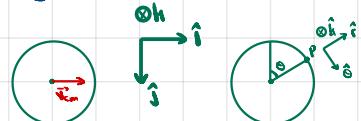


## 20.2 Constrained Motion: Translation and Rotation

Rolling wheel



$$\vec{v}_{ip} = \vec{v}_{g,cm} + \vec{v}_{cm,p}$$

$$\vec{v}_{g,cm} = v_{g,cm} \hat{i}$$

$$\vec{v}_{cm,p}(t) = (v_{g,cm,t} + v_{g,cm,t})\hat{i}$$

$$\omega_{cm} = \omega_{wheel}$$

In CM frame P has uniform circular motion

$$\vec{v}_{cm,p} = R\omega_{cm} \hat{\theta}$$

We want to know  $\vec{v}_{ip}$ , but  $\vec{v}_{cm,p}$  is in polar,  $\vec{v}_{cm,p}$  is in cartesian.

Note the mixed choice for measuring  $\theta$  and  $\hat{i}, \hat{j}$ .

$$\Rightarrow \hat{i} \cdot \sin\theta - \cos\theta$$

$$\hat{\theta} \cdot \cos\theta \hat{i} + \sin\theta \hat{j}$$

$$\vec{v}_{cm,p} = R\omega_{cm} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\Rightarrow \vec{v}_{ip} = \vec{v}_{g,cm} - R\omega_{cm} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\vec{v}_{ip} = \hat{i} (v_{g,cm} + R\omega_{cm} \cos\theta) + \hat{j} R\omega_{cm} \sin\theta$$

$$\theta = \pi \Rightarrow P \text{ is contact point w/ ground}$$

$$v_{ip,0}(\theta = \pi) = \hat{i} (v_{g,cm} - R\omega_{cm})$$

If the wheel is not slipping,  $v_{g,cm} = R\omega_{cm}$

$$\Rightarrow v_{ip}(\theta = \pi) = \vec{0}$$

### Example 20.1 Bicycle Wheel Rolling w/o Slipping



$$a) \vec{r}_{ip} = \vec{r}_{g,cm} + \vec{r}_{cm,p}$$

$$= (x_{cm,0} + v_{g,cm}t)\hat{i} + b(\sin\theta \hat{i} - \cos\theta \hat{j})$$

$$+ \hat{i} (x_{cm,0} + v_{g,cm}t + b\sin\theta) - b\cos\theta \hat{j}$$

$$\vec{v}_{ip} = \hat{i} (v_{g,cm} + b\theta'(t) \cos\theta) + b\theta'(t) \sin\theta \hat{j}$$

as expected given formula derived for  $\vec{v}_{ip}$  in general.

$$\vec{a}_{ip} = \hat{i} (b\theta''(t) \cos\theta - b\theta'^2(t) \sin\theta) + \hat{j} (b\theta''(t) \sin\theta + b\theta'^2(t) \cos\theta)$$

in the CM frame

$$\vec{r}_{cm,p} = b\hat{f}$$

$$\vec{v}_{cm,p} = b\omega \hat{\theta}$$

$$\vec{a}_{cm,p} = -b\omega^2 \hat{f}$$

uniform circular motion

Let's obtain these expressions from our results for the ground frame.

$$\vec{r}_{cm,p} = \vec{r}_{g,ip} - \vec{r}_{g,cm}$$

$$= \hat{i} (x_{g,cm,0} + v_{g,cm,t} + b\sin\theta) - b\cos\theta \hat{j} - (\hat{x}_{g,cm,t} + v_{g,cm,t})\hat{i}$$

$$= \hat{i} b\sin\theta - \hat{j} b\cos\theta - b\hat{f}$$

$$\vec{v}_{cm,p} = \vec{v}_{ip} - \vec{v}_{g,cm}$$

$$= \hat{i} (v_{g,cm} + b\omega_{cm} \cos\theta) + \hat{j} b\omega_{cm} \sin\theta$$

$$= -v_{g,cm} \hat{i}$$

$$= \hat{i} (b\omega_{cm} \cos\theta) + \hat{j} b\omega_{cm} \sin\theta$$

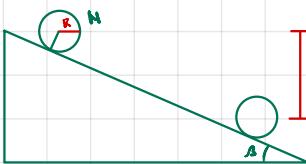
$$= b\omega_{cm} \hat{\theta}$$

$$\vec{a}_{cm,p} = b\omega_{cm} \cdot \frac{d\hat{\theta}}{dt}$$

$$\frac{d\hat{\theta}}{dt} = \omega_{cm} (-\sin\theta \hat{i} + \cos\theta \hat{j})$$

$$\Rightarrow \vec{a}_{cm,p} = -b\omega_{cm}^2 \hat{f}$$

### Example 20.2



W rolls w/o slipping

The ball starts at zero linear and angular momentum. To roll, a torque must act to change the angular momentum.

Gravity doesn't provide torque, we know that torque due to gravity on a rigid body is equivalent to a torque applied to a point particle at the center of mass.



$$\vec{T}_{sh} = \vec{r}_{s,i} \times (m_i g \hat{j})$$

$$= (\vec{r}_{s,i} \cdot m_i) \times g \hat{j}$$

$$\sum \vec{T}_{sh} = \sum (\vec{r}_{s,i} \cdot m_i) \times g \hat{j}$$

$$= m_i \vec{r}_{g,cm} \times g \hat{j}$$

$$= \vec{r}_{g,cm} \times \vec{F}_g$$

$$= \vec{T}_{sig}$$

About the CM gravity applies no torque.

Friction at the contact point provides torque relative to CM.

$$\vec{T}_{cm,f} = R\hat{j} \times (-f_s \hat{i})$$

$= R\hat{j} \cdot \hat{k}$ , the net torque about CM

CMSS

$$= I_{cm} \alpha \hat{k}$$

$$\Rightarrow \frac{MR^2}{2} \alpha = Rf_s$$

$$\alpha = \frac{2f_s}{MR}$$

Using 2nd law we can find the acceleration of the CM. Recall we can do this because the wheel is a system of particles

$$\vec{p}_{sys} = \sum m_i v_i = m_{sys} \vec{v}_{cm}$$

$$\frac{dp_{sys}}{dt} = \vec{F} = m_{sys} \vec{a}_{cm}$$

$$\hat{i}: mg \sin\beta - f_s = M \cdot a_{cm}$$

$$\hat{j}: mg \cos\beta - N = 0$$

The torque equation told us how much angular acceleration there is  $\propto$  a function of friction.

2nd law tells us what friction is  $\propto$  a function of acceleration of center of mass

If the ball rolls w/o slipping then

$$a_{cm} = R\alpha$$

Therefore we have

$$mg \sin\beta - f_s = M \cdot a_{cm}$$

$$\alpha = \frac{2f_s}{MR}$$

$$a_{cm} = R\alpha$$

$$f_s = N \cdot \mu$$

$$N = mg \cos\beta$$

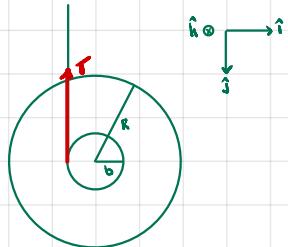
5 equations

unknowns:  $N, f_s, \alpha, a_{cm}$

$$a_{cm} = \frac{3N \cos\beta}{M}$$

$$\alpha = \frac{3N \cos\beta}{MR}$$

### Example 20.3 Falling Hoop



net torque eq. relates torques to angular acceleration

$$bT = \frac{mR^2}{2} \alpha \Rightarrow \alpha = \frac{2bT}{mR^2}$$

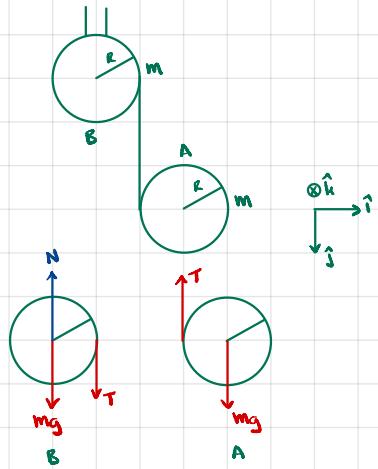
2nd law relates torques to linear acceleration

$$mg - T = ma$$

2 equations, 3 unknowns:  $T, a, \alpha$

no slipping  $\Rightarrow a = b\alpha$

### Example 20.4 - Uncoiling Drum



The metal tape applies a torque on each drum.

$$RT = I_{cm}\alpha_B$$

$$RT = I_{cm}\alpha_A \Rightarrow \alpha_A = \alpha_B = \alpha$$

$$mg + T - N = 0$$

$$mg - T = m \cdot a_n$$

The tape has a fixed length. The vertical change in position of A equals the change in unrolled tape. Unrolling is either because of drum B or drum A

$$R\Delta\theta_B + R\Delta\theta_A = \Delta y$$

$$\frac{\Delta y}{\Delta t} = R \frac{d\theta_A}{dt} + R \frac{d\theta_B}{dt}$$

$$\therefore \ddot{y} = R\ddot{\theta}_A + R\ddot{\theta}_B$$

$$= R(\alpha_A + \alpha_B)$$

### 20.3 Angular Momentum for System of Particles Undergoing Translation and Rotation

System of N particles

$$\vec{L}_s^{\text{total}} = \sum_i \vec{L}_{s,i} = \sum_i \vec{r}_{s,i} \times m_i \vec{v}_{s,i}$$

$$= \sum_i (\vec{r}_{s,cm} + \vec{r}_{cm,i}) \times m_i (\vec{v}_{s,cm} + \vec{v}_{cm,i})$$

$$= \sum_i \vec{r}_{s,cm} \times m_i \vec{v}_{s,cm}$$

$$+ \sum_i \vec{r}_{s,cm} \times m_i \vec{v}_{cm,i}$$

$$+ \sum_i \vec{r}_{cm,i} \times m_i \vec{v}_{s,cm}$$

$$+ \sum_i \vec{r}_{cm,i} \times m_i \vec{v}_{cm,i}$$

$$= \vec{r}_{s,cm} \times (\sum_i m_i) \vec{v}_{s,cm}$$

$$+ \vec{r}_{s,cm} \times \sum_i m_i \vec{v}_{cm,i}$$

$$+ (\sum_i \vec{r}_{cm,i} + m_i) \vec{v}_{cm,i}$$

$$+ \sum_i \vec{r}_{cm,i} \times m_i \vec{v}_{cm,i}$$

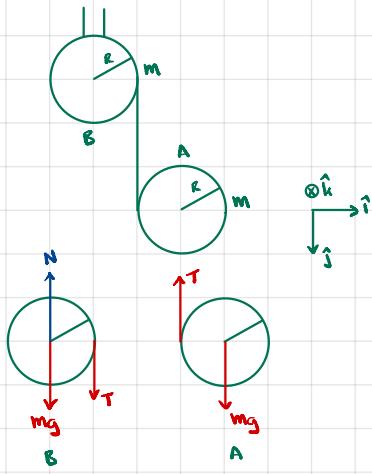
$$= \vec{r}_{s,cm} \times m_T \vec{v}_{s,cm} + \sum_i \vec{r}_{cm,i} \times m_i \vec{v}_{cm,i}$$

$$= \vec{r}_{s,cm} \times \vec{p}_{cm} + \sum_i \vec{r}_{cm,i} \times m_i \vec{v}_{cm,i}$$

orbital angular momentum spin angular momentum: total ang mom. of the system w.r.t. CM frame

$\vec{L}_s^{\text{total}} = \vec{r}_{s,cm} \times \vec{p}_{cm}$ , angular momentum about S of point respect to the CM in the particle with mass of the system at center of mass. CM frame

$$\vec{L}_s^{\text{total}} = \vec{L}_s^{\text{orbital}} + \vec{L}_s^{\text{spin}}$$



System:  $\text{dcm}_A + \text{dcm}_B$

net torque about CM<sub>B</sub>

$$\vec{\tau}_{\text{cm}_B, B, \perp} = R\hat{i} \times T\hat{j} = RT\hat{k}$$

$$\begin{aligned}\vec{\tau}_{\text{cm}_B, A, \perp} &= (2R\hat{i} + R\hat{j}) \times mg\hat{j} + (R\hat{i} + R\hat{j}) \times -T\hat{j} \\ &= 2Rmg\hat{i} - RT\hat{k} \\ &= (2Rmg - RT)\hat{i}\end{aligned}$$

$$\vec{\tau}_{\text{cm}_B, S, \perp} = 2Rmg\hat{i}$$

Drum B

$$2^{\text{nd}} \text{ law: } mg + T - N = m \cdot a_B = 0$$

$$\text{torque: } \vec{\tau}_{\text{cm}_B, B, \perp} = RT\hat{k} = I_{\text{cm}_B, B} \cdot \alpha_B\hat{k}$$

$$\Rightarrow RT = I_{\text{cm}_B, B} \cdot \alpha_B$$

Drum A

$$2^{\text{nd}} \text{ law: } mg - T - N = m \cdot a_A = 0$$

$$\text{torque: } \vec{\tau}_{\text{cm}_A, A, \perp} = -R\hat{i} \times (-T\hat{j})$$

$$= RT\hat{i} = I_{\text{cm}_A, A} \cdot \alpha_A\hat{i}$$

$$\Rightarrow RT = I_{\text{cm}_A, A} \cdot \alpha_A$$

We also have the equation:  $a_A = R(\alpha_A + \alpha_B)$

$$\text{Assume } I_{\text{cm}_B, B} = I_{\text{cm}_A, A} = I_{\text{cm}}$$

$$N = mg + T$$

$$RT = I_{\text{cm}} \cdot \alpha_B$$

$$RT = I_{\text{cm}} \cdot \alpha_A$$

$$T = m(g - a_A)$$

$$a_A = R(\alpha_A + \alpha_B)$$

unknowns:  $\alpha_A, \alpha_B, T, N, a_A$

$$\Rightarrow \alpha_A = \alpha_B = \alpha = \frac{RT}{I_{\text{cm}}}$$

$$\Rightarrow T = m(g - \frac{2R^2T}{I_{\text{cm}}})$$

$$T(1 + \frac{2R^2}{I_{\text{cm}}}) = mg$$

$$T = \frac{mg}{1 + \frac{2R^2m}{I_{\text{cm}}}}$$

$$= \frac{mg I_{\text{cm}}}{I_{\text{cm}} + 2R^2m}$$

$$\alpha = \frac{mgR}{I_{\text{cm}} + 2R^2m}$$

$$a_A = \frac{2mgR^2}{I_{\text{cm}} + 2R^2m}$$

$$N = \frac{mg I_{\text{cm}} + m^2 g^2 R^2 + mg I_{\text{cm}}}{I_{\text{cm}} + 2R^2m}$$

$$= \frac{2mg I_{\text{cm}} + 2m^2 g R^2}{I_{\text{cm}} + 2R^2m} = \frac{2mg(I_{\text{cm}} + R^2m)}{I_{\text{cm}} + 2R^2m}$$

$$\text{If } I = \frac{mR^2}{2}$$

$$RT = \frac{mR^2}{2} \alpha$$

$\uparrow$  means  $\uparrow I$ , linearly, slope  $\frac{R^2}{2}$

$$T = \frac{mg}{s} \text{ increases linearly, slope } \frac{g}{5}$$

$$\alpha = \frac{2g}{5R} \text{ doesn't change}$$

$$\frac{Rmg}{s} = \frac{mR^2}{2} \cdot \frac{2g}{5R} = \frac{mgR}{s}$$

$$\frac{Rg}{s} = \frac{R^2}{2} \cdot \frac{2g}{5R} = \frac{Rg}{5}$$

$$T = \frac{mg}{s}$$

$$\alpha = \frac{2g}{5R}$$

$$a_A = \frac{4g}{5}$$

$$N = \frac{6mg}{5}$$

$$\alpha = \frac{2g}{5R}$$

$$\omega = \frac{2gt}{5R}$$

$$\theta = \frac{gt^2}{5R}$$

$$s = R\theta = \frac{gt^2}{5}$$

$$\vec{\tau}_{\text{cm}_B, B} = \frac{mR^2}{2} \cdot \frac{2gt}{5R} \hat{k} = \frac{Rgmt}{5} \hat{k}$$

$$\vec{\tau}_{\text{cm}_B, A} = \vec{\tau}_{\text{cm}_B, \text{cm}_A} + \vec{\tau}_{\text{cm}_A, A}$$

$$= (2R^2 + R^2) \times m \cdot \frac{4gt}{5} \hat{j} + \frac{mR^2}{2} \cdot \frac{2gt}{5} \hat{k}$$

$$\cdot \frac{8gmt}{5} \hat{k} = \frac{Rgmt}{5} \hat{k}$$

$$\cdot \frac{9gmt}{5} \hat{k}$$

$$\vec{\tau}_{\text{cm}_B} = 2Rgmt \hat{k}$$

$$\frac{d\vec{\tau}_{\text{cm}_B}}{dt} = 2Rgmt \hat{i} - \vec{\tau}_{\text{cm}_B, \text{cm}_A}$$

## Ex 20.5 Earth's Motion Around the Sun

$$m_e = 5.97 \cdot 10^{24} \text{ kg}$$

$$R_e = 6.38 \cdot 10^6 \text{ m}$$

$$\text{circular orbit, } r_{s,e} = 1.50 \cdot 10^8 \text{ m}$$

$$T_{\text{orbit}} = 365.25 \text{ days}$$

$$T_{\text{spin}} = 23 \text{ h } 56 \text{ min} = 23 \cdot 3600 + 56 \cdot 60 \text{ s}$$

axis of spin is tilted  $23.5^\circ$



$$\text{Earth is sphere} \Rightarrow I_{\text{cm},e} = \frac{2m_e R_e^2}{5}$$

$$\vec{L}_{s,e}^{\text{orbital}} = \vec{r}_{s,\text{cm},e} \times m_e v_{\text{cm},e} \hat{\theta}$$

$$= \vec{r}_{s,e} \hat{r} \times m_e v_{\text{cm},e} \hat{\theta}$$

$$= \vec{r}_{s,e} m_e v_{\text{cm},e} \hat{k}$$

$$= \vec{r}_{s,e}^2 m_e \omega_{\text{orbit}} \hat{k}$$

$$\omega_{\text{orbit}} = \frac{2\pi}{T_{\text{orbit}}} = \frac{2\pi}{365.25 \cdot 24 \cdot 3600}$$

$$= 1.991 \cdot 10^{-7} \text{ rad.s}^{-1}$$

$$\Rightarrow \vec{L}_{s,e}^{\text{orbital}} = (1.50 \cdot 10^8)^2 \cdot 5.97 \cdot 10^{24} \cdot 1.991 \cdot 10^{-7} \hat{k}$$

$$= 2.68 \cdot 10^{30} \hat{k}$$

$$\vec{L}_{s,e}^{\text{spin}} = I_{\text{cm},e} \times \vec{\omega}_{\text{spin}} = I_{\text{cm},e} \times \omega_{\text{spin}} \hat{n}$$

$$= \frac{2m_e R_e^2}{5} \cdot \frac{2\pi}{8.616 \cdot 10^4} \hat{n}$$

$$= 7.10 \cdot 10^{31} \hat{n}$$

$$\frac{L_{s,e}^{\text{total}}}{L_{s,e}^{\text{spin}}} = \frac{5 \vec{r}_{s,e}^2 T_{\text{spin}}}{2 R_e T_{\text{orbit}}} = 3.77 \cdot 10^6$$

## 20.4 Kinetic Energy of System of Particles

$$K = \frac{\sum m_i v_{s,i}^2}{2} = \frac{\sum m_i (v_{s,\text{cm}} + v_{\text{cm},i})^2}{2}$$

$$= \frac{\sum m_i v_{s,\text{cm}}^2 + \sum m_i v_{\text{cm},i}^2 + \sum m_i 2v_{s,\text{cm}} v_{\text{cm},i}}{2}$$

$$= K_{s,\text{cm}} + K_{\text{cm}}$$

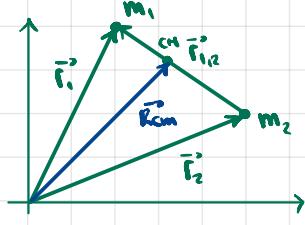
## 20.5 Rigid Body Undergoing Fixed Axis Rotation

$$K_{\text{sys}} = K_{s,\text{cm}} + K_{\text{cm}} = K_{\text{trans}} + K_{\text{rot}}$$

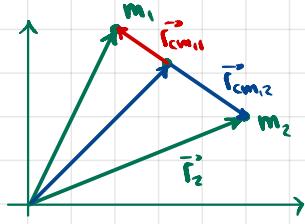
$$= \frac{M_r v_{s,\text{cm}}^2}{2} + \frac{I_{\text{cm}} \omega_{\text{cm}}^2}{2}$$

## Appendix: Charles' Theorem

"motion of any rigid body consists of a translation of the CM and rotation about the CM"



$$\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$



$$\vec{r}_{cm,1} = \vec{r}_1 - \vec{R}_{cm} = \vec{r}_1 - \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$= \frac{m_2(\vec{r}_1 - \vec{r}_2)}{m_1 + m_2} = \frac{N}{m_1} \vec{r}_{1,2} \quad N = \frac{m_1 m_2}{m_1 + m_2}$$

$$\vec{r}_{1,2} = \vec{r}_1 - \vec{r}_2 = (\vec{r}_{cm,1} + \vec{R}_{cm}) - (\vec{r}_{cm,2} + \vec{R}_{cm})$$

$$= \vec{r}_{cm,1} - \vec{r}_{cm,2}$$

relative position vectors are the same in any reference frame.

$$\text{In CM frame } \frac{m_1 \vec{r}_{cm,1} + m_2 \vec{r}_{cm,2}}{m_1 + m_2} = \vec{0}$$

$$\Rightarrow m_1 \vec{r}_{cm,1} = -m_2 \vec{r}_{cm,2}$$

$$\Rightarrow m_1 |\vec{r}_{cm,1}| = m_2 |\vec{r}_{cm,2}|$$

$$d\vec{r}_{cm,1} = \frac{N}{m_1} d\vec{r}_{1,2}$$

Analogous calculations for  $\vec{r}_{cm,2}$  yield

$$\vec{r}_{cm,2} = \vec{r}_2 - \vec{R}_{cm} = \vec{r}_2 - \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$= \frac{m_1(\vec{r}_2 - \vec{r}_1)}{m_1 + m_2} = \frac{N}{m_2} \vec{r}_{1,2}$$

$$d\vec{r}_{cm,2} = -\frac{N}{m_2} d\vec{r}_{1,2}$$

Arbitrary displacement of  $i^{\text{th}}$  particle

$$d\vec{r}_i = d\vec{r}_{cm,i} + d\vec{R}_{cm}$$

At this point we have positions of  $m_1$  and  $m_2$  from CM frame in terms of positions in initial frame. Same for position displacement vectors  $d\vec{r}_{cm,i}$ .

We have position in CM in terms of initial frame, and the change in the positions.

constraint: distance between particles is constant, ie rigid body

$$|\vec{r}_{1,2}|^2 = (\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}_1 - \vec{r}_2)$$

Differentiate

$$0 = (\vec{r}'_1 - \vec{r}'_2) \cdot (\vec{r}_1 - \vec{r}_2) + (\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}'_1 - \vec{r}'_2)$$

$$= 2(\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}'_1 - \vec{r}'_2)$$

$$0 = 2(\vec{r}_1 - \vec{r}_2) \cdot (d\vec{r}_1 - d\vec{r}_2)$$

$$= 2 \vec{r}_{1,2} \cdot d\vec{r}_{1,2} \quad (\text{rigid body condition})$$

But we're dealing with a specific kind of system, that of a rigid body.

So we reach a condition reflecting that fact.

If  $d\vec{r}_{1,2} = \vec{0}$  then the condition is satisfied.

$$\Rightarrow d\vec{r}_{cm,1} = d\vec{r}_{cm,2} = \vec{0}$$

$$\Rightarrow d\vec{r}_i = d\vec{R}_{cm}$$

(rigid body undergoes pure translation)

Suppose  $d\vec{r}_{1,2} \neq \vec{0}$

rigid body condition rewritten

$$0 = d\vec{r}_{cm,1} \cdot \frac{m_1}{N} \vec{r}_{1,2} + d\vec{r}_{cm,2} \cdot \frac{m_2}{N} \vec{r}_{1,2}$$

$$= 2 \frac{m_1}{N} \vec{r}_{1,2} \cdot d\vec{r}_{cm,1}$$

$$= 2 \frac{m_1}{N} \vec{r}_{cm,1,2} \cdot d\vec{r}_{cm,1}$$

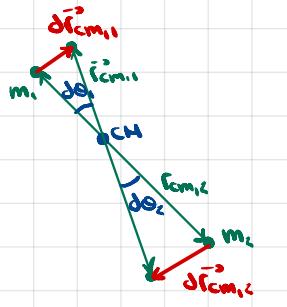
$$\Rightarrow 2 \vec{r}_{cm,1,2} \cdot d\vec{r}_{cm,1}$$

(rigid body condition in CM frame)

$$\vec{r}_{cm,1,2} \perp d\vec{r}_{cm,1}$$

we can rewrite the rigid body condition differently and reach

$$\vec{r}_{cm,1,2} \perp d\vec{r}_{cm,2}$$



Furthermore, we conclude that  $m_1$  and  $m_2$  are rotating about the center of mass. The angle in position is always  $\perp$  to position from center of mass and the angular displacement is the same for both masses.

$$\delta\theta_1 = \frac{|\vec{\delta r}_{cm,1}|}{|\vec{r}_{cm,1}|}$$

$$\delta\theta_2 = \frac{|\vec{\delta r}_{cm,2}|}{|\vec{r}_{cm,2}|}$$

$$\text{But } \vec{\delta r}_{cm,1} = \frac{N}{m_1} \cdot \vec{r}_{1,2}$$

$$\vec{\delta r}_{cm,2} = -\frac{N}{m_2} \vec{r}_{1,2}$$

$$\Rightarrow \delta\theta_1 = \frac{N}{m_1} \frac{|\vec{r}_{1,2}|}{|\vec{r}_{cm,1}|}$$

$$\delta\theta_2 = \frac{N}{m_2} \frac{|\vec{r}_{1,2}|}{|\vec{r}_{cm,2}|}$$

We've also derived that

$$m_1 |\vec{r}_{cm,1}| = m_2 |\vec{r}_{cm,2}|$$

$$\Rightarrow \delta\theta_1 = \delta\theta_2$$

$$\vec{dr}_i = \vec{dR}_{cm} + \vec{d\alpha}_{cm,i}$$

$\uparrow$  CM translation  
 $\downarrow$  rotation about CM