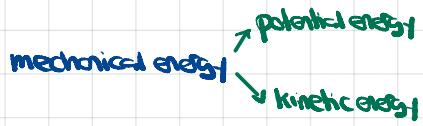


14.2 Conservative and Non-conservative Forces



conservative force with done is indep. of path

$$\Rightarrow \oint \vec{F}_c \cdot d\vec{r} = 0$$

14.3 Changes in Potential Energies of a System

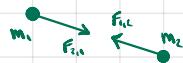
\rightarrow When only internal conservative forces act in a closed system, the sum of DK and DV of the system

is zero

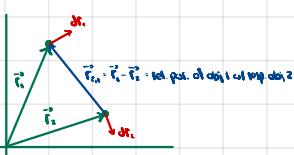
Setup

- closed system, $\Delta E_{sys} = 0$, two masses m_1, m_2

- there is a single pair of internal forces, and they are conservative.



choose coord. system



relative displacement = $d\vec{r}_{1,11} = \vec{r}'_1 - \vec{r}_1$

$$\Delta K_{sys} = \Delta K_1 + \Delta K_2 = W_c = \int_A^B \vec{F}_{1,11} \cdot d\vec{r}_1 + \int_A^B \vec{F}_{2,11} \cdot d\vec{r}_2, \text{ where } A, B \text{ are initial and final states}$$

$$\text{Third Law} \Rightarrow \vec{F}_{1,12} = -\vec{F}_{2,11} \Rightarrow \Delta K_{sys} = \int_A^B \vec{F}_{1,11} \cdot d\vec{r}_1 - \int_A^B \vec{F}_{1,12} \cdot d\vec{r}_1$$

$$= \int_A^B \vec{F}_{1,11} \cdot (d\vec{r}_1 - d\vec{r}_2) = \int_A^B \vec{F}_{1,11} \cdot d\vec{r}_{1,11}$$

$$+ d\vec{r}_{2,1} = -d\vec{r}_{1,12} \Rightarrow \int_A^B \vec{F}_{1,11} \cdot d\vec{r}_{1,11} - \int_A^B \vec{F}_{1,12} \cdot d\vec{r}_{1,12}$$

we now define change in internal potential energy of the system

to be neg. of work done by the conservative force that the objects undergo a relative displacement from initial state A to final state B along any displacement that changes the initial state A to the final state B.

$$\Delta U_{sys} = -W_c = - \int_A^B \vec{F}_{1,11} \cdot d\vec{r}_{1,11} - \int_A^B \vec{F}_{1,12} \cdot d\vec{r}_{1,12} = -\Delta K_{sys}$$

this def. is only valid for conservative forces.

14.4.1 Chg in Gravit. Pot. En. near Earth Surface

System: Earth, object

gravitational force: internal conservative force acting inside the system

coord. system: origin on Earth surface, +z points away from center of Earth

assumption: displacement of Earth is negligible, only need to consider object displacement.

$$\vec{F}_g = -mg\hat{j}$$

$$d\vec{r} = dz\hat{j}$$

$$\vec{F}_g \cdot d\vec{r} = -mgdz$$

$$\Delta U_g = -W_g = -(-mg(z_f - z_i)) = mg(z_f - z_i)$$

\rightarrow chg in pot. en. is difference in pot. en. fin. - init. due to change of z in eval. of the work integral.

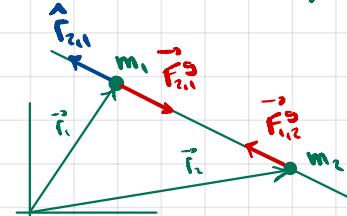
choose = zero reference potential:

$$U_g(0) = 0 \Rightarrow U_g(z) = mgz$$

14.4.3 Inverse Square Gravitation Force

System: two objects of masses m_1, m_2 separated by center-to-center distance $r_{1,11}$

coord. system:



displacement vector: $d\vec{r}_{1,11} = d\vec{r}_{1,11} \hat{r}_{1,11}$

$$\vec{F}_{1,11}^g = -\frac{Gm_1m_2}{r_{1,11}^2} \hat{r}_{1,11}$$

$$\vec{F}_{1,11}^g \cdot d\vec{r}_{1,11} = -\frac{Gm_1m_2}{r_{1,11}^2} d\vec{r}_{1,11}$$

$$\Delta U_g = - \int_A^B \vec{F}_{1,11}^g \cdot d\vec{r}_{1,11} = - \int_{r_i}^{r_f} -\frac{Gm_1m_2}{r_{1,11}^2} dr_{1,11} = -\frac{Gm_1m_2}{r_{1,11}} \Big|_{r_i}^{r_f}$$

$$= -\frac{Gm_1m_2}{r_i} + \frac{Gm_1m_2}{r_f}$$

choose ref. point for zero of potential energy

$$r_i = \infty \quad U(\infty) = 0 \Rightarrow U_g = -\frac{Gm_1m_2}{r}$$