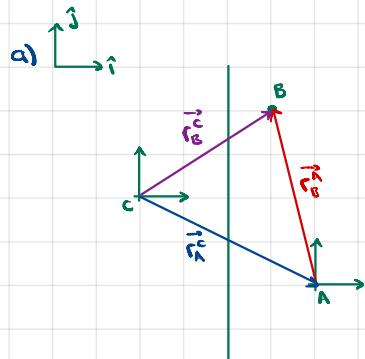
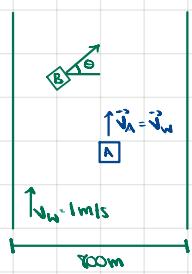


Pset6

Problem 1



$$\begin{aligned}\vec{r}_B^C &= \vec{r}_A^C + \vec{r}_B^A \\ \vec{v}_B^C &= \vec{v}_A^C + \vec{v}_B^A \\ \vec{v}_B^A &= \langle 0.5, 0 \rangle \\ \vec{v}_A^C &= \langle 0, 1 \rangle\end{aligned}$$

$\vec{v}_B^A$  direction  
 $\Rightarrow v_{B,i} = v_{A,i} + 0 \Rightarrow v_{B,i} - v_{A,i} = 1$   
 i.e. to A, B isn't moving in the  $\hat{i}$  dir  $\Rightarrow$  A and B have some velocity in  $\hat{j}$  dir

$\vec{v}_A^C$  direction

$$v_{B,i} = v_{A,i} + v_{B,i} = 0 + 0.5 \Rightarrow v_{B,i} = 0.5$$

$$\Rightarrow \vec{v}_B^C = \langle 0.5, 1 \rangle$$

b)  $\vec{r}_B^C = \int \vec{v}_B^C dt = \langle 0.5t, t \rangle$

Alternative, i.  $\tan \theta = 2 = \frac{d_1}{500} \Rightarrow d_1 = 1600 \text{ m}$

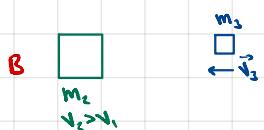
c) 1600 s

Problem 2

System: two carts with ball



$$P_A \quad m_2 v_2 + (m_1 + m_3) v_1$$



$$P_B \quad m_2 v_2 + m_3 v_3 + m_1 v_{1,f}$$

$$v_3 = v_1 - u$$

$$\Rightarrow m_2 v_2 + m_2 v_1 - m_3 u + m_1 v_{1,f}$$

$$P_A = P_B \Rightarrow \cancel{m_2 v_2 + (m_1 + m_3) v_1} = \cancel{m_2 v_2 + m_2 v_1 - m_3 u + m_1 v_{1,f}}$$

$$m_1 v_1 = -m_3 u + m_1 v_{1,f}$$

a)

$$v_{1,f} = \frac{m_1 v_1 + m_3 u}{m_1} = v_{1,f} = v_1 + \frac{m_3}{m_1} u$$

b)



$$P_C = (m_2 + m_3) v_{3,2} + m_1 v_{1,1} = m_2 v_2 + m_2 v_1 - m_3 u + m_1 v_{1,f}$$

$$v_{3,2} = v_1 + \frac{m_3}{m_1} u$$

$$\Rightarrow (m_2 + m_3) v_1 + \frac{(m_2 + m_3) m_3}{m_1} u = m_2 v_2 + m_2 v_1 - m_3 u$$

$$u \left[ \frac{(m_2 + m_3) m_3 + m_1 m_3}{m_1} \right] = m_2 v_2 + m_2 v_1 - m_2 v_1 - m_3 u$$

$$u \left[ \frac{m_2 (m_1 + m_2 + m_3)}{m_1} \right] = m_2 v_2 - m_2 v_1$$

$$u = \frac{m_2 (m_2 v_2 - m_2 v_1)}{m_2 (m_1 + m_2 + m_3)} = u = \frac{m_1 m_2 (v_2 - v_1)}{m_3 (m_1 + m_2 + m_3)}$$

### Problem 3

zero gravitational field

$$m_{\text{r,i}} = m_{\text{r,d}} + m_{\text{r,f}}$$

$$m_{\text{r,i}} = 2.81 \cdot 10^7 \text{ kg}$$

$$m_{\text{f,i}} = 2.46 \cdot 10^7 \text{ kg}$$

$$m_{\text{r,d}} = 0.35 \cdot 10^7 \text{ kg}$$

$$v = 3000 \text{ m/s}$$

$$\text{Fuel burned at constant rate} \Rightarrow \frac{dm_{\text{r,i}}}{dt} = \text{constant}$$

$$\text{Burn time} = 510 \text{ s}$$

a) model of change in momentum between  $t$  and  $t+\Delta t$

$$t: m(t)v(t)$$

$$t+\Delta t: (m(t) + \Delta m)v(t+\Delta t) - \Delta m(v(t+\Delta t) + u)$$

$$\Delta p = 0 = (m(t) + \Delta m)v(t+\Delta t) - \Delta m(v(t+\Delta t) + u) - m(t)v(t)$$

$$\frac{\Delta p}{\Delta t} = m(t) \frac{\Delta v}{\Delta t} - \frac{\Delta m}{\Delta t} u = 0 \Rightarrow m(t) \frac{\Delta v}{\Delta t} = \frac{\Delta m}{\Delta t} u$$

$$\Rightarrow v(t) = v(0) - u \ln\left(\frac{m}{m_0}\right)$$

After 510 s, mass is  $M_{\text{r,f}}$

$$v(510) = -3000 \ln\left(\frac{M_{\text{r,f}}}{M_{\text{r,i}}}\right) = -3000 \ln\left(\frac{0.35}{2.81}\right) = 6249.019 \text{ m/s}$$

b) Two burn stages

stage 1

$$u = 3000 \text{ m/s}$$

$$m_{\text{r,i,1}} = 2.03 \cdot 10^7 \text{ kg}$$

$$\text{burn time} = 150 \text{ s}$$

After stage 1, total rocket mass is  $0.78 \cdot 10^7 \text{ kg}$

$$v(t) - v(0) = -u \ln\left(\frac{m_{\text{r,f}}}{m_0}\right) = -3000 \ln\left(\frac{0.78}{2.81}\right) = 3844.94 \text{ m/s}$$

c) Discarded parts have mass  $1.4 \cdot 10^6 \text{ kg}$

$$\text{Initial stage two dry mass} = 2.1 \cdot 10^6 \text{ kg}$$

$$\therefore \text{ " fuel mass} = 4.3 \cdot 10^6 \text{ kg}$$

$$\text{burn time} = 360 \text{ s}$$

$$v(t) - v(0) = -3000 \ln\left(\frac{2.1 \cdot 10^6}{6.4 \cdot 10^6}\right) = 3343.08 \text{ m/s}$$

$$d) \text{ final speed} = 7188.02 \text{ m/s}$$

$$(2.46 \cdot 10^7 \text{ kg})$$

e) In a) all the fuel was used to accelerate a dry mass of  $0.35 \cdot 10^7 \text{ kg}$ .

In the two stage setup,  $2.03 \cdot 10^7 \text{ kg}$  of fuel was used to accelerate  $0.35 \cdot 10^7 \text{ kg}$  of dry mass, then  $0.43 \cdot 10^7 \text{ kg}$  of fuel was used to accelerate a substantially lighter  $0.21 \cdot 10^7 \text{ kg}$  of dry mass.

$\Rightarrow$  more acceleration  $\Rightarrow$   $\uparrow$  final velocity

## Problem 4 - Falling Drop

Raindrop mass  $m_r$



$$\frac{dm_r}{dt} = km_r v_r$$

$m_r$ : instantaneous mass

$k$  [ $\text{m}^{-1}$ ]

a)

time

momentum

$$t \quad m_r(t) v_r(t)$$

$t + \Delta t$

$$(m_r(t) + \Delta m_r) v_r(t + \Delta t)$$

$$\Delta p = (m_r(t) + \Delta m_r) v_r(t + \Delta t) - m_r(t) v_r(t)$$

$$= m_r(t) \Delta v_r + \Delta m_r v_r(t + \Delta t)$$

$$\frac{\Delta p}{\Delta t} = m_r(t) \frac{\Delta v_r}{\Delta t} + \frac{\Delta m_r}{\Delta t} v_r(t + \Delta t)$$

$$\text{Fext} = \frac{dp}{dt} = m_r(t) \frac{dv_r}{dt} + \frac{dm_r}{dt} v_r(t)$$

$$m_r(t) g = m_r(t) \frac{dv_r}{dt} + \frac{dm_r}{dt} v_r(t)$$

$$\frac{dm_r}{dt} = km_r v_r \quad \Rightarrow \quad m_r(t) g = m_r(t) \frac{dv_r}{dt} + km_r(t) v_r^2(t) \Rightarrow \frac{dv_r}{dt} = g - kv_r^2(t)$$

$$= \frac{1}{g - kv_r^2} dv_r \cdot dt$$

$$= \int \frac{1}{k(g/k - v_r^2)} dv_r = \frac{1}{k} \int \frac{(g/k)^{1/2} \cos \theta d\theta}{(g/k)(1 - \sin^2 \theta)} = \frac{1}{k} \left(\frac{g}{k}\right)^{1/2} \left(\frac{k}{g}\right) \int \frac{\cos \theta}{1 - \sin^2 \theta} d\theta = \left(\frac{k}{g}\right)^{1/2} \frac{1}{k} \ln(1 - \sin^2 \theta) (-\frac{1}{2}) = -\frac{\ln(1 - \sin^2 \theta)}{2\sqrt{gk}} = -\frac{\ln(1 - (kv_r)^2)}{2\sqrt{gk}}$$

$$v_r = \sqrt{\frac{g}{k}} \sin \theta$$

$$dv_r = (g/k)^{1/2} \cos \theta d\theta$$

$$\Rightarrow v_r(t) = -\frac{e^{-2\sqrt{gk}t} - 1}{\sqrt{mg}} \quad \Rightarrow \quad v_r(t) = -\frac{e^{-2\sqrt{gk}t} - 1}{\sqrt{k}}$$

$$v_r(0) = 0 \Rightarrow C = 0$$

Interpretation: mass of raindrop increases exponentially. At time  $t$  mass is  $m(t)$ , gravitational force is  $m(t)g$ . At time  $t + \Delta t$  mass is  $m + \Delta m$ , where  $\Delta m$  is proportional to  $k m(t) v(t)$ .  $\Delta m$  is converted to  $v(t + \Delta t)$ .

Final speed by  $\Delta m g$

$$\text{b) } \lim_{t \rightarrow \infty} v_r(t) = \lim_{t \rightarrow \infty} -\frac{e^{-2\sqrt{gk}t} - 1}{\sqrt{mg}} \cdot \sqrt{\frac{g}{k}} = \sqrt{\frac{g}{k}}$$

## Example

$$g = 9.8 \text{ m/s}^2$$

$$k = 0.1 \text{ kg}^{-1}$$

$$v(t) = -9.89 e^{-1.9t} + 9.89$$

### Problem 5 - Moving Vehicle and Falling Rain



\* in the  $\hat{y}$  direction  $t: \Delta m_r v_b$

$$t+\Delta t: 0$$

$$\frac{\Delta p}{\Delta t} = -\frac{\Delta m_r}{\Delta t} v_b = -b v_b \Rightarrow \frac{dp}{dt} = -b v_b = F_{ext}$$

Force exerted by the car on  
the rain to stop the rain in  $\hat{y}$  dir.

time	momentum
------	----------

$$t: m_0 v(t)$$

$$t+\Delta t: (m_0 + \Delta m_r) v(t+\Delta t) = (m_0 + b\Delta t) v(t+\Delta t)$$

a)

$$\Delta p = m_0 \Delta v + b \Delta t v(t+\Delta t)$$

$$\frac{\Delta p}{\Delta t} = m_0 \frac{\Delta v}{\Delta t} + b v(t+\Delta t)$$

$$\frac{dp}{dt} = m_0 \frac{dv}{dt} + b v(t) = 0$$

YES! Similar eq. to the one from problem 4. There,  $\Delta m/dt$  modeled exponential growth. Here  $\Delta m/dt$  is constant.  
Also, there are no forces exerted by car + rain in the  $\hat{x}$  direction here.

exponential decay

$$\frac{dv}{dt} = -\frac{b}{m_0} v(t) \Rightarrow \frac{1}{v} dv = -\frac{b}{m_0} dt$$

$$\ln(v/v_0) = -\frac{b}{m_0}(t_0 - t_0)$$

$$b) v(t) = v_0 + e^{-\frac{bt}{m_0}}$$