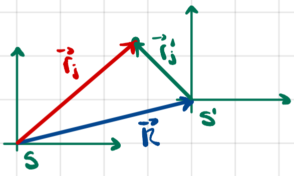
15.2 Reference Frames, Relative Velocity



$\vec r_j = \vec R + \vec r_j'$

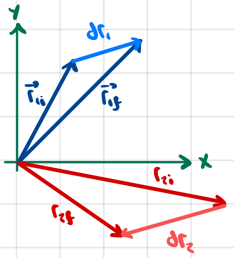
relative velocity (aka boost velocity) $\vec V = \frac{d\vec R}{dt}$

$\vec V$ constant ⇒ $\vec A = \frac{d\vec V}{dt} = \vec 0$ ⇒ zero relative acceleration

⇒ S and S' are relatively inertial reference frames

$\vec v_j = \vec v_j' + \vec V$

15.2.1 Relative Velocities



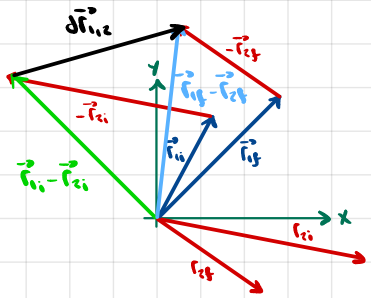
$\vec r_{1,2} = \vec r_1 - \vec r_2$ is relative position of body 1 w/ respect to body 2

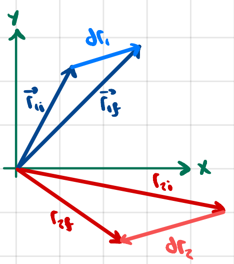
$d\vec r_{1,2} = d\vec r_1 - d\vec r_2$ is relative displacement of the two bodies.

$= \vec r_{1f} - \vec r_{1i} - (\vec r_{2f} - \vec r_{2i})$

$= (\vec r_{1f} - \vec r_{2f}) - (\vec r_{1i} - \vec r_{2i})$

Let's see $d\vec r_{1,2}$ in two ways





$d\vec r_{1,2}$ is the change in the relative position.
we either calculate it directly by calculating the difference between position vectors. or we sum the two changes in position to obtain the total vector change.

relative velocity of body 1 w/ respect to body 2 is

$$\vec v_{1,2} = \frac{d\vec r_{1,2}}{dt} = \frac{d\vec r_1}{dt} - \frac{d\vec r_2}{dt} = \vec v_1 - \vec v_2$$

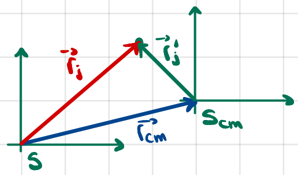
note that our calculations of $\vec r_{1,2}$, $d\vec r_{1,2}$, and $\vec v_{1,2}$ were all from reference frame S.

$$\vec v_{1,2} = \vec v_1 - \vec v_2 = (\vec v_1' + \vec V) - (\vec v_2' + \vec V) = \vec v_1' - \vec v_2' = \vec v_{1,2}'$$

⇒ rel. velocity between the two bodies is independent of reference frame if the latter are relatively inertial.

15.2.2 Center-of-Mass Reference Frame

Consider a system of particle masses.
As previously we have ref. frame S. $\vec r_{cm}$ denotes the center of mass of the system, and this is now the origin of the second ref. frame S_{cm}.



$\vec r_j = \vec r_j' + \vec r_{cm}$ ⇒ $\vec r_j' = \vec r_j - \vec r_{cm}$

$\vec v_j = \vec v_j' + \vec v_{cm}$ ⇒ $\vec v_j' = \vec v_j - \vec v_{cm}$

consider two particles. Their velocities in CM ref. frame are:

$$\vec v_1' = \vec v_1 - \vec v_{cm} = \vec v_1 - \frac{m_1\vec v_1 + m_2\vec v_2}{m_1+m_2} = \frac{m_2\vec v_1 - m_2\vec v_2}{m_1+m_2}$$

$$= \frac{m_2}{m_1+m_2}(\vec v_1 - \vec v_2) \Rightarrow \vec v_1' = \frac{\mu}{m_1}\vec v_{1,2}, \quad \mu = \frac{m_1 m_2}{m_1+m_2}$$

similarly

$$\vec v_2' = \vec v_2 - \vec v_{cm} = \vec v_2 - \frac{m_1\vec v_1 + m_2\vec v_2}{m_1+m_2} = \frac{-m_1\vec v_1 + m_1\vec v_2}{m_1+m_2}$$

$$= \frac{m_1}{m_1+m_2}(-\vec v_1 + \vec v_2) \Rightarrow \vec v_2' = \frac{-\mu}{m_2}\vec v_{1,2}$$

momentum of system is zero in CM frame

$$m_1\vec v_1' + m_2\vec v_2' = \cancel{m_1}\frac{\mu}{\cancel{m_1}}\vec v_{1,2} + \cancel{m_2}\frac{(-\mu)}{\cancel{m_2}}\vec v_{1,2} = \vec 0$$

15.2.3 Kinetic Energy in CM Frame

$$K_{cm} = \frac{1}{2} m_1 \vec{v}_1' \cdot \vec{v}_1' + \frac{1}{2} m_2 \vec{v}_2' \cdot \vec{v}_2'$$

if we sub in expr. for $\vec{v}_1'$ and $\vec{v}_2'$ in terms of $\vec{v}_{1,2}$ we obtain

$$K_{cm} = \frac{1}{2} \mu v_{1,2}^2 \ , \ \mu = \frac{m_1 m_2}{m_1 + m_2}$$

15.2.4 ΔK and Rel. Inertial Ref. Frames

K in frame S

$$K_s = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} m_1 (\vec{v}_1' + \vec{v}_{cm}) \cdot (\vec{v}_1' + \vec{v}_{cm}) + \frac{1}{2} m_2 (\vec{v}_2' + \vec{v}_{cm}) \cdot (\vec{v}_2' + \vec{v}_{cm})$$

$$= \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 + \frac{1}{2}(m_1 + m_2) v_{cm}^2 + \cancelto{0}{(m_1 v_1' + m_2 v_2')} v_{cm}$$

system momentum = 0 in CM frame

$$\Rightarrow K_s = K_{cm} + \frac{1}{2}(m_1 + m_2) v_{cm}^2$$

$$\Rightarrow K_s = \frac{1}{2} \mu v_{1,2}^2 + \frac{1}{2}(m_1 + m_2) v_{cm}^2$$

v_{cm} is constant. The frames are relatively inertial, they don't accelerate rel. to each other. The second term is constant. Therefore **the change in kinetic energy is the same in both frames**

$$\Delta K_s = \Delta K_{CM} = \frac{1}{2} \mu (v_{1,2,f}^2 - v_{1,2,i}^2)$$

15.3 Characterizing Collisions

coeff. of restitution: $e = \dfrac{\text{rel. vel. } f}{\text{rel. vel. } i}$

Elastic Collisions: $\Delta K = 0 \iff v_{1,2,f} = v_{1,2,i} \iff e = 1$

Inelastic coll.: $\Delta K < 0 \iff e < 1$

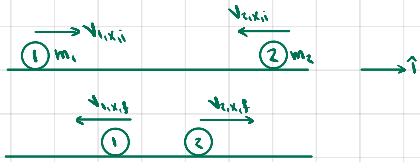
Totally Inelastic coll.: $e = 0$ because final rel. vel. is zero.

$$\Delta K = -\frac{1}{2} \mu v_{rel,i}^2 = -\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_{rel,i}^2$$

Superelastic coll.: $e > 1 \iff \Delta K > 0$

15.4 One Dimensional Collision Between Two Objects

setup



assumptions

- -> no forces during collision except interaction forces between the two objects
- -> reference frame: "laboratory"

-> $F_{ext} = 0 \Rightarrow \Delta p = 0$

$$\Rightarrow m_1 v_{1,x,i} + m_2 v_{2,x,i} = m_1 v_{1,x,f} + m_2 v_{2,x,f} \qquad (1)$$

-> elastic collision $\Rightarrow \Delta K = 0$

$$\Rightarrow \frac{1}{2} m_1 v_{1,x,i}^2 + \frac{1}{2} m_2 v_{2,x,i}^2 = \frac{1}{2} m_1 v_{1,x,f}^2 + \frac{1}{2} m_2 v_{2,x,f}^2 \qquad (2)$$

rewrite (1) and (2) $\qquad m_1 \cdot 2 v_{1,x,i} = - m_2 \cdot 2 v_{2,x,i}$

$$m_1 (v_{1,x,i} - v_{1,x,f}) = m_2 (v_{2,x,f} - v_{2,x,i}) \qquad (3)$$

$$m_1 (v_{1,x,i}^2 - v_{1,x,f}^2) = m_2 (v_{2,x,f}^2 - v_{2,x,i}^2)$$

$$m_1 (v_{1,x,i} + v_{1,x,f})(v_{1,x,i} - v_{1,x,f}) = m_2 (v_{2,x,f} + v_{2,x,i})(v_{2,x,f} - v_{2,x,i}) \qquad (4)$$

Divide (4) by (3)

$$v_{1,x,i} + v_{1,x,f} = v_{2,x,f} + v_{2,x,i}$$

$$\Rightarrow v_{1,x,i} - v_{2,x,i} = -(v_{1,x,f} - v_{2,x,f})$$

$$\Rightarrow v_{1,2,i} = - v_{1,2,f}$$

$$\Rightarrow v_i^{rel} = - v_f^{rel}$$ (1D energy-momentum principle)

recap/interpretation

Relative speed changes direction by 180°. We derived this result by using conservation of momentum and energy.

Previously we had also calculated the ΔK in a frame that is rel. inertial w/ respect to the CM frame. The result showed that $\Delta K = 0 \Leftrightarrow v_{1,2,i}^2 = v_{1,2,f}^2$.

solve for $v_{1,x,f}$ and $v_{2,x,f}$

$$v_{2,x,f} = v_{1,x,f} + v_{1,x,i} - v_{2,x,i}$$

sub $v_{2,f,x} = v_{1,x,f} + v_{1,x,i} - v_{2,x,i}$ into (1)

$$m_1 v_{1,x,i} + m_2 v_{2,x,i} = m_1 v_{1,x,f} + m_2 (v_{1,x,f} + v_{1,x,i} - v_{2,x,i})$$

solve for $v_{1,x,f} \Rightarrow v_{1,x,f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1,x,i} + \dfrac{2 m_2}{m_1 + m_2} v_{2,x,i}$

then solve for $v_{2,x,f} \Rightarrow v_{2,x,f} = \dfrac{m_2 - m_1}{m_2 + m_1} v_{2,x,i} + \dfrac{2 m_1}{m_1 + m_2} v_{1,x,i}$

what happens if $m_1 >> m_2$?

$$\lim_{m_1 \to \infty} v_{1,x,f} = v_{1,x,i}$$

$$\lim_{m_1 \to \infty} v_{2,x,f} = 2 v_{1,x,i} - v_{2,x,i}$$

$$= v_{1,x,i} + (v_{1,x,i} - v_{2,x,i}) = v_{1,x,i} + v_{1,2,i}$$

$$v_{1,x,f} - v_{2,x,f} = - v_{1,x,i} - v_{2,x,i}$$

ie, m_1 basically keeps going at original speed, m_2 had its original veloc. reversed and then added to double m_1's initial velocity.

$$v_{2,x,f} = \frac{m_1 (v_{1,x,i} - v_{1,x,f}) + m_2 v_{2,x,i}}{m_2}$$

$$m_1 (v_{1,x,i}^2 - v_{1,x,f}^2) = m_2 \left[\left(\frac{m_1 (v_{1,x,i} - v_{1,x,f}) + m_2 v_{2,x,i}}{m_2} \right)^2 - v_{2,x,i}^2 \right]$$

15.4.2 1D Collision Between Two objects, Center-of-Mass Reference Frame

Let's analyze the same collision from the CM reference.

$$v_{x,cm} = \frac{m_1 v_{1,x,i} + m_2 v_{2,x,i}}{m_1 + m_2}$$

From CM ref frame,

$$v'_{1,x,i} = v_{1,x,i} - v_{x,cm} = \frac{m_2(v_{1,x,i} - v_{2,x,i})}{m_1 + m_2}$$

$$v'_{2,x,i} = v_{2,x,i} - v_{x,cm} = \frac{m_1(v_{1,x,i} - v_{2,x,i})}{m_1 + m_2}$$

As noted previously, momentum from CM frame is zero before and after the collision.

$$m_1 v'_{1,x,i} + m_2 v'_{2,x,i} = 0 \Rightarrow m_1 v'_{1,x,i} = -m_2 v'_{2,x,i}$$

$$m_1 v'_{1,x,f} + m_2 v'_{2,x,f} = 0 \Rightarrow m_1 v'_{1,x,f} = -m_2 v'_{2,x,f}$$

elastic collision $\Rightarrow \Delta K = 0$

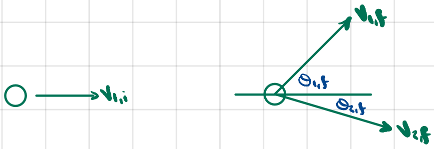
$$\frac{1}{2} m_1 v_{1,x,i}^2 + \frac{1}{2} m_2 v_{2,x,i}^2 = \frac{1}{2} m_1 v_{1,x,f}^2 + \frac{1}{2} m_2 v_{2,x,f}^2$$

$$m_1(v_{1,x,i}^2 - v_{1,x,f}^2) = m_2(v_{2,x,f}^2 - v_{2,x,i}^2)$$

$$m_1(v_{1,x,i} + v_{1,x,f})(v_{1,x,i} - v_{1,x,f})$$

$$= m_2(v_{2,x,f} + v_{2,x,i})(v_{2,x,f} - v_{2,x,i})$$

15.6 2D Elastic Collisions



$$\vec{p}_i^{\,sys} = m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i}$$

$$\vec{v}_{1,i} = v_{1,i}\hat{i} \qquad \Rightarrow \qquad \vec{p}_i^{\,sys} = m_1 v_{1,i}\hat{i}$$
$$v_{2,i} = 0$$

$$\vec{p}_f^{\,sys} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}$$

$$\vec{v}_{1,f} = v_{1,f}\cos\theta_{1,f}\,\hat{i} + v_{1,f}\sin\theta_{1,f}\,\hat{j}$$
$$\vec{v}_{2,f} = v_{2,f}\cos\theta_{2,f}\,\hat{i} - v_{2,f}\sin\theta_{2,f}\,\hat{j}$$

$$\vec{p}_f^{\,sys} = (m_1 v_{1,f}\cos\theta_{1,f} + m_2 v_{2,f}\cos\theta_{2,f})\hat{i} + (m_1 v_{1,f}\sin\theta_{1,f} - m_2 v_{2,f}\sin\theta_{2,f})\hat{j}$$

no external forces $\Rightarrow \Delta\vec{p} = \vec{p}_f^{\,sys} - \vec{p}_i^{\,sys} = 0$

$$m_1 v_{1,f}\cos\theta_{1,f} + m_2 v_{2,f}\cos\theta_{2,f} = m_1 v_{1,i}$$
$$m_1 v_{1,f}\sin\theta_{1,f} - m_2 v_{2,f}\sin\theta_{2,f} = 0$$

Elastic $\Rightarrow \Delta K = 0 \qquad \dfrac{m_1 v_{1,i}^2}{2} = \dfrac{m_1 v_{1,f}^2}{2} + \dfrac{m_2 v_{2,f}^2}{2}$

Let's rewrite the momentum conditions

$$m_2 v_{2,f}\cos\theta_{2,f} = m_1(v_{1,i} - v_{1,f}\cos\theta_{1,f})$$
$$m_2 v_{2,f}\sin\theta_{2,f} = m_1 v_{1,f}\sin\theta_{1,f}$$

Square and add

$$m_2^2 v_{2,f}^2 = m_1^2 v_{1,f}^2 + m_1^2 v_{1,i}^2 - 2m_1^2 v_{1,i} v_{1,f}\cos\theta_{1,f}$$

$$v_{2,f}^2 = \frac{m_1^2}{m_2^2}\left(v_{1,f}^2 + v_{1,i}^2 - 2v_{1,i}v_{1,f}\cos\theta_{1,f}\right)$$

Sub into K eq

$$\frac{m_1 v_{1,i}^2}{\cancel{2}} = \frac{m_1 v_{1,f}^2}{\cancel{2}} + \frac{\cancel{m_2}}{\cancel{2}}\cdot\frac{m_1^2}{m_2^{\cancel{2}}}\left(v_{1,f}^2 + v_{1,i}^2 - 2v_{1,i}v_{1,f}\cos\theta_{1,f}\right)$$

$$v_{1,f}^2\left(\frac{m_1^2}{m_2} + m_1\right) + v_{1,i}\left(\frac{m_1^2}{m_2} - m_1\right) - \frac{m_1^2}{m_2}\cdot 2v_{1,i}v_{1,f}\cos\theta_{1,f}$$

$$= v_{1,f}^2\left(1 + \frac{m_1}{m_2}\right) + v_{1,i}\left(\frac{m_1}{m_2} - 1\right) - 2\frac{m_1}{m_2}v_{1,i}v_{1,f}\cos\theta_{1,f}$$

$$\alpha = \frac{m_1}{m_2} \Rightarrow v_{1,f}^2(1+\alpha) - v_{1,i}(1-\alpha) - 2\alpha v_{1,i}\cos\theta_{1,f}\, v_{1,f} = 0$$

A quadratic eq. in $v_{1,f}$

$$v_{1,f}^2(1+\alpha) - 2\alpha v_{1,i}\cos\theta_{1,f}\, v_{1,f} - v_{1,i}(1-\alpha) = 0$$

$$v_{1,f} = \frac{2\alpha v_{1,i}\cos\theta_{1,f} \pm \sqrt{4\alpha^2 v_{1,i}^2\cos^2\theta_{1,f} + 4(1+\alpha)(1-\alpha)v_{1,i}}}{2(1+\alpha)}$$

$$= \frac{\alpha v_{1,i}\cos\theta_{1,f} \pm \sqrt{(1-\alpha)^2 v_{1,i} + \alpha^2 v_{1,i}^2\cos^2\theta_{1,f}}}{1+\alpha}$$

Divide the momentum eq.

$$\tan\theta_{2,f} = \frac{v_{1,f}\sin\theta_{1,f}}{v_{1,i} - v_{1,f}\cos\theta_{1,f}}$$

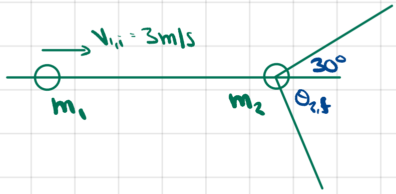
From one of the momentum eq. we have

$$v_{2,f} = \frac{m_1 v_{1,f}\sin\theta_{1,f}}{m_2\sin\theta_{2,f}} = \frac{v_{1,f}\sin\theta_{1,f}}{\sin\theta_{2,f}}\cdot\alpha$$

We had three equations, four unknowns:

$v_{1,f}, v_{2,f}, \theta_{1,f}, \theta_{2,f}$. We need to specify one.

Example 15.5 Elastic 2D Collision, Identical Particles



$$m_1 v_{1,i} = m v_{1,f}\cos 30° + m v_{2,f}\cos\theta_{2,f}$$

$$0 = m v_{1,f}\sin 30° + m v_{2,f}\sin\theta_{2,f}$$

$$\frac{m v_{1,i}^2}{2} = \frac{m v_{1,f}^2}{2} + \frac{m v_{2,f}^2}{2} \Rightarrow v_{1,i}^2 = v_{1,f}^2 + v_{2,f}^2$$

$$v_{1,i} = \frac{v_{1,f}\cdot\sqrt{3}}{2} + v_{2,f}\cos\theta_{2,f} \Rightarrow v_{2,f}\cos\theta_{2,f} = v_{1,i} - v_{1,f}\frac{\sqrt{3}}{2}$$

$$0 = \frac{v_{1,f}}{2} - v_{2,f}\cdot\sin\theta_{2,f} \Rightarrow v_{2,f}\sin\theta_{2,f} = \frac{v_{1,f}}{2}$$

$$v_{2,f}^2\cos^2\theta_{2,f} = v_{1,i}^2 + \frac{3}{4}v_{1,f}^2 - v_{1,i}v_{1,f}\sqrt{3}$$

$$v_{2,f}^2\sin^2\theta_{2,f} = \frac{1}{4}v_{1,f}^2$$

$$\Rightarrow v_{2,f}^2 = \cancel{v_{1,i}^2} + v_{1,f}^2 - v_{1,i}v_{1,f}\sqrt{3} = \cancel{v_{1,i}^2} - v_{1,f}^2$$

$$2v_{1,f}^2 - v_{1,f}v_{1,i}\sqrt{3} = 0 \Rightarrow v_{1,f}(2v_{1,f} - \sqrt{3}v_{1,i}) = 0$$

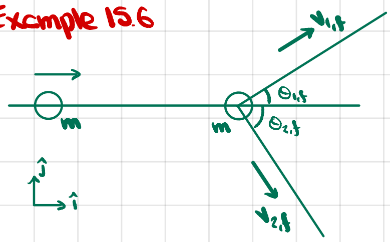
$$\Rightarrow v_{1,f} = \frac{\sqrt{3}}{2}v_{1,i} = \frac{3\sqrt{3}}{2} = 2.6\,\text{m/s}$$

$$v_{1,i}^2 = \frac{3}{4}v_{1,i}^2 + v_{2,f}^2 \Rightarrow v_{2,f} = \pm\frac{1}{2}v_{1,i}$$

$$\frac{v_{1,i}}{2}\sin\theta_{2,f} = \frac{\sqrt{3}}{4}v_{1,i} \Rightarrow \sin\theta_{2,f} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta_{2,f} = \pi/3$$

Example 15.6



$$E_i = \frac{mv_1^2}{2} + \frac{mv_2^2}{2}$$

$$E_f = \frac{mv_{1,f}^2}{2} + \frac{mv_{2,f}^2}{2}$$

$$\Rightarrow v_1^2 + v_2^2 = v_{1,f}^2 + v_{2,f}^2$$

$$\vec{p}_i = mv_1\hat{\imath} + mv_2\hat{\imath}$$

$$\vec{p}_f = mv_{1,f}(\cos\theta_1\hat{\imath} + \sin\theta_1\hat{\jmath}) + mv_{2,f}(\cos\theta_2\hat{\imath} - \sin\theta_2\hat{\jmath})$$

$$\Delta\vec{p} = 0 \Rightarrow mv_1 = m(v_{1,f}\cos\theta_1 + v_{2,f}\cos\theta_2)$$
$$mv_2 = m(v_{1,f}\sin\theta_1 - v_{2,f}\sin\theta_2)$$

$$v_1 = v_{1,f}\cos\theta_1 + v_{2,f}\cos\theta_2$$

$$v_2 = v_{1,f}\sin\theta_1 - v_{2,f}\sin\theta_2$$

$$v_1^2 + v_2^2 = v_{1,f}^2 + v_{2,f}^2$$

unknowns: $v_{1,f}, v_{2,f}, \theta_1, \theta_2$

$$v_1^2 = v_{1,f}^2\cos^2\theta_1 + v_{2,f}^2\cos^2\theta_2 + 2v_{1,f}v_{2,f}\cos\theta_1\cos\theta_2$$

$$v_2^2 = v_{1,f}^2\sin^2\theta_1 + v_{2,f}^2\sin^2\theta_2 - 2v_{1,f}v_{2,f}\sin\theta_1\sin\theta_2$$

$$v_1^2 + v_2^2 = v_{1,f}^2 + v_{2,f}^2 + 2v_{1,f}v_{2,f}\cos(\theta_1 + \theta_2)$$

$$\Rightarrow \cos(\theta_1 - \theta_2) = 0$$

$$\Rightarrow \theta_1 + \theta_2 = \frac{\pi}{2}$$

Solution using vectors not components

$$\vec{p}_i = \vec{p}_f \Rightarrow m\vec{v}_1 = m\vec{v}_{1,f} + m\vec{v}_{2,f}$$

note we use a reference frame in which $\vec{v}_2 = 0$.

$$\vec{v}_1 = \vec{v}_{1,f} + \vec{v}_{2,f}$$

$$\frac{m\vec{v}_1\cdot\vec{v}_1}{2} = \frac{m\vec{v}_{1,f}\cdot\vec{v}_{1,f}}{2} + \frac{m\vec{v}_{2,f}\cdot\vec{v}_{2,f}}{2}$$

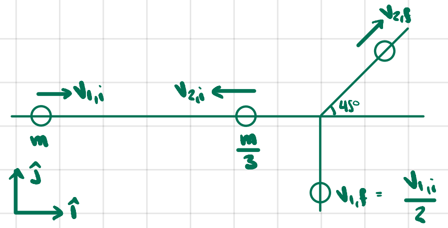
$$\Rightarrow v_1^2 = v_{1,f}^2 + v_{2,f}^2$$

$$\vec{v}_1\cdot\vec{v}_1 = (\vec{v}_{1,f} + \vec{v}_{2,f})\cdot(\vec{v}_{1,f} + \vec{v}_{2,f})$$

$$v_1^2 = v_{1,f}^2 + v_{2,f}^2 + 2\vec{v}_{1,f}\cdot\vec{v}_{2,f}$$

$$\Rightarrow \vec{v}_{1,f}\cdot\vec{v}_{2,f} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$$

Ex 15.7



→ Do not assume elastic collision.

$$\cancel{m}v_{1,i} - \frac{\cancel{m}v_{2,i}}{3} = \frac{\cancel{m}v_{2,f}}{3}\cos 45°$$

$$0 = -\frac{\cancel{m}v_{1,i}}{2} + \frac{\cancel{m}v_{2,f}}{3}\sin 45°$$

2 equation, 2 unknowns: $v_{2,f}, v_{2,i}$

$$3v_{1,i} - v_{2,f}\cos 45° = v_{2,i}$$
$$-3v_{1,i} + 2v_{2,f}\sin 45° = 0$$

$$\Rightarrow v_{2,f} = \frac{3v_{1,i}}{\sqrt{2}}$$

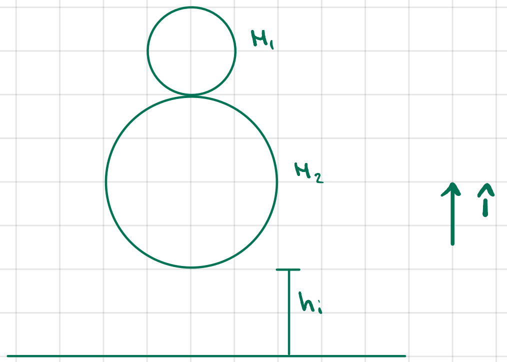
$$v_{2,i} = 3v_{1,i} - \frac{3v_{1,i}}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2} = \frac{3}{2}v_{1,i}$$

$$K_i = \frac{mv_{1,i}^2}{2} + \frac{m}{3}\cdot\frac{1}{2}\cdot\frac{9}{4}v_{1,i}^2 = \frac{21mv_{1,i}^2}{24}$$

$$K_f = \frac{m}{2}\frac{v_{1,i}^2}{4} + \frac{m}{3}\cdot\frac{1}{2}\cdot\frac{9v_{1,i}^2}{2} = \frac{3mv_{1,i}^2}{24} + \frac{18mv_{1,i}^2}{24}$$
$$= \frac{21mv_{1,i}^2}{24}$$

The collision is elastic.

Ex 15.3



$M_2 >> M_1$

elastic collision

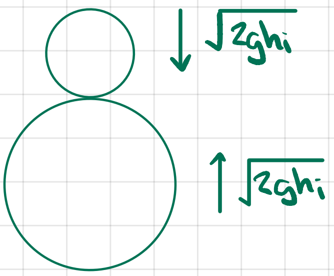
Both balls fall. Right before M_2 hits the ground it has kinetic energy

$$M_2 g h_i = \frac{M_2 v_2^2}{2} \Rightarrow v_2 = \sqrt{2gh_i}$$

When M_2 rebounds it has the same v_2 speed, and an elastic collision occurs w/ M_1.

$$M_1 g(h_i + \cancel{2R}) = \cancel{M_1 g 2R} + \frac{M_1 v_1^2}{2}$$

$$\Rightarrow v_1 = \sqrt{2gh_i}$$



$\Delta p = 0$ during collision

$$M_2\sqrt{2gh_i} - M_1\sqrt{2gh_i} = M_2 V_{2f} + M_1 V_{1f}$$

$$V_{2f} = \frac{M_1}{M_2}V_{1f} + \sqrt{2gh_i} - \frac{M_1}{M_2}\sqrt{2gh_i}$$

$\Delta K = 0$

$$\frac{M_2 \cdot 2gh_i}{2} + \frac{M_1 \cdot 2gh_i}{2} = \frac{M_2 V_{2f}^2}{2} + \frac{M_1 V_{1f}^2}{2}$$

$$\Rightarrow 2M_2 g h_i + 2M_1 g h_i = M_2 V_{2f}^2 + M_1 V_{1f}^2$$

From these eq. we obtain 1D-energy-momentum principle

$$V_{1,f} - V_{2,f} = -(V_{1,i} - V_{2,i}) = 2\sqrt{2gh_i}$$

$$V_{2,f} = V_{1,f} - 2\sqrt{2gh_i}$$

$$M_2\sqrt{2gh_i} - M_1\sqrt{2gh_i} = M_2(V_{1,f} - 2\sqrt{2gh_i}) + M_1 V_{1,f}$$

$$= V_{1,f}(M_2 + M_1) - 2M_2\sqrt{2gh_i}$$

$$V_{1,f} = \frac{\sqrt{2gh_i}\,(3M_2 - M_1)}{M_1 + M_2}$$

$$\lim_{M_2 \to \infty} V_{1,f} = 3\sqrt{2gh_i}$$

$$V_{2,f} = \frac{\sqrt{2gh_i}\,(3M_2 - M_1)}{M_1 + M_2} - 2\sqrt{2gh_i}$$

$$= \frac{\sqrt{2gh_i}\,(3M_2 - M_1 - 2M_1 - 2M_2)}{M_1 + M_2}$$

$$= \frac{\sqrt{2gh_i}\,(M_2 - 3M_1)}{M_1 + M_2}$$

$$\lim_{M_2 \to \infty} V_{2,f} = \sqrt{2gh_i}$$

$$\lim_{M_1 \to 0} V_{2,f} = \sqrt{2gh_i}$$

From M_2 reference frame

$$M_1 \cdot (-2\sqrt{2gh_i}) = M_1 V_{1,f} + M_2 V_{2,f}$$

1D-E-M-principle: $V_{2,f} = V_{1,f} - (V_{1,i} - V_{2,i})$
$= V_{1,f} - 2\sqrt{2gh_i}$

$$-2M_1\sqrt{2gh_i} = M_1 V_{1,f} + M_2 V_{1,f} - 2M_2\sqrt{2gh_i}$$

$$V_{1,f} = \frac{2\sqrt{2gh_i}\,(M_2 - M_1)}{M_1 + M_2}$$

$$\lim_{M_2 \to \infty} V_{1,f} = 2\sqrt{2gh_i}$$