# PSet 1

## Problem 1 (same as Example 4.6)

$v_{c,0} = 12\ m \cdot s^{-1}$ $\quad v_c(t_2) = 0$ $\quad a_c(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ b(t-t_1) & t_1 < t \le t_2 \end{cases}$ $\quad b = -6\ m \cdot s^{-3}$

a)

$$v_c(t) = \begin{cases} v_{c,0} & 0 \le t \le t_1 \\ v_{c,0} + \dfrac{b(t-t_1)^2}{2} & t_1 < t \le t_2 \end{cases}$$

$$x_c(t) = \begin{cases} v_{c,0}t + x_{c,0} & 0 \le t \le t_1 \\ v_{c,0}t + b(t-t_1)^3/6 + x_{c,0} & t_1 < t \le t_2 \end{cases}$$

inserting initial data

$$a_c(t) = \begin{cases} 0 & 0 \le t \le 1 \\ -6(t-1) & 1 < t \le t_2 \end{cases}$$

$$v_c(t) = \begin{cases} 12 & 0 \le t \le 1 \\ 12 - 3(t-1)^2 & 1 < t \le t_2 \end{cases}$$

$$x_c(t) = \begin{cases} 12t & 0 \le t \le 1 \\ 12t - (t-1)^3 & 1 < t \le t_2 \end{cases}$$

b) BIKE

$v_{b,0}$ constant speed throughout

$x_b(0) = -17$

$x_b(t_2) = x_c(t_2)$

$v_c(t_2) = 0$

$\Rightarrow$

$a_b(t) = 0$

$v_b(t) = v_{b,0}$

$x_b(t) = -17 + v_{b,0}t$

same position at $t = t_2$

$x_b(t_2) = x_c(t_2)$

$-17 + v_{b,0}t_2 = 12t_2 - (t_2 - 1)^3$

what is $t_2$?

$v_c(t_2) = 0 = 12 - 3(t_2^2 - 2t_2 + 1)$

$t_2^2 - 2t_2 + 1 = 4 \Rightarrow t_2^2 - 2t_2 - 3 = 0$

$\Delta = 4 - 4 \cdot 1 \cdot (-3) = 16$

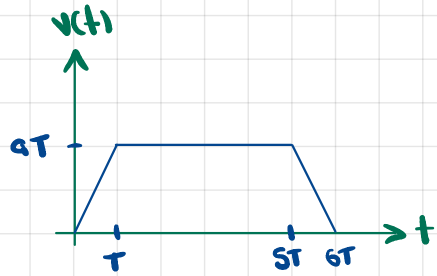
$t_2 = \dfrac{2 \pm 4}{2} \begin{matrix} \nearrow 3 \\ \searrow -1 \end{matrix}$

$-17 + 3v_{b,0} = 36 - (2)^3$

$\Rightarrow 3v_{b,0} = 28 + 17 = 45$

$\Rightarrow v_{b,0} = 15\ m \cdot s^{-1}$

# Problem 2




## a)

Total interval of time: $[0, 6T]$

$$a(t) = \begin{cases} a & 0 \le t < T \\ 0 & T < t \le 5T \\ -a & 5T < t \le 6T \end{cases}$$

$$v(t) = \begin{cases} at & 0 \le t < T \\ aT & T < t \le 5T \\ 6aT - at = a(6T - t) & 5T < t \le 6T \end{cases}$$

$$x(t) = \begin{cases} at^2/2 & 0 \le t < T \\ aT(1-T) + aTt = aT(1-T+t) & T < t \le 5T \\ \left(\frac{9}{2}aT^2 + aT\right) - \frac{a(6T-t)^2}{2} & 5T < t \le 6T \end{cases}$$

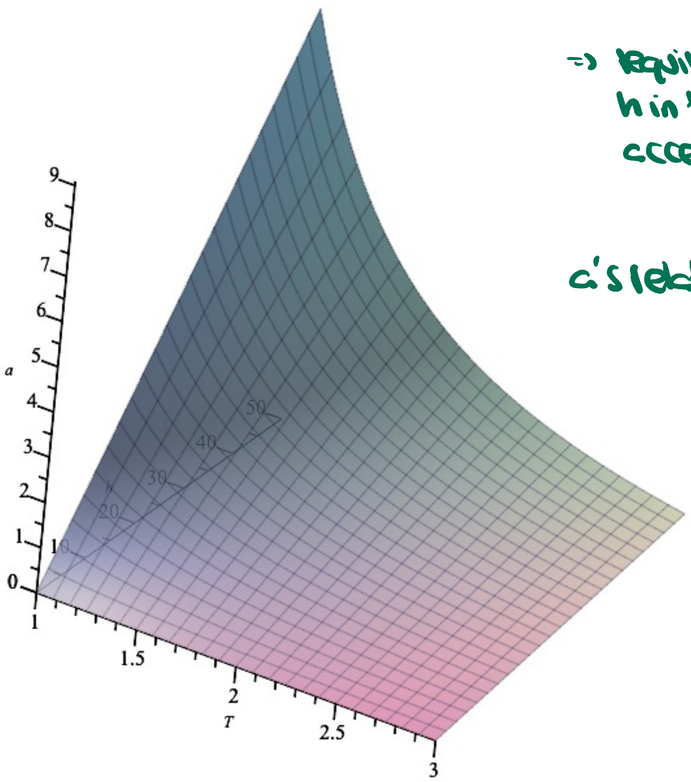
## b)

$x(6T) = h \Rightarrow \frac{9}{2}aT^2 + aT = h \Rightarrow 9aT^2 + 2aT = 2h \Rightarrow aT(9T+2) = 2h$

$\Rightarrow a(T,h) = \dfrac{2h}{T(9T+2)}$

**Example:**

$h = 30\,m,\ T = 2\,s \Rightarrow a(2,30) = \frac{60}{40} = 1.5\ m/s^2$

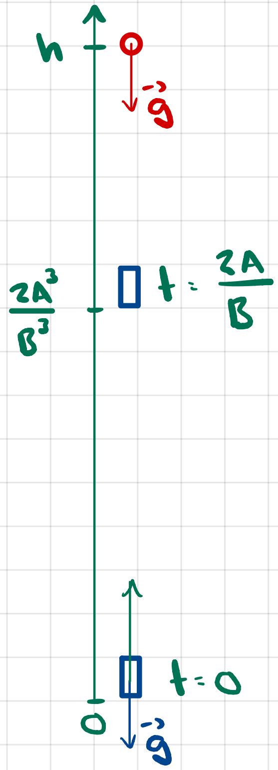



a is linear in height: floors double as tall

$\Rightarrow$ required acceleration doubles (to achieve position h in time T, with the specified pattern of acceleration and deceleration.

a's relationship with T is nonlinear.

# Problem 3




$a_r(t) = A - Bt \quad A, B > 0 \quad t > 0$

$v_r(t) = At - \frac{Bt^2}{2}$

$y_r(t) = \frac{At^2}{2} - \frac{Bt^3}{6}$

max height: $v_r(t) = 0 \Rightarrow t\left(A - \frac{Bt}{2}\right) \Rightarrow t = \frac{2A}{B}$

$$y_r\left(\frac{2A}{B}\right) = \frac{A}{\cancel{2}} \cdot \frac{\overset{2}{\cancel{4}}A^2}{B^2} - \frac{\cancel{B}}{\underset{3}{\cancel{6}}} \cdot \frac{\overset{4}{\cancel{8}}A^3}{B^{\cancel{3}\,2}} = \frac{2A^3}{B^2} - \frac{4A^3}{3B^2} = \frac{2A^3}{3B^2}$$

The stone falls for $\frac{2A}{B}$ seconds and $|\Delta y_s| = \frac{2A^3}{B^3} - h$

$a_s(t) = -gt \qquad v_s(t) = -\frac{gt^2}{2} \qquad y_s(t) = h - \frac{gt^3}{6}$

$$y_s(2A/B) = \frac{2A^3}{B^3} = h - \frac{g}{\underset{3}{\cancel{6}}} \cdot \frac{\overset{4}{\cancel{8}}A^3}{B^3} \Rightarrow h = \frac{A^3}{B^3}\left[2 + \frac{4}{3}g\right]$$

# Problem 4


## Person

$\vec{a}_p(t) = \langle Bt, 0 \rangle, \ B > 0$

$\vec{v}_p(t) = \langle Bt^2/2, 0 \rangle$

$\vec{r}_p(t) = \langle Bt^3/6 + d, 0 \rangle$

## Ball

$\vec{a}_b(t) = \langle 0, -g \rangle$

$\vec{v}_b(t) = \langle v_0\cos\theta, v_0\sin\theta - gt \rangle$

$\vec{r}_b(t) = \langle v_0\cos\theta\, t, v_0\sin\theta\, t - \frac{gt^2}{2} \rangle$

## Catching the Ball

$\vec{r}_b(t) = \vec{r}_p(t)$

$$\begin{cases} t v_0\cos\theta = Bt^3/6 + d \Rightarrow \\ t v_0\sin\theta - gt^2/2 = 0 \quad \Rightarrow t(v_0\sin\theta - gt/2) = 0 \rightarrow t = \dfrac{2v_0\sin\theta}{g} \end{cases}$$

sub. into first eq.

$$\frac{2v_0\sin\theta}{g} \cdot v_0\cos\theta = \frac{B}{\cancel{6}_3} \cdot \frac{\cancel{8}^4 v_0^3\sin^3\theta}{g^3} + d$$

$$\Rightarrow \left[\frac{\sin(2\theta)\, v_0^2}{g} - d\right] \cdot \frac{3}{4}\,\frac{g^3}{v_0^3\sin^3\theta} = B$$

$$\Rightarrow B(\theta, v_0, d, g) = \left[\sin(2\theta)\, v_0^2 - dg\right] \cdot \frac{3g^2}{4v_0^3\sin^3\theta}$$

## Maple Plots

# Problem 5 - Vertical Collision

## First can

$\vec{F} = -mg$

$a_1(t) = -g$

$v_1(t) = v_0 - gt$

$y_1(t) = v_0 t - \frac{gt^2}{2}$

## Second can

$$a_2(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ -g & t_1 < t \end{cases}$$

$v_2(t)$

$0 \le t \le t_1 \Rightarrow v_2(t) = 0$

$t > t_1 \Rightarrow v_2(t) = C - gt$

$v_2(t_1) = v_0 = C - gt_1 \Rightarrow C = v_0 + gt_1$

$$\Rightarrow v_2(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ v_0 + gt_1 - gt & t > t_1 \end{cases}$$

$y_2(t)$

$0 \le t \le t_1 \Rightarrow y_2(t) = 0$

$t > t_1 \Rightarrow y_2(t) = C + (v_0 + gt_1)t - \frac{gt^2}{2}$

$y_2(t_1) = 0 = C + v_0 t_1 + gt_1^2 - gt_1^2/2$

$= C + v_0 t_1 + gt_1^2/2 \Rightarrow C = -(v_0 t_1 + gt_1^2/2)$

$\Rightarrow y_2(t)$

$$= \begin{cases} 0 & 0 \le t \le t_1 \\ -(v_0 t_1 + gt_1^2/2) + (v_0 + gt_1)t - \frac{gt^2}{2} & t > t_1 \end{cases}$$

## Collision at $y = y_c$

→ calculate time at which can 2 reaches $y_c$, given $v_0, t_1$.

→ given $v_0, t_1$ calculate time for can 1 to drop to $y_c$.

→ equate times to obtain $v_0$ as function of $t_1$

a) $y_c = -(v_0 t_1 + gt_1^2/2) + (v_0 + gt_1)t - \frac{gt^2}{2}$

$\Rightarrow \frac{g}{2}t^2 - (v_0 + gt_1)t + (v_0 t_1 + \frac{gt_1^2}{2} + y_c)$

$\Delta = (v_0 + gt_1)^2 - \cancel{4}^{2} \cdot \frac{g}{\cancel{2}}(v_0 t_1 + \frac{gt_1^2}{2} + y_c)$

$= v_0^2 + \cancel{2v_0 g t_1} + \cancel{g^2 t_1^2} - \cancel{2v_0 g t_1} - \cancel{g^2 t_1^2} - 2gy_c$

$= v_0^2 - 2gy_c \Rightarrow v_0^2 \ge 2gy_c$

$\Rightarrow t = \frac{v_0 + gt_1 \pm [v_0^2 - 2gy_c]^{1/2}}{g}$ → can 2 reaches $y_c$

$v_0 t - \frac{gt^2}{2} = y_c \Rightarrow \frac{g}{2}t^2 - v_0 t + y_c = 0$

$\Delta = v_0^2 - \cancel{4}^{2} \cdot \frac{g}{\cancel{2}} \cdot y_c = v_0^2 - 2y_c g \Rightarrow v_0^2 \ge 2y_c g$

$t = \frac{v_0 \pm [v_0^2 - 2y_c g]^{1/2}}{g}$ → can 1 reaches $y_c$

$$\frac{\cancel{v_0} + [v_0^2 - 2y_c g]^{1/2}}{\cancel{g}} = \frac{\cancel{v_0} + gt_1 - [v_0^2 - 2gy_c]^{1/2}}{\cancel{g}}$$

$\Rightarrow 2[v_0^2 - 2y_c g]^{1/2} = gt_1$

$4v_0^2 - 8y_c g = g^2 t_1^2$

$\Rightarrow v_0^2(y_c, g, t_1) = \frac{g^2 t_1^2 + 8y_c g}{4}$

$\Rightarrow v_0(y_c, g, t_1) = \left[\frac{g^2 t_1^2 + 8y_c g}{4}\right]^{1/2}$

b) $v_0(5, 9.8, 4) = 21.95$ m/s

$t_{collision} = 4.24$ s