

## PSet 11

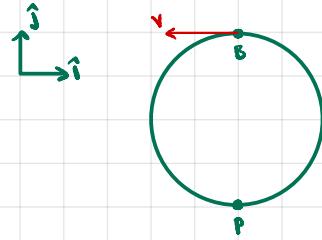
### Problem 1 - Bug Walking on Pivoted Ring

Setup: ring mass  $m_1$ , on frictionless table, pivoted at certain point on the rim. A bug mass  $m_2$  walks around ring from  $\vec{v}$  to constant speed relative to the ring.

External forces are gravity and normal force from table. Both apply torque but cancel each other. The resultant force in  $\hat{h}$  direction is zero.

The bug moves by applying a force on the ring. The ring reacts w/ a force on the bug, and about pivot are there a pair of internal torques.

$$a) \tau_{p, \text{ring}} = 0$$



$$\vec{\tau}_{p, \text{bug}, i} = \vec{0} \times m_{\text{bug}} \vec{v}_i = \vec{0}$$

$$\vec{\tau}_{p, \text{ring}, i} = I_{p, \text{ring}} \times m_i \cdot \vec{0} = \vec{0}$$

$$\vec{\tau}_{p, \text{sys}, i} = \vec{0}$$

$$\vec{v}_{B,P} = \vec{v} + \vec{v}_{r,p}$$

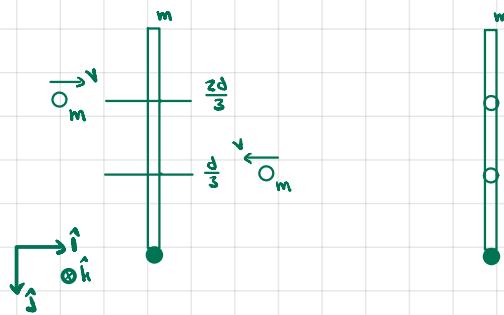
$$\vec{\tau}_{p, \text{bug}, f} = I_{p, \text{bug}} \cdot \vec{\omega}_{p,i} = m_2 \cdot 4R^2 \cdot \omega_{p,i} \hat{h}$$

$$\text{Atten. } (2R\hat{j}) \times m_2 \vec{v}_{p,i} \\ \cdot (2R\hat{j}) \times m_2 (\vec{v}_{r,B} + \vec{v}_{r,C})$$

The ring has a certain  $\vec{\omega}_{p,i}$ .  $\vec{v}_{r,C} = 2R \cdot \vec{\omega}_{p,i}$  at the halfway point

$$= 2R\hat{j} \times m_2 (-4\hat{i})$$

## Problem 2 - A Rigid Rod



$$a) \vec{L}_{\text{sys},0} = \left(-\frac{2d}{3}\hat{j}\right) \times m\vec{v} + \left(-\frac{d}{3}\hat{j}\right) \times m(-\vec{v})$$

$$= \frac{2dm\vec{v}}{3}\hat{k} - \frac{m\vec{v}\hat{i}}{3} = \frac{md\vec{v}}{3}\hat{k}$$

$$\vec{L}_{\text{sys},f} = I_{\text{sys},p} \omega_3 \hat{k} = \frac{\omega_3 \cdot 8md^2}{9} \hat{k}$$

$$I_{\text{sys},p} = \left[ \frac{md^2}{12} + m\frac{d^2}{4} \right] + m\frac{d^2}{9} + m\frac{4d^2}{9}$$

$$= \frac{3md^2 + md^2 + 4md^2}{9} = \frac{8md^2}{9}$$

$$\vec{L}_{\text{sys},0} = \vec{L}_{\text{sys},f} \rightarrow \frac{m\vec{v}}{3} = \omega_3 \frac{8md^2}{9}\hat{k}$$

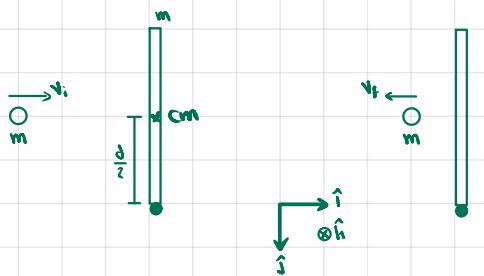
$$\Rightarrow \omega_3 = \frac{3\vec{v}}{8d}$$

$$b) K_i = \frac{1}{2} \frac{mv^2}{2} = mv^2$$

$$K_f = \frac{I_{\text{sys},p} \omega_3^2}{2} = \frac{8md^2}{9} \cdot \frac{9\vec{v}^2}{64d^2} \cdot \frac{1}{2} = \frac{mv^2}{16}$$

$$\frac{K_f - K_i}{K_i} = -\frac{15}{16}$$

### Problem 3 - Elastic Collision Between Ball and Pivoted Rod



elastic collision

$$K_i = \frac{mv_i^2}{2}$$

$$K_f = \frac{mv_f^2}{2} + \frac{I_p \omega_f^2}{2} = \frac{mv_f^2}{2} + \frac{md^2}{3} \cdot \omega_f^2 \cdot \frac{1}{2}$$

$$I_p = \frac{md^2}{3} + m \frac{d^2}{4} = \frac{md^2}{3}$$

$$K_i = K_f \Rightarrow \cancel{\frac{mv_i^2}{2}} = \cancel{\frac{mv_f^2}{2}} + \cancel{\frac{md^2}{3} \cdot \omega_f^2 \cdot \frac{1}{2}}$$

$$v_i^2 = v_f^2 + \frac{d^2 \omega_f^2}{3}$$

$$\vec{L}_{sp,i} = \left(-\frac{d}{2}\hat{j}\right) \times mv_i\hat{i} = \frac{mdv_i}{2}\hat{k}$$

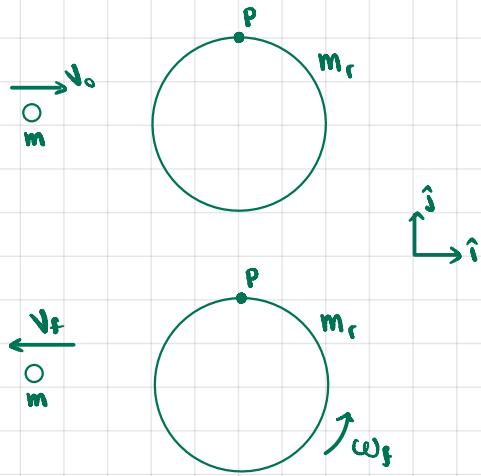
$$\begin{aligned} \vec{L}_{sp,f} &= \left(-\frac{d}{2}\hat{j}\right) \times (-mv_f\hat{i}) + \frac{md^2}{3}\omega_f\hat{k} \\ &= -\frac{mdv_f}{2}\hat{k} + \frac{md^2\omega_f}{3}\hat{k} \end{aligned}$$

$$\begin{aligned} \Delta L \cdot 0 &\Rightarrow \cancel{\frac{mdv_i}{2}} = \cancel{\frac{md^2\omega_f}{3}} - \cancel{\frac{mdv_f}{2}} \\ &\Rightarrow 3dv_i = 2d^2\omega_f - 3dv_f \end{aligned}$$

2 eq., 2 unknowns:  $\omega_f, v_f$

$$\Rightarrow v_f = \frac{v_i}{7} \quad \omega_f = \frac{12v_i}{7d}$$

### Problem 4 - Elastic Collision of Object and Pivoted Ring



$$I_{p,i} = m_r R^2 + m_r R^2 = 2m_r R^2$$

$$\Delta E = 0 \Rightarrow \frac{mv_0^2}{2} = \frac{mv_f^2}{2} + \frac{1}{2}m_r R^2 \omega_f^2$$

$$\frac{mv_0^2}{2} = \frac{mv_f^2}{2} + m_r R^2 \omega_f^2$$

$$\vec{L}_{p,m,i} = (-R\hat{i} - R\hat{j}) \times mv_0\hat{i} = -Rmv_0(-\hat{k}) \\ = Rmv_0\hat{k}$$

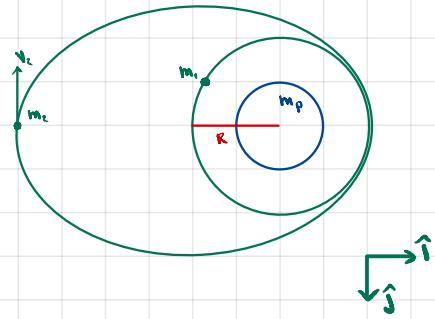
$$\vec{L}_{p,m,f} = -Rmv_f\hat{k} + 2m_r R^2 \omega_f \hat{k}$$

$$\Delta L = 0 \Rightarrow Rmv_0 = -Rmv_f + 2m_r R^2 \omega_f$$

$$\Rightarrow v_f = \frac{v_0(2m_r - m)}{2m_r + m}$$

$$\omega_f = \frac{2mv_0}{R(2m_r + m)}$$

## 5. A Spaceship and a Planet



spaceship 1

$$\vec{F}_{p,1} = -\frac{Gm_p v_i}{R^2} \hat{r} = m_1 \vec{a}_R = m_1 (-R\omega^2 \hat{r})$$

$$\frac{Gm_p}{R^2} = R\omega^2 \quad \Rightarrow \quad \cancel{R} \cdot \frac{v_i^2}{R} = \text{uniform circular motion.}$$

$$\Rightarrow v_i = \sqrt{\frac{Gm_p}{R}}$$

spaceship 2

$$\vec{F}_{p,2} = (-3R\hat{i}) \times m_2 v_2 (-\hat{j})$$

$$= 3Rm_2 v_2 \hat{k}$$

$$\vec{F}_{p,2} = R\hat{i} \times m_2 v_p \hat{j} = Rm_2 v_p \hat{k}$$

$$\Delta L = 0 \Rightarrow 3Rv_2 v_2 - Rv_2 v_p = v_p = 3v_2$$

$$\int_{r_i}^{r_e} -\frac{Gm_2 m_p}{r^2} dr = Gm_2 m_p \left( \frac{1}{r_e} - \frac{1}{r_i} \right) = -\Delta U$$

$$U(r) = -\frac{Gm_2 m_p}{r}$$

$$E_{m,i} = -\frac{Gm_2 m_p}{3R} + \frac{m_2 v_i^2}{2}$$

$$E_{m,f} = -\frac{Gm_2 m_p}{R} + \frac{m_2 v_p^2}{2}$$

$$\Delta E_m = 0 \Rightarrow -\frac{Gm_2 m_p}{3R} + \frac{m_2 v_i^2}{2} = -\frac{Gm_2 m_p}{R} + \frac{m_2 v_p^2}{2}$$

$$-\frac{Gm_2 m_p}{3R} + \frac{3Gm_2 m_p}{3R} = \frac{m_2 9v_i^2}{2} - \frac{m_2 v_i^2}{2}$$

$$\frac{16Gm_2 m_p}{3R} = \frac{8m_2 v_i^2}{2} \Rightarrow v_p = \sqrt{\frac{Gm_p}{6R}}$$

$$v_p = \sqrt{\frac{3Gm_p}{2R}}$$

Therefore, at the point of  $m_2$ 's nearest approach,

$$v_i = \sqrt{\frac{Gm_p}{R}}$$

$$v_p = \sqrt{\frac{3Gm_p}{2R}}$$

$$\Rightarrow v_p = \sqrt{3/2} v_i \Rightarrow v_p > v_i$$

Spaceship 2 has to slow down.

The change in speed is  $v_i - v_p$

$$= \sqrt{\frac{Gm_p}{R}} (1 - \sqrt{3/2})$$

$$= -0.22 \sqrt{\frac{Gm_p}{R}}$$