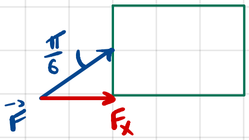
Example 6.1

a)



$F_x = F\cos(\pi/6)$

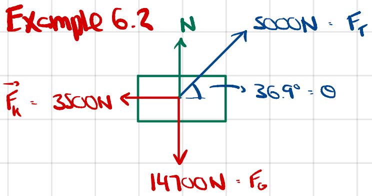
$W = F_x \cdot \Delta x = \frac{210\cdot\sqrt{3}}{2}\cdot 18 = 1890\sqrt{3}\ J \approx 3.3\cdot 10^3\ J$

b) $\vec{F} = \langle 160, -40\rangle$

$\vec{s} = \langle 14, 11\rangle$

$W = \vec{F}\cdot\vec{s} = 1.8\cdot 10^3\ J$

Example 6.2



* $N + 5000\sin\theta - F_G = 0$

$\Rightarrow N = 11697.9\ N$

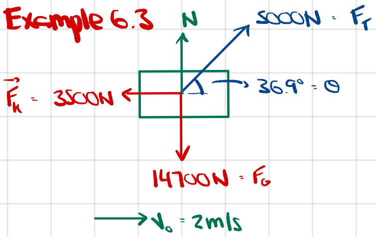
$F_T = \langle 5000\cos\theta, 5000\sin\theta\rangle$

$W_T = \vec{F}_T\cdot\vec{s} = 5000\cdot\cos 36.9°\cdot 20 = 79968.5\ J$

$\vec{s} = \langle 20, 0\rangle$

$W_k = \vec{F}_k\cdot\vec{s} = \langle -3500, 0\rangle\langle 20, 0\rangle = -70000\ J$

Example 6.3



$W = \Delta K \Rightarrow 9968.5 J = \frac{1}{2}\cdot\frac{14700}{9.8}\cdot(v_f^2 - 2^2)$

$\Rightarrow \left(\frac{2\cdot 9.8\cdot 9968.5}{14700} + 4\right)^{1/2} = v_f = 4.16\ m/s$

Altern.

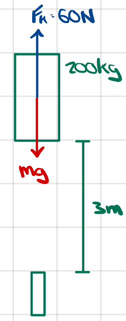
$F_R = m\cdot a \Rightarrow 498.42 = \frac{14700}{9.8}\cdot a_x \Rightarrow a_x = 0.33\ m/s^2$

$v_x = v_{0x} + a_x t$

$x = v_{0x}t + \frac{a_x t^2}{2}$ $\Rightarrow$ $x = 20 = 2t + 0.33t^2/2 \Rightarrow t = 6.50$

$\Rightarrow v_f = 2 + 0.33\cdot 6.50 \Rightarrow v_f = 4.16\ m/s$

Example 6.4



$(F_G - F_k)\cdot 3 = W_R = 3(200\cdot 9.8 - 60) = 5700\ J$

$5700 = \Delta K = \frac{1}{2}\cdot 200\, v_f^2 \Rightarrow v_f = \sqrt{\frac{2\cdot 5700}{200}} = \sqrt{57} \approx 7.55\ m/s$

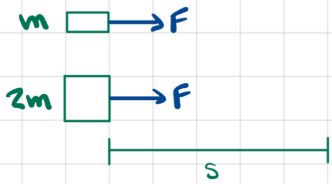
Work to stop the hammerhead

$(F_G - F_{up} - F_k)\ 0.074m = W = \Delta K = \frac{1}{2}200\cdot(-7.55^2)$

$200\cdot 9.8\cdot 0.074 - F_{up}\cdot 0.074 - 60 = -7.55^2\cdot 100$

$F_{up} = \frac{200\cdot 9.8\cdot 0.074 + 7.55^2\cdot 100 - 60}{0.074} = 78176.21\ N$

Example 6.5

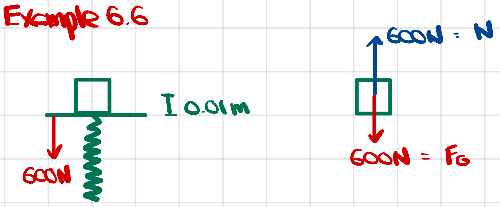


$W = F\cdot s$ in both cases

$\Delta K = \frac{m v_{f_1}^2}{2}$

$\Delta K = \frac{2m v_{f_2}^2}{2}$ $\Rightarrow v_{f_1}^2 = 2v_{f_2}^2 \Rightarrow v_{f_1} = \sqrt{2}\, v_{f_1}$

Example 6.6



$F_s = -kx \Rightarrow -600 = -k\cdot 0.01 \Rightarrow k = 60000$

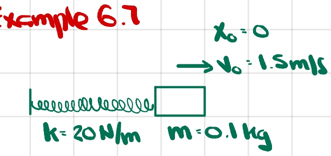
The resultant force varies in time as the spring is compressed.

$F_R = F_G - F_s = 600 - kx$

$W = \int_0^{0.01}(600 - kx)dx = 600x - 60000\frac{x^2}{2}\Big|_0^{0.01}$

$= 600\cdot\frac{1}{100} - 60000\cdot\frac{1}{10000}\cdot\frac{1}{2} = 6 - 3 = 3\ J$

Example 6.7



$$W = \int_0^{x_f} (-20x)\,dx = \frac{-20x_f^2}{\cancel{2}} = \frac{1}{\cancel{2}}\cdot 0.1\cdot(-1.5^2)$$

$$\Rightarrow x_f^2 = \frac{1.5^2}{200} \Rightarrow x_f = \frac{1.5}{\sqrt{200}} \approx 0.1060 = 10.6\,cm$$

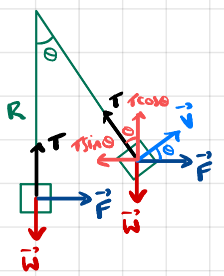
now with friction, $\mu_k = 0.47$

$$\int_0^{x_f} (-20x - 0.1\cdot 9.8\cdot 0.47)\,dx = -10x_f^2 - 0.4606x_f = \frac{1}{2}0.1(-1.5^2)$$

$$10x_f^2 + 0.4606x_f - 2.25\cdot 0.1\cdot 0.5 \Rightarrow x_f = 0.0855 \approx 8.55\,cm$$

Example 6.8

$$\vec{F} = \langle F_x(t), 0\rangle$$



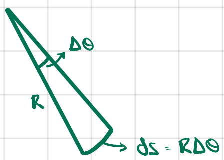
$$|\vec{v}| = R\sqrt{\theta'^2(\cos^2\theta + \sin^2\theta)} = |\theta'|$$

$\vec{F}$ varies such that the person and swing are always in "equilibrium", meaning the net force is always zero.

$$F_x - T\sin\theta = 0 \qquad \Rightarrow F_x = \frac{w}{\cos\theta}\cdot\sin\theta = w\tan\theta$$

$$T\cos\theta - w = 0 \Rightarrow T = \frac{w}{\cos\theta}$$

$\Rightarrow F_x$ increases w/ θ.



$$W = \int_c \vec{F}_x \cos\theta \cdot ds$$

$$= \int_0^{\theta_f} w\tan\theta\cos\theta\, R\,d\theta$$

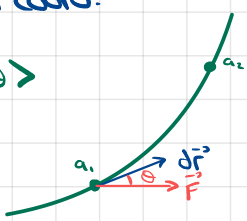
$$= wR\int_0^{\theta_f} \sin\theta\,d\theta = wR(-\cos\theta_f + \cos 0)$$

$$= wR(1 - \cos\theta_f)$$

Altern., let's use rectangular coord.

$$d\vec{r} = \langle ds\cos\theta, ds\sin\theta\rangle$$

$$\int_c \vec{F}\cdot d\vec{r} = \int_0^{\theta_f} F_x\cos\theta\,ds$$



Example 6.9

4 engines, each with forward thrust of $322\cdot 10^3\,N$

plane flying at 250 m/s

in one second: $\dfrac{F\cdot\Delta s}{\Delta t} = \dfrac{322\cdot 10^3\cdot 250}{1}\,W$

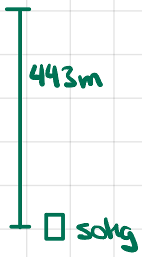
$$\frac{322\cdot 250\cdot 10^3\,W}{746\,W} = 107908\,Hp$$

Altern. formula

$$P_{AVG} = \frac{F\cdot\Delta s}{\Delta t} = F\cdot V_{AVG}$$

$$\lim_{\Delta t\to 0} P_{AVG} = F\cdot v$$

Example 6.10



$$W = 50\cdot 9.8\cdot 443$$

$$P_{AVG} = \frac{50\cdot 9.8\cdot 443}{15\cdot 60} = 241\,W$$

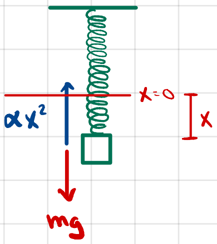
Altern. to reach top in 15min, $V_{AVG} = \dfrac{443}{15\cdot 60}$

$$\Rightarrow P_{AVG} = F\cdot V = 50\cdot 9.8\cdot\frac{443}{15\cdot 60} = 241\,W$$

$$241\,W = 0.241\,kW = 0.32\,Hp$$

Bridging Problem

$F_s = -\alpha x^2$



Between 0 and x_1, $W = \Delta K$

$$W = \int_0^{x_1} (mg - \alpha x^2)\,dx = mgx_1 - \frac{\alpha x_1^3}{3}$$

$$\Delta K = \frac{1}{2} m v_1^2$$

$$\Rightarrow \frac{1}{2} m v_1^2 = mgx_1 - \frac{\alpha x_1^3}{3}$$

$$\Rightarrow v_1 = \sqrt{2\left(gx_1 - \frac{\alpha x_1^3}{3m}\right)}$$

From x_1 on, work done and power achieved by spring

$$P = F_s \cdot v = -\alpha x_1^2 \cdot \sqrt{2\left(gx_1 - \frac{\alpha x_1^3}{3m}\right)}$$

alltern. $W = \int_{x_1}^{x_f} -\alpha x^2\,dx = -\frac{\alpha}{3}(x_f^3 - x_1^3)$

$mg - \alpha x^2 = m\ddot{x}$, we would need to solve for $x(t)$, calculate the Δt between x_1 and x_f, and then we'd have avg power.

max distance stretched

$$v_1 = 0 \Rightarrow x_2\left(g - \frac{\alpha x_2^2}{3m}\right) = 0 \Rightarrow x_2 = \sqrt{\frac{3mg}{\alpha}}$$

what happens at x_2?

$$F_s = \alpha \cdot \frac{3mg}{\alpha} = 3mg$$

The block does not remain at x_2.

6.49 $F(x) = 18.0\,N - 0.530\,\frac{N}{m}\cdot x$

$m = 6.00$ kg

speed after 14m?

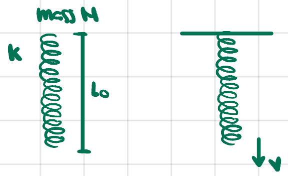
$W = \int_0^x (18 - 0.530x)\,dx = 18x - 0.265x^2 = \frac{1}{2}6v_f^2$

$\Rightarrow v_f = 8.166$ m/s

6.62 $F_x = -20 - 3x$

$W = \int_0^{6.9} (-20-3x)\,dx = -209.415$ J

6.93



$v(\ell) = a\ell$

$v(L) = aL = v$

$\Rightarrow a = \frac{v}{L}$

a) $\square\ d\ell$ $\quad dm = \frac{d\ell}{L}M$ $\quad v(\ell) = \frac{v}{L}\ell$

$\int \frac{1}{2}v_{d\ell}^2\,dm = \frac{1}{2}\frac{v^2}{L^2}M\int_0^L \ell^2\,d\ell = \frac{v^2M}{2\cancel{L^2}}\cdot\frac{\cancel{L^3}}{3} = \frac{mv^2}{6}$

b) $M = 0.243$ kg

$k = 3200$ N/m

$x_0 = -0.025$ m

$W = \frac{1}{2}3200\,(0.025)^2 = \frac{1}{2}\cdot 0.053\,v_f^2$

$\Rightarrow v_f = 6.14$ m/s (ignoring spring mass)

c) Assume the end of the spring (and ball) move at speed v at L_0.
we know $K_s = \frac{1}{2}Mv^2\cdot\frac{1}{4L_0}$ and $K_B = \frac{1}{2}mv^2$

the spring force is doing work

work done on ball + spring is just $\int_{x_0}^{0} -kx\,dx = \frac{kx_0^2}{2} = \frac{3200\cdot 0.025^2}{2} = 1$

$W = \Delta K \Rightarrow \frac{0.243v^2}{6} + \frac{0.053v^2}{2} = 1 \Rightarrow v = 3.86$ m/s

The individual kinetic energies

$\frac{1}{2}0.053\cdot 3.86^2 \approx 0.40$

$\frac{1}{6}0.243\cdot 3.86^2 \approx 0.60$

6.94 air resistance ~ v^2

$F_{air} = \alpha v^2 + \frac{\beta}{v^2}$, $\alpha, \beta > 0$

a) $\alpha = 0.30 \ \frac{Ns^2}{m^2}$ $\quad \beta = 3.5 \cdot 10^5 \ N\frac{m^2}{s^2}$

Airplane engine generates force that counterbalances F_{air}.

$F_{air} = 0.3v^2 + \frac{3.5 \times 10^5}{v^2}$

a) Assumption: fuel contains W_0 amount of energy: W_0 Joules of work can be done

$W_0 = Range \cdot F = R \cdot (0.3v^2 + \frac{3.5 \cdot 10^5}{v^2})$

Range $= R = vt$

$\Rightarrow W_0 = vt(0.3v^2 + \frac{3.5 \cdot 10^5}{v^2}) = t(0.3v^3 + \frac{3.5 \times 10^5}{v})$

$\cancel{\frac{dW_0}{dv}}^{0} = R(0.6v - \frac{7 \cdot 10^5}{v^3}) + \frac{dR}{dv}(0.3v^2 + \frac{3.5 \cdot 10^5}{v^2})$

$\frac{dR}{dv} = 0 \Rightarrow R(0.6v - \frac{7 \cdot 10^5}{v^3}) = 0 \Rightarrow v = 32.87 m/s$

Altern.

$W = F \cdot d = W_0$

min $F \Rightarrow$ max d

min F = min F_{air} $\Rightarrow$ $0.6v - \frac{7 \cdot 10^5}{v^3} = 0 \Rightarrow 0.6v^4 - 7 \cdot 10^5 = 0$

$\Rightarrow v = \sqrt[4]{\frac{7 \times 10^5}{0.6}} = 32.87 \ m/s$

b) $W_0 = vtF = (Fv)t = Power \cdot t$

$t = \frac{W_0}{Power} \Rightarrow t(v) = \frac{W_0}{0.3v^3 + \frac{3.5 \times 10^5}{v}}$ $\Rightarrow$ max $t(v)$ $\Leftrightarrow$ min Fv

minimize rate of work, ie minim. power $\Rightarrow$ slowest use of energy in fuel, max endurance (time airborne)

$\Rightarrow$ min $Fv \Rightarrow$ min $0.3v^3 + 3.5 \cdot 10^5 v^{-1} \Rightarrow 0.9v^2 - \frac{3.5 \cdot 10^5}{v^2} = 0$

$0.9v^4 = 3.5 \times 10^5 \Rightarrow v = 24.97 m/s$

$W_0 = vt(\alpha v^2 + \frac{\beta}{v^2})$

$0 = t(\alpha v^2 + \frac{\beta}{v^2}) + vt(2\alpha v - \frac{2\beta}{v^3})$

$$F_{air} = \alpha v^2 + \frac{\beta}{v^2}$$

$$W_0 = R(v) \cdot F_{air}(v)$$

$$R = v \cdot f(v)$$

$$\cancelto{0}{\frac{dW_0}{dv}} = \cancelto{0}{\frac{dR}{dv}} F_{air}(v) + R(v) \frac{dF_{air}}{dv} \Rightarrow R(v) F'_{air}(v) = 0 \Rightarrow F'_{air}(v) = 0$$

$$\frac{dR}{dv} = f(v) + v f'(v) = 0 \Rightarrow$$

$$W_0 = v f(v) F_{air}(v) \Rightarrow f(v) = \frac{W_0}{v \cdot F_{air}(v)}$$

$$f'(v) = \frac{-W_0 (v \cdot F_{air}(v))'}{(v \cdot F_{air}(v))^2} = 0 \Rightarrow (v \cdot F_{air}(v))' = 0$$

The two conditions

$$F_{air} + v F'_{air}$$

$$\frac{d F_{air}(v)}{dv} = 0 \qquad \frac{d^2 F_{air}(v)}{dv} = F''_{air}$$

$$\frac{d (v F_{air}(v))}{dv} = 0 \qquad \frac{d^2}{dv^2}(v F_{air}(v)) = F'_{air} + v F''_{air} + F'_{air} = 2F'_{air} + v F''_{air}$$