Rightarrow \vec{L}_{S,f} = \vec{L}_{S,i}$

example 19.4



$$\vec{F}_{E,m}^G = - \frac{GM_e m}{r^2} \hat{r}$$

$$\vec{\tau}_S = \vec{r}_S \times \vec{F}_{E,m}^G = \vec{0} \Rightarrow \Delta \vec{L}_S = 0$$

$$\vec{L}_{S,i} = \vec{r}_{S,i} \times m \vec{v}_i = (x_i \hat{i} + h \hat{j}) \times m v_i \hat{i} = -m v_i h \hat{k}$$

$$\vec{L}_{S,e} = (r_e \hat{i}) \times m v_p (-\hat{j}) = -r_e m v_p \hat{k}$$

speed of meteor at nearest approach

$$\Delta L_S = 0 \Rightarrow \cancel{m} v_i h = \cancel{m} v_p r_e \quad \text{unknowns: } v_p, h$$

$$\Delta E = 0 \quad \frac{\cancel{m} v_i^2}{2} = \frac{\cancel{m} v_p^2}{2} - \frac{GM_e m}{r_e}$$

$$\Rightarrow v_i^2 = v_p^2 - \frac{2GM_e}{r_e}$$

$$v_p = \frac{v_i h}{r_e} \Rightarrow v_i^2 = v_i^2 \cdot \frac{h^2}{r_e^2} - \frac{2GM_e}{r_e}$$

$$r_e^2 v_i^2 + 2GM_e r_e = v_i^2 \cdot h^2$$

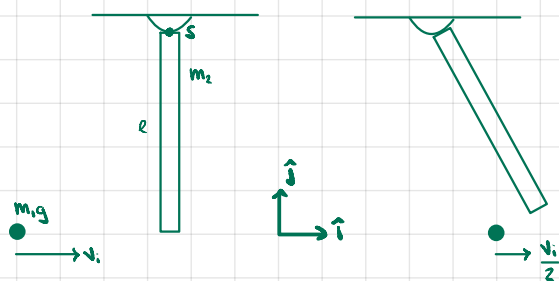
$$h = \sqrt{r_e^2 + \frac{2GM_e r_e}{v_i^2}}$$

19.5 Angular Impulse

If there is a total applied torque $\vec{\tau}_S$ about a point S over time interval Δt then the torque applies an angular impulse about a point S, given by

$$\vec{J}_S = \int_{t_i}^{t_f} \vec{\tau}_S dt = \vec{L}_{S,f} - \vec{L}_{S,i}$$

19.8 Principle of Conservation of Angular Momentum

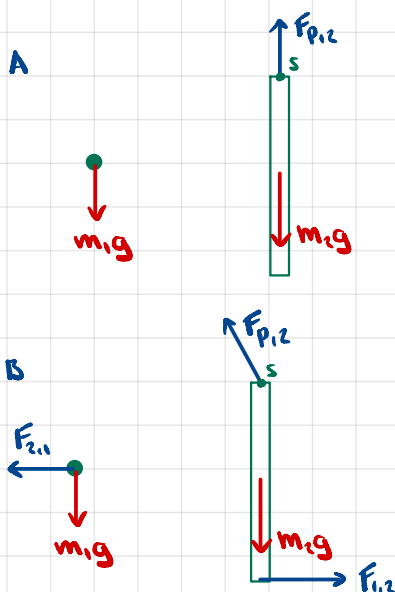


$$I_{cm,rod} = \frac{m_2 l^2}{12}$$

system: rod + m_1

three states: A (immediately before collision), B (immediately after collision), C (rod touches ceiling with zero angular speed)

fundamental quantities: momentum, energy, angular momentum



we don't know if the collision is elastic or not, so we don't know if mechanical energy is constant.

$$\vec{L}_{S,p,A} = \vec{0} \times \vec{F}_{p,2} + (-l\hat{j}) \times (\vec{F}_{2,1} - m_1 g \hat{j}) + (-\frac{l}{2}\hat{j}) \times (-m_2 g \hat{j}) + (-l\hat{j}) \times \vec{F}_{1,2} = \vec{0}$$

$$\Rightarrow \vec{L}_{S,p,A} = \vec{L}_{S,p,B}$$

$$\vec{L}_{S,p,A} = (-l\hat{j}) \times m_1 v_i \hat{i} = m_1 l v_i \hat{k}$$

$$\vec{L}_{S,p,B} = (-l\hat{j}) \times m_1 \frac{v_i}{2} \hat{i} + I_S \vec{\omega}_B$$

$$= \frac{l m_1 v_i}{2} \hat{k} + I_S \omega_B \hat{k}$$

$$\Rightarrow m_1 l v_i = \frac{m_1 l v_i}{2} + I_S \omega_B$$

$$\Rightarrow \omega_B = \frac{m_1 l v_i}{2 I_S}$$

$$I_S = m_2 d_{cm}^2 + I_{cm} = m_2 \frac{l^2}{4} + \frac{m_2 l^2}{12} = \frac{m_2 l^2}{3}$$

$$\Rightarrow \omega_B = \frac{3 m_1 l v_i}{2 m_2 l^2} = \frac{3 m_1 v_i}{2 m_2 l}$$

B to C

gravitational force is conservative $\Rightarrow \Delta E_m = 0$



$$E_B = \frac{I_S \omega_B^2}{2} + \frac{m_1 (v_i/2)^2}{2}$$

$$= \frac{1}{2} \cdot \frac{m_2 l^2}{3} \cdot \frac{9 m_1^2 v_i^2}{4 m_2^2 l^2} + \frac{m_1 v_i^2}{8}$$

$$= \frac{3 m_1^2 v_i^2}{8 m_2} + \frac{m_1 v_i^2}{8}$$

$$E_C = m_2 g \frac{l}{2} + \frac{m_1 (v_i/2)^2}{2}$$

$$\Delta E_m = 0 \Rightarrow E_B = E_C$$

$$\frac{3 m_1^2 v_i^2}{8 m_2} = m_2 g \frac{l}{2}$$

$$\Rightarrow v_i = \frac{\sqrt{m_2 g l}}{\sqrt{3} m_1}$$

$$\Rightarrow v_i = \frac{4 m_2^2 g l}{3 m_1^2}$$

what condition is necessary for the collision to be elastic?

$$E_A = \frac{m_1 v_i^2}{2}$$

$$E_A = E_B \Rightarrow \frac{3 m_1^2 v_i^2}{8 m_2} + \frac{m_1 v_i^2}{8} = \frac{m_1 v_i^2}{2}$$

$$\frac{\cancel{3 m_1^2} v_i^2}{\cancel{8 m_2}} = \frac{\cancel{3 m_1^2} v_i^2}{\cancel{8}} \Rightarrow \frac{m_1}{m_2} = 1$$