18.1 Introduction to Static Equilibrium

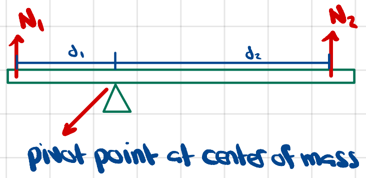
static equilibrium of an extended body: in an inertial reference frame, both CM is at rest and the body does not undergo any rotation.

→ sufficient and necessary conditions:

$$\sum_i \vec{F}_i = \vec{0}$$

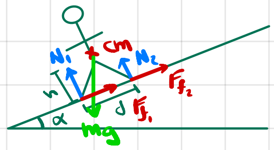
$$\vec{\tau}_S = \sum_i \vec{\tau}_{S,i} = \vec{0}$$

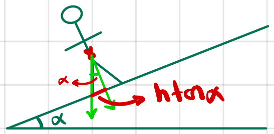
18.2 Lever Law



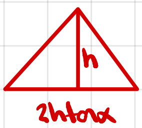
no rotation if $\tau_2 = N_2 d_2 = N_1 d_1 = -\tau_1$

Example 18.3 Person Standing on a Hill





The person is in static equil.



$$\sum_i \vec{F}_i = 0 \Rightarrow N_1 + N_2 - mg\cos\alpha = 0$$
$$mg\sin\alpha - f_1 - f_2 = 0$$

$$\sum_i \vec{\tau}_{cm,i} = 0 \Rightarrow \frac{d}{2}N_2 - \frac{d}{2}N_1 + hf_1 + hf_2 = 0$$

unknowns: N_1, N_2, f_1, f_2

eliminate an unknown if $f_1 = N_1\mu_s$
$f_2 = N_2\mu_s$

$N_1 + N_2 = mg\cos\alpha$
$\mu_s(N_1+N_2) = mg\sin\alpha$

$$2h\mu_s(N_1+N_2) = d(N_1-N_2)$$

$\mu_s mg\cos\alpha = mg\sin\alpha \Rightarrow \tan\alpha = \mu_s$

$$2h = \frac{d(N_1-N_2)}{mg\sin\alpha} \Rightarrow N_1 = N_2 + \frac{2hmg\sin\alpha}{d}$$

$$N_1 = mg\cos\alpha - N_1 + \frac{2hmg\sin\alpha}{d}$$

$$N_1 = \frac{mg\cos\alpha}{2} + \frac{hmg\sin\alpha}{d}$$

$$N_2 = mg\cos\alpha - N_1$$

$$= mg\cos\alpha - \frac{mg\cos\alpha}{2} - \frac{hmg\sin\alpha}{d}$$

$$= \frac{mg\cos\alpha}{2} - \frac{hmg\sin\alpha}{d} = 0$$

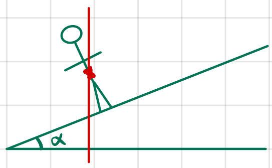
$$\Rightarrow d = \frac{2h\cancel{mg}\sin\alpha}{\cancel{mg}\cos\alpha} = 2h\tan\alpha$$

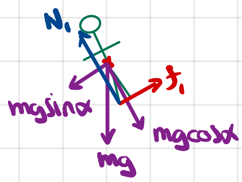
$N_2 = 0 \Rightarrow N_1 = mg\cos\alpha$

N_2 cannot be smaller than 0. For $d < 2h\tan\alpha$, the required N_2 to keep static equilibrium isn't possible.

torque: $\frac{d}{2}N_1 = h\mu_s N_1$ at d_{min}

$h\tan\alpha = h\mu_s \Rightarrow \tan\alpha = \mu_s$





$$\tau_{cm} = -\frac{d}{2}N_1 + hf_1$$

Interpretation $N_1 + N_2 = mg\cos\alpha$
increasing one normal decreases the other.

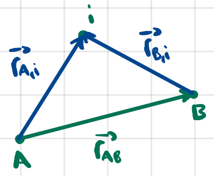
$mg\sin\alpha = \mu_s(N_1+N_2)$ same principle.

$\frac{d}{2}(N_1 - N_2) = h\mu_s(N_1+N_2)$

net torque due to normal = torque due to friction.

For this to balance, $N_1 > N_2$.

Appendix 18A : The Torques About Any Two Points Are Equal For a Body in Static Equilibrium



Force acts at position of point i.
origin is irrelevant.

$\vec{r}_{A,i} = \vec{r}_{A,B} + \vec{r}_{B,i}$

sum of torques about point A: $\vec{\tau}_A = \sum \vec{r}_{A,i} \times \vec{F}_i$

" " " " " B: $\vec{\tau}_B = \sum \vec{r}_{B,i} \times \vec{F}_i$

$$\vec{\tau}_A = \sum (\vec{r}_{A,B} + \vec{r}_{B,i}) \times \vec{F}_i = \sum r_{A,B} \times F_i + \sum r_{B,i} \times F_i$$

$$= \vec{r}_{A,B} \times \sum \vec{F}_i + \vec{\tau}_B$$

By assumption, $\sum F_i = 0 \Rightarrow \vec{\tau}_A = \vec{\tau}_B$

In static equilibrium, net torque about any point is the same, zero.