Kinematics of Rotational Motion

$\theta = \frac{s}{r} \Rightarrow s = r\theta$

θ measured in radians.

$\frac{2\pi r}{r} = 2\pi$ radians in circular circumference

$1 \text{ rad} = \frac{360°}{2\pi} = 57.3°$

Avg Ang Vel. $= \omega_{avg,z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

$\Delta\theta$ = angular displacement

instant. ang. vel. $= \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega_z$

ang. vel. vector $= \vec{\omega} = \omega_z \hat{k}$

avg. ang. accel. $= \alpha_{avg,z} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

instant. ang. accel. $= \alpha_z = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega_z}{dt}$

ang. accel. vector $= \vec{\alpha} = \alpha_z \hat{k}$

rotation, constant angular acceleration

α_z = constant

$\alpha_z(t) = \alpha_z$
$\omega_z(t) = \omega_0 + \alpha_z t$
$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_z t^2$ $\theta(\alpha_z, t)$

Altern. calc. w/o integrating

$\alpha_z = \text{constant} = \alpha_{avg,z} = \frac{\omega_z - \omega_{0,z}}{t - 0}$

$\Rightarrow \omega_z = \omega_{0,z} + \alpha_z t$

$\omega_{avg,z} = \frac{\omega_{0,z} + \omega_z}{2}$

$\omega_{avg,z} = \frac{\theta - \theta_0}{t - 0}$

$\Rightarrow \theta = \theta_0 + \frac{1}{2}(\omega_{0,z} t + \omega_z t)$ $\theta(\omega_z, t)$
$= \theta_0 + \frac{1}{2}(\omega_{0,z} t + \omega_{0,z} t + \alpha_z t^2)$
$= \theta_0 + \omega_{0,z} t + \frac{1}{2}\alpha_z t^2$ $\theta(\alpha_z, t)$

$t = \frac{\omega_z - \omega_{0,z}}{\alpha_z}$

$\Rightarrow \theta = \theta_0 + \frac{\omega_z - \omega_{0,z}}{2\alpha_z}(\omega_{0,z} + \omega_z)$

$2\alpha_z(\theta - \theta_0) = \omega_z^2 - \omega_{z,0}^2$

$\omega_z^2 = \omega_{z,0}^2 + 2\alpha_z(\theta - \theta_0)$

$s = r\theta$

linear speed

$\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow v = r\omega$

angular speed

$a_T = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$

note α and α_z are slightly different.
α is magnitude of acceleration. α_z is the component of $\vec{\alpha}$ in z-dir. $\alpha = |\vec{\alpha}|$.

$a_{rad} = r\omega^2 = \frac{v^2}{r}$

$I = \sum m_i r_i^2$

"moment" means I depends on how mass is distributed in space.

$K_{i,rot} = \frac{m_i v_i^2}{2} = \frac{m_i r_i^2 \omega^2}{2}$

For same ω, more distance from axis of rotation $\Rightarrow \uparrow K_i$

$K_{rot} = \sum K_i = \frac{\omega^2 \sum m_i r_i^2}{2} = \frac{I\omega^2}{2}$