

14.6 Spring Force Energy Diagram

Ex 14.1 pot. en. for a particle of mass m

$$U(x) = -U_1 \left[\left(\frac{x}{x_1} \right)^3 - \left(\frac{x}{x_1} \right)^2 \right]$$

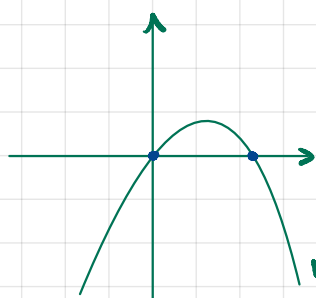
a) $H(x|x_1) = \frac{U(x)}{U_1} = \left(\frac{x}{x_1} \right)^2 - \left(\frac{x}{x_1} \right)^3$

$$H(y) = y^2 - y^3$$

$$H'(y) = 2y - 3y^2 = 0 \Rightarrow 3y^2 - 2y = 0 \Rightarrow y(3y - 2) = 0 \Rightarrow y = 0 \text{ or } y = \frac{2}{3}$$

$$F(x) = -U'(x) = -U_1 \left[3 \cdot \frac{1}{x_1} \left(\frac{x}{x_1} \right)^2 - \frac{2x}{x_1^2} \right] = 0$$

$$\Rightarrow \frac{3x^2}{x_1^3} - \frac{2x}{x_1^2} = \frac{3x^2 - 2xx_1}{x_1^3} = 0 \Rightarrow x(3x - 2x_1) = 0 \Rightarrow x = 0 \text{ or } x = \frac{2}{3}x_1$$

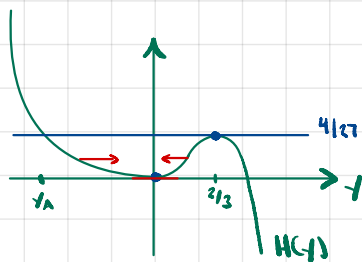


b) The force on the particle is zero when $y = \frac{x}{x_1} = 0$ or $y = \frac{x}{x_1} = \frac{2}{3}$.
 $y = 0$ is stable, $y = \frac{2}{3}$ is unstable.

$$H(0) = 0 \quad H(2/3) = \frac{4}{9} - \frac{8}{27} = \frac{4}{27}$$

U is a potential energy fn. H is also a pot. en. fn.

c)



$$K_A = 0 \quad \frac{U_A}{U_1} = H(-1/3) = \frac{4}{27} = y^2 - y^3 \Rightarrow y = -\frac{1}{3} \text{ or } y = \frac{2}{3}$$

$$y^3 - y^2 + \frac{4}{27} = 0$$

Between $y = -\frac{1}{3}$ and $y = \frac{2}{3}$ the mass has periodic motion

d) In the periodic motion, energy is conserved.

$$H(-1/3) = \frac{4}{27}, \quad -1/3 = -\frac{1}{3}$$

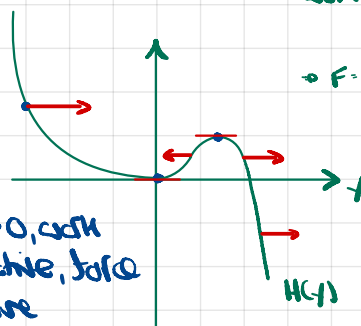
$$-1/3 = \frac{x}{x_1} = -\frac{1}{3} \quad H(-1/3) = \frac{U(-x_1/3)}{U_1} = \frac{4}{27} \Rightarrow U(-x_1/3) = \frac{U_1 \cdot 4}{27} = \frac{1}{2} m v_0^2$$

$$\Rightarrow v_0 = \pm \sqrt{\frac{8U_1}{27}}$$

$$\text{Work} = \int_{x_1}^{x_2} F dx = -\Delta U = U_1 - U_2$$

$$\Rightarrow F = \frac{dU}{dx}$$

$\Delta U > 0$, work negative, force positive



from $y=0$ to here $\Delta U < 0$, so work is positive

