

23.1 Intro to Potential Energy

conservative force \vec{F}_c

$$W = \int_A^B \vec{F}_c \cdot d\vec{s} \quad (\text{path indep.}) = \text{function}(\vec{r}_B) - \text{function}(\vec{r}_A)$$

let's call the function $-U(\vec{r})$

$$\Rightarrow W = -U(\vec{r}_B) + U(\vec{r}_A) = -U_B + U_A = -\Delta U$$

$$\Rightarrow \Delta U = -W$$

Work-Kinetic Energy theorem

$$W = -U_B + U_A = \Delta K = K_B - K_A = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$\Rightarrow \frac{1}{2}mv_A^2 + U_A = \frac{1}{2}mv_B^2 + U_B$$

total mechanical energy

$$\Rightarrow K + U = E_{\text{mech}} = \text{constant}$$

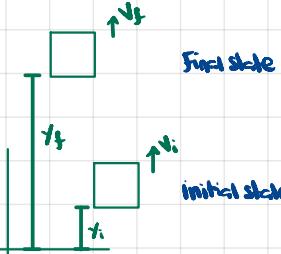
U - potential energy

$$\Delta K + \Delta U - \Delta E_{\text{mech}} = 0 \quad (\text{for a conservative force})$$

total work

$$W = W_c + W_{nc} = \Delta K \Rightarrow W_{nc} = \Delta K + \Delta U$$

23.2 Potential Energy for Gravity near surface of Earth



$$\text{Def: } U(h_f) - U(h_i) = - \int_{h_i}^{h_f} \vec{F}_g \cdot d\vec{s}$$

$$\vec{F}_g = -mg\hat{j}, \text{ conservative force}$$

$$\Rightarrow \Delta U = -(-mg)(h_f - h_i) = mg(h_f - h_i) = mg\Delta h$$

$$\Rightarrow \Delta h > 0 \Rightarrow \Delta U > 0$$

23.3 Potential Energy, Reference State

t_p : reference point

$U(t_p)$: reference potential

$$U(t) - U(t_p)$$

$$U(t) - U(t_p)$$

$$U(t) - U(t_p) = (U(t) - U(t_p)) - (U(t_i) - U(t_p)) = U(t) - U(t_i) = \Delta U$$

$$t_p = 0, U(t_p) = 0 = 0$$

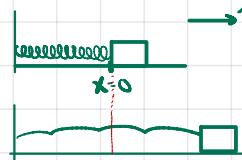
$$U(t_i) - U(t_p) = mg(t_i - t_p) = mgh_i$$

$$U(t) = mg\cdot t_f$$

$$\text{For any height } t_f, U(t_f) = mgy \text{ when } U(0) = 0$$

potential energy function for gravitational force

23.4 Potential Energy of Spring



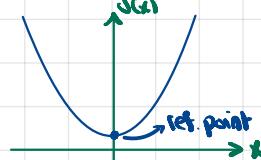
$$\vec{F} = -kx\hat{i} \quad d\vec{s} = dx\hat{i}$$

$$\vec{F} \cdot d\vec{s} = -kxdx$$

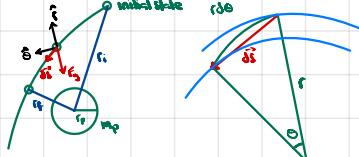
$$\text{Def: } U(x_f) - U(x_i) = -W_c = - \int_{x_i}^{x_f} (-kx) dx = \frac{k(x_f^2 - x_i^2)}{2}$$

Def: reference state $x = 0, U(x=0) = U_0$.

$$\text{stability state: } x, U(x) = U_0 + \frac{kx^2}{2}$$



23.3 Potential Energy of Gravitation



$$\vec{r}(t) = r(t)\hat{r}$$

$$d\vec{s} = dr\hat{r} + r d\theta \hat{\theta}$$

$$\vec{F}_g = -\frac{Gmmp}{r^2} \hat{r}$$

$$* \hat{r} \cdot \hat{r} = 1$$

$$\hat{r} \cdot \hat{\theta} = 0$$

$$\vec{F}_g \cdot d\vec{s} = -\frac{Gmmp}{r^2} dr$$

$$\begin{aligned} U(r_f) - U(r_i) &= - \int_{r_i}^{r_f} \vec{F}_g \cdot d\vec{s} = - \int_{r_i}^{r_f} -\frac{Gmmp}{r^2} dr \\ &= Gmmp \left(-\frac{1}{r} \right) \Big|_{r_i}^{r_f} \\ &= -Gmmp \left(\frac{1}{r_f} - \frac{1}{r_i} \right) \end{aligned}$$

reference state: at ∞ $U(\infty) = 0$

arbitrary state: $r \geq r_i$ $U(r) - U(\infty)$

$$= -\frac{Gmmp}{r}, r \geq r_i$$

24.1 Mechanical Energy and Energy Conservation

$$E_{\text{mech}} = K + U$$

$$\Delta K + \Delta U = \Delta E_{\text{mech}}$$

$$W_c = -\Delta U$$

$$\text{But, } W = W_c + W_{nc} = \Delta K$$

$$W_{nc} = \Delta U - \Delta K$$

$$\Rightarrow W_{nc} = \Delta K + \Delta U = \Delta E_{\text{mech}}$$

24.2 Energy/State Diagrams

1. choose initial and final states, and coord. system

initial state: final state:



2. choose zero potential energy

3. identify K and U for each state

$$K_i = 0$$

$$K_f = \frac{1}{2}mv_f^2$$

$$U_i = mgh$$

$$U_f = mgR\cos\theta_f$$

4. W_{nc} ?

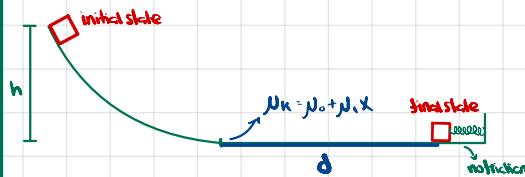
\rightarrow assume dome is frictionless $\Rightarrow W_{nc} = 0$

5. apply energy principle

$$W_{nc} = (K_f - K_i) + (U_f - U_i)$$

$$0 = \frac{1}{2}mv_f^2 + mg(R\cos\theta_f - R)$$

24.3 Worked Example - Block Sliding Down Circular Slope



Work = ΔE_{mech}

choose zero point for each potential function

$$U_g(y=0) = 0$$

$$U_s(x=0) = 0$$

$$E_i = U_{gi} = mgh$$

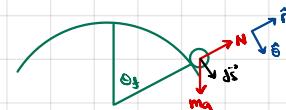
$$E_f = \frac{1}{2}kx_f^2$$

$$-\int_0^h (\rho_0 + \rho_1 x) mg dx = \frac{1}{2}kx_f^2 - mgh$$

$$-\rho_0 mgd - \frac{\rho_1 mgd^2}{2} = \frac{1}{2}kx_f^2 - mgh$$

$$x_f = \sqrt{\frac{1}{k} (2mgh - 2\rho_0 mgd - \rho_1 mgd^2)}$$

24.4 2nd Law and Energy conservation



$$dW_n = \vec{F}_n \cdot d\vec{s} = 0 \text{ because } \vec{F}_n \perp \vec{d}s$$

$$W_g = mgs \sin\theta ds$$

$$W_g = -\Delta U$$

This eq. is not present in the energy condition

$$\text{2nd law in dir } \perp \text{ to } d\vec{s}: \hat{r} \cdot N - mg\cos\theta = -\frac{mv^2}{R}$$

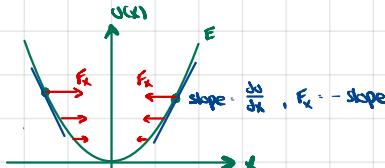
$$\hat{r} \cdot mgs \sin\theta = mR \frac{d^2\theta}{dt^2}, \text{ this is integrated w.r.t. } ds \text{ to obtain the energy principle}$$

25.1 Force is the derivative of potential

$$U(x_f) - U(x_i) = - \int_{x_i}^{x_f} F_x^c dx = \int_{x_i}^{x_f} \frac{dU}{dx} dx$$

$$F_x^c = -\frac{dU}{dx} \quad (\text{indep. of reference point})$$

$$\text{Ex: } U(x) = \frac{1}{2}kx^2, U(0) = 0$$



$$F_x = -kx$$

25.2 Stable and Unstable Equilibrium Points

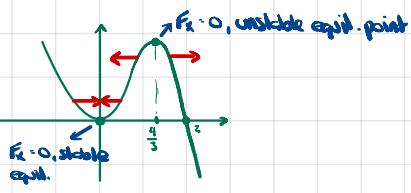
$$W_{nc} = 0$$

$$U(x) = ax^2 - bx^3$$

$$F_x = -\frac{dU}{dx} = -(2ax - 3bx^2)$$

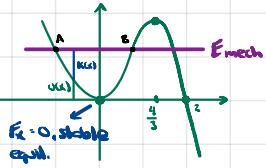
zero points: $x = 0$ and $x = \frac{2a}{3b}$

ex: $a = 4 \text{ J m}^{-2}$, $b = 2 \text{ J m}^{-3} \Rightarrow x = \frac{4}{3} \text{ m}$



25.3 Receding Potential Energy Diagrams

$$W_{nc} = 0 \Leftrightarrow \Delta E_{\text{mech}}$$



A,B: turning points, $K(x_A) = K(x_B) = 0$

