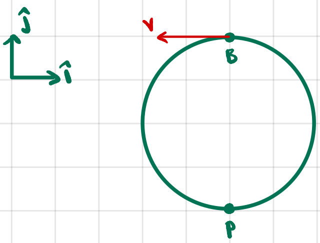
Pset 11

Problem 1 - Bug Walking on Pivoted Ring

setup: ring mass m_1 on frictionless table, pivoted at certain point on the rim. A bug mass m_2 walks around ring rim at constant speed relative to the ring.

External forces are gravity and normal force from table. Both apply torque but cancel each other. The resultant force in $\hat{n}$ direction is zero.

The bug moves by applying a force on the ring. The ring reacts w/ a force on the bug, and about pivot we have a pair of internal torques.



a) $\tau_p^{sys} = 0$

$\vec{L}_{p,Bug,i} = \vec{0} \times m_{Bug,i} = \vec{0}$

$\vec{L}_{p,ring,i} = I_{p,ring} \times m_1 \cdot \vec{0} = \vec{0}$

$\vec{L}_{p,sys,i} = \vec{0}$

$v_{B,p} = v + v_{r,p}$

$\vec{L}_{P,Bug,f} = I_{p,Bug} \cdot \vec{\omega}_{p,B} = m_2 \cdot 4R^2 \cdot \omega_{p,B} \hat{n}$

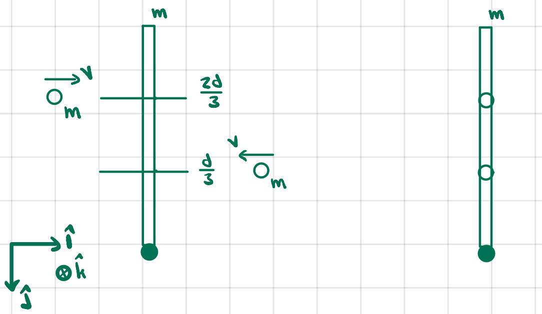
Altern. $(2R\hat{j}) \times m_2 \vec{v}_{p,B}$

$= (2R\hat{j}) \times m_2 (\vec{v}_{r,B} + \vec{v}_{p,r})$

The ring has a certain $\vec{\omega}_{p,r}$. $v_{p,r} = 2R \cdot \omega_{p,r}$ at the halfway point

$= 2R\hat{j} \times m_2 (-v\hat{i}$

Problem 2 - A Rigid Rod



a) $\vec{L}_{sys,i} = \left(-\frac{2d}{3}\hat{j}\right) \times mv\hat{i} + \left(-\frac{d}{3}\hat{j}\right) \times m(-v\hat{i})$

$= \frac{2dmv}{3}\hat{k} - \frac{mdv}{3}\hat{k} = \frac{mdv}{3}\hat{k}$

$\vec{L}_{sys,f} = I_{sys,p} \cdot \omega_f \hat{k} = \frac{\omega_f \cdot 8md^2}{9}\hat{k}$

$I_{sys,p} = \left[\frac{md^2}{12} + m\frac{d^2}{4}\right] + m\frac{d^2}{9} + m\frac{4d^2}{9}$

$= \frac{3md^2 + md^2 + 4md^2}{9} = \frac{8md^2}{9}$

$\vec{L}_{sys,i} = \vec{L}_{sys,f} \Rightarrow \frac{\cancel{m}\cancel{d}v}{\cancel{3}} = \omega_f \frac{8\cancel{m}d^{\cancel{2}}}{\cancel{9}\,3}$

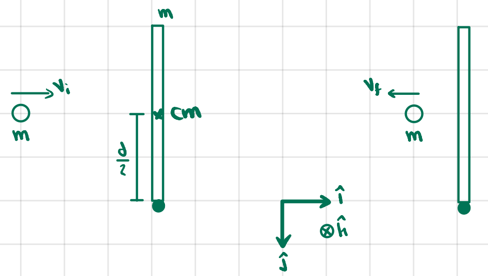
$\Rightarrow \omega_f = \frac{3v}{8d}$

b) $K_i = 2 \cdot \frac{mv^2}{2} = mv^2$

$K_f = \frac{I_{sys,p}\,\omega_f^2}{2} = \frac{\cancel{8}m\cancel{d^2}}{\cancel{9}} \cdot \frac{\cancel{9}v^2}{\cancel{64}\,8\,\cancel{d^2}} \cdot \frac{1}{2} = \frac{mv^2}{16}$

$\frac{K_f - K_i}{K_i} = -\frac{15}{16}$

Problem 3 - Elastic Collision Between Ball and Pivoted Rod



elastic collision

$$K_i = \frac{m v_i^2}{2}$$

$$K_f = \frac{m v_f^2}{2} + \frac{I_p \omega_f^2}{2} = \frac{m v_f^2}{2} + \frac{m d^2}{3} \cdot \omega_f^2 \cdot \frac{1}{2}$$

$$I_p = \frac{m d^2}{12} + m \frac{d^2}{4} = \frac{m d^2}{3}$$

$$K_i = K_f \Rightarrow \frac{\cancel{m} v_i^2}{\cancel{2}} = \frac{\cancel{m} v_f^2}{\cancel{2}} + \frac{\cancel{m} d^2}{3} \cdot \omega_f^2 \cdot \frac{1}{\cancel{2}}$$

$$v_i^2 = v_f^2 + \frac{d^2 \omega_f^2}{3}$$

$$\vec{L}_{sys,i} = \left(-\frac{d}{2}\hat{j}\right) \times m v_i \hat{i} = \frac{m d v_i}{2} \hat{k}$$

$$\vec{L}_{sys,f} = \left(-\frac{d}{2}\hat{j}\right) \times \left(-m v_f \hat{i}\right) + \frac{m d^2}{3} \omega_f \hat{k}$$

$$= -\frac{m d v_f}{2} \hat{k} + \frac{m d^2 \omega_f}{3} \hat{k}$$

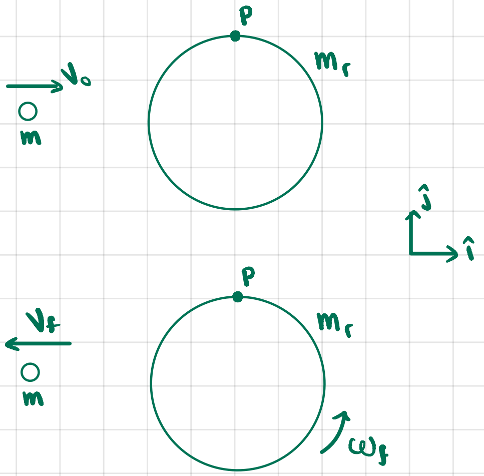
$$\Delta \vec{L} = 0 \Rightarrow \frac{\cancel{m} d v_i}{2} = \frac{\cancel{m} d^2 \omega_f}{3} - \frac{\cancel{m} d v_f}{2}$$

$$\Rightarrow 3 d v_i = 2 d^2 \omega_f - 3 d v_f$$

2 eq., 2 unknowns: ω_f, v_f

$$\Rightarrow v_f = \frac{v_i}{7} \qquad \omega_f = \frac{12 v_i}{7 d}$$

Problem 4 - Elastic Collision of Object and Pivoted Ring



$$I_{p,r} = m_r R^2 + m_r R^2 = 2 m_r R^2$$

$$\Delta K = 0 \Rightarrow \frac{m v_0^2}{2} = \frac{m v_f^2}{2} + \frac{\cancel{2} m_r R^2 \cdot \omega_f^2}{\cancel{2}}$$

$$\frac{m v_0^2}{2} = \frac{m v_f^2}{2} + m_r R^2 \omega_f^2$$

$$\vec{L}_{p,m,i} = (-R\hat{i} - R\hat{j}) \times m v_0 \hat{i} = -Rmv_0(-\hat{k})$$
$$= Rmv_0 \hat{k}$$

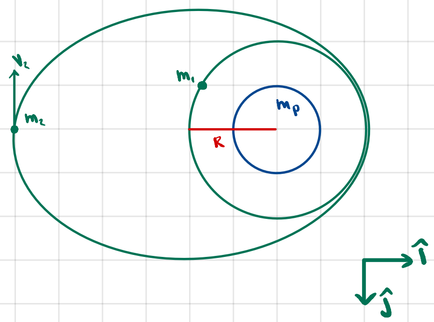
$$\vec{L}_{p,m,f} = -Rmv_f \hat{k} + 2 m_r R^2 \omega_f \hat{k}$$

$$\Delta L = 0 \Rightarrow Rmv_0 = -Rmv_f + 2 m_r R^2 \omega_f$$

$$\Rightarrow v_f = \frac{v_0(2m_r - m)}{2m_r + m}$$

$$\omega_f = \frac{2mv_0}{R(2m_r + m)}$$

5. A Spaceship and a Planet



Spaceship 1

$$\vec{F}_{p,1} = -\frac{Gm_p\cancel{m_1}}{R^2}\hat{r} = m_1\vec{a}_R = \cancel{m_1}\cdot(-R\omega_1^2\hat{r})$$

$$\frac{Gm_p}{R^2} = R\omega_1^2 = \cancel{R}\cdot\frac{v_1^2}{R^{\cancel{2}}}$$

uniform circular motion.

$$\Rightarrow v_1 = \sqrt{\frac{Gm_p}{R}}$$

Spaceship 2

$$\vec{L}_{p,i} = (-3R\hat{i})\times m_2v_2(-\hat{j})$$
$$= 3Rm_2v_2\hat{k}$$

$$\vec{L}_{p,f} = R\hat{i}\times m_2v_p\hat{j} = Rm_2v_p\hat{k}$$

$$\Delta L = 0 \Rightarrow 3\cancel{R}\cancel{m_2}v_2 = \cancel{R}\cancel{m_2}v_p \Rightarrow v_p = 3v_2$$

$$\int_{r_1}^{r_2} -\frac{Gm_2m_p}{r^2}\cdot dr = Gm_2m_p\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = -\Delta U$$

$$U(r) = -\frac{Gm_2m_p}{r}$$

$$E_{m,i} = -\frac{Gm_2m_p}{3R} + \frac{m_2v_2^2}{2}$$

$$E_{m,f} = -\frac{Gm_2m_p}{R} + \frac{m_2v_p^2}{2}$$

$$\Delta E_m = 0 \Rightarrow -\frac{Gm_2m_p}{3R} + \frac{m_2v_2^2}{2} = -\frac{Gm_2m_p}{R} + \frac{m_2v_p^2}{2}$$

$$-\frac{Gm_2m_p}{3R} + \frac{3Gm_2m_p}{3R} = \frac{m_2\cdot 9v_2^2}{2} - \frac{m_2v_2^2}{2}$$

$$\frac{\cancel{2}G\cancel{m_2}m_p}{3R} = \frac{\cancel{8}^{\cancel{4}\,2}\cancel{m_2}v_2^2}{\cancel{2}} \Rightarrow v_2 = \sqrt{\frac{Gm_p}{6R}}$$

$$v_p = \sqrt{\frac{3Gm_p}{2R}}$$

Therefore, at the point of m_2's nearest approach,

$$v_1 = \sqrt{\frac{Gm_p}{R}} \qquad v_p = \sqrt{\frac{3Gm_p}{2R}}$$

$$\Rightarrow v_p = \sqrt{3/2}\,v_1 \Rightarrow v_p > v_1$$

Spaceship 2 has to slow down.

The change in speed is $v_1 - v_p$

$$= \sqrt{\frac{Gm_p}{R}}\left(1-\sqrt{3/2}\right)$$

$$= -0.22\sqrt{\frac{Gm_p}{R}}$$