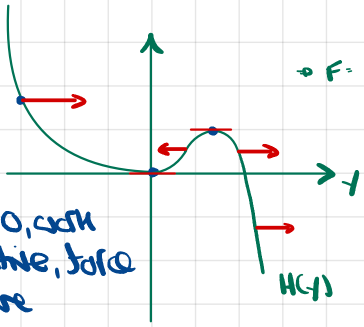
14.6 Spring Force Energy Diagram

Work $= \int_{x_1}^{x_2} F\,dx = -\Delta U = U_1 - U_2$

$\Rightarrow F = \frac{dU}{dx}$

Ex 14.1 pot. en. for a particle of mass m

$$U(x) = -U_1\left[\left(\frac{x}{x_1}\right)^3 - \left(\frac{x}{x_1}\right)^2\right]$$

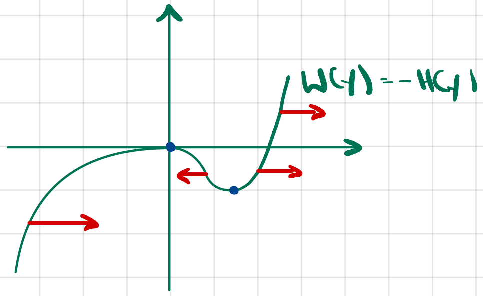


$\Delta U > 0$, work negative, force positive

from $y=0$ to here $\Delta U < 0$, so work is positive

a) $$H(x/x_1) = \frac{U(x)}{U_1} = \left(\frac{x}{x_1}\right)^2 - \left(\frac{x}{x_1}\right)^3$$

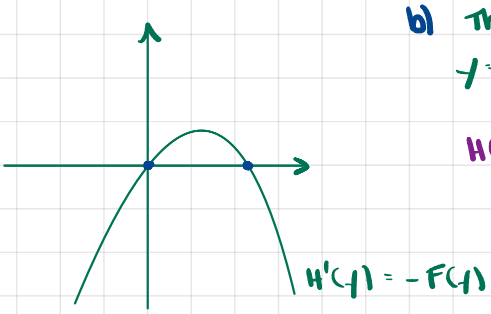
$$H(y) = y^2 - y^3$$



$$H'(y) = 2y - 3y^2 = 0 \Rightarrow 3y^2 - 2y = 0 \Rightarrow y(3y-2) = 0 \quad \begin{cases} y = 0 \\ y = \frac{2}{3} \end{cases}$$

$$F(x) = U'(x) = -U_1\left[3 \cdot \frac{1}{x_1}\left(\frac{x}{x_1}\right)^2 - \frac{2x}{x_1^2}\right] = 0$$

$$\Rightarrow \frac{3x^2}{x_1^3} - \frac{2x}{x_1^2} = \frac{3x^2 - 2x x_1}{x_1^3} = 0 \Rightarrow x(3x - 2x_1) = 0 \quad \begin{cases} x = 0 \\ x = \frac{2}{3}x_1 \end{cases}$$

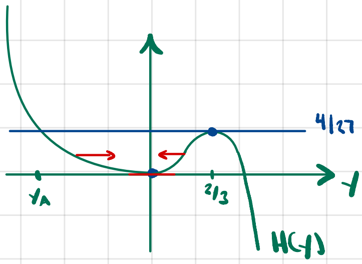


b) The force on the particle is zero when $y = \frac{x}{x_1} = 0$ or $y = \frac{x}{x_1} = \frac{2}{3}$.
$y = 0$ is stable, $y = 2/3$ is unstable.

$H(0) = 0 \qquad H(2/3) = \frac{4}{9} - \frac{8}{27} = \frac{4}{27}$

U is a potential energy fn. H is also a pot. en. fn.

c)



$K_A = 0$

$$\frac{U_A}{U_1} = H(y_A) = \frac{4}{27} = y^2 - y^3 \quad \begin{cases} y = -\frac{1}{3} \\ y = 2/3 \end{cases}$$

$$y^3 - y^2 + \frac{4}{27} = 0$$

Between $y = -\frac{1}{3}$ and $y = \frac{2}{3}$ the mass has periodic motion

$$E_{m_A} = \frac{4}{27} = E_{m_0} = \frac{mv^2}{2} \Rightarrow v = \pm\sqrt{\frac{8}{27m}}$$

d) In the periodic motion, energy is conserved.

$H(y_A) = \frac{4}{27}, \; y_A = -\frac{1}{3}$

$$y_A = \frac{x}{x_1} = -\frac{1}{3} \qquad H(y_A) = \frac{U(-x_1/3)}{U_1} = \frac{4}{27} \Rightarrow U(-x_1/3) = U_1 \cdot \frac{4}{27} = \frac{1}{2}mv_0^2$$

$$\Rightarrow v_0 = +\sqrt{\frac{8U_1}{27}}$$