12.1 Mass Flow Problems

Ex 12.1 - Filling a coal car

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = \dot{m}v + m\dot{v} = bv + m\dot{v}$$

$$\dot{v} + \frac{b}{m}v = \frac{F}{m} \Rightarrow \dot{v} + \frac{b}{m_0+bt}v = \frac{F}{m_0+bt}$$

$$m(t) = m_0 + bt$$

$$u(t) = e^{\int \frac{b}{m_0+bt}dt} = e^{\ln(m_0+bt)} = m_0+bt$$

$$\dot{v}(m_0+bt) + bv = F$$

$$(v(m_0+bt))' = F$$

$$v(m_0+bt) = \int F\,dt + C = Ft + C$$

$$v(t) = \frac{Ft}{m_0+bt} + \frac{C}{m_0+bt}$$

$$v(0) = \frac{C}{m_0} = 0 \Rightarrow C = 0$$

$$\Rightarrow v(t) = \frac{Ft}{m_0+bt}$$

$$m_c = m_0 + bt_c \Rightarrow t_c = \frac{m_c - m_0}{b}$$

$$v(t_c) = \frac{F(m_c - m_0)}{bm_c}$$

Solution w/ Momentum

$$p_x(0) = 0$$

At $t = t_f$, coal of mass $m_c = bt_f$ has been transferred into the cart, now moving at v_f.

$$p_x(t_f) = (m_0 + m_c)v_f = (m_0 + bt_f)v_f$$

momentum principle: $\int_0^{t_f} F_x\,dt = p_x(t_f) - p_x(0)$

$$Ft_f = (m_0 + bt_f)v_f$$

$$\Rightarrow v_f = \frac{Ft_f}{m_0 + bt_f}$$