

## 20.1 Kinetic Energy

$$\text{Kinetic Energy} = K = \frac{1}{2}mv^2 \geq 0, \text{ a scalar}$$

$$\text{SI [J]} = \text{kg}(\text{m/s})^2$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}m(v_{x_f}^2 + v_{y_f}^2 + v_{z_f}^2 - v_{x_i}^2 - v_{y_i}^2 - v_{z_i}^2)$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

## 20.2 Work by a Constant Force

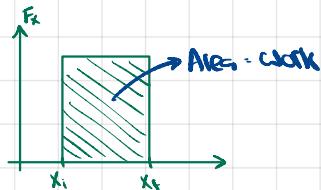


$$\vec{F} = F_x \hat{i}$$

$$\vec{dx} = dx \hat{i} \quad \Delta x = x_f - x_i$$

$$F_x = \text{constant}$$

$$\text{Work} = W = F_x \Delta x = F_x(x_f - x_i) \quad \text{SI units [N·m] - [J]}$$



## 20.3 Work by Non-constant Force

$$\vec{F}(x) = F_x(x) \hat{i}$$

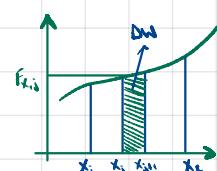
$$\text{Ex: (Spring Force)} \quad F_x = -kx$$



$$\Delta x = x_{fin} - x_i$$

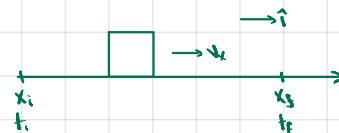
$$\Delta W = F_x \Delta x = F_{x,i}(x_{fin} - x_i)$$

$$W = \lim_{n \rightarrow \infty} \sum_{j=1}^n F_{x,j} \Delta x_j = \int_{x_i}^{x_f} F_x(x) dx$$



## 20.4 Integrate Accel. w/r to Time and Position

So far we've seen



$$a_x(t) = \frac{dx}{dt}$$

$$\int_{t_i}^{t_f} a_x(t) dt = \int_{x_i}^{x_f} \frac{dx}{dt} dt = \int dx = v_{xf} - v_{xi} = \Delta x$$

now let's think of  $a \approx \frac{\Delta v}{\Delta t}$

$$\int_{x_i}^{x_f} a_x(x) dx = \int_{x_i}^{x_f} \frac{dv}{dt} dx = \int_{x_i}^{x_f} dv \frac{dx}{dt} = \int_{x_i}^{x_f} v dx = \frac{1}{2}(v_{xf}^2 - v_{xi}^2)$$

## 20.5 Work-Kinetic Energy Theorem

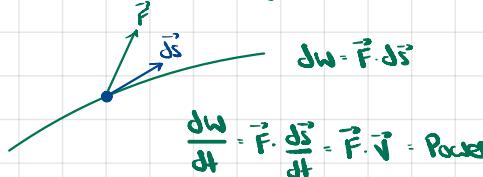
$$\text{2nd law: } F_x = m a_x$$

$$W = \int_{x_i}^{x_f} F_x dx = m \int_{x_i}^{x_f} a_x dx = \frac{1}{2}m(v_{xf}^2 - v_{xi}^2) = \Delta K$$

$$\Rightarrow W = \Delta K$$

## 20.6 Power

$$\text{Def (Power)} \quad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} \quad \text{SI unit Watt} = W = 1 \text{ J/s}$$

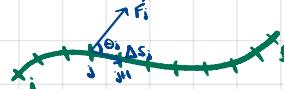


## 21.3 Kinetic Energy as a Scalar Product

$$K = \frac{1}{2}mv^2 \text{ scalar} \quad * \text{ref. frame dependent}$$

$$= \frac{1}{2}m\vec{v} \cdot \vec{v} = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$$

## 21.4 Work in 2D and 3D



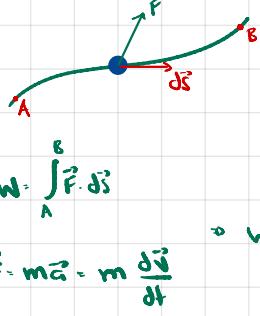
$$\vec{F}_i = \text{comp of } \vec{F}_i \text{ in dir of } \Delta s_i = \vec{F}_i \cos \theta_i$$

$$\vec{F}_i \cdot \Delta s_i = \vec{F}_i |\Delta s_i|$$

$$W = \lim_{n \rightarrow \infty} \sum_{i=1}^n W_i = \lim_{n \rightarrow \infty} \sum_{i=1}^n \vec{F}_i \cdot \Delta s_i = \int \vec{F} \cdot \vec{s}$$

line integral

## 21.5 Work-Kinetic Energy theorem in 2D and 3D



$$W = \int_A^B \vec{F} \cdot d\vec{s}$$

$$\Rightarrow W = \int_A^B m \frac{d\vec{v}}{dt} \cdot d\vec{s} = \int_A^B m \frac{d\vec{v}}{dt} \cdot \vec{ds} = \int_A^B m d\vec{v} \cdot \vec{v}$$

$$\vec{F} \cdot m\vec{a} = m \frac{d\vec{v}}{dt}$$

$$d\vec{s} = \sqrt{dx^2 + dy^2}$$

To calculate, let's choose a coord. system



$$d\vec{v} = dx\hat{i} + dy\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow d\vec{v} \cdot \vec{v} = (dv_x)v_x + (dv_y)v_y$$

$$W = \int_A^B (m dv_x v_x + m dv_y v_y) = \int_A^B m dv_x v_x + \int_A^B m dv_y v_y$$

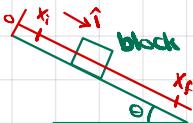
$$= \frac{m}{2} (v_{x,f}^2 - v_{x,i}^2) + \frac{m}{2} (v_{y,f}^2 - v_{y,i}^2)$$

$$= \frac{m}{2} (v_{x,f}^2 + v_{y,f}^2) - \frac{m}{2} (v_{x,i}^2 + v_{y,i}^2)$$

$$\Rightarrow K_f - K_i = \Delta K$$

$$\Rightarrow W = \Delta K$$

## 21.6 - Worked Example - Block Going Down Ramp



<u>W</u>	<u>ΔK</u>
free body diagram physics	description $\frac{1}{2} m(v_f^2 - v_i^2)$
$\int_{x_i}^{x_f} f_k \cdot dx + \int_{x_i}^{x_f} mg \sin\theta \cdot dx$	

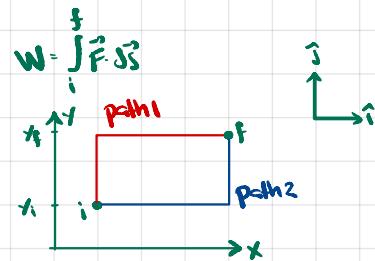
$$f_k: N - mg \cos\theta = 0 \\ f_k \cdot N \cdot N = \mu_k mg \cos\theta$$

$$- \int_{x_i}^{x_f} f_k \cdot dx + \int_{x_i}^{x_f} mg \sin\theta \cdot dx \\ - \int_{x_i}^{x_f} mg (\sin\theta - \mu_k \cos\theta) \cdot dx$$

$$- mg (\sin\theta - \mu_k \cos\theta) (x_f - x_i)$$

$$\Rightarrow mg (\sin\theta - \mu_k \cos\theta) (x_f - x_i) = \frac{1}{2} m(v_f^2 - v_i^2)$$

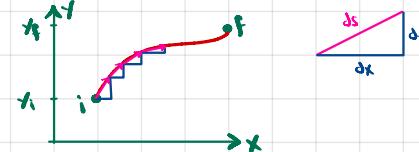
## 22.1 Path Independence



$$\vec{F}_g = -mg\hat{j}$$

$$W_1 = -mg(y_f - y_i)$$

$$W_2 = -mg(y_f - y_i)$$

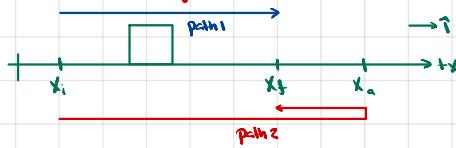


$$d\vec{s} = dx\hat{i} + dy\hat{j}$$

$$\vec{F}_g = -mg\hat{j}$$

$$W = \int_A^B \vec{F}_g \cdot d\vec{s} = \int_A^B -mg\hat{j} \cdot (dx\hat{i} + dy\hat{j}) = \int_{y_i}^{y_f} -mg dy = -mg(y_f - y_i)$$

## 22.2 Path Dependence - Friction



$$\vec{F}_k = -\mu_k mg\hat{i}$$

$$W_1 = \int_{x_i}^{x_f} (-\mu_k mg\hat{i})(dx\hat{i}) = -\mu_k mg \int_{x_i}^{x_f} dx = -\mu_k mg(x_f - x_i)$$

$$W_2 = \int_{x_i}^{x_a} -\mu_k mg dx + \int_{x_a}^{x_f} -\mu_k mg dx = -\mu_k mg(x_a - x_i) + \mu_k mg(x_f - x_a)$$

## 22.3 Conservative Forces

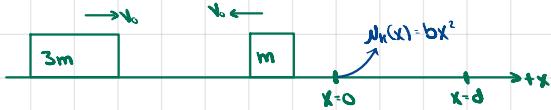
$$W = \int_A^B \vec{F} \cdot d\vec{s} \text{ path indep.} \Leftrightarrow \vec{F} \text{ conservative force}$$

$$\Leftrightarrow \int_A^B \vec{F} \cdot d\vec{s} + \int_B^A \vec{F} \cdot d\vec{s} = 0$$

$$W_C = \oint \vec{F}_C \cdot d\vec{s} = 0$$

If  $W$  is path dep then  $\vec{F}$  is non-conservative.

### PS.7.1 - Worked Example - Collision and Sliding on a Rough Surface



using conservation of momentum, we can calculate velocity right before entering the friction area.

$$3mv_0 - mv_0 = 4mv \Rightarrow 4mv = 2mv_0 \Rightarrow v = \frac{v_0}{2}$$

The work done by friction as the blocks are decelerated equals change in kinetic energy.

$$\int_0^d -N_k(x) \cdot 4mg dx = \frac{1}{2} m(0^2 - \frac{v_0^2}{2}) = -\frac{v_0^2}{2}$$

$$v_0^2 = 2 \int_0^d bx^2 \cdot 4mg dx = 2b \cdot 4mg \int_0^d x^2 dx = 8bm \frac{d^3}{3}$$