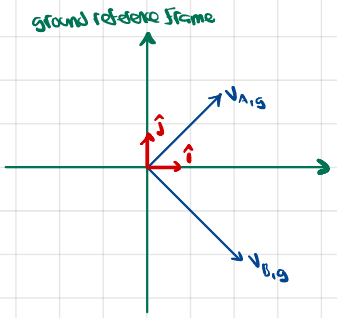
Ex 11.1

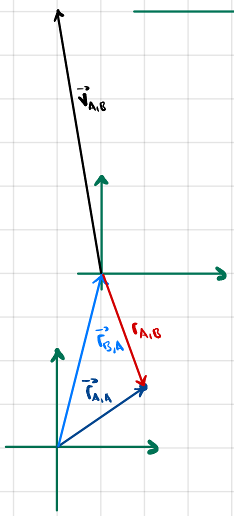


$|V_{A,g}| = 160 = (V_{A_x}^2 + V_{A_y}^2)^{1/2} = (2V_A^2)^{1/2} = \sqrt{2}\, V_A \Rightarrow V_A = \dfrac{160}{\sqrt{2}}$

$|V_{B,g}| = 200 \Rightarrow V_B = \dfrac{200}{\sqrt{2}}$

$V_{A,A} = \dfrac{160}{\sqrt{2}}(\hat{\imath} + \hat{j})$

$V_{B,A} = \dfrac{200}{\sqrt{2}}(\hat{\imath} - \hat{j})$



$\vec{r}_{A,A} = \vec{r}_{B,A} + \vec{r}_{A,B}$

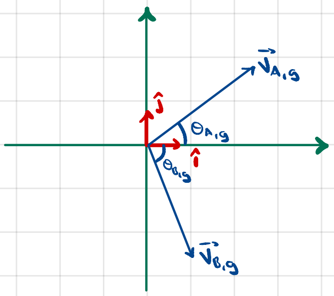
$\vec{r}_{A,B} = \vec{r}_{A,A} - \vec{r}_{B,A}$

$\vec{V}_{A,B} = \vec{V}_{A,A} - \vec{V}_{B,A} \Rightarrow \vec{V}_{A,B} = \dfrac{160}{\sqrt{2}}(\hat{\imath} + \hat{j}) - \dfrac{200}{\sqrt{2}}(\hat{\imath} - \hat{j})$

$= -\dfrac{40}{\sqrt{2}}\hat{\imath} + \dfrac{360}{\sqrt{2}}\hat{j}$

$|V_{A,B}| = \left(\dfrac{1600 + 129600}{2}\right)^{1/2}$

$= 40\sqrt{41} \approx 256.12$



$V_{A,g,x} = V_{A,g} \cos\theta_{A,g}$

$V_{A,g,y} = V_{A,g} \sin\theta_{A,g}$

$V_{B,g,x} = V_{B,g} \cos\theta_{B,g}$

$V_{B,g,y} = V_{B,g} \sin\theta_{B,g}$

$$\vec{V}_{A,B} = (V_{A,g}\cos\theta_{A,g} - V_{B,g}\cos\theta_{B,g})\hat{\imath} + (V_{A,g}\sin\theta_{A,g} - V_{B,g}\sin\theta_{B,g})\hat{j}$$

$\theta_{A,g} = \dfrac{\pi}{4}$

$\theta_{B,g} = -\dfrac{\pi}{4}$

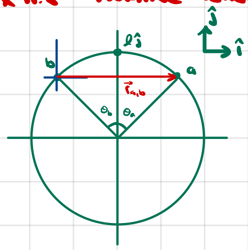
$\Rightarrow \vec{V}_{A,B} = \left(160 \cdot \dfrac{\sqrt{2}}{2} - 200\,\dfrac{\sqrt{2}}{2}\right)\hat{\imath} + \left(160\,\dfrac{\sqrt{2}}{2} + 200\,\dfrac{\sqrt{2}}{2}\right)\hat{j}$

$= -\dfrac{40}{\sqrt{2}}\hat{\imath} + \dfrac{360}{\sqrt{2}}\hat{j}$

note: Let α be the angle of $\vec{V}_{A,B}$, ie the angle of A's velocity vector as seen from B's reference frame.

$\tan\alpha = \dfrac{360/\sqrt{2}}{-40/\sqrt{2}} \Rightarrow \alpha = \tan^{-1}(-9) = (-1.46 + \pi)\,\text{rad} = 96.34°$

Ex 11.2 - Relative Motion and Polar Coord.



a) $\vec{r}_{A,A} = \vec{r}_{B,A} + \vec{r}_{A,B} = \vec{R} + \vec{r}_{A,B}$

$\vec{v}_{A,A} = \vec{v}_R + \vec{v}_{A,B}$

b) $\vec{r}_{a,b} = \vec{r}_a - \vec{r}_b$

$\vec{v}_{a,b} = \vec{v}_a - \vec{v}_b$

$\vec{r}_b = \ell \cdot \hat{r}_b$
$\vec{r}_a = \ell \cdot \hat{r}_a$

$\hat{r}_b = -\sin\theta_b \hat{i} + \cos\theta_b \hat{j}$
$\hat{r}_a = \sin\theta_b \hat{i} + \cos\theta_b \hat{j}$

$$\vec{v}_b = \ell \frac{d\hat{r}_b}{dt} = \ell(-\cos\theta_b \theta_b' \hat{i} - \sin\theta_b \theta_b' \hat{j})$$
$$= -\ell\theta_b'(\cos\theta_b \hat{i} + \sin\theta_b \hat{j})$$

$$\vec{v}_a = \ell \frac{d\hat{r}_a}{dt} = \ell(\cos\theta_b \theta_b' \hat{i} - \sin\theta_b \theta_b' \hat{j})$$
$$= \ell\theta_b'(\cos\theta_b \hat{i} - \sin\theta_b \hat{j})$$

$$\vec{v}_a - \vec{v}_b = \ell\theta_b'(\cos\theta_b \hat{i} - \sin\theta_b \hat{j}) + \ell\theta_b'(\cos\theta_b \hat{i} + \sin\theta_b \hat{j})$$

$$= \hat{i}\,\ell\theta_b'(\cos\theta_b + \cos\theta_b) + \hat{j}\theta_b'(\sin\theta_b - \sin\theta_b)$$

$$= 2\ell\omega\cos\theta_b \hat{i}$$

Ex 11.3 Recoil in Different Frames

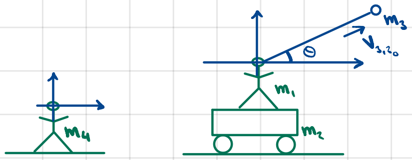
a) $\vec{v}_{3,4} = \vec{v}_{c,4} + \vec{v}_{3,c}$ law of addition of velocities

In the x direction no external forces act : we have $\Delta p = 0$.

In the y direction, a normal force prevents the cart from changing momentum.

$\vec{p}_{sys,4,0} = \vec{0}$

$\vec{p}_{sys,4,f,x} = m_3 v_{3,4,f,x} + (m_1+m_2) v_{c,4,f,x}$



$v_{3,4,f,x} = v_{c,4,f,x} + v_{3,c,f,x}$
$= v_{c,4,f,x} + v_0\cos\theta$

$\Rightarrow p_{sys,4,f,x} = m_3 v_{3,4,f,x} + (m_1+m_2) v_{c,4,f,x}$

$= m_3 v_{3,4,f,x} + (m_1+m_2)(v_{3,4,f,x} - v_0\cos\theta)$

$\Delta p_x = 0 \Rightarrow v_{3,4,f,x}(m_1+m_2+m_3) = (m_1+m_2)v_0\cos\theta$

$$\Rightarrow v_{3,4,f,x} = \frac{(m_1+m_2)v_0\cos\theta}{m_1+m_2+m_3}$$

$v_{c,4,f,x} = v_{3,4,f,x} - v_0\cos\theta$

$$\Rightarrow v_{c,4,f,x} = -\frac{m_3 v_0\cos\theta}{m_1+m_2+m_3}$$

$\Rightarrow p_{sys,4,f,x} = 0$

In the y direction,

$p_{sys,4,f,y} = m_3 v_{3,4,f,y} + (m_1+m_2) \cdot 0$

But a normal force acts on the cart.

$\Delta p_{sys,4,f,y} = m_3 v_{3,4,f,y}$

$v_{3,4,f,y} = v_{c,4,f,y} + v_{3,c,f,y}$
$= 0 + v_0\sin\theta$

$\Rightarrow v_{3,4,f,y} = v_0\sin\theta$

Note that momentum in the y direction increased

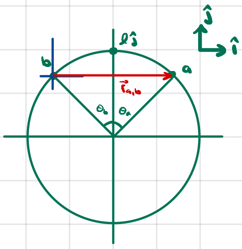
$\Delta p_{sys,4,f,y} = m_3 v_0 \sin\theta$

Because we know the velocity vector from ground reference, we know the angle an observer in this reference sees.

$$\vec{v}_{3,4,f} = \frac{(m_1+m_2)v_0\cos\theta}{m_1+m_2+m_3}\hat{i} + v_0\sin\theta\hat{j}$$

$$\tan\alpha = \frac{(m_1+m_2+m_3)v_0\sin\theta}{(m_1+m_2)v_0\cos\theta}$$

$$\alpha = \tan^{-1}\left[\frac{(m_1+m_2+m_3)\tan\theta}{m_1+m_2}\right]$$



$\hat{j}$
$\hat{i}$
$\ell\hat{j}$
b
a
$\vec{r}_{a,b}$
θ_b
θ_a