PSet 7

Problem 1

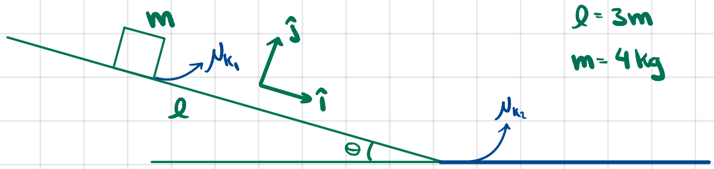
$\theta = 30°$

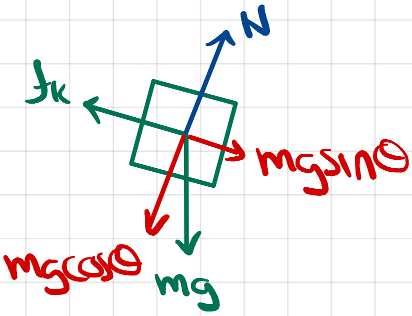
$\mu_{k_1} = 0.2$

$\mu_{k_2} = 0.3$

$l = 3m$

$m = 4kg$





a) while sliding down the plane, friction does negative work.

$f_k = -\mu_k N \hat{i}$

2nd law, $\hat{j}$ dir: $N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$

$\Rightarrow f_k = -\mu_k mg\cos\theta\,\hat{i}$

$W_k = \int_c \vec{f_k}\cdot d\vec{s} = -\mu_k mg\cos\theta \int_0^l dx = -l\mu_k mg\cos\theta = -3\cdot 0.2\cdot 4\cdot 9.8\cdot \frac{\sqrt{3}}{2} = -20.369\ J$

b) $\vec{F^g} = mg\sin\theta\,\hat{i} + mg\cos\theta\,\hat{j}$

$W_g = \int_0^l mg\sin\theta\,dx = l\,mg\sin\theta = 3\cdot 4\cdot 9.8\cdot 0.5 = 58.80\ J$

c) $\vec{F} = (mg\sin\theta - \mu_{k_1} N)\hat{i} + (N - mg\cos\theta)\hat{j}$

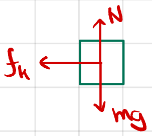
$\Rightarrow \vec{F} = (mg\sin\theta - \mu_{k_1} mg\cos\theta)\hat{i}$

$d\vec{s} = dx\,\hat{i} + dy\,\hat{j}$

$W_1 = \int_c \vec{F}\cdot d\vec{s} = mg(\sin\theta - \mu_{k_1}\cos\theta)\int_0^l dx = l\,mg(\sin\theta - \mu_k\cos\theta) = 58.80 - 20.369 = 38.431$

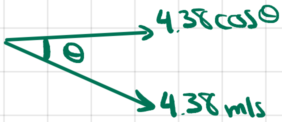
$W_1 = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) \Rightarrow 38.431 = \frac{1}{2}\cdot 4(v_f^2 - 0) \Rightarrow v_f^2 = (38.431/2)^{1/2} = 4.38\ m/s$

d)



$W_k = \int_c \vec{f_k}\,d\vec{s} = \int_0^d (-\mu_{k_2} mg\,\hat{i})(dx\,\hat{i}) = -\mu_k mg \int_0^d dx = -\mu_k mgd$

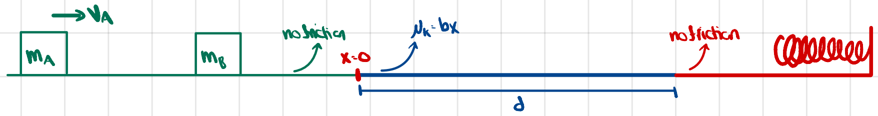
e) $W_k = \Delta K = \frac{1}{2}m(0^2 - v_i^2) \Rightarrow -\mu_k \cancel{m} gd = -\frac{1}{2}\cancel{m} v_i^2 \Rightarrow d = \dfrac{v_i^2}{2\mu_k g} = \dfrac{(3.7964)^2}{2\cdot 0.3\cdot 9.8} = 2.45\ m$



$4.38\cos\theta = 4.38\cdot\sqrt{3}/2$

$= 3.7964\ m/s = V_i$

Problem 2 – Collision and Sliding on a Rough Surface



1. conservation of momentum $\Rightarrow$ determine velocity of (m_A+m_B) at $x=0$

2. work done by friction = Δ kinetic energy $\Rightarrow$ velocity at d found

3. work done by spring = Δ kinetic energy $\Rightarrow$ compression distance found

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1. $m_A v_A = (m_A+m_B)v_0 \Rightarrow v_0 = \dfrac{m_A}{m_A+m_B} v_A$

2. $\displaystyle\int_0^d -bx(m_A+m_B)g\,dx = \frac{1}{2}(m_A+m_B)(v_d^2 - v_0^2)$

$$-bg\cancel{(m_A+m_B)}\frac{d^2}{\cancel{2}} = \cancel{\frac{1}{2}}\cancel{(m_A+m_B)}(v_d^2 - v_0^2)$$

$$\Rightarrow v_d^2 = v_0^2 - bgd^2 = \frac{m_A^2}{(m_A+m_B)^2} v_A - bgd^2$$

3. $\displaystyle\int_0^D -kx\,dx = \frac{1}{2}(m_A+m_B)(-v_d^2)$

$$\Rightarrow -\frac{kD^2}{2} = \frac{1}{2}(m_A+m_B)(-v_d^2)$$

$$D = \sqrt{\frac{(m_A+m_B)v_d^2}{k}} = \sqrt{\frac{(m_A+m_B)}{k}\left[\frac{m_A^2}{(m_A+m_B)^2} v_A - bgd^2\right]}$$

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momentum principle applied to 2.

Δ momentum $= \int f_k\,dt$

$$\cancel{(m_A+m_B)}(v_d - v_0) = \int_0^{t_d} -bx(t)\cancel{(m_A+m_B)}g\,dt$$

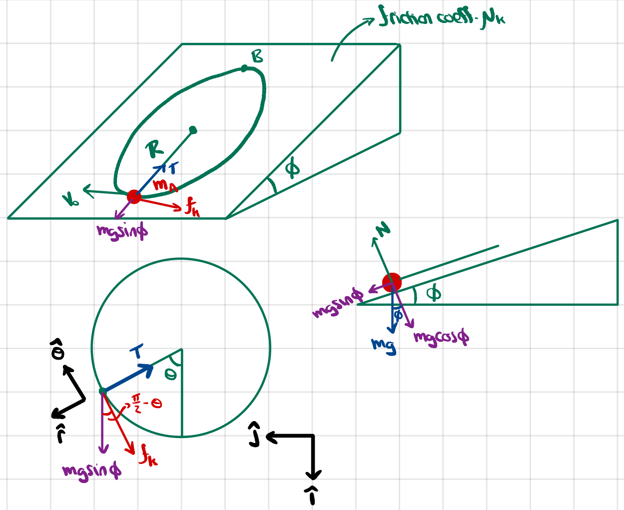
$$v_d - v_0 = -bg\int_0^{t_d} x(t)\,dt$$

time	momentum
t	$m_T v(t)$
$t+\Delta t$	$m_T v(t+\Delta t)$

$$\Delta p = m_T \Delta v \Rightarrow \frac{\Delta p}{\Delta t} = \frac{\Delta v}{\Delta t} m_T \Rightarrow \frac{dp}{dt} = (m_A+m_B)\frac{dv}{dt}$$

$$F = \frac{dp}{dt} \Rightarrow -bx(t) = (m_A+m_B)\frac{dv}{dt}$$

Problem 3 - Inclined Plane



* $\vec r(t) = R\hat r$ $\quad \frac{d\vec r(t)}{dt} = R\cdot\frac{d\hat r}{dt} = \vec v(t)$

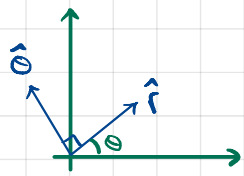
$$\frac{d\hat r}{dt} = -\sin\theta(t)\,\theta'(t)\,\hat r + \cos\theta(t)\,\theta'(t)\,\hat\theta$$

$$\vec v(t) = -R\sin\theta\,\theta'\hat r + R\cos\theta\,\theta'\hat\theta$$
$$= R\theta'(-\sin\theta\,\hat r + \cos\theta\,\hat\theta) = R\theta'\hat\theta$$

$$\vec a(t) = (-R\cos\theta\cdot\theta'^2 - R\sin\theta\cdot\theta'')\hat r + (-R\sin\theta\,\theta'^2 + R\cos\theta\,\theta'')\hat\theta$$
$$= -R\theta'^2\cos\theta\,\hat r - R\theta'^2\sin\theta\,\hat\theta - R\sin\theta\,\theta''\hat r + R\cos\theta\,\theta''\hat\theta$$
$$= -R\theta'^2\hat r + R\theta''\hat\theta = a_r\hat r + a_T\hat\theta$$

$\vec r(t) = R\hat r \quad \vec v(t) = R\theta'\hat\theta \quad \vec a(t) = -R\theta'^2\hat r + R\theta''\hat\theta$

a) mass A has circular motion ⇒ acceleration both tangentially and radially
we want to calculate work along a curve, a line integral $\int_c \vec f_k\cdot d\vec s$.



$\hat r = \langle\cos\theta, \sin\theta\rangle$
$\hat\theta = \langle-\sin\theta, \cos\theta\rangle$

$$\vec f_k = -\mu_k N\hat\theta = -\mu_k mg\cos\phi\,(-\sin\theta(t)\,\hat\imath + \cos\theta(t)\,\hat\jmath)$$
$$= \mu_k mg\cos\phi\sin\theta(t)\,\hat\imath - \mu_k mg\cos\phi\cos\theta(t)\,\hat\jmath$$

$$\int_c \vec f_k\cdot d\vec s = \int_c \vec f_k\cdot\vec v\,dt$$

$$\int_c(-\mu_k N\hat\theta)(R\omega\hat\theta)\,dt = -\mu_k NR\int_0^{t_f}\theta'(t)\,dt = -\mu_k NR\,\theta(t_f) = -\mu_k NR\frac{\pi}{2}$$

$$\int \vec f_k\cdot\frac{\vec v}{v}\,v\,dt = \int \vec f_k\cdot\hat T\,ds = \int_0^{\pi/2}(-\mu_k N\hat\theta)\frac{\cancel{(R\omega\hat\theta)}}{\cancel{R\omega}}\cdot R\,d\theta = -\mu_k NR\int_0^{\pi/2}d\theta = -\mu_k NR\frac{\pi}{2}$$

b) At B $\vec v = \vec 0$. Motion is no longer circular, and the only force causing acceleration is mgcosφ.

$$\vec f_k + \vec F^g + \vec T = m\vec a$$

$$\vec f_k = -\mu_k mg\cos\phi\,\hat\theta$$

$$\vec F^g = mg\sin\phi\cos\theta(t)\,\hat r + mg\sin\phi\sin\theta(t)\,\hat\theta$$

$$\vec T = -T\hat r$$

$$-\mu_k mg\cos\phi + mg\sin\phi\sin\theta(t) = ma_T = mR\theta''(t)$$

$$\Rightarrow -T + mg\sin\phi\cos\theta(t) = -mR\theta'(t)^2$$