Angular Momentum, particle mass

Defined $\vec{L}_s \equiv \vec{r}_s \times \vec{p} = \vec{r}_s \times m\vec{v}$

$|\vec{L}_s| = rmv\sin\theta$
$= (r\sin\theta)\, mv$
↳ distance to line of action of $m\vec{v}$

we calculated angular momentum for particle masses on different trajectories.

→ circular motion $\vec{L}_s = I_s\vec{\omega}$, $I_s = mR^2$ the moment of inertia of the particle mass about the center axis.

→ circular motion, L about point on central axis, distance h below the trajectory.

$\vec{L}_s = mR^2\omega_z\hat{k} - hmR\omega_z\hat{r}$

note that the z-comp. is independent of h, the location of the point S on the z-axis.

Angular Momentum of objects Symmetric about an Axis.

A symmetric object can be thought of as a set of pairs of point particles, diametrically separated, undergoing circular motion.

A pair of such point particles has ang. mom. about point S:

$\vec{L}_s = mR^2\omega_z\hat{k} - \cancel{hmR\omega_z\hat{r}} + mR^2\omega_z\hat{k} + \cancel{hmR\omega_z\hat{r}}$
$= 2mR^2\omega_z\hat{k}$

A ring is composed of a set of pairs of point particles.

$\vec{L}_s = \sum_{pairs} 2\cdot\Delta m_i R^2\vec{\omega}$, where each point particle in a pair has mass Δm_i.
$= (\sum_{pairs} m_{pair})R^2\vec{\omega} = mR^2\vec{\omega} = I_{axis}\vec{\omega}$

* mR^2 is the I_s of the ring

$dm = \frac{Rd\theta}{2\pi R}\cdot m$

$I_s = \int_0^{2\pi} \frac{m\,d\theta}{2\pi}\cdot R^2 = \frac{mR^2}{2\pi}\cdot(2\pi - 0) = mR^2$

A solid, symmetric object is composed of rings and pairs of point particles.

→ $\vec{L}_s = I_{axis}\vec{\omega}$, I_{axis} the moment of inertia of the entire object about point on the axis.

$\vec{L}_s$ is independent of the point we choose on the axis.

some interesting and important results

→ if $\vec{p}_{sys} = 0$ then $\vec{L}_{A_{sys}} = \vec{L}_{B_{sys}}$ for any two points A, B.

→ in linear motion, $K = \frac{mv^2}{2} = \frac{p^2}{2m}$

in rotational motion, $K^{rot} = \frac{I_{axis}\,\omega^2}{2}$

For a symmetric object, $\vec{L}_{axis} = I_{axis}\vec{\omega} \Rightarrow \omega = \frac{L_{axis}}{I_{axis}}$

$\Rightarrow K^{rot} = \frac{L_{axis}^2}{2I_{axis}}$

Torque and Angular Momentum

$\vec{L}_s = \vec{r}_{s,m} \times \vec{p}$

$\frac{d\vec{L}_s}{dt} = \frac{d\vec{r}_{s,m}}{dt} \times \vec{p} + \vec{r}_{s,m} \times \frac{d\vec{p}}{dt}$

$= \cancelto{0}{\vec{v} \times m\vec{v}} + \vec{r}_{s,m} \times \vec{F}$

$= \vec{\tau}_s = \vec{\tau}_s^{ext}$

$\Rightarrow \vec{\tau}_s^{ext} = \frac{dL_s}{dt}$