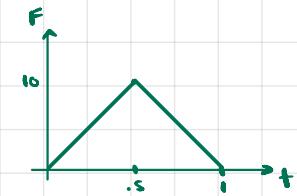


### Ex 10.1 $\Delta t = 1\text{s}$

$$F(t) = \begin{cases} b\hat{i} & 0 \leq t \leq 0.5 \\ (d - b)t\hat{i} & 0.5 \leq t \leq 1 \end{cases} \quad b = 2 \cdot 10 \quad d = 2 \cdot 10$$



$$I = \int_0^1 10 dt + \int_{0.5}^1 (20 - 20t) dt = 10\left(\frac{1}{4} - 0\right) + 20\left(\frac{1}{2}\right) - 10\left(1 - \frac{1}{4}\right) = 2.5 + 10 - 10 + 2.5 = 5$$

### Ex 10.5 - Exploding Projectile

$$\vec{F} = m \frac{d\vec{v}}{dt} = \int_0^t \vec{F}_i dt + \int_0^t m \frac{d\vec{v}}{dt} dt = m\vec{v}_i - m\vec{v}_f$$

$$\int_0^t \vec{F}_i dt = \int_0^t -mg\hat{j} dt = -mg(t_i - t_0)\hat{j}$$

$$= (m\vec{v}_y(t_i) - m\vec{v}_y(t_0))\hat{j} + (m\vec{v}_x(t_i) - m\vec{v}_x(t_0))\hat{i}$$

$$\Rightarrow \vec{v}_y(t_0) = g(t_i - t_0), \quad \vec{v}_x(t_0) = \vec{v}_x(t_i)$$

Gravitational force acted during  $t_0$  to  $t_i$  and reduced momentum in  $y$ -direction to 0.

Momentum in  $x$ -dir. stayed the same.

$$\vec{p}_i = m_i(\vec{v}_x(t_0)\hat{i} + \vec{v}_y(t_0)\hat{j}) = m_i(\vec{v}_x(t_0)\hat{i} + g(t_i - t_0)\hat{j})$$

$$\vec{p}_i = m_i\vec{v}_x(t_0)\hat{i} + m_i\vec{v}_y(t_0)\hat{j} = m_i\vec{v}_{i,x}$$

From kinematics perspective,  $y$ -direction

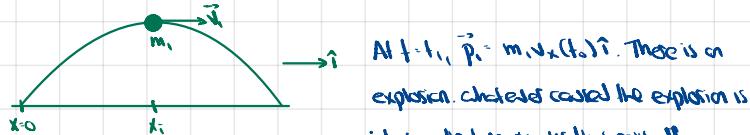
$$a_y = -g$$

$$\vec{v}_y = \vec{v}_{y,i} - gt$$

$$t = \sqrt{\frac{2v_{y,i}}{g}}$$

Note that impulse is a definite integral and is the  $\Delta$  of momentum.

From  $t_0$  to  $t_i$ , speed went from  $v_{y,i}$  to 0. we calculated this change as  $-g(t_i - t_0)$ .



$$\vec{p}_i = (-m_2 \vec{v}_{2,x} + m_3 \vec{v}_{3,x})\hat{i}$$

we can equate momentum before and after explosion

$$m_i \vec{v}_{i,x} = -\frac{1}{4} m_2 \vec{v}_{2,x} + \frac{3}{4} m_3 \vec{v}_{3,x}$$

but  $m_2$  falls to the initial position.  $m_2$  starts with a zero vertical speed on the descent, therefore it will reach the ground in the same time it took for  $m_3$  to reach the explosion point. Since this is also the time it took to cover the horizontal distance,  $v_{2,x}$  must equal  $v_{3,x}$ .

$$v_{2,x} = \frac{-v_{i,x}}{4} + \frac{3v_{3,x}}{4} = 5v_{i,x} - 3v_{3,x} \Rightarrow v_{3,x} = \frac{5}{3} v_{i,x}$$

$$\int_0^{t_i} \vec{v}_{3,x} dt = \frac{5}{3} \int_0^{t_i} \vec{v}_{i,x} dt = \frac{5}{3} x_i$$

The larger piece traveled  $x_i + \frac{5}{3} x_i$ .

### Solution using center of mass

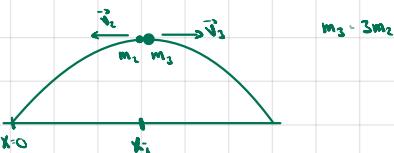
$$\vec{r}_{cm} = \frac{m_i \vec{r}_i(t)}{m_i} = \vec{r}_i(t) \text{ so up to the explosion the center of mass is the position of the mass. There is an external force acting on the system: } \vec{F}_g. \text{ It accelerates the center of mass, but this only actually has an effect in the } y \text{-direction.}$$

The center of mass has an  $x$ -dir velocity that does not change:  $a_{cm,x} = 0$ . Internal forces (like those involved in the explosion) do not accelerate the center of mass. Therefore, the  $x$ -dir center of mass is the same whether or not the explosion occurs.

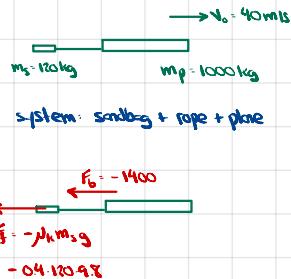
With the explosion, the center of mass is at  $2x_i$ ; then the mass hits the ground. With the explosion:

$$x_{cm} = \frac{m_2 \cdot 0 + m_3 \cdot x_f}{m_2 + m_3} = \frac{\frac{3}{4} m_i x_f}{m_i} = \frac{3x_f}{4} = 2x_i$$

$$\Rightarrow x_f = \frac{8}{3} x_i$$



### Ex 10.6



When the plane snags the sled, the sled loses some momentum.

$$m_p v_{0,p} + m_s v_{0,s} = (m_p + m_s) v_i$$

$$1000 \cdot 40 + 120 \cdot 0 = 1120 v_i \Rightarrow v_i = \frac{350}{1120} \approx 35.71 \text{ m/s}$$

This calculation assumes the snag collision is instantaneous.

$$\text{After collision, initial momentum is } p_i = (1000+120) \cdot 35.71 = 39995.20$$

Fiction and the brakes provide impulse that reduces momentum to zero.

$$\int_{t_0}^t (F_B + f_f) dt = -39995.20$$

here we calc time to reach momentum of zero under impulse from friction and brakes

$$-1120 t_f - 39995.20 \Rightarrow t_f = 35.71 \text{ s}$$

$$F_B = m_p \cdot a \Rightarrow -1120 \cdot 40 = 1120 \cdot a \Rightarrow a = -1.67 \text{ m/s}^2$$

$$v(t) = 35.71 - 1.67t$$

here we calc time to reach zero velocity under an acceleration due to friction and braking forces.

$$v = 0 \Rightarrow t = 35.71 \text{ s}$$

$$x(t) = 35.71t - \frac{1.67t^2}{2}$$

$$x(35.71) = 381.79 \text{ m}$$

Note:

$$W = \int_{t_0}^t F_B dx = \frac{-1120 \cdot 35.71^2}{2} = -1120 \times 5$$

$$\Rightarrow x_f = 381.79 \text{ m}$$

### Ex 10.2

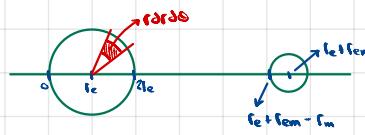
$$r_{cm} = 3.84 \cdot 10^8 \text{ m}$$

$$m_e = 5.98 \cdot 10^{24} \text{ kg}$$

$$m_m = 7.34 \cdot 10^{26} \text{ kg}$$

$$r_e = 6.37 \cdot 10^6 \text{ m}$$

$$r_m = 1.74 \cdot 10^6 \text{ m}$$



$$\vec{r}_{cm} = \frac{1}{m_e + m_m} [m_e \vec{r}_{e,cm} + m_m \vec{r}_{m,cm}]$$

$$\vec{r}_{e,cm} = \frac{\int_{r_e}^{r_e+r_m} r \cos \theta_e \hat{i} \sin \theta_e \hat{j} dr \hat{k}}{\int_{r_e}^{r_e+r_m} r dr}$$

$$(x - r_e)^2 + j^2 = r_e^2$$

$$x^2 + j^2 - 2xr_e + r_e^2 = r_e^2$$

$$x^2 + j^2 = 2xr_e = 2r_e r \cos \theta_e = r^2$$

$$\Rightarrow r = 2r_e \cos \theta_e$$

$$\Rightarrow x_{e,cm} = \frac{\int_{r_e}^{r_e+r_m} r^2 \cos^2 \theta_e dr \hat{i}}{\int_{r_e}^{r_e+r_m} r dr} = r_e \hat{i}$$

$$\vec{r}_{m,cm} = 0$$

This calculation was just for "Jen".

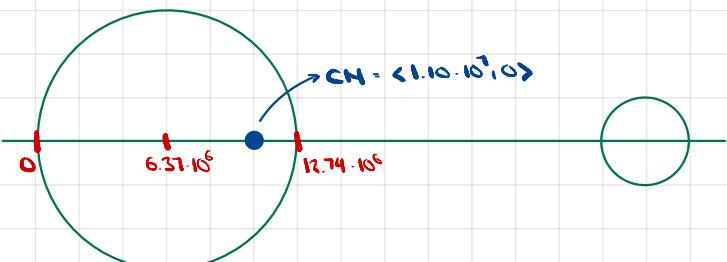
$$\vec{r}_{cm} = r \hat{i}$$

$$\vec{r}_{m,cm} = (r_e + r_m) \hat{i}$$

$$\vec{r}_{cm} = \frac{1}{m_e + m_m} [m_e r \hat{i} + m_m (r_e + r_m) \hat{i}]$$

$$= \frac{r_e (m_e + m_m) + m_m r_m}{m_e + m_m} \hat{i} = 1.10 \cdot 10^7 \text{ m}$$

Center of mass is within the Earth.



### Ex 10.3



uniform rod : constat density  $\lambda = \frac{M}{L}$

$$\vec{r}_{cm} = \frac{\int_0^L x \hat{i} dx}{\int_0^L dx} = \frac{\frac{1}{2} L^2 \hat{i}}{L} = \frac{1}{2} L \hat{i}$$

b) non-uniform rod  $\lambda(x) = \frac{\lambda_0}{L^2} x^2$

$$\vec{r}_{cm} = \frac{\frac{\lambda_0}{L^2} \int_0^L x^3 \hat{i} dx}{\frac{\lambda_0}{L^2} \int_0^L x^2 dx} = \frac{\frac{1}{4} L^4 \hat{i}}{\frac{1}{3} L^2} = \frac{3}{4} L \hat{i}$$

### Ex 10.4