4.6 1D Kinematics and Integration

$$\frac{dv(t)}{dt} = a(t)$$

$$\int \left[\frac{dv(t)}{dt}\right] dt = \int a(t)\,dt = v(t) + C$$

Alternatively

$$dv(t) = a(t)\,dt$$

$$\int dv(t) = \int a(t)\,dt = v(t) + C$$

"integral of the differential of function equals function plus constant"

Area as the Indefinite Integral of Acceleration

Ex4.6 Initial Data CAR

$v_{c,0} = 12\ m \cdot s^{-1}$ $\quad v_c(t_2) = 0 \quad a_c(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ b(t-t_1) & t_1 < t \le t_2 \end{cases}$

$b = -6\ m \cdot s^{-3}$

a)
$$v_c(t) = \begin{cases} v_{c,0} & 0 \le t \le t_1 \\ v_{c,0} + \frac{b(t-t_1)^2}{2} & t_1 < t \le t_2 \end{cases}$$

$$x_c(t) = \begin{cases} v_{c,0}t + x_{c,0} & 0 \le t \le t_1 \\ v_{c,0}t + b(t-t_1)^3/6 + x_{c,0} & t_1 < t \le t_2 \end{cases}$$

inserting initial data

$$a_c(t) = \begin{cases} 0 & 0 \le t \le 1 \\ -6(t-1) & 1 < t \le t_2 \end{cases}$$

$$v_c(t) = \begin{cases} 12 & 0 \le t \le 1 \\ 12 - 3(t-1)^2 & 1 < t \le t_2 \end{cases}$$

$$x_c(t) = \begin{cases} 12t & 0 \le t \le 1 \\ 12t - (t-1)^3 & 1 < t \le t_2 \end{cases}$$

b) BIKE

$v_{b,0}$ constant speed throughout

$x_b(0) = -17$

$x_b(t_2) = x_c(t_2)$

$v_c(t_2) = 0$

$\Rightarrow$

$a_b(t) = 0$

$v_b(t) = v_{b,0}$

$x_b(t) = -17 + v_{b,0}t$

same position at $t = t_2$

$x_b(t_2) = x_c(t_2)$

$-17 + v_{b,0}t_2 = 12t_2 - (t_2-1)^3$

what is t_2?

$v_c(t_2) = 0 = 12 - 3(t_2^2 - 2t_2 + 1)$

$t_2^2 - 2t_2 + 1 = 4 \Rightarrow t_2^2 - 2t_2 - 3 = 0$

$\Delta = 4 - 4 \cdot 1 \cdot (-3) = 16$

$t_2 = \frac{2 \pm 4}{2} \begin{matrix} \nearrow 3 \\ \searrow -1 \end{matrix}$

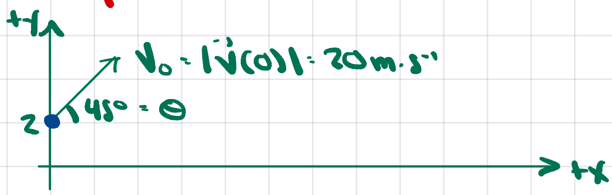
$-17 + 3v_{b,0} = 36 - (2)^3$

$\Rightarrow 3v_{b,0} = 28 + 17 = 45$

$\Rightarrow v_{b,0} = 15\ m \cdot s^{-1}$

5.2 Projectile Motion

Example 5.1



a) **Stone Motion**

$\downarrow \vec{g} = \langle 0, -mg \rangle$

$\vec{a}(t) = \langle 0, -g \rangle$

$\vec{v}(t) = \langle V_0 \cos\theta, V_0 \sin\theta - gt \rangle \quad t \geq 0$

$\vec{r}(t) = \langle V_0 \cos\theta\, t, y_0 + V_0 \sin\theta\, t - gt^2/2 \rangle$

highest point of trajectory $\Leftrightarrow r_y'(t) = 0$

$$r_y'(t) = V_0 \sin\theta - gt = 0 \Rightarrow t = \frac{V_0 \sin\theta}{g}$$

$$r_y(V_0 \sin\theta / g) = y_0 + \frac{V_0^2 \sin^2\theta}{g} - \frac{V_0^2 \sin^2\theta}{g^{\cancel{2}}} \cdot \frac{\cancel{g}}{2} = y_0 + \frac{V_0^2 \sin^2\theta}{2g}$$

maximum vertical displacement

$$= r_y(V_0 \sin\theta / g) - r_y(0) = \frac{V_0^2 \sin^2\theta}{2g}$$

inserting values

$\sin\theta = \frac{\sqrt{2}}{2}$

$V_0 = 20$

$g = 9.8$

$y_0 = 2$

$$t_{max} = \frac{20 \cdot \sqrt{2}}{2 \cdot 9.8} \approx \sqrt{2} \approx 1.4$$

$$\Rightarrow r_{y_{max}} - y_0 = \frac{\cancel{400}^{100} \cdot \cancel{2}}{\cancel{4} \cdot \cancel{2} \cdot 9.8} = 10.20$$