20.1 Kinetic Energy

Kinetic Energy $= K = \frac{1}{2}mv^2 \geq 0$, a scalar

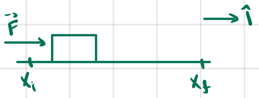
SI $[J] = kg\,(m/s)^2$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}m(v_{x_f}^2 + v_{y_f}^2 + v_{z_f}^2 - v_{x_i}^2 - v_{y_i}^2 - v_{z,i}^2)$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

20.2 Work by a Constant Force

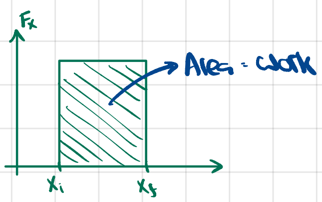


$\vec{F} = F_x\hat{\imath}$

$\Delta\vec{x} = \Delta x\,\hat{\imath}$ $\quad\Delta x = x_f - x_i$

F_x = constant

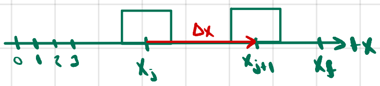
Work $= W = F_x\Delta x = F_x(x_f - x_i)$ $\quad$ SI units $[N\cdot m] = [J]$



20.3 Work by Non-constant Force

$\vec{F}(x) = F_x(x)\hat{\imath}$

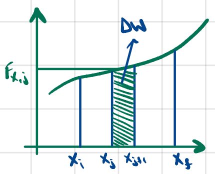
Ex: (Spring force) $F_x = -kx$



$\Delta x = x_{j+1} - x_j$

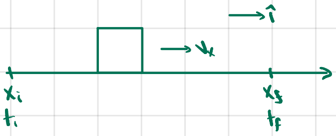
$\Delta W = F_x\Delta x = F_{x,j}(x_{j+1} - x_j)$

$$W = \lim_{n\to\infty}\sum_{j=1}^{n} F_{x,j}\Delta x_j = \int_{x_i}^{x_f} F_x(x)\,dx$$



20.4 Integrate Accel. w/r to Time and Position

So far we've seen



$$a_x(t) = \frac{dv_x}{dt}$$

$$\int_{t_i}^{t_f} a_x(t)\,dt = \int_{t_i}^{t_f}\frac{dv_x}{dt}dt = \int_{v_i}^{v_f} dv_x = v_{x,f} - v_{x,i} = \Delta v_x$$

now lets think of a as fn of x

$$\int_{x_i}^{x_f} a_x(x)\,dx = \int_{x_i}^{x_f}\frac{dv_x}{dt}dx = \int_{x_i}^{x_f} dv_x\underbrace{\frac{dx}{dt}}_{v_x} = \int_{v_{x,i}}^{v_{x,f}} v_x\,dv_x = \frac{1}{2}(v_{x,f}^2 - v_{x,i}^2)$$

20.5 Work-Kinetic Energy Theorem

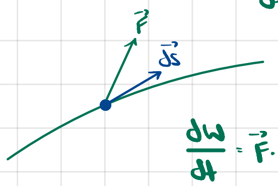
2nd law $F_x = ma_x$

$$W = \int_{x_i}^{x_f} F_x\,dx = m\int_{x_i}^{x_f} a_x\,dx = \frac{1}{2}m(v_f^2 - v_i^2) = \Delta K$$

$\Rightarrow W = \Delta K$

20.6 Power

Def (Power) $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$ $\quad$ SI unit Watt $= W = 1\,J/s$



$dW = \vec{F}\cdot\vec{ds}$

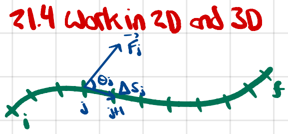
$$\frac{dW}{dt} = \vec{F}\cdot\frac{\vec{ds}}{dt} = \vec{F}\cdot\vec{v} = \text{Power}$$

21.3 Kinetic Energy as a Scalar Product

$K = \frac{1}{2}mv^2$ scalar $\quad$ * ref. frame dependent

$$= \frac{1}{2}m\vec{v}\cdot\vec{v} = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$$

21.4 Work in 2D and 3D



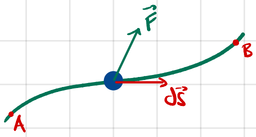
$F_j^{\parallel}$ = comp. of F_j in dir of $\Delta s_j = \vec{F}_j\cos\theta_j$

$\vec{F}_j\cdot\Delta s_j = F_j^{\parallel}|\Delta s_j|$

line integral

$$W = \lim_{n\to\infty}\sum_{j=1}^{n} W_j = \lim_{n\to\infty}\sum_{j=1}^{n}\vec{F}_j\cdot\Delta s_j = \int_i^f \vec{F}\cdot\vec{ds}$$

21.5 Work-Kinetic Energy Theorem in 2D and 3D



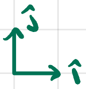
$$W = \int_A^B \vec{F} \cdot d\vec{s}$$

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\Rightarrow W = \int_A^B m\frac{d\vec{v}}{dt} \cdot d\vec{s} = \int_A^B m\frac{d\vec{v}}{dt}\vec{v}\,dt = \int_A^B m\,d\vec{v} \cdot \vec{v}$$

$$d\vec{s} = \vec{v}\,dt$$

To calculate, lets choose a coord. system



$$d\vec{v} = dv_x\hat{i} + dv_y\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow d\vec{v} \cdot \vec{v} = (dv_x)v_x + (dv_y)v_y$$

$$W = \int_A^B (m\,dv_x\,v_x + m\,dv_y\,v_y) = \int_A^B m\,dv_x\,v_x + \int_A^B m\,dv_y\,v_y$$

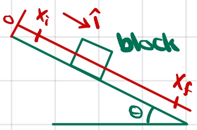
$$= \frac{m}{2}(v_{x,f}^2 - v_{x,i}^2) + \frac{m}{2}(v_{y,f}^2 - v_{y,i}^2)$$

$$= \frac{m}{2}(v_{x,f}^2 + v_{y,f}^2) - \frac{m}{2}(v_{x,i}^2 + v_{y,i}^2)$$

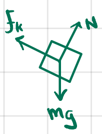
$$= K_f - K_i = \Delta K$$

$$\Rightarrow W = \Delta K$$

21.6 - Worked Example - Block Going Down Ramp



W	ΔK
free body force diagram	description
physics	$\frac{1}{2}m(v_f^2 - v_i^2)$



$\hat{j}$: $N - mg\cos\theta = 0$

$f_k = \mu_k N = \mu_k mg\cos\theta$

$$* \int_A^B \vec{f}_k \cdot d\vec{s} = \int_A^B -f_k\hat{i} \cdot dx\hat{i} = \int_{x_i}^{x_f} -f_k\,dx$$

$$\int_A^B m\vec{g} \cdot d\vec{s} = \int_A^B (mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j})(dx\,\hat{i})$$

$$= \int_{x_i}^{x_f} mg\sin\theta\,dx$$

$$-\int_{x_i}^{x_f} f_k\,dx + \int_{x_i}^{x_f} mg\sin\theta\,dx$$

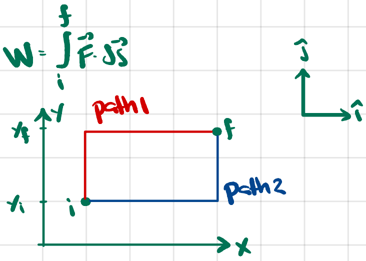
$$= \int_{x_i}^{x_f} mg(\sin\theta - \mu_k\cos\theta)\,dx$$

$$= mg(\sin\theta - \mu_k\cos\theta)(x_f - x_i)$$

$$\Rightarrow mg(\sin\theta - \mu_k\cos\theta)(x_f - x_i) = \frac{1}{2}m(v_f^2 - v_i^2)$$

22.1 Path Independence

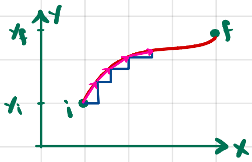
$$W = \int_i^f \vec{F} \cdot d\vec{s}$$

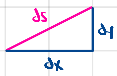


$$\vec{F}^g = -mg\hat{j}$$

$$W_1 = -mg(y_f - y_i)$$

$$W_2 = -mg(y_f - y_i)$$



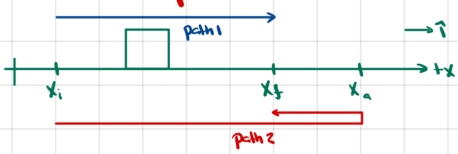


$$d\vec{s} = dx\hat{i} + dy\hat{j}$$

$$\vec{F}^g = -mg\hat{j}$$

$$W = \int_i^f \vec{F}^g \cdot d\vec{s} = \int_i^f -mg\hat{j} \cdot (dx\hat{i} + dy\hat{j}) = \int_{y_i}^{y_f} -mg\,dy = -mg(y_f - y_i)$$

22.2 Path Dependence - Friction



$$\vec{F}_k = -\mu_k mg\hat{i}$$

$$W_1 = \int_{x_i}^{x_f} (-\mu_k mg\hat{i})(dx\hat{i}) = -\mu_k mg\int_{x_i}^{x_f} dx = -\mu_k mg(x_f - x_i)$$

$$\updownarrow \neq$$

$$W_2 = \int_{x_i}^{x_a} -\mu_k mg\,dx + \int_{x_a}^{x_f} \mu_k mg\,dx = -\mu_k mg(x_a - x_i) + \mu_k mg(x_f - x_a)$$

22.3 Conservative Forces

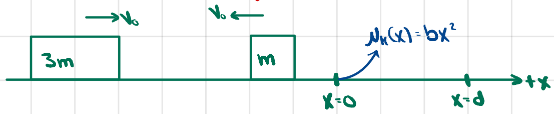
$$W = \int_i^f \vec{F} \cdot d\vec{s} \text{ path indep.} \Leftrightarrow \vec{F} \text{ conservative force}$$

$$\Leftrightarrow \int_i^f \vec{F} \cdot d\vec{s} + \int_f^i \vec{F} \cdot d\vec{s} = 0$$

$$W_c = \oint \vec{F}_c \cdot d\vec{s} = 0$$

If W is path dep then $\vec{F}$ is non-conservative.

PS.7.1 - Worked Example - Collision and Sliding on a Rough Surface



using conservation of momentum, we can calculate velocity right before entering the friction area.

$$3mv_0 - mv_0 = 4mv \Rightarrow 4mv = 2mv_0 \Rightarrow v = \frac{v_0}{2}$$

The work done by friction as the blocks are decelerated equals change in kinetic energy.

$$\int_0^d -\mu_k(x)\cdot 4mg\,dx = \frac{1}{\cancel{2}}\cdot \cancelto{2}{4}m\left(0^2 - \frac{v_0^2}{\cancelto{2}{4}}\right) = -\frac{v_0^2}{2}$$

$$v_0^2 = 2\int_0^d bx^2\cdot 4mg\,dx = 2b\cdot 4mg\int_0^d x^2\,dx = 8bmg\frac{d^3}{3}$$