# Recap of Results

we can always write the eq. for conservation of momentum

$$m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}$$

we can work with the component in x-direction

$$m_1 v_{1,x,i} + m_2 v_{2,x,i} = m_1 v_{1,x,f} + m_2 v_{2,x,f}$$

for elastic collisions, we write $\Delta K = 0 \Rightarrow K_i = K_f$

$$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

we can solve for $v_{1,f}$ and $v_{2,f}$ to obtain the 1D energy momentum principle

$$v_{1,x,i} - v_{2,x,i} = -(v_{1,x,f} - v_{2,x,f}) \Rightarrow v_{1,2,i} = -v_{1,2,f}$$

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for totally inelastic collisions, $v_{1,2,f} = 0$ ie $v_{1,x,f} = v_{2,x,f}$

again, conservation of momentum

$$m_1v_{1,x,i} + m_2v_{2,x,i} = (m_1+m_2)v_{x,f}$$

$$\Rightarrow v_{x,f} = \frac{m_1v_{1,x,i} + m_2v_{2,x,i}}{m_1+m_2}$$

we haven't imposed any explicit condition on $\Delta K$ but implicitly we have

$$\Delta K = \frac{(m_1+m_2)v_{x,f}^2}{2} - \frac{m_1v_{1,x,i}^2}{2} - \frac{m_2v_{2,x,i}^2}{2}$$

simplifying $= \dfrac{-m_1m_2(v_{1,x,i} - v_{2,x,i})^2}{2m_1+2m_2} < 0$

$\Rightarrow$ Kinetic Energy is lost in totally inelastic collision.

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next we derived results concerning the center of mass frame
we know $\vec{r}_{cm}$, so we can obtain any position from CM frame given position in lab frame

$$\vec{r}_1' = \frac{\mu}{m_1}\vec{r}_{12} \qquad \vec{v}_1' = \frac{\mu}{m_1}\vec{v}_{1,2}$$

$$\Rightarrow$$

$$\vec{r}_2' = \frac{-\mu}{m_2}\vec{r}_{12} \qquad \vec{v}_2' = -\frac{\mu}{m_2}\vec{v}_{1,2}$$

where $\mu$ = reduced mass = $\dfrac{m_1m_2}{m_1+m_2}$

But $\vec{v}_{1,2} = \vec{v}_{1,2}$, ie the relative velocity vector is the same in every reference frame (that is relatively inertial)

This gives us a new result in the 1D elastic collision case.

1D energy momentum principle $\Rightarrow v_{1,2,i} = -v_{1,2,f}$

$$\vec{v}_1' = \frac{\mu}{m_1}\vec{v}_{1,2} \qquad \vec{v}_2' = -\frac{\mu}{m_2}\vec{v}_{1,2}$$

$$\Rightarrow \vec{v}_{1,i}' = \frac{\mu}{m_1}\vec{v}_{1,2,i}$$

$$\vec{v}_{1,f}' = \frac{\mu}{m_1}\vec{v}_{1,2,f} = \frac{\mu}{m_1}(-\vec{v}_{1,2,i})$$

$$\Rightarrow \vec{v}_{1,f}' = -\vec{v}_{1,i}'$$

likewise $\vec{v}_{2,f}' = -\vec{v}_{2,i}'$

In CM frame, in 1D elastic collision, velocities of objects change direction but not magnitude (speed).

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Next we look at kinetic energy. Start with the definitions

$$K_{cm} = \frac{m_1v_{1,i}'^2}{2} + \frac{m_2v_{2,i}'^2}{2}$$

$$K_{ground} = \frac{m_1v_{1,i}^2}{2} + \frac{m_2v_{2,i}^2}{2}$$

we know the relationship between $v_{1,i}$ and $v_{1,i}'$, and between $v_{2,i}$ and $v_{2,i}'$.

$$\vec{v}_1' = \frac{\mu}{m_1}\vec{v}_{1,2} \qquad \vec{v}_2' = -\frac{\mu}{m_2}\vec{v}_{1,2}$$

so we can find $K_{ground}$ in terms of $K_{cm}$ and vice versa
sub $v_1'$ and $v_2'$ into $K_{ground}$, obtain

$$K_{ground} = K_{cm} + \frac{m_1+m_2}{2}v_{cm}^2$$

sub $v_1'$ and $v_2'$ into $K_{cm}$, obtain

$$K_{cm} = \frac{\mu v_{1,2}^2}{2}$$

in an elastic collision, $v_{1,2,i} = -v_{1,2,f}$ so

$$\Delta K_{cm} = 0$$

in a totally inel. collision $v_{1,2,f} = 0$

$$\Delta K_{cm} = -\frac{\mu v_{1,2,i}^2}{2} < 0$$

Finally, note that $K_{ground} = K_{cm} + \text{constant}$

$$\Rightarrow \Delta K_{ground} = \Delta K_{cm}$$