

Kinematics of Rotational Motion

$$\theta = \frac{s}{r} \Rightarrow s = r\theta$$

θ measured in radians.

$$\frac{2\pi r}{r} = 2\pi \text{ radians in circular circumference}$$

$$1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

$$\text{Avg Angular Vel.} = \omega_{\text{avg},z} = \frac{\theta_z - \theta_1}{t_z - t_1} = \frac{\Delta\theta}{\Delta t}$$

$\Delta\theta$ = angular displacement

$$\text{Instant. ang. vel.} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega_z$$

$$\text{ang. vel. vector} = \vec{\omega} = \omega_z \hat{k}$$

$$\text{avg. ang. accel.} = \alpha_{\text{avg},z} = \frac{\omega_z - \omega_1}{t_z - t_1} = \frac{\Delta\omega}{\Delta t}$$

$$\text{Instant. ang. accel.} = \alpha_z = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega_z}{dt}$$

$$\text{ang. accel. vector} = \vec{\alpha} = \alpha_z \hat{k}$$

Rotation, constant angular acceleration

$\alpha_z = \text{constant}$

$$\alpha_z(t) = \alpha_z$$

$$\omega_z(t) = \omega_0 + \alpha_z t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_z t^2 \quad \theta(\alpha_z, t)$$

Altern. calc. w/o integrating

$$\alpha_z = \text{constant} = \alpha_{\text{avg},z} = \frac{\omega_z - \omega_{0,z}}{t - 0}$$

$$\Rightarrow \omega_z = \omega_{0,z} + \alpha_z t$$

$$\omega_{\text{avg},z} = \frac{\omega_{0,z} + \omega_z}{2}$$

$$\omega_{\text{avg},z} = \frac{\theta - \theta_0}{t - 0}$$

$$\begin{aligned} \Rightarrow \theta - \theta_0 &= \frac{1}{2} (\omega_{0,z} t + \omega_z t) \quad \theta(\omega_z, t) \\ &= \theta_0 + \frac{1}{2} (\omega_{0,z} t + \omega_{0,z} t + \alpha_z t^2) \\ &= \theta_0 + \omega_{0,z} t + \frac{1}{2} \alpha_z t^2 \quad \theta(\alpha_z, t) \end{aligned}$$

$$t = \frac{\omega_z - \omega_{0,z}}{\alpha_z}$$

$$\Rightarrow \theta = \theta_0 + \frac{\omega_z - \omega_{0,z}}{2\alpha_z} (\omega_{0,z} + \omega_z)$$

$$2\alpha_z(\theta - \theta_0) = \omega_z^2 - \omega_{0,z}^2$$

$$\omega_z^2 = \omega_{0,z}^2 + 2\alpha_z(\theta - \theta_0)$$

$$s = r\theta$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt} \Rightarrow v = r\omega$$

↑ linear speed
↓ angular speed

$$a_T = \frac{dv}{dt} = r \frac{d\omega}{dt} = r\alpha$$

note α and α_z are slightly different.

α is magnitude of acceleration. α_z is the component of $\vec{\alpha}$ in z-dir. $\alpha = |\vec{\alpha}|$.

$$a_{\text{rad}} = r\omega^2 = \frac{v^2}{r}$$

$$I = \sum m_i r_i^2$$

"moment" means I depends on how mass is distributed in space.

$$K_{i,\text{rot}} = \frac{m_i v_i^2}{2} = \frac{m_i r_i^2 \omega^2}{2}$$

For same ω , more distance from axis of rotation = 1H.

$$K_{i,\text{rot}} \sum K_i = \frac{\omega^2 \sum m_i r_i^2}{2} = \frac{I \omega^2}{2}$$