2nd Law $\vec{F} = \dfrac{d\vec{p}}{dt}$

impulse $\displaystyle\int_{t_i}^{t_f} \vec{F}\,dt = \int_{t_i}^{t_f} \dfrac{d\vec{p}}{dt}\,dt = \Delta p$

avg force $\dfrac{impulse}{\Delta t} = \dfrac{\displaystyle\int_{t_i}^{t_f}\vec{F}\,dt}{\Delta t}$

momentum, system of particles

$\vec{p}_{sys} = \displaystyle\sum_{j=1}^{n} \vec{p}_j$

$\vec{F} = \displaystyle\sum_{j=1}^{n} \vec{F}_j$, F_j = force acting on j^{th} particle

2nd Law : $\vec{F}_j = \dfrac{d}{dt}\vec{p}_j \Rightarrow \vec{F} = \displaystyle\sum_{j=1}^{n}\dfrac{d}{dt}\vec{p}_j = \dfrac{d}{dt}\left(\displaystyle\sum_{j=1}^{n}\vec{p}_j\right) = \dfrac{d}{dt}\vec{p}_j$

Resultant Force on System = rate of change of system momentum

$\vec{F}_j^{\,T} = \vec{F}_j^{\,ext} + \vec{F}_j^{\,int} = \vec{F}_j^{\,ext} + \displaystyle\sum_{\substack{k=1\\k\neq j}}^{n}\vec{F}_{k,j}^{\,int}$

$\vec{F} = \displaystyle\sum_{j=1}^{n}\left[\vec{F}_j^{\,ext} + \displaystyle\sum_{\substack{k=1\\k\neq j}}^{n}\vec{F}_{k,j}^{\,int}\right]$

3rd Law $\Rightarrow \vec{F}_{k,j}^{\,int} = -\vec{F}_{j,k}^{\,int}$

$\Rightarrow \displaystyle\sum_{j=1}^{n}\vec{F}_j^{\,int} = 0$ (internal forces cancel)

$\Rightarrow \vec{F} = \displaystyle\sum_{j=1}^{n}\vec{F}_j^{\,ext} = \vec{F}^{\,ext} = \dfrac{d}{dt}\vec{p}^{\,sys}$

Resultant Force on System is sum of (only) external forces

If $\vec{F}^{ext} = 0$ then $\dfrac{d}{dt}\vec{p}^{sys} = 0 \Rightarrow \vec{p}(t_f) = \vec{p}(t_i)$

Kinetic Energy $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(mv)^2\cdot\dfrac{1}{m} = \dfrac{p^2}{2m}$

$\Delta K = \dfrac{1}{2}m(v_f^2 - v_i^2) = \dfrac{1}{2}m(v_{x_f}^2 + v_{y_f}^2 - v_{x_i}^2 - v_{y_i}^2)$

Necessary to integrate acceleration
Think of a as fn of x, not t

$\displaystyle\int_{x_i}^{x_f} a_x(x)\,dx = \int_{x_i}^{x_f}\dfrac{dv_x}{dt}\,dx = \int_{v(x_i)}^{v(x_f)} dv_x\, v_x = \dfrac{v_{x_f}^2 - v_{x_i}^2}{2}$

Work - KE Theorem

$F_x = ma_x$

$W = \displaystyle\int_{x_i}^{x_f} F_x\,dx = m\int_{x_i}^{x_f} a_x\,dx = \dfrac{1}{2}m(v_{x_f}^2 - v_{x_i}^2)$

$\Rightarrow W = \Delta K$

Power is defined as $P = \dfrac{dW}{dt} = \vec{F}\cdot\vec{v}$

F unit = $N = kg\,\dfrac{m}{s^2}$

$\Rightarrow P$ unit is $J\,s^{-1}$

v unit = $\dfrac{m}{s}$

W unit is $kg\,\dfrac{m^2}{s^2} = J$