

18.1 Introduction to Static Equilibrium

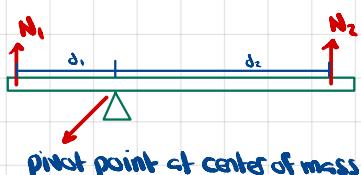
static equilibrium of an extended body: in an inertial reference frame, both CM is at rest and the body does not undergo any rotation.

→ sufficient and necessary conditions:

$$\sum \vec{F}_i = \vec{0}$$

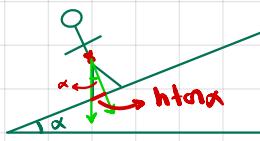
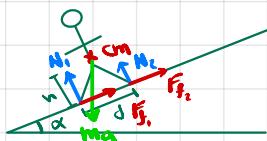
$$\vec{T}_s = \sum \vec{T}_{s,i} = \vec{0}$$

18.2 Levers Law



no rotation if $T_s = N_2 d_2 - N_1 d_1 = T_1$

Example 18.3 Person Standing on a Hill



The person is in static equil.

$$\sum \vec{F}_i = \vec{0} \Rightarrow N_1 + N_2 - mg \cos \alpha = 0$$

$$mg \sin \alpha - f_1 - f_2 = 0$$

$$\sum \vec{T}_{cm,i} = 0 \Rightarrow \frac{d}{2} N_2 - \frac{d}{2} N_1 + hf_1 + hf_2 = 0$$

unknowns: N_1, N_2, f_1, f_2

eliminate one unknown $f_1 = N_1 \mu_s$
 $f_2 = N_2 \mu_s$

$$N_1 + N_2 = mg \cos \alpha$$

$$\mu_s (N_1 + N_2) = mg \sin \alpha$$

$$2\mu_s (N_1 + N_2) = d(N_1 - N_2)$$

$$\mu_s mg \cos \alpha - mg \sin \alpha \Rightarrow \tan \alpha = \mu_s$$

$$2h = \frac{d(N_1 - N_2)}{mg \sin \alpha} \Rightarrow N_1 = N_2 + \frac{2hmg \sin \alpha}{d}$$

$$N_1 = mg \cos \alpha - N_2 + \frac{2hmg \sin \alpha}{d}$$

$$N_1 = \frac{mg \cos \alpha}{2} + \frac{hmg \sin \alpha}{d}$$

$$N_2 = mg \cos \alpha - N_1$$

$$= mg \cos \alpha - \frac{mg \cos \alpha}{2} - \frac{hmg \sin \alpha}{d}$$

$$= \frac{mg \cos \alpha}{2} - \frac{hmg \sin \alpha}{d} = 0$$

$$\Rightarrow d = \frac{2hmg \sin \alpha}{mg \cos \alpha} = 2htan \alpha$$

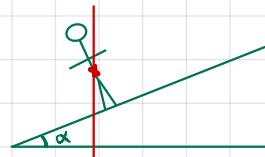
$$N_2 = 0 \Rightarrow N_1 = mg \cos \alpha$$

N_2 cannot be smaller than 0. For $d < 2htan \alpha$, the required N_2 to keep static equilibrium isn't possible.

$$\text{torque} \cdot \frac{d}{2} N_1 = h \mu_s N_1$$

at d_{\min}

$$htan \alpha = h \mu_s \Rightarrow \tan \alpha = \mu_s$$



$$T_{cm} = -\frac{d}{2} N_1 + hf_1$$



Interpretation $N_1 + N_2 = mg \cos \alpha$
increasing one normal decreases the other.

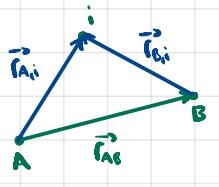
$mg \sin \alpha = \mu_s (N_1 + N_2)$ same principle.

$$\frac{d}{2} (N_1 - N_2) = h \mu_s (N_1 + N_2)$$

net torque due to normal = torque due to friction.

For this to balance, $N_1 > N_2$.

Appendix 18A · The Torques About Any Two Points Are Equal For a Body in Static Equilibrium



Force acts at position of point i.
origin is irrelevant.

$$\vec{r}_{Ai} = \vec{r}_{AB} + \vec{r}_{Bi}$$

sum of torques about point A: $\vec{\tau}_A = \sum \vec{r}_{Ai} \times \vec{F}_i$

" " " " B: $\vec{\tau}_B = \sum \vec{r}_{Bi} \times \vec{F}_i$

$$\vec{\tau}_A = \sum (\vec{r}_{AB} + \vec{r}_{Bi}) \times \vec{F}_i = \sum (\vec{r}_{AB} \times \vec{F}_i + \vec{r}_{Bi} \times \vec{F}_i)$$

$$= \vec{r}_{AB} \times \sum \vec{F}_i + \vec{\tau}_B$$

By assumption, $\sum \vec{F}_i = 0 \Rightarrow \vec{\tau}_A = \vec{\tau}_B$

In static equilibrium, net torque about any point is the same, zero.