Week 2 Notes

# Lecture 4 - Newton's Laws of Motion

## Newton's 1st law tells us about motion of isolated bodies

$\underbrace{\text{isolated bodies}}_{\text{net Force} = 0}$

$\vec{F}_{TOTAL} = 0$

- "an isolated body moves in a straight line at constant velocity and will continue to do so as long as it remains undisturbed"
- "inertial coordinate systems exist"

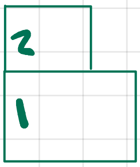
## Newton's Second Law

non-isolated bodies

$\vec{F} = m\vec{a}$ special case of point mass

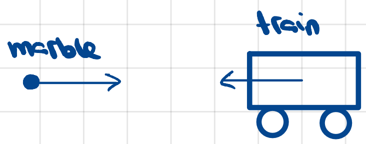
more generally, $\vec{F} = \frac{d\vec{p}}{dt}$. For point mass, $\vec{p} = m\vec{v}$ = momentum

## Newton's Third Law




$\vec{F}_{12} = -\vec{F}_{21}$

Ex:

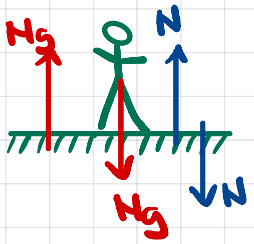


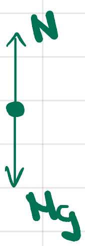

$a_m = \frac{F}{m_m}$ $a_T = \frac{-F}{m_T}$

$M_{marble} \lll M_{train}$

$\left|\frac{a_m}{a_T}\right| = \frac{m_T}{m_m}$

Ex:



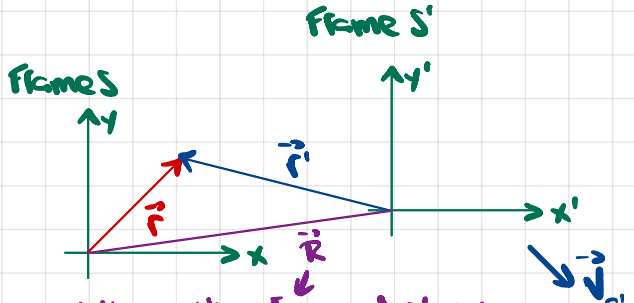



given coord. system can be considered a reference frame within which we can describe the kinematics of an object

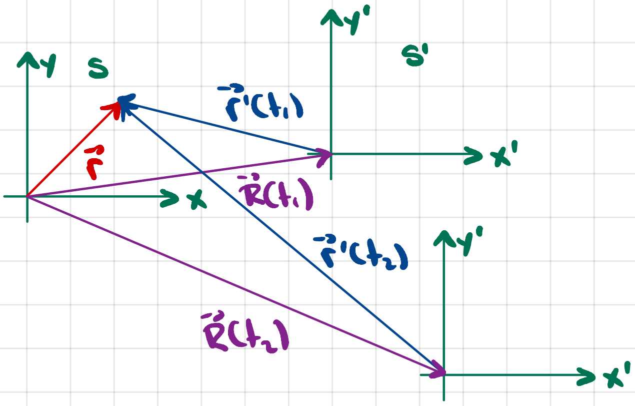
↳ $r(t), v(t), a(t)$

inertial reference frame: one in which Newton's laws apply, i.e.:

"an isolated body moves in a straight line at constant velocity and will continue to do so as long as it remains undisturbed"




relative position of origin of S' rel. to S




→ Two observers, one at each origin

$\vec{r} = \vec{R} + \vec{r}' \Rightarrow \vec{r}' = \vec{r} - \vec{R}$

Assume S' is moving at constant velocity w/ respect to frame S with velocity $\vec{v}$ = constant

→ $\vec{R}$ is a function of time $\Rightarrow \vec{R}(t) = \vec{R}_0 + \vec{v}_{s'} t$

→ $\vec{r}'$ is now a function of time, i.e. for the observer S', $\vec{r}'$ is changing.

$\vec{r}'(t) = \vec{r}(t) - \vec{R}(t)$

$$\frac{d\vec{r}'(t)}{dt} = \frac{d\vec{r}(t)}{dt} - \frac{d\vec{R}(t)}{dt} \Rightarrow \vec{v}'(t) = \vec{v} - \vec{v}_{s'}$$

different velocities will be measured in different frames

$$\frac{dv'(t)}{dt} = \frac{d\vec{v}(t)}{dt} - \cancelto{0}{\frac{d\vec{v}_{s'}}{dt}} \Rightarrow \vec{a}'(t) = \vec{a}(t)$$

$\vec{F} = m\vec{a}$

$\vec{F}' = m\vec{a}' = m\vec{a}$

$\Rightarrow$ Forces are identical

→ "If one of these reference frames is an inertial frame, then any other frame moving w/ constant velocity w/ respect to the first frame will also be an inertial frame."

→ "you're always free to transform from one inertial frame to another"

What if $\vec{V}_{s'}$ is not constant? ie, S' has acceleration $\vec{A}$ relative to S.

recall:

$$\vec{r}'(t) = \vec{r}(t) - \vec{R}(t) \quad \Rightarrow \frac{d}{dt}\vec{r}'(t) = \frac{d}{dt}\vec{r}(t) - \frac{d}{dt}\vec{R}(t) \quad \Rightarrow \vec{v}'(t) = v(t) - \vec{V}_{s'}(t)$$

$$\Rightarrow \frac{d}{dt}\vec{v}'(t) = \frac{d}{dt}\vec{v}(t) - \frac{d}{dt}\vec{V}_{s'}(t) \quad \Rightarrow \vec{a}'(t) = \vec{a}(t) - \vec{A}(t)$$

$\Rightarrow$ different accelerations measured in different frames

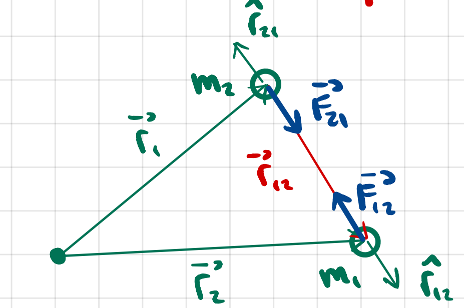
$$\Rightarrow \vec{F}' = m\vec{a}' = m\vec{a} - m\vec{A}$$

arises from choice of coord. system

$$= \vec{F}_{physical} + \vec{F}_{fictitious}, \quad F_{Fictitious} = -m\vec{A}$$

* in this course we will confine ourselves to inertial reference frames

# Lecture 5 - Gravity




$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

$$\hat{r}_{21} = -\hat{r}_{12}$$

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$

$$\vec{F}_{21} = \frac{-Gm_1m_2}{r_{21}^2}\hat{r}_{21} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$$

$$G = 6.57\cdot 10^{-11}\ \mathrm{N\,kg^{-2}\,m^2}$$

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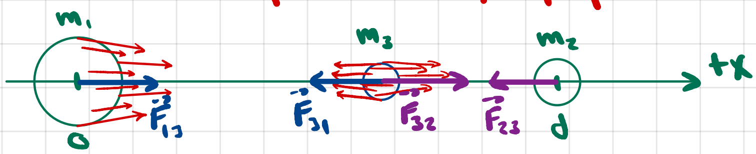
## 5.2 Worked Example - Gravity Superposition




* if we consider the masses as 2D (or 3D) objects, we would need to calculate the gravitational force at each infinitesimal piece of each mass.
Assuming constant density and symmetry about the x-axis, the components of gravitational force in this problem are 0 in y (and z) directions.

$$\vec{F}_{31} = \frac{-Gm_1m_3}{r_{31}^2}\cdot\hat{r}_{31} = \frac{-Gm_1m_3}{r_{31}^2}\hat{\imath} \qquad \hat{r}_{31} = \frac{\vec{r}_3-\vec{r}_1}{|\vec{r}_3-\vec{r}_1|} = 1\hat{\imath}$$

* $F_{31}$: force exerted on 3 by 1.
$\vec{r}_{32} = \vec{r}_3 - \vec{r}_2$

$$\vec{F}_{32} = \frac{-Gm_2m_3}{r_{32}^2}\hat{r}_{32} = \frac{Gm_2m_3}{r_{32}^2}\hat{\imath} \qquad \hat{r}_{32} = \frac{\vec{r}_3-\vec{r}_2}{|\vec{r}_3-\vec{r}_2|} = -1\hat{\imath}$$

$$\vec{F}_{31}+\vec{F}_{32} = 0 \Rightarrow \frac{\cancel{G}m_1\cancel{m_3}}{r_{31}^2} = \frac{\cancel{G}m_2\cancel{m_3}}{r_{32}^2} \Rightarrow m_1r_{32}^2 = m_2r_{31}^2 \Rightarrow \frac{m_1}{r_{31}^2} = \frac{m_2}{r_{32}^2}$$

The larger the mass, the larger the distance to mass 3.

* $r_{21} = d$ = distance from $m_1$ to $m_2$

$$\Rightarrow r_{31}+r_{23} = d,\quad x = r_{31} \Rightarrow m_1(d-x)^2 = m_2x^2$$

masses are given, and $r_{31}$ and $r_{32}$ are dependent on each other. ⇒ there are specific values of x that solve $\vec{F}_{31}+\vec{F}_{32}=0$.

$$\Rightarrow m_1d^2 - 2m_1dx + m_1x^2 = m_2x^2 \Rightarrow x^2(m_2-m_1) + 2m_1dx - m_1d^2 = 0$$

$$\Delta = 4m_1^2d^2 - 4(m_2-m_1)(-m_1d^2) \Rightarrow \cancel{4m_1^2d^2} + 4m_2m_1d^2 - \cancel{4m_1^2d^2} = 4m_2m_1d^2$$

$$x = \frac{-2m_1d \pm (4m_2m_1d^2)}{2(m_2-m_1)} = \frac{-2m_1d \pm 2d\sqrt{m_1m_2}}{2(m_2-m_1)} = \frac{d(m_1 \pm \sqrt{m_1m_2})}{m_1-m_2}$$

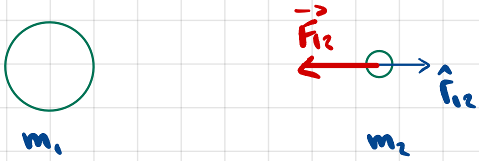
**Note**: we obtained $m_1r_{32}^2 = m_2r_{31}^2$ with the assumption that $\hat{r}_{32} = -\hat{\imath}$ and $\hat{r}_{31} = \hat{\imath}$. If $\hat{r}_{32} = \hat{\imath}$, for example, or $\hat{r}_{31} = -\hat{\imath}$, we would reach $m_1r_{32}^2 = -m_2r_{31}^2$ which has no solution.

$$\hat{r}_{32} = \frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|} = -1\hat{\imath} \Rightarrow x - d = -|x-d| \Rightarrow x - d < 0 \Rightarrow x < d$$

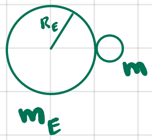
$$\Rightarrow 0 < x < d$$

$$\hat{r}_{31} = \frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|} = 1\hat{\imath} \Rightarrow x - 0 = |x| \Rightarrow x > 0$$

## 5.3 - Gravity on the surface of the earth



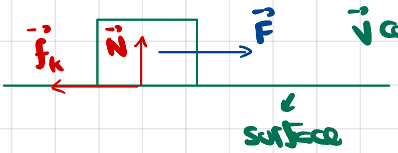

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$




$$mg = \frac{Gm_Em}{R_E^2} \Rightarrow g = \frac{Gm_E}{R_E^2}$$

$$g = \frac{6.67 \cdot 10^{-11}\ N \cdot kg^{-2} m^2 \cdot 5.97 \cdot 10^{24}\ kg}{(6.37 \cdot 10^{6})^2\ m^2} = 9.81\ m/s^2$$

## 6.1 Contact Forces




$\vec{v}$ constant

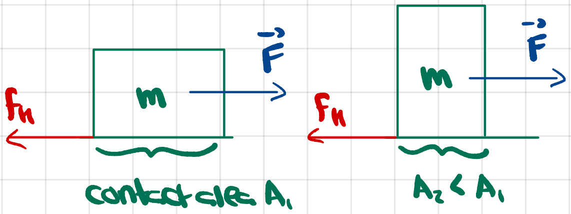
$\vec{F}$ an external force

$\vec{C} = \vec{N} + \vec{f}_k$ ↳ kinetic friction

Empirically, we have following properties of $\vec{f}_k$:

(1) $|\vec{f}_k| = \mu_k |\vec{N}|$

(2) independent of contact area



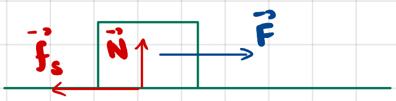

$\vec{v}$ constant, both masses

$\vec{F}$ necessary for constant velocity $\vec{v}$ is same for both masses $\Rightarrow$ kinetic friction is the same

$\Rightarrow \vec{f}_k$ indep. of contact area

(3) independent of the velocity

## 6.2 Static Friction



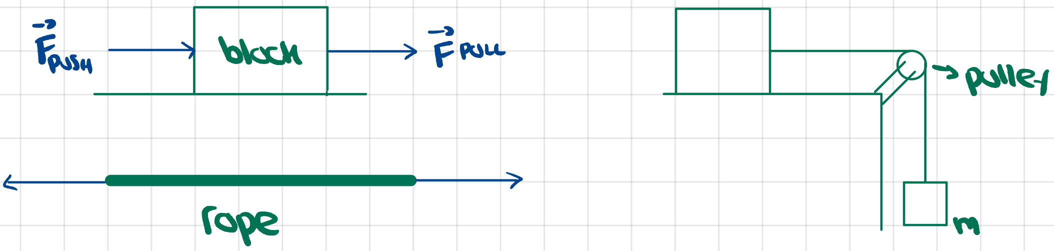

$\vec{C} = \vec{N} + \vec{f}^s$ ↳ static friction

$0 \le |\vec{f}^s| \le |\vec{f}^s_{max}|$

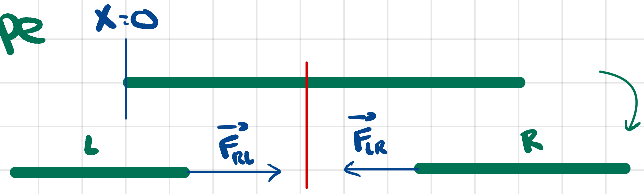
$|\vec{f}^s_{max}| = \mu_s |\vec{N}| \qquad \mu_s > \mu_k$

## 7.1 Pushing, Pulling, and Tension

Examples




Rope




take an imaginary slice of the rope

3rd Law: $\vec{F}_{RL} = -\vec{F}_{LR}$

Tension Force: $T \equiv |\vec{F}_{RL}| = |\vec{F}_{LR}|$, at each point on the rope

$T(x) = |\vec{F}_{LR}(x)|$

## 7.2 Ideal Rope

Under what conditions can we say the tension is uniform?

case 1: $\vec{a} = 0$

$\vec{T}_A = \vec{F}_A \longleftarrow$ ▬▬▬▬ $\longrightarrow \vec{F}_B = \vec{T}_B$

$\vec{F} = m\vec{a}$

$T_B - T_A = m_r \cdot 0 \quad \Rightarrow T_B = T_A$

case 2: $\vec{a} \neq 0$

$T_B - T_A = m_r a \Rightarrow T_B = T_A + m_r a$

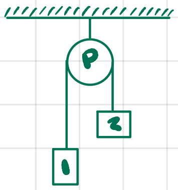
special case: $a \neq 0$, $m_r \simeq 0$

$T_B \simeq T_A$ very light rope

$\Rightarrow \vec{T}$ is uniform in the rope

* we will often refer to this case in problems involving pulling on ropes with pulleys, etc.

# 7.3 Solving Pulley Systems

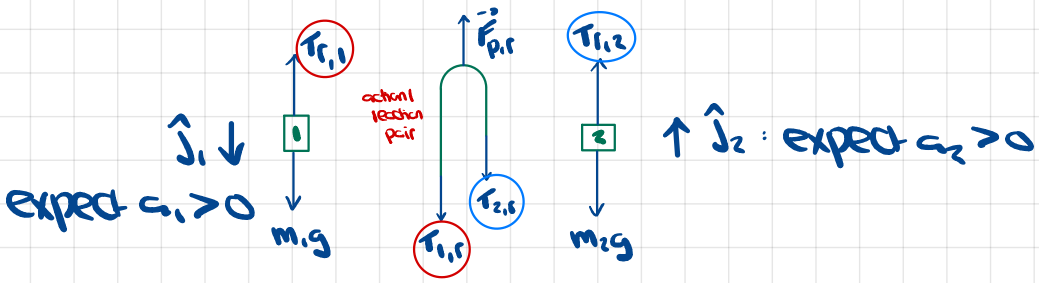



**Assumption**: rope/pulley is frictionless ⇒ pulley will remain at rest

(1) identify all the moving objects

(2) draw free body force diagrams for each moving object

* identify action-reaction pairs




**Assumption**: $m_r \simeq 0$ ⇒ tension approx uniformly distributed on the rope ⇒ $T_{2,r} = T_{1,r}$

→ need to choose unit vectors

assume momentarily that $m_1 > m_2$ : we expect $m_1$ to move vertically downwards

this is how we choose the unit vector $\hat{j}_1$ : $\downarrow$

(3) choose separate unit vectors (coord. system for each object

→ our accelerations for each object will be relative to each coord. system

## Apply Newton's 2nd Law

→ on 1: $\vec{F}_1 = m\vec{a}_1 \Rightarrow m_1 g - T = m_1 a_1$

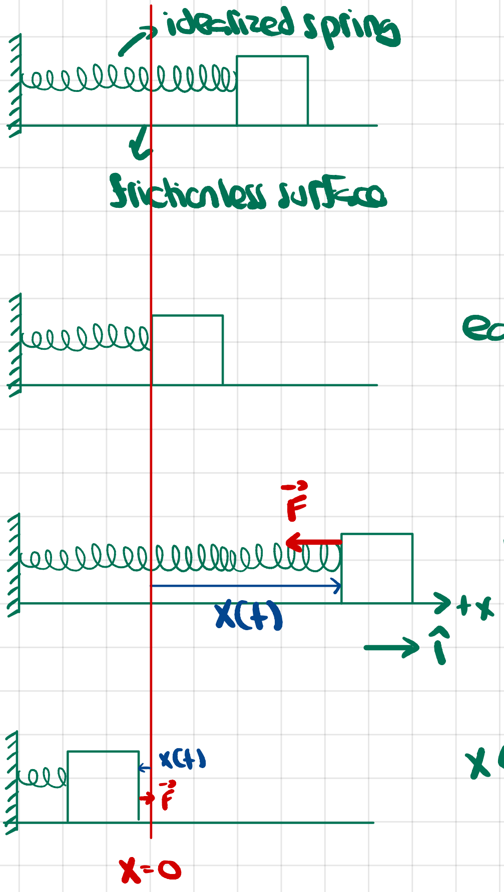
→ on 2: $\vec{F}_2 = m\vec{a}_2 \Rightarrow T - m_2 g = m_2 a_2 \Rightarrow T = m_2(a+g)$

* There are three unknowns: $T, a_1, a_2$. However, we have a constraint: $a_1 = a_2$. This follows because both masses move together, and because of the chosen coord. systems.

sub T from 2nd eq into 1st eq: $m_1 g - m_2 a - m_2 g = m_1 a$

$$\Rightarrow a = \frac{g(m_1 - m_2)}{m_1 + m_2}$$

# 7.4 Hooke's Law




equilibrium

dynamic picture time $t$, $x>0$, extension

$x<0$, compression

$\vec{F} = -kx\hat{i}$ example of a linear restoring force, direction of force towards equilibrium

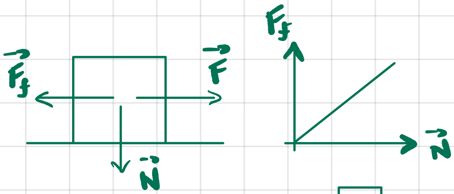
$k \equiv$ spring constant, SI units: [N/m]

# DD.1.1 Friction at the nanoscale

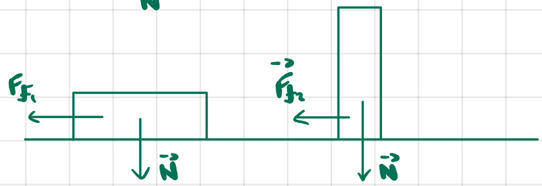
## macroscopic objects

Da Vinci ($15^{th}$ century), Amontons ($17^{th}$ century)

1)




2)




Friction independent of area

3) Coulomb: Dry Friction: Friction independent of velocity

→ static friction ≠ dynamic friction

we cannot predict this, only measure it

→ $F_f = \mu N$ (from (1)), typically $0.3 \leq \mu \leq 0.6$, $\mu > 1$ possible (e.g. rubber)

→ estimated that ~3% of GDP "wasted" on friction

## How bad is friction at the nanoscale?

→ Tire of a car


1 atomic layer ~ $1Å = 10^{-10}$ m

wear: ~10mm every 50000 km

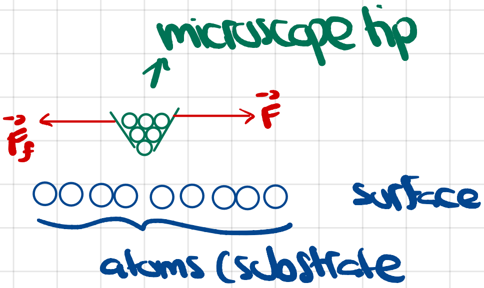
$10mm = 10^{-2} m = 10^{8}$ atomic layers

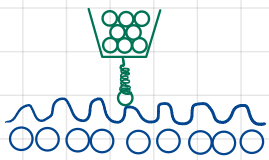
$2\pi R \sim 1m$ ⇒ $5 \cdot 10^{7} m = 50000 km = 5 \cdot 10^{7}$ revolutions

⇒ 1-2 atomic layers lost per revolution

## Experiments at Nanoscale

→ Atomic Force Microscope

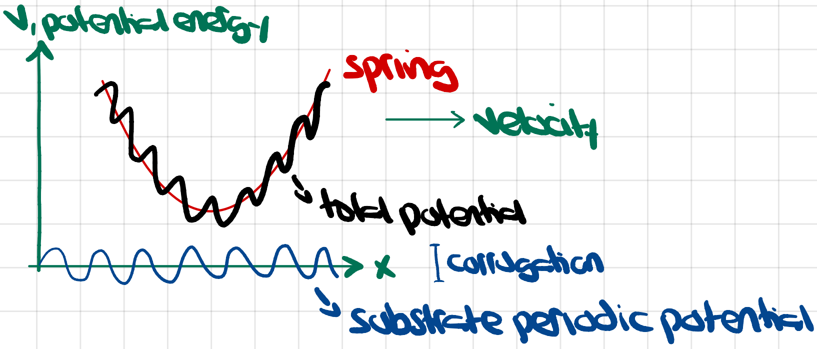


periodic arrangement of atoms

⇒ assume associated potential is also periodic at atomic scale

→ model friction as spring plus periodic potential (Prandtl, Tomlinson model, ~1928, 1929) energy as function of potential



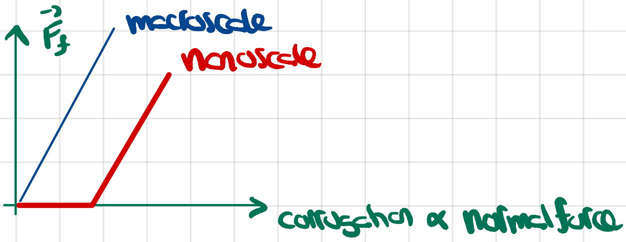

How to model the spring?

→ spring has linear force, latter proportional to displacement ⇒ potential quadratic in displacement

As we translate the spring across the surface with velocity $\vec{v}$ ⇒ time-varying potential for the object

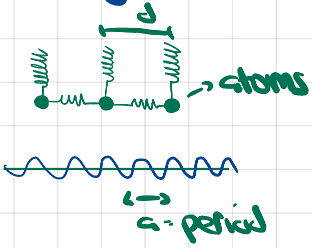
* substrate fixed

In this model we can understand friction as the external force pulling the object over successive maxima.




→ we've considered a contact of a single atom

→ with several atoms:




→ simple case: $d = a$ or $d = na$, $n$ integer (commensurate case)

golden ratio, the "most irrational" number in some specific mathem. sense

→ incommensurate: $\frac{d}{a}$ = irrational number, e.g. $\sqrt{2}$, $\frac{1}{2}(\sqrt{5}+1)$

→ friction is dramatically reduced

"superlubricity"

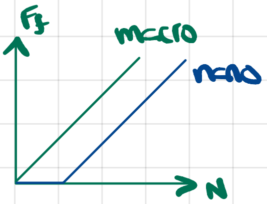
So far we've considered only the mechanics of friction: Temperature = T = 0

What happens for T > 0?

thermolubricity: when you heat up the surface, friction is reduced because atom can find the new minimum of potential without having to be pulled over the barrier.

Recap / Summary

Laws of Friction

1) 


2) Friction indep. of contact area

nano: depends on commensurability

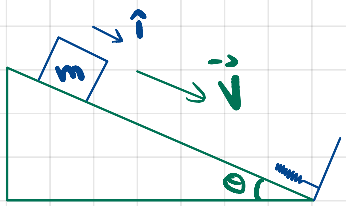
3) Friction indep. of velocity

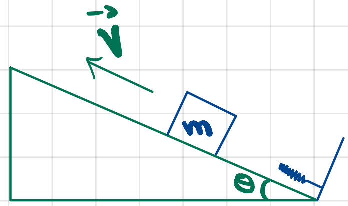
nano: only true in some finite temperature range

many open questions remain

→ in particular, point 2: what does the contact area really look like between two macro objects

# PS.2.1 Sliding Block

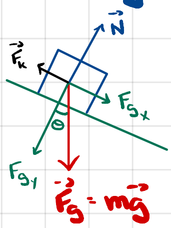



## Problem data

coefficient of kinetic friction = $\mu_k$

## a) sliding down the incline




$$\vec{F}_{g_x} = |\vec{F}_g| \cdot \sin\theta = mg\sin\theta\,\hat{\imath}$$

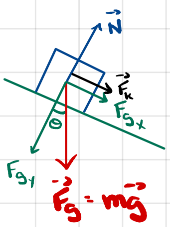
$$\vec{N} + \vec{F}_{g_y} = 0 \Rightarrow \vec{N} = -F_{g_y} = |\vec{F}_g|\cos\theta\,\hat{\imath} = mg\cos\theta\,\hat{\imath}$$

$$\vec{F}_k = -\mu_k\vec{N} = -\mu_k mg\cos\theta\,\hat{\imath}$$

$$\vec{F}_{g_x} + \vec{F}_k = m\vec{a} \Rightarrow \left[\cancel{m}g\sin\theta + (-\mu_k\cancel{m}g\cos\theta)\right]\hat{\imath} = \cancel{m}a\,\hat{\imath}$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

## b) sliding up the incline




$$\vec{F}_{g_x} = mg\sin\theta\,\hat{\imath}$$

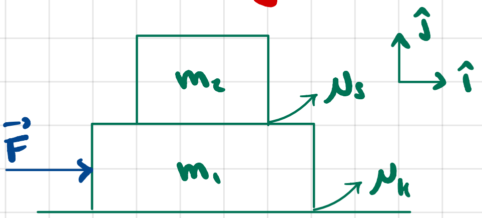
$$\vec{N} = mg\cos\theta\,\hat{\imath}$$

$$\vec{F}_k = \mu_k mg\cos\theta$$

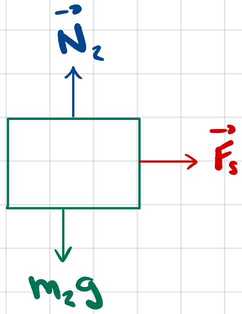
$$\vec{F}_{g_x} + \vec{F}_k = m\cdot\vec{a} \Rightarrow \cancel{m}g\sin\theta + \mu_k\cancel{m}g\cos\theta = \cancel{m}a$$

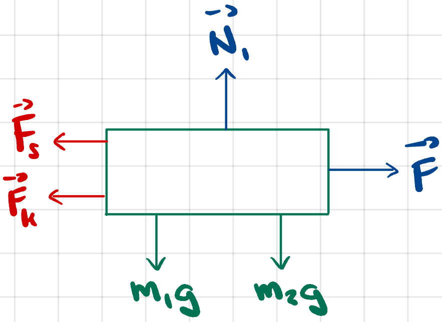
$$\Rightarrow a = g(\sin\theta + \mu_k\cos\theta)$$

# PS.2.2 Pushing Stacked Blocks




$m_2 < m_1$

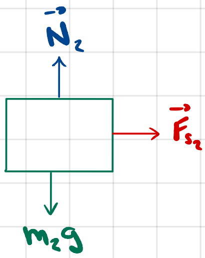


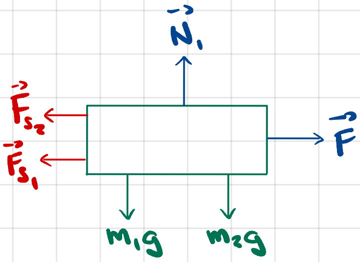




## Description of the motion:

→ initially, blocks are static, $\vec F = \vec 0$

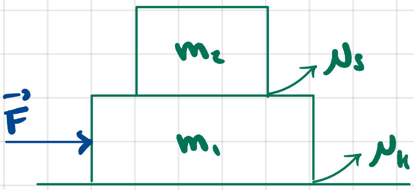
→ $\vec F$ is impressed upon $m_1$. Contact forces come into play



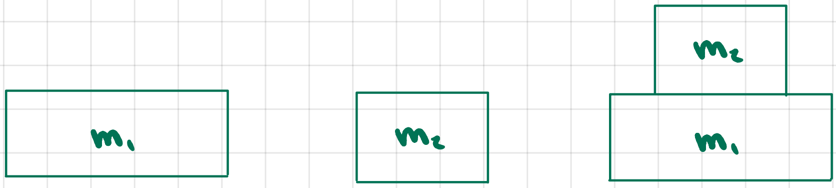




→ $m_1$ and $m_2$ will initially not move, due to the contact forces. When $|\vec F| > |\vec F_{s_1}| + |\vec F_{s_2}|$ $m_1$ and $m_2$ move together. Lets assume $F_k = F_{s_1}$. After some small acceleration to obtain velocity $\neq 0$, keeping $\vec F = \vec F_{s_2} + \vec F_k$ will keep velocity constant.

# PS.2.2 Pushing Stacked Blocks

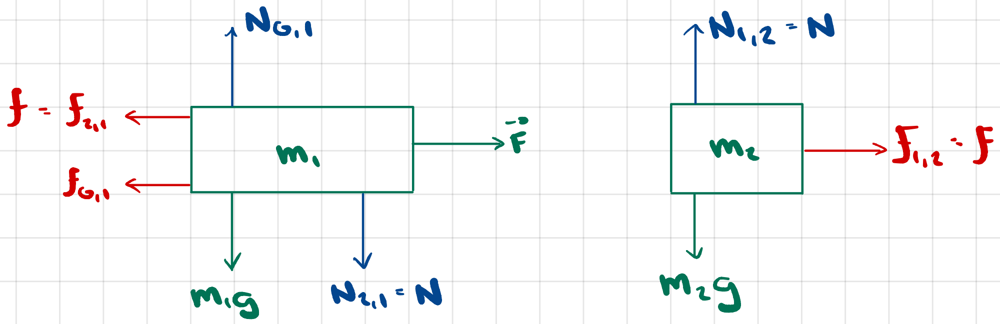



(1) what is the system?

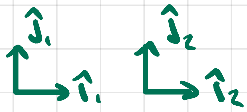



(a) (b) (c)

(2) Free-body force diagrams




(3) choose unit vectors (coord. systems) for each object




(4) apply 2nd Law

| | | $\vec{F} = m\vec{a}$ |
|---|---|---|
| block 1 | $\hat{i}$ | $F - f - f_k = m_1 a_1$ |
| | $\hat{j}$ | $N_{G,1} - N - m_1 g = 0 \Rightarrow N_{G,1} = g(m_1 + m_2)$ |
| block 2 | $\hat{i}$ | $f = m_2 a_2$ |
| | $\hat{j}$ | $N - m_2 g = 0 \Rightarrow N = m_2 g$ |

constraints

no slipping $\Leftrightarrow$ $a_1 = a_2 = a$

$f_{s_{max}} = \mu_s N$

$f_k = \mu_k N_{G,1}$

$\Rightarrow F - \mu_s N - \mu_k N_{G,1} = m_1 a$

$f = m_2 a$

$\Rightarrow a = \dfrac{f}{m_2} = \dfrac{\mu_s N}{m_2} = \mu_s g$

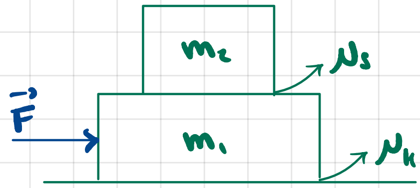
$\Rightarrow F - \mu_s m_2 g - \mu_k g(m_1 + m_2) = m_1 a$

$F = g(\mu_s m_2 + \mu_k (m_1 + m_2) + m_1 \mu_s)$

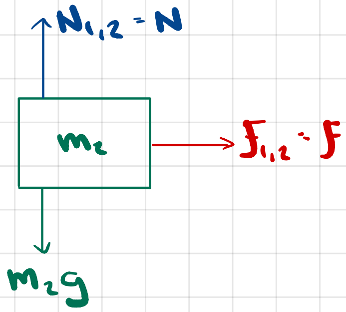
$= g(m_1 + m_2)(\mu_s + \mu_k)$

# Note

→ Assume $\mu_k = 0$ ie no ground friction. Lets go through the sequence of events when we apply a force $\vec{F}$ to $m_1$.



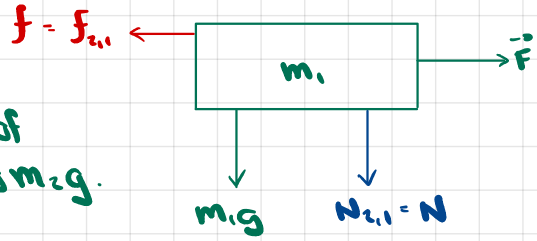

1. The system $m_1 + m_2$ accelerates no matter the magnitude of $\vec{F}$.




$$F = (m_1+m_2)a \Rightarrow a = \frac{F}{m_1+m_2}$$

$$f = m_2 \cdot \frac{F}{m_1+m_2} \Rightarrow \frac{f}{F} = \frac{m_2}{m_1+m_2}$$

So F accelerates $m_1+m_2$ and F is divided between the two blocks in proportion to their masses. $m_2$ feels the force through friction. $m_1$ feels an offset to F in the form of friction.




We know the max acceleration that $m_2$ can have (through the action of friction alone) because we know the max magnitude of friction, $\mu_s m_2 g$.

$$f_{max} = \mu_s m_2 g = m_2 a_{2_{max}} \Rightarrow a_{2_{max}} = \mu_s g$$

F starts acceleration at $\frac{F}{m_1+m_2}$ and we can increase F until $\frac{F}{m_1+m_2} = \mu_s g \Rightarrow F_{max} = \mu_s g(m_1+m_2)$

As we are increasing F, $f$ is also increasing from $m_2 \frac{F}{m_1+m_2}$ to $a_{2_{max}}$.

$f$ reaches its max value when $F = F_{max}$.

At this point, additional increases in F accelerate $m_1$ but not $m_2$. We can't apply 2nd law to the $m_1 + m_2$ system anymore.

$$\begin{cases} F - f_{max} = m_1 a \\ f_{max} = m_2 \cdot a_{2_{max}} \end{cases}$$ are our equations now.

Suppose $a_1 = 2a_{2_{max}}$. $F - f_{max} = m_1 2a_{2_{max}} = m_1 \cdot 2\frac{f_{max}}{m_2} \Rightarrow F = (1 + 2\frac{m_1}{m_2})f_{max}$

Note that if $m_1 = m_2$, F required for $2a_{2_{max}}$ is simply $3f_{max}$.

Suppose F now goes from $F = (1+2\frac{m_1}{m_2})f_{max}$ to $F_{max}$, ie $m_1$'s acceleration drops to the same acceleration $a_{2_{max}}$ as that of $m_2$. What happens? Because $m_1$ was previously accelerating more, it has a higher velocity than $m_2$. Because both now have same accel., relative velocity stays constant. (Note we are assuming friction exists throughout these events, ie one of the blocks is very long).

$$\begin{cases} F - f_{max} = m_1 a_{2max} \\ f_{max} = m_2 \cdot a_{2max} \end{cases}$$ we still have these same equations.

$$v_1 = v_{0_1} + a_{2max} t$$
$$\Rightarrow v_1 - v_2 = v_{0_1} - v_{0_2},$$ where $v_{0_1}$ and $v_{0_2}$ are the velocities
$$v_2 = v_{0_2} + a_{2max} t$$

when we drop $F$ to $F_{max}$ again.

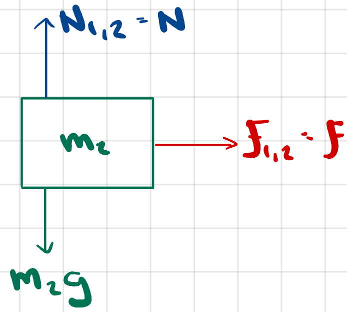
Note that $m_2$ is sliding now even though accelerations are the same. It is different velocities that create slippage. $m_1$ is still causing a friction force on $m_2$ that accelerates the latter. $F - f_k$ is just enough to accelerate $m_1$ to $a_{2max}$.

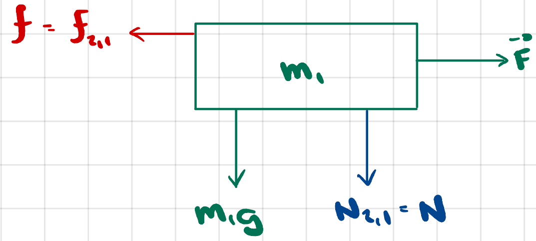
Now decrease magnitude of $F$ to below $F_{max}$.

Note that when we started off and pushed $m_1$ with $F$, we were trying to change the relative velocity between $m_1$ and $m_2$. Any attempt, i.e. any force that tries to do that, causes a frictional force to come into play. Once there is a rel. velocity $\neq 0$, then friction will exist at $f_{max}$.

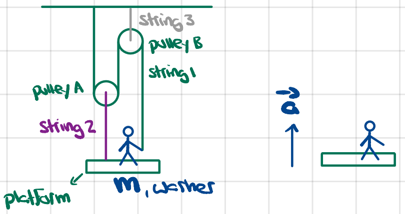
Therefore, because at this point in our sequence of events $m_1$ and $m_2$ have rel. vel. $\neq 0$, there is friction $f_{max}$. It accelerates $m_2$ at $a_{2max}$. Note however that $F < F_{max}$ means $m_1$ accelerates at $a < a_{2max}$.

$$v_1 - v_2 = \underbrace{(v_{0_1} - v_{0_2})}_{>0} + \underbrace{(a_1 - a_{2max})}_{<0} t \quad \Rightarrow$$



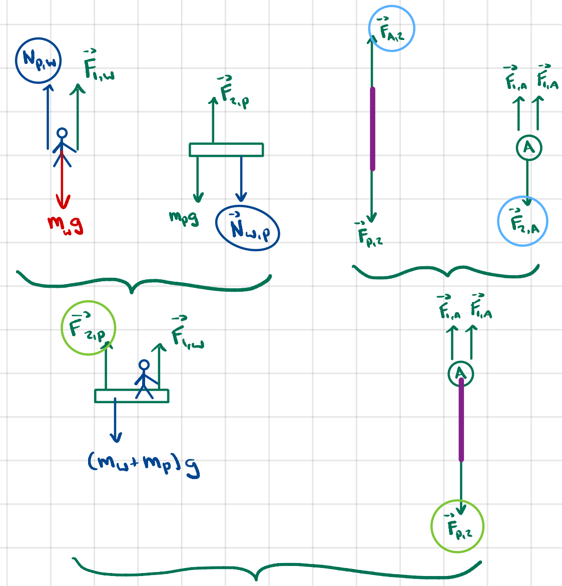



# P.S. 2.3 Window Washer Problem

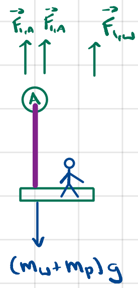



(1) What is the system?, (2) Free body force diagrams

First let's consider each individual object.




$|\vec{F}_{1,A}| = |\vec{F}_{w,1}| = T_1$




$$\frac{2m_w g - m_w a - m_p(g+a)}{3}$$

(3) unit vectors (coord. system) for each object

we are expecting acceleration upwards.

Let's use $\uparrow \hat{j}$ for every object.

(4) apply second law

$\vec{F}_{1,A} = \vec{F}_{1,w} = T_1$

$$\Rightarrow 3T_1 - (m_w+m_p)g = (m_w+m_p)a$$

$$\Rightarrow T_1 = \frac{(m_w+m_p)(g+a)}{3}$$

For the person to hold the system suspended in the air, $a=0$, so $T_1 = \dfrac{(m_w+m_p)g}{3}$

a) True statements

Window washer accelerates upwards
$\Rightarrow$ Resultant force is upwards.

He/she pulls rope 1 down. Rope 1 pulls him/her up. The forces are pairs in the sense of the 3rd law, so:

$$|\vec{F}_{w,1}| = |\vec{F}_{1,w}|$$

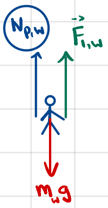
$\vec{F}_{1,w}$ has magnitude $T_1$, ie the tension on rope 1. With $a=0$, we have

$$3T_1 - (m_w+m_p)g = 0$$

$$\Rightarrow T_1 = \frac{g(m_w+m_p)}{3}$$

That tension is the same at any point on rope 1 is due to there not being any acceleration at any point on the rope.

b)




These are all external forces. In isolation we don't have enough information to determine their magnitudes in terms of known parameters.

c) washer, platform, rope 1, pulley A

d) $T_1 = \dfrac{g(m_w+m_p)}{3}$

e) There are multiple ways to find $F_{2,p}$ by applying 2nd law on individual objects.

3rd law: $F_{2,p} = F_{p,2}$

rope 2: $\vec{F}_{A,2} - \vec{F}_{p,2} = 0 \Rightarrow \vec{F}_{p,2} = \vec{F}_{A,2}$

pulley A: $F_{A,2} = 2F_{1,A} = 2T_1 = \frac{2}{3}(m_p+m_w)(g+a)$

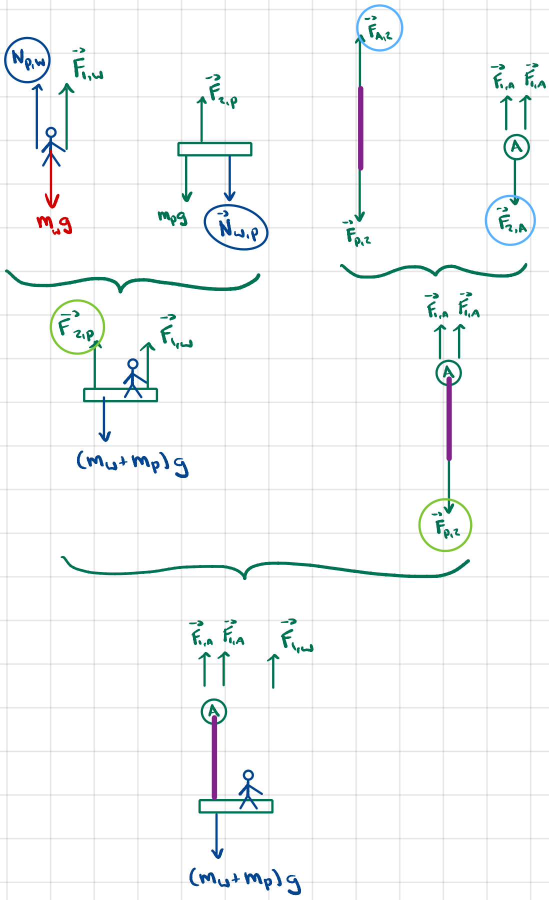
Altern., 2nd law for platform

$$F_{2,p} - m_p g - N_{w,p} = m_p a$$

$N_{w,p} = [2m_w g - m_w a - m_p(g+a)]/3$

$\Rightarrow F_{2,p} = (3m_p(a+g) + 2m_w g - m_w a - m_p(g+a))/3$

$= (2m_p(a+g) + m_w(2g-a))/3$




$|\vec{F}_{1,A}| = |\vec{F}_{w,1}| = T_1$

$$T_1 = \frac{(m_w+m_p)(g+a)}{3}$$

we can now determine what the internal forces are

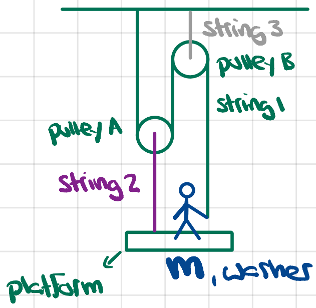
→ washer

$F_{1,w} + N_{p,w} - m_w g = m_w a$ , $F_{1,w} = T_1$

$\Rightarrow N_{p,w} = m_w(a+g) - T_1$

$= [3m_w(g+a) - m_w(g+a) - m_p(g+a)]/3$

$= [2m_w(g+a) - m_p(g+a)]/3$




## e) Find magnitude of $F_{2,p}$

There are multiple ways to find $F_{2,p}$ by applying 2nd law on individual objects.

$F_{2,p} = F_{p,2}$ (3rd law)

rope 2: $F_{A,2} - F_{p,2} = \cancel{m}^{0} \cdot a \Rightarrow F_{A,2} = F_{p,2} = F_{2,p}$

pulley A: $2T_1 - F_{2,A} = \cancel{m}^{0} a \Rightarrow 2T_1 = F_{2,A} = \frac{2}{3}(m_p+m_w)(g+a)$

Alternatively, from 2nd law on platform

$$F_{2,p} - m_p g - N_{w,p} = m_p a$$

$$F_{2,p} = \frac{3m_p(a+g) + 2m_w(g+a) - m_p(g+a)}{3}$$

$$= \frac{2}{3}(m_p+m_w)(g+a) = \frac{2}{3}T_1$$