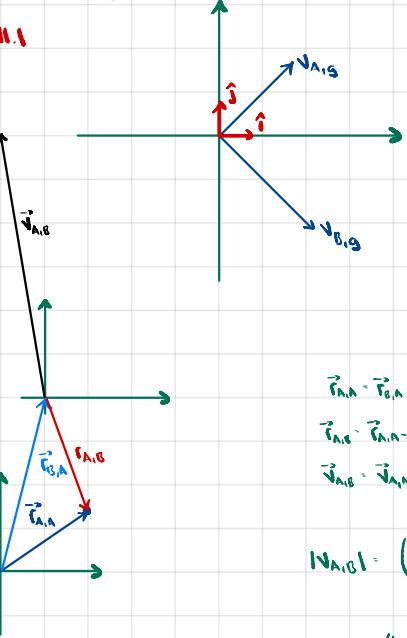


Ex 1.1

ground reference frame



$$|v_{A,g}| = 160 = (\sqrt{A_x^2 + \sqrt{A_y^2}})^{1/2} = (2\sqrt{A})^{1/2} = \sqrt{2} \sqrt{A} \Rightarrow \sqrt{A} = \frac{160}{\sqrt{2}}$$

$$|v_{B,g}| = 200 = \sqrt{B} = \frac{200}{\sqrt{2}}$$

$$v_{A,A} = \frac{160}{\sqrt{2}} (\hat{i} + \hat{j})$$

$$v_{B,A} = \frac{200}{\sqrt{2}} (\hat{i} - \hat{j})$$

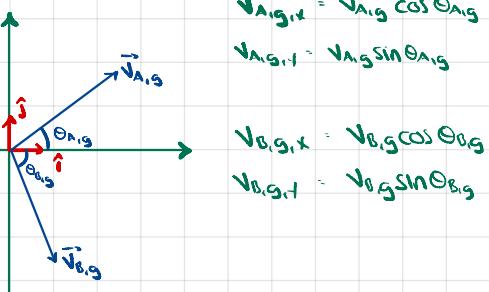
$$\vec{r}_{AA} = \vec{r}_{OA} + \vec{r}_{AB}$$

$$\vec{r}_{AB} = \vec{r}_{OA} - \vec{r}_{OB}$$

$$\vec{r}_{AB} = \vec{r}_{OA} - \vec{r}_{OB} \Rightarrow \vec{v}_{A,B} = \frac{160}{\sqrt{2}} (\hat{i} + \hat{j}) - \frac{200}{\sqrt{2}} (\hat{i} - \hat{j})$$

$$= -\frac{40}{\sqrt{2}} \hat{i} + \frac{360}{\sqrt{2}} \hat{j}$$

$$|v_{A,B}| = \left( \frac{1600 + 129600}{2} \right)^{1/2} = 40\sqrt{41} \approx 256.12$$



$$v_{A,g,x} = v_{A,g} \cos \theta_{A,g}$$

$$v_{A,g,y} = v_{A,g} \sin \theta_{A,g}$$

$$v_{B,g,x} = v_{B,g} \cos \theta_{B,g}$$

$$v_{B,g,y} = v_{B,g} \sin \theta_{B,g}$$

$$\vec{v}_{A,B} = (v_{A,g} \cos \theta_{A,g} - v_{B,g} \cos \theta_{B,g}) \hat{i} + (v_{A,g} \sin \theta_{A,g} - v_{B,g} \sin \theta_{B,g}) \hat{j}$$

$$\theta_{A,g} = \frac{\pi}{4} \Rightarrow \vec{v}_{A,g} = (160 \cdot \frac{\sqrt{2}}{2} - 200 \cdot \frac{\sqrt{2}}{2}) \hat{i} + (160 \cdot \frac{\sqrt{2}}{2} + 200 \cdot \frac{\sqrt{2}}{2}) \hat{j}$$

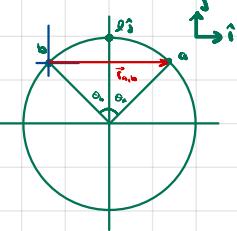
$$\theta_{B,g} = -\frac{\pi}{4}$$

$$= -\frac{40}{\sqrt{2}} \hat{i} + \frac{360}{\sqrt{2}} \hat{j}$$

Note: Let  $\alpha$  be the angle of  $\vec{v}_{A,B}$ , i.e. the angle of A's velocity vector  $v_A$  from B's reference frame.

$$\tan \alpha = \frac{360/\sqrt{2}}{-40/\sqrt{2}} \Rightarrow \alpha = \tan^{-1}(-9) = (-146 + \pi) \text{ rad} = 96.34^\circ$$

## Ex 11.2 - Relative Motion and Polar Coord.



a)  $\vec{r}_{A,A} = \vec{r}_{B,A} + \vec{r}_{A,B} = \vec{R} + \vec{r}_{A,B}$

$$\vec{v}_{A,A} = \vec{v}_R + \vec{v}_{A,B}$$

b)  $\vec{r}_{a,b} = \vec{r}_a - \vec{r}_b$

$$\vec{v}_{a,b} = \vec{v}_a - \vec{v}_b$$

$$\vec{r}_b = l \cdot \hat{r}_b$$

$$\vec{r}_a = l \cdot \hat{r}_a$$

$$\hat{r}_b = -\sin\theta_b \hat{i} + \cos\theta_b \hat{j}$$

$$\hat{r}_a = \sin\theta_a \hat{i} + \cos\theta_a \hat{j}$$

$$\vec{v}_b = l \frac{d\hat{r}_b}{dt} = l(-\cos\theta_b \theta'_b \hat{i} - \sin\theta_b \theta'_b \hat{j}) \\ = -l\theta'_b (\cos\theta_b \hat{i} + \sin\theta_b \hat{j})$$

$$\vec{v}_a = l \frac{d\hat{r}_a}{dt} = l(\cos\theta_a \theta'_a \hat{i} - \sin\theta_a \theta'_a \hat{j}) \\ = l\theta'_a (\cos\theta_a \hat{i} - \sin\theta_a \hat{j})$$

$$\vec{v}_a - \vec{v}_b = l\theta'_a (\cos\theta_a \hat{i} - \sin\theta_a \hat{j}) + l\theta'_b (\cos\theta_b \hat{i} + \sin\theta_b \hat{j})$$

$$= \hat{i}l\theta'_a (\cos\theta_a + \cos\theta_b) + \hat{j}\theta'_b (\sin\theta_b - \sin\theta_a)$$

$$= 2l\omega \cos\theta_b \hat{i}$$

## Ex 11.3 Recoil in Different Frames

a)  $\vec{v}_{3,4} = \vec{v}_{c,4} + \vec{v}_{3,c}$  law of addition of velocities

In the x direction no external forces act: we have  $\Delta p = 0$ .

In the y direction, a normal force prevents the cart from changing momentum.

$$\vec{P}_{3,4,4,0} = \vec{0}$$

$$\vec{P}_{3,4,4,0} = m_3 \vec{v}_{3,4,4,0} + (m_1 + m_2) \vec{v}_{c,4,4,0}$$

$$\begin{aligned} \vec{v}_{3,4,4,0} &= \vec{v}_{c,4,4,0} + \vec{v}_{3,c,4,0} \\ &= \vec{v}_{c,4,4,0} + \vec{v}_0 \cos\theta \end{aligned}$$

$$\Rightarrow P_{3,4,4,0,x} = m_3 v_{3,4,4,0,x} + (m_1 + m_2) v_{c,4,4,0,x}$$

$$= m_3 v_{3,4,4,0,x} + (m_1 + m_2)(v_{3,4,4,0,x} - v_0 \cos\theta)$$

$$\Delta p_x = 0 \Rightarrow v_{3,4,4,0,x} (m_1 + m_2 + m_3) = (m_1 + m_2) v_0 \cos\theta$$

$$\Rightarrow v_{3,4,4,0,x} = \frac{(m_1 + m_2) v_0 \cos\theta}{m_1 + m_2 + m_3}$$

$$v_{c,4,4,0} = v_{3,4,4,0,x} - v_0 \cos\theta$$

$$\Rightarrow v_{c,4,4,0} = -\frac{m_3 v_0 \cos\theta}{m_1 + m_2 + m_3}$$

$$\Rightarrow P_{3,4,4,0,y} = 0$$

In the y direction,

$$P_{3,4,4,0,y} = m_3 v_{3,4,4,0,y} + (m_1 + m_2) \cdot 0$$

But a normal force acts on the cart.

$$\Delta P_{3,4,4,0,y} = m_3 v_{3,4,4,0,y}$$

$$\begin{aligned} v_{3,4,4,0,y} &= v_{c,4,4,0} + v_{3,c,4,0} \\ &= 0 + v_0 \sin\theta \end{aligned}$$

$$\Rightarrow v_{3,4,4,0,y} = v_0 \sin\theta$$

Note that momentum in the y direction increased

$$\Delta P_{3,4,4,0,y} = m_3 v_0 \sin\theta$$

Because we know the velocity vector from ground reference, we know the angle on observes in this reference sees.

$$\vec{v}_{3,4,4} = \frac{(m_1 + m_2) v_0 \cos\theta}{m_1 + m_2 + m_3} \hat{i} + v_0 \sin\theta \hat{j}$$

$$\tan\alpha = \frac{(m_1 + m_2 + m_3) v_0 \sin\theta}{(m_1 + m_2) v_0 \cos\theta}$$

$$\alpha = \tan^{-1} \left[ \frac{(m_1 + m_2 + m_3) v_0 \sin\theta}{(m_1 + m_2) v_0 \cos\theta} \right]$$

