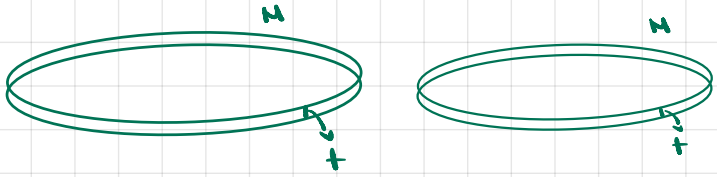
1145



density ρ_1 ρ_2

$\rho_1 < \rho_2$

moment of inertia

→ both disks are uniform so the CM is in the middle.

$$I_{cm_1} = \int_0^{2\pi}\int_0^{R_1}\int_0^{t} r^2 \rho_1 r \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{R_1}\int_0^{t} r^3 \rho_1 \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{R_1} r^3 \rho_1 t \, dr\, d\theta$$

$$= \rho_1 t \int_0^{2\pi} \frac{R_1^4}{4} \, d\theta$$

$$= \frac{\rho_1 t R_1^4 \cdot 2\pi}{4}$$

$$= \frac{\rho_1 R_1^4 \cdot \pi t}{2}$$

$$\rho_1 = \frac{M}{\pi R_1^2 t} \Rightarrow R_1^2 = \frac{M}{\pi t \rho_1}$$

$$I_{cm_1} = \cancel{\rho_1} \cdot \frac{M^2}{\pi^{\cancel{2}} t^{\cancel{2}} \rho_1^{\cancel{2}}} \cdot \frac{\cancel{\pi}\cancel{t}}{2} = \frac{M^2}{\pi t \rho_1}$$

$$\rho_1 < \rho_2 \Rightarrow I_{cm_1} > I_{cm_2}$$

Alternatively

volume disk $= \pi R^2 t$

mass disk $= \rho \pi R^2 t$

$M_1 = M_2$

$$\rho_1 \pi R_1^2 t = \rho_2 \pi R_2^2 t \Rightarrow \rho_1 R_1^2 = \rho_2 R_2^2$$

$$I_{disk} = \int_0^{2\pi}\int_0^{R}\int_0^{t} \frac{M}{\pi R^2 t} r^3 \, dz\, dr\, d\theta$$

$$= \frac{M}{\pi R^2 t} \int_0^{2\pi}\int_0^{R} t r^3 \, dr\, d\theta$$

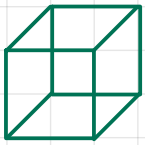
$$= \frac{M}{\cancel{\pi}\cancel{R^2}\cancel{t}} \cdot \frac{R^{\cancel{4}2}}{\cancel{4}_2} \cdot \cancel{2}\cancel{\pi} = \frac{MR^2}{2}$$

$$I_1 = \frac{MR_1^2}{2}$$

$$I_2 = \frac{MR_2^2}{2}$$

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{\rho_2}{\rho_1} > 1 \Rightarrow I_1 > I_2$$

1146



moment of inertia about each of x-, y-, and z-axis is I_0.