## Ex 7.1

$\uparrow v_0 = 20$ m/s

0.145 kg

$$W_g = \int_0^h -mg\,dx = -mg(h-0) = -(U_h - U_0) = -U_h = -mgh$$

$$-\cancel{m}gh = \frac{1}{2}\cancel{m}(0^2 - v_0^2) \Rightarrow h = \frac{v_0^2}{2g} = \frac{400}{2\cdot 9.8} = 20.40\,\text{m}$$

Note that $y=0$ corresponds to where the ball was thrown from. If instead $y_0 = 10$ then

$$W_g = \int_{10}^h -mg\,dx = -\cancel{m}g(h-10) = \frac{1}{2}\cancel{m}(-v_0^2) \Rightarrow h = 10 + \frac{v_0^2}{2g}$$

$\downarrow$

$-\Delta U$ is still equal to the same quantity $\Delta K$

## Ex 7.2

a)

$\uparrow 20$ m/s

0.5 m

The ball starts at rest. $v_0 = K_0 = 0$.
We can define any height to be zero height. Let's use the starting point. $y_0 = 0$, $U_0 = mgy_0 = 0$
Person's hand accelerates the ball to 20 m/s over 0.5 m. Gravity is still doing work too, albeit negative work.

$$W_g = -mg(0.5-0) = -\frac{mg}{2} = -0.7105\,\text{J}$$

$W_h = \int_0^{0.5} F_h\,dx = F_h \cdot \frac{1}{2}$ Between 0 and 0.5, total work done equals change in kinetic energy.

$$W_g + W_h = \frac{1}{2}mv^2 \Rightarrow \frac{F_h}{2} - 0.7105 = \frac{1}{2}0.145\cdot 400 \Rightarrow F_h = 59.42\,\text{N}$$

Note a few ways of looking at the previous calculation.

$$W_g = -\Delta U_g \Rightarrow W_h = \Delta K + \Delta U_g = (K_2 + U_{g_2}) - (K_1 + U_{g_1}) = (29 + 0.7105) - (0+0)$$

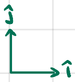
ie work done by hand, since $>0$, increases $E_{mech}$ of the system ball + earth. Mech. En. started at zero. w/o an external force it wouldn't be possible to increase $K_2$ w/o also decreasing $U_{g_2} = mgh_2$. Because the coord. system is oriented up away from Earth, $h_2$ must decrease ie the ball is falling.

When the hand releases the ball, $E_{mech}$ is 29.7105 J. Since only gravity is doing work from this point on, mech. energy is conserved. At height h we have

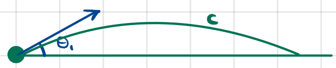
$$E_{mech_h} = \frac{1}{2}mv_2^2 + mgy_2 = 29.7105 = \frac{1}{2}mv_h^2 + mgh \Rightarrow v_h = \pm\sqrt{\frac{2}{m}(29.7105 - mgh)}$$

$v_h(15.5) = 10.29$ m/s

## Ex 7.3




There is only the conservative force gravity acting on the ball. Mechanical energy is the same at each point in the trajectory.




$E_{mech_0} = U_0 + K_0 = 0 + \frac{1}{2}mv_0^2 = \frac{mv_0^2}{2}$, where $\vec{v}_0 = \langle v_{0x}, v_{0y} \rangle$, $v_0 = |\vec{v}_0| = (v_{0x}^2 + v_{0y}^2)^{1/2}$

Assump. is that no matter $\theta$, $v_0$ is constant, ie balls launched with same initial speed.

At any other height h from initial point potential energy is higher, speed in y direction is lower, because work is doing negative work.

$E_{mech_h} = \frac{\cancel{m}v^2}{2} + \cancel{m}gh = \frac{\cancel{m}v_0^2}{2} \Rightarrow v = \pm\sqrt{v_0^2 - 2gh}$

note that $v^2 = v_x^2 + v_y^2$ is speed, h is vertical displacement.

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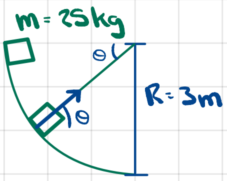
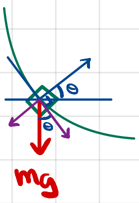
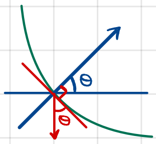
## Ex 7.4




a) $W_g = \int_{y_i}^{y_f} -mg\,dy = -mg(y_f - y_i) = -25 \cdot 9.8(0-3) = 735\text{ J}$

$\Rightarrow \Delta U = -735\text{ J}$

Intuitively, we know that at the bottom velocity has only an x component, $\vec{v} = \langle v_x, 0 \rangle$. Therefore, $E_{mech_1} = \frac{1}{2}mv_{1x}^2$, because $U_1 = mg \cdot 0 = 0$. Also, $E_{mech_0} = mg \cdot 3$.

Can we equate $E_{mech_0}$ and $E_{mech_1}$?

There is another force, N, acting on the slider. The work done by this force is the difference $\Delta E_{mech}$.

$|N| = |mg\cos\theta|$

$N = -mg\sin\theta \cdot \hat{r} = -mg(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

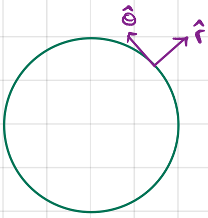
Trajectory: $\hat{r} = 3$, $\theta = 0$ to $\pi/2$

$W_N = \int_C \underbrace{-mg\langle\cos\theta, \sin\theta\rangle}_{[N]_{ij}} \underbrace{\langle -\sin\theta, \cos\theta \rangle}_{[\hat{\theta}]_{ij}} ds = 0$ because N is perpend. to trajectory.

$\Rightarrow \Delta E_{mech} = 0$

$\Rightarrow 3mg = \frac{1}{2}mv_{1x}^2 \Rightarrow v_{1x} = \pm 7.66\text{ m/s}$ The entire 735 J of pot. energy became kinetic energy.

b) $W_g = -\Delta U = mgR \Rightarrow \Delta U = -\cancel{m}gR = \Delta K = \frac{1}{2}\cancel{m}v_f^2 \Rightarrow v_f = \sqrt{2gR}$




$\hat{r} = \langle \cos\theta, \sin\theta \rangle$

$\hat{\theta} = \langle -\sin\theta, \cos\theta \rangle$

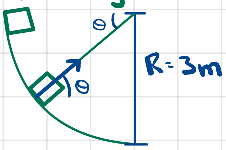
$$\frac{d\hat{\theta}}{dt} = \theta' \langle -\cos\theta, -\sin\theta \rangle = -\theta' \hat{r}$$

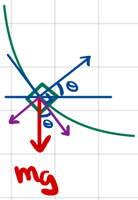
$\vec{r} = r\hat{r} = r\langle \cos\theta, \sin\theta \rangle$

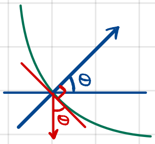
$$\vec{v} = \frac{d\vec{r}}{dt} = r\langle -\sin\theta \cdot \theta', \cos\theta \cdot \theta' \rangle = r\theta'\hat{\theta}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\theta''\hat{\theta} + r\theta'(-\theta'\hat{r}) = -r\theta'^2\hat{r} + r\theta''\hat{\theta}$$

$m = 25\,kg$




$$N - mg\sin\theta = m \cdot a_R = m \cdot (-r\theta'^2) \Rightarrow mr\theta'^2 = mg\sin\theta(t) - N(t)$$

$$\cancel{m}g\cos\theta = m \cdot a_T = \cancel{m}r\theta'' \Rightarrow \frac{d^2\theta}{dt} = \frac{g}{r}\cos\theta(t)$$

note that $v(t) = \theta'(t) \cdot R$

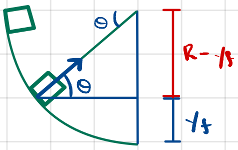
also $W_g = \int_{y_i}^{y_f} -mg\,dy = -\cancel{m}g(y_f - y_i) = -\Delta U = \Delta K = \frac{1}{2}\cancel{m}v_f^2$

$$\Rightarrow v_f = \sqrt{2g(y_i - y_f)}$$

$$a_R = -R\theta'^2 = -R\left(\frac{v}{R}\right)^2 = \frac{-v^2}{R} = \frac{-2g(R - y_f)}{R}$$

At the bottom, $a_r = -2g$

$mg - N = m(-2g) \Rightarrow N = 3mg$




$$\frac{R - y_f}{R} = \sin\theta$$

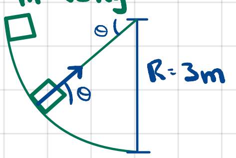
At other points,

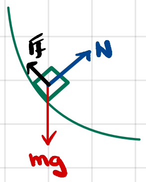
$$mg\sin\theta - N = m\frac{(-2g(R - y_f))}{R}$$

$$N = mg\left(\frac{R - y_f}{R}\right) + \frac{2mg}{R}(R - y_f) = 3mg\left(\frac{R - y_f}{R}\right)$$ this is the normal as fn of height.

## Ex 7.5

$m = 25\,kg$




$$W_f = \int_C F_f\, ds = \int_0^{\pi/2} F_f R\, d\theta = F_f R \cdot \frac{\pi}{2}$$

$$W_f + W_g = \Delta K \Rightarrow \frac{F_f R \pi}{2} + mgR = \frac{m v_f^2}{2}$$

$$F_f = \frac{m v_f^2 - 2mgR}{R\pi} = \frac{m(v_f^2 - 2gR)}{R\pi}$$

$$\Rightarrow F_f = \frac{25(6^2 - 2\cdot 9.8\cdot 3)}{3\cdot\pi} = -60.47\,N$$

$$\Rightarrow W_f = -60.47\cdot 3\cdot\frac{\pi}{2} = -285\,J$$

$$W_g = 25\cdot 9.8\cdot 3 = 735$$

Friction reduced mechanical energy.

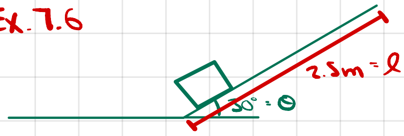
$$W_f = \Delta K - W_g = \Delta K + \Delta U = E_{mech_f} - E_{mech_0}$$

$$W_f < 0 \Rightarrow \Delta E_{mech} < 0$$

$U_f = mg\cdot 0 = 0$
$U_0 = mg\cdot 3 = 735$

$K_f = \frac{m\cdot 6^2}{2} = 450$
$K_i = 0$

## Ex. 7.6




$U_0 = mg\cdot 0$
$K_0 = \frac{m v_0^2}{2}$

$U_f = mgh$
$K_f = 0$

For $m = 12$, $\ell = 1.6$, $\sin\theta = 1/2$, $v_0 = 5$, $g = 9.8$ $\Rightarrow h = 1.6\cdot 0.5 = 0.8$

$U_0 = 0$
$K_0 = 12\cdot 25/2 = 150\,J$
$U_f = 12\cdot 9.8\cdot 1.6\cdot 0.5 = 94.08\,J$
$K_f = 0$

$$W_f = \Delta E_{mech} = 94.08 - 150 = -55.92\,J$$

a) $W_f = \int_0^{1.6} F_f\, dx = F_f\cdot 1.6 = -55.92 \Rightarrow F_f = -34.95\,N$

b) $U_0 = 94.08$
$K_0 = 0$

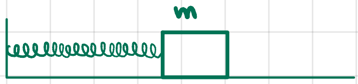
$U_f = 0$
$K_f = \frac{m v_f^2}{2}$

$$W_f = \int_{1.6}^{0} 34.95\, dx = 34.95(-1.6) = -55.92\,J$$

$$-55.92 = \frac{12\cdot v_f^2}{2} - 94.08$$

$v_f = 2.52\,m/s$

## Ex. 7.7




$m = 0.2\,kg$
no friction
massless spring
$k = 5\ N/m$

Pull mass, stretch spring to $x = 0.1m$
Before pulling

$K_0 = U_{el_0} = 0$

At stretched position

$K_1 = 0$

$W_F = \int_0^{x_1} F\,dx$

$W_{el} = \int_0^{x_1} -kx\,dx = -k\frac{x_1^2}{2} = -\Delta U_{el} \Rightarrow U_{el_1} > 0$

potential energy is stored in the spring

$W_F + W_{el} = \Delta K = 0 \Rightarrow \int_0^{x_1} F\,dx = k\frac{x_1^2}{2} = U_{el_1}$

ie $W_F = U_{el_1}$

Note $\frac{d}{dx}\int_0^{x_1} F\,dx = F(x_1) = kx_1$

release the spring

$W_{el} = \int_{x_1}^{x_2} -kx\,dx = -\frac{k(x_2^2 - x_1^2)}{2} = -\Delta U_{el} = \Delta K$

$-\frac{k}{2}(x_2^2 - x_1^2) = \frac{1}{2}mv_2^2$

$\Rightarrow v_2 = \pm\sqrt{-\frac{k}{m}(x_2^2 - x_1^2)}$

$v_2 = \pm\sqrt{-\frac{2\cdot5}{0.2}(0.08^2 - 0.1^2)} = 0.3\ m/s$

note that this is all just $\Delta E_{mech} = 0$

$(U_2 + K_2) - (U_1 + K_1)$

$= \underbrace{5\cdot\frac{0.08^2}{2}}_{0.016} + \frac{1}{2}\cdot0.2\cdot v_2^2 - \underbrace{5\cdot\frac{0.1^2}{2}}_{0.025} - 0$

$K_2 = \frac{1}{2}\cdot0.2\cdot v_2^2 = 0.009 \Rightarrow v_2 = \sqrt{\frac{0.009}{0.1}}$

$\Rightarrow v_2 = 0.3\,m/s$

## Ex 7.8

Initial mech. en. is zero. Ext. force does pos. work, increases system energy. $W_F$ is the chg in mech en.

$W_F + W_{el} = \Delta K$

$W_F = \Delta K + \Delta U = \Delta E_{mech} = \left(k\frac{x_2^2}{2} + \frac{1}{2}mv_2^2\right) - 0$

$W_F = \int_0^{0.1} 0.61\,dx = 0.61\cdot0.1 = 0.061\ J$

$0.061 = \frac{5\cdot0.1^2}{2} + \frac{0.2v_2^2}{2} \Rightarrow v_2 = \pm0.6\ m/s$

what does the $\pm$ mean?

At $x_2 = 0.1$ there is $U_2$ and $K_2$, they sum to 0.061.
If the force immed. stops acting, only the conservative elastic force does work. $F_{el}$ does neg. work at first, ie the potential goes up (K goes down), then the opposite occurs as the spring returns towards $x = 0$. When it passes $x = 0.08$, pot. energy has been converted back to kinetic, and v is $-0.6\,m/s$.

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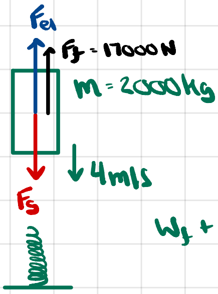
## Ex 7.9




Right before collision $K_1 = \frac{2000\cdot16}{2} = 16000\,J$, $U_{el_1} = 0$, $U_{g_1} = 0$

grav. pot. decreases (g does pos work), elastic potential increases as spring is compressed

$W_f + W_g + W_{el} = W_f - \Delta U_g - \Delta U_{el} = \Delta K \Rightarrow \Delta E_{mech} = W_f$

At $x = -2$, $W_g = -mg(-2-0) = 2mg = -\Delta U_g$

$\Rightarrow \Delta U_g = -2\cdot2000\cdot9.8 = -39200\ J$

$W_{el} = -\frac{k((-2)^2 - 0^2)}{2} = -2k = -\Delta U_{el} \Rightarrow \Delta U_{el} = 2k$

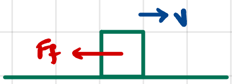
$W_f = \int_0^{-2} 17000\,dx = 17000(-2-0) = -34000\,J$

$K_2 = 0 \Rightarrow \Delta K = 0 - 16000 = -16000$

$\Rightarrow -34\cdot10^3 = -16000 - 39200 + \frac{1}{2}k(-2)^2 \Rightarrow k = 10600\,N$

The elevator has kinetic energy, and gravity does work to increase that energy by reducing potential. Were there no friction force, to stop the elevator, the spring would have to do all the negative work to reduce K to zero (so, 16000 J of increased elastic potential) and to absorb gravitational potential that would otherwise become kinetic energy (so, another 39200 J absorbed $U_{el}$).
With friction, total mech energy is reduced, and the spring has to do less work (ie absorb less elastic potential).

what happens if we try to define potential energy for friction?




$W_f = \int_{x_1}^{x_2} -F_f\,dx = U_f(x_2) - U_f(x_1) = \Delta U_f = \Delta K$ — $F_f$ is constant, but is $<0$ if $x_2 > x_1$, $>0$ if $x_2 < x_1$

$-F_f(x_2 - x_1) = -\Delta U_f \Rightarrow \Delta U_f = F_f x_2 - F_f x_1 \Rightarrow U_f(x) = F_f x$

on the trajectory to the right, $W_f = \Delta K \Rightarrow U_{f_1} - U_{f_2} = K_2 - K_1 \Rightarrow E_{m_1} = E_{m_2}$

potential increases.

to the left now

$W_f = \int_{x_2}^{x_1} F_f\,dx = F_f(x_1 - x_2) = -\Delta U_f \Rightarrow \Delta U_f = F_f(x_2 - x_1) = U_{f_2} - U_{f_1}$

potential increases again. potential keeps going up, kinetic keeps going down.

---

with gravity


$\int_{x_1}^{x_2} -mg\,dx = -mg(x_2 - x_1) < 0 = -\Delta U \Rightarrow U_g = mgx$

$-mg(x_2 - x_1) = \frac{1}{2}m(-v_1^2)$

on the way up $U_1 - U_2 = \Delta K \Rightarrow U_2 + K_2 = U_1 + K_1 \Rightarrow$ we know v at each one.

on the way down

$\int_{x_2}^{x_3} -mg\,dx = -mg(x_3 - x_2) > 0 = -\Delta U \Rightarrow U_g = mgx$

$-mg(x_3 - x_2) = \frac{1}{2}m(v_3^2 - v_2^2)$

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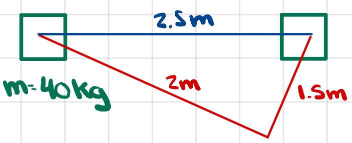
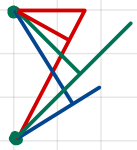
Ex 7.10



$\mu_k = 0.2$

straight path

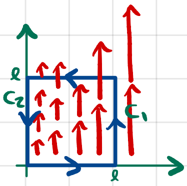
$W_f = \int_0^{2.5} -(40 \cdot 9.8) \cdot 0.2\,dx = -196\,J$

$W_F + W_f = \Delta K = 0 \Rightarrow W_F = -W_f = 196\,J$

Dogleg Path

$W_f = \int_0^{2} -(40 \cdot 9.8) \cdot 0.2\,dx + \int_2^{3.5} -(40 \cdot 9.8) \cdot 0.2\,dx = -274\,J$

Ex 7.11 $\vec{F} = Cx\hat{j}$, $C > 0$




$$W_F = \int_{C_1} (C\ell\hat{j}) \cdot (dy\,\hat{j}) + \int_{C_2} 0\,(-dy\,\hat{j})$$

$$= \int_0^\ell C\ell\,dy = \frac{C\ell^2}{2}$$

* $\vec{F} = \langle 0, Cx \rangle$ is a vector field.

Is $\vec{F} = \nabla f$?

$f_x = 0 \qquad f_y = Cx$

$f_{xy} = 0 \qquad f_{yx} = C$

F is not a gradient ⇒ non-conservative