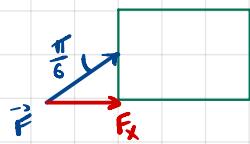


### Example 6.1

a)



$$F_x = F \cos(\pi/6)$$

$$W = F_x \cdot \Delta x = \frac{210 \cdot \sqrt{3}}{2} \cdot 18 = 1890\sqrt{3} \text{ J} \approx 33 \cdot 10^3 \text{ J}$$

$$\text{b) } \vec{F} = \langle 160, -40 \rangle$$

$$\vec{s} = \langle 14, 11 \rangle$$

$$W = \vec{F} \cdot \vec{s} = 18 \cdot 10^3 \text{ J}$$

### Example 6.2



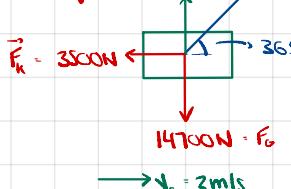
$$\vec{F}_f = \langle 5000 \cos 36.9^\circ, 5000 \sin 36.9^\circ \rangle$$

$$W_f = \vec{F}_f \cdot \vec{s} = 5000 \cos 36.9^\circ \cdot 20 = 79968.5 \text{ J}$$

$$\vec{s} = \langle 20, 0 \rangle$$

$$W_k = \vec{F}_g \cdot \vec{s} = \langle -3500, 0 \rangle \cdot \langle 20, 0 \rangle = -70000 \text{ J}$$

### Example 6.3



$$W = \Delta K \Rightarrow 9968.5 \text{ J} = \frac{1}{2} \cdot \frac{14700}{9.8} \cdot (V_f^2 - V_i^2)$$

$$\Rightarrow \left( \frac{2 \cdot 9.8 \cdot 9968.5}{14700} + 4 \right)^{1/2} = V_f = 4.16 \text{ m/s}$$

Attempt.

$$F_R \cdot m \cdot a = 498.42 = \frac{14700}{9.8} \cdot a_x \Rightarrow a_x = 0.33 \text{ m/s}^2$$

$$v_x = v_{0,x} + a_{x,t} t \Rightarrow x = 20 - 2t + 0.33t^2/2 \Rightarrow t = 6.50$$

$$x = v_{0,x} t + \frac{a_{x,t} t^2}{2} \Rightarrow$$

$$\Rightarrow V_f = 2 + 0.33 \cdot 6.50 \Rightarrow V_f = 4.16 \text{ m/s}$$

### Example 6.4

$$(F_G - F_k) \cdot 3 \cdot W_R = 3(200 \cdot 9.8 - 60) = 5700 \text{ J}$$

$$5700 = \Delta K = \frac{1}{2} \cdot 200 \cdot V_f^2 \Rightarrow V_f = \sqrt{\frac{5700}{200}} \cdot \sqrt{57} = 7.55 \text{ m/s}$$

clock to stop the hammerhead

$$(F_G - F_{up} - F_k) \cdot 0.074 \text{ m} = W \cdot \Delta K = \frac{1}{2} \cdot 200 \cdot (-7.55^2)$$

$$200 \cdot 9.8 \cdot 0.074 - F_{up} \cdot 0.074 - 60 = -7.55^2 \cdot 100$$

$$F_{up} = \frac{200 \cdot 9.8 \cdot 0.074 + 7.55^2 \cdot 100 - 60}{0.074} = 78176.21 \text{ N}$$

### Example 6.5

$$m \quad \vec{F}$$

$$2m \quad \vec{F}$$

$W = F \cdot s$  in both cases

$$\Delta K = \frac{mv_f^2}{2} \Rightarrow V_{f_1}^2 = 2V_{f_2}^2 \Rightarrow V_{f_1} = \sqrt{2} V_{f_2}$$

$$\Delta K = \frac{2mV_{f_2}^2}{2}$$

### Example 6.6



$$F_s = -kx \Rightarrow -600 = -k \cdot 0.01 \Rightarrow k = 60000$$

The resultant force varies in time as the spring is compressed.

$$F_R = F_g - F_s = 600 - kx$$

$$W = \int_0^{0.01} (600 - kx) dx = 600x - 60000 \frac{x^2}{2} \Big|_0^{0.01}$$

$$= 600 \cdot 1 - 60000 \cdot \frac{1}{10000} \cdot \frac{1}{2} = 6 - 3 = 3 \text{ J}$$



### Example 6.7

$x_0 = 0$   
 $\rightarrow v_0 = 1.5 \text{ m/s}$   
  
 $k = 20 \text{ N/m}$     $m = 0.1 \text{ kg}$

$$W = \int_0^{x_f} (-20x) dx = -\frac{20x^2}{2} = \frac{1}{2} \cdot 0.1 \cdot (-1.5^2)$$

$$\Rightarrow x_f^2 = \frac{1.5^2}{200} \Rightarrow x_f = \frac{1.5}{\sqrt{200}} = 0.1060 = 10.6 \text{ cm}$$

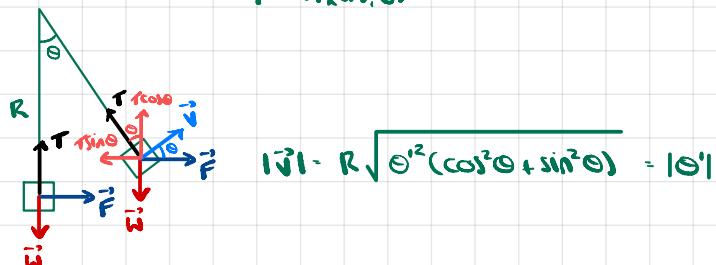
now with friction,  $\mu_h = 0.47$

$$\int_0^{x_f} (-20x - 0.198 \cdot 0.47) dx = -10x_f^2 - 0.4606x_f = \frac{1}{2} \cdot 0.1 \cdot (-1.5^2)$$

$$10x_f^2 + 0.4606x_f - 2.25 \cdot 0.1 \cdot 0.5 \Rightarrow x_f = 0.0855 \approx 8.55 \text{ cm}$$

### Example 6.8

$$\vec{F} = \langle F_x(t), 0 \rangle$$



$\vec{F}$  varies such that the particle swings at  $\omega$  in "equilibrium", meaning the net force is always zero.

$$F_x - T \sin \theta = 0 \Rightarrow F_x = \frac{W}{\cos \theta} \sin \theta = w \tan \theta$$

$$T \cos \theta - w = 0 \Rightarrow T = \frac{w}{\cos \theta}$$

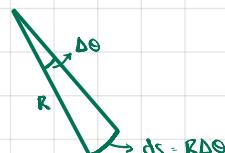
$\Rightarrow F_x$  increases w/  $\theta$ .

$$W = \int_0^\theta F_x \cos \theta \cdot d\theta$$

$$= \int_0^\theta w \tan \theta \cos \theta R d\theta$$

$$= wR \int_0^\theta \sin \theta d\theta = wR(-\cos \theta) + C$$

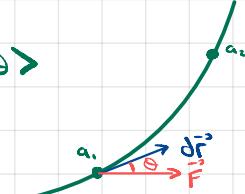
$$= wR(1 - \cos \theta)$$



Altan, let's use rectangular coord.

$$d\vec{r} = \langle ds \cos \theta, ds \sin \theta \rangle$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^\theta F_x \cos \theta \cdot ds$$



### Example 6.9

4 engines, each with forced thrust of  $322 \cdot 10^3 \text{ N}$

plane flying at  $250 \text{ m/s}$

$$\text{in one second: } \frac{F \cdot \Delta s}{\Delta t} = \frac{322 \cdot 10^3 \cdot 250}{1} \text{ N}$$

$$\frac{322 \cdot 250 \cdot 10^3 \text{ N}}{746 \text{ W}} = 107908 \text{ Hp}$$

Altan. formula

$$P_{\text{ANG}} = \frac{F \cdot \Delta s}{\Delta t} = F \cdot V_{\text{ANG}}$$

$$\lim_{\Delta t \rightarrow 0} P_{\text{ANG}} = F \cdot V$$

### Example 6.10

$$W = 50 \cdot 9.8 \cdot 443$$

$$P_{\text{ANG}} = \frac{50 \cdot 9.8 \cdot 443}{15 \cdot 60} = 241 \text{ W}$$

50m

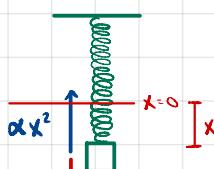
$$\text{Altan. to reach top in 15min, } V_{\text{ANG}} = \frac{443}{15 \cdot 60}$$

$$\Rightarrow P_{\text{ANG}} = F \cdot V = 50 \cdot 9.8 \cdot \frac{443}{15 \cdot 60} = 241 \text{ W}$$

$$241 \text{ W} = 0.241 \text{ kW} = 0.32 \text{ Hp}$$

## Bridging Problem

$$F_s = -\alpha x^2$$



Between 0 and  $x_1$ ,  $W \cdot \Delta K$

$$W = \int_0^{x_1} (mg - \alpha x^2) dx = mgx_1 - \frac{\alpha x_1^3}{3}$$

$$\Delta K = \frac{1}{2}mv_i^2$$

$$\Rightarrow \frac{1}{2}mv_i^2 = mgx_1 - \frac{\alpha x_1^3}{3}$$

$$\Rightarrow v_i = \sqrt{2 \left( gx_1 - \frac{\alpha x_1^3}{3m} \right)}$$

From  $x, v_i$ , calculate initial and power delivered by spring

$$P = F_s \cdot v = -\alpha x_1 \cdot \sqrt{2 \left( gx_1 - \frac{\alpha x_1^3}{3m} \right)}$$

$$\text{altern. } W = \int_{x_1}^{x_2} -\alpha x^2 dx = -\frac{\alpha}{3} (x_2^3 - x_1^3)$$

$mg - \alpha x^2 = mx^2$ , we would need to solve for  $x(t)$ , calculate the  $\Delta t$  between  $x_1$  and  $x_2$ , and then we'd have only passes.

max distance stretched

$$v_i = 0 \Rightarrow x_2 \left( g - \frac{\alpha x_2^2}{3m} \right) = 0 \Rightarrow x_2 = \sqrt{\frac{3mg}{\alpha}}$$

What happens at  $x_2$ ?

$$F_s = \alpha \cdot \frac{3mg}{\alpha} = 3mg$$

The block does not remain at  $x_2$ .

$$6.49 F(x) = 180N - 0.530 \frac{N}{m} \cdot x$$



$$m = 6.00 \text{ kg}$$

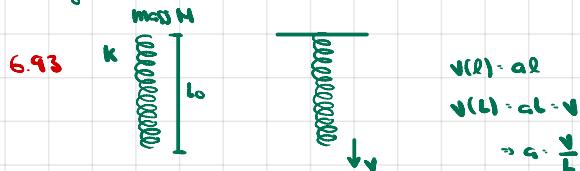
speed after 14m?

$$W = \int_0^x (18 - 0.530x) dx = 18x - 0.265x^2 = \frac{1}{2} 6v_f^2$$

$$\Rightarrow v_f = 8.166 \text{ m/s}$$

$$6.62 F_x = -20 - 3x$$

$$W = \int_0^{6.9} (-20 - 3x) dx = -209.415 \text{ J}$$



$$\begin{aligned} v(L) &= \alpha L \\ v(L) &= \alpha L - v \\ \Rightarrow \alpha &= \frac{v}{L} \end{aligned}$$

$$a) \quad \boxed{dx} \quad dm \cdot \frac{\partial E}{\partial x} N \quad v(L) = \frac{v}{L} x$$

$$\int \frac{1}{2} v_{\partial x}^2 dm + \frac{1}{2} \frac{v^2}{L^2} N \int x^2 dx = \frac{v^2 N}{2L} \cdot \frac{L}{3} \cdot \frac{mv^2}{6}$$

$$b) \quad M = 0.243 \text{ kg}$$

$$k = 3200 \text{ N/m}$$

$$x_0 = 0.025 \text{ m}$$

$$W = \frac{1}{2} 3200 (0.025)^2 = \frac{1}{2} \cdot 0.053 v_f^2$$

$$\Rightarrow v_f = 6.14 \text{ m/s} \quad (\text{ignoring spring mass})$$

c) Assume the end of the spring (and ball) move at speed v at L.

$$\text{we know } K_s = \frac{1}{2} N v^2 \cdot \frac{1}{4} L \quad \text{and} \quad K_B = \frac{1}{2} m v^2$$

$\nearrow$  the spring force is doing work

$$\text{work done on ball + spring is just } \int_{x_0}^L kx dx = \frac{kx_0^2}{2} = \frac{3200 \cdot 0.025^2}{2} = 1$$

$$W = \Delta K \Rightarrow \frac{0.243v^2}{6} + \frac{0.053v^2}{2} - 1 \Rightarrow v = 3.86 \text{ m/s}$$

The individual kinetic energies

$$\frac{1}{2} 0.053 \cdot 3.86^2 \approx 0.40$$

$$\frac{1}{6} 0.243 \cdot 3.86^2 \approx 0.60$$

694 air resistance  $\sim v^2$

$$F_{air} = \alpha v^2 + \frac{\beta}{v^2}, \alpha, \beta > 0$$

$$\text{a) } \alpha = 0.30 \frac{Ns^2}{m^2} \quad \beta = 3.5 \cdot 10^5 N \frac{m^2}{s^2}$$

Airplane engine generates force that counterbalances  $F_{air}$ .

$$F_{air} = 0.3v^2 + \frac{3.5 \cdot 10^5}{v^2}$$

a) Assumption: fuel contains  $W_0$  amount of energy.  $W_0$ : Joules of work can be done

$$W_0 = \text{Power} \cdot t = R \cdot (0.3v^2 + \frac{3.5 \cdot 10^5}{v^2})$$

$$\text{Range} = R = \sqrt{t}$$

$$\Rightarrow W_0 = \sqrt{t} (0.3v^2 + \frac{3.5 \cdot 10^5}{v^2}) = t \left( 0.3v^3 + \frac{3.5 \cdot 10^5}{v} \right)$$

$$\frac{\partial W_0}{\partial v} = R \left( 0.6v - \frac{7 \cdot 10^5}{v^3} \right) + \frac{\partial R}{\partial v} \left( 0.3v^2 + \frac{3.5 \cdot 10^5}{v^2} \right)$$

$$\frac{\partial R}{\partial v} = 0 \Rightarrow R \left( 0.6v - \frac{7 \cdot 10^5}{v^3} \right) = 0 \Rightarrow v = 32.87 \text{ m/s}$$

Allgemein:

$$W = F \cdot d = W_0$$

$$\min F = \max d$$

$$\min F = \min F_{air} \Rightarrow 0.6v - \frac{7 \cdot 10^5}{v^3} = 0 \Rightarrow 0.6v^4 - 7 \cdot 10^5 = 0$$

$$\Rightarrow v = \sqrt[4]{\frac{7 \cdot 10^5}{0.6}} = 32.87 \text{ m/s}$$

b)  $W_0 = \sqrt{t} F = (Fv)t = \text{Power} \cdot t$

$$t = \frac{W_0}{\text{Power}} \Rightarrow t(v) = \frac{W_0}{0.3v^3 + \frac{3.5 \cdot 10^5}{v}} \Rightarrow \max t(v) \Leftrightarrow \min Fv$$

minimize rate of work, i.e. minim. power  $\Rightarrow$  slowest use of energy/fuel, max endurance (time airborne)

$$\Rightarrow \min Fv \Rightarrow \min 0.3v^3 + 3.5 \cdot 10^5 v^{-1} \Rightarrow 0.9v^2 - \frac{3.5 \cdot 10^5}{v^2} = 0$$

$$0.9v^4 = 3.5 \cdot 10^5 \Rightarrow v = 24.97 \text{ m/s}$$

$$W_0 = \sqrt{t} (\alpha v^2 + \frac{\beta}{v^2})$$

$$0 = t (\alpha v^2 + \frac{\beta}{v^2}) + \sqrt{t} (2\alpha v - \frac{2\beta}{v^3})$$

$$F_{\text{air}} = \alpha v^2 + \frac{\beta}{v^2}$$

$$W_0 = R(v) \cdot F_{\text{air}}(v)$$

$$R \cdot v + t(v)$$

$$\frac{\partial W_0}{\partial v} = \frac{\partial R(v)}{\partial v} F_{\text{air}}(v) + R(v) \frac{\partial F_{\text{air}}}{\partial v} \Rightarrow R(v) F'_{\text{air}}(v) = 0 \Rightarrow F'_{\text{air}}(v) = 0$$

$$\frac{\partial R}{\partial v} = t(v) + v t'(v) = 0 \Rightarrow$$

$$W_0 = v t(v) F_{\text{air}}(v) \Rightarrow t(v) = \frac{W_0}{v \cdot F_{\text{air}}(v)}$$

$$t'(v) = -\frac{W_0 (\cancel{v} \cdot F_{\text{air}}(v))'}{(\cancel{v} \cdot F_{\text{air}}(v))^2} = 0 \Rightarrow (\cancel{v} \cdot F_{\text{air}}(v))' = 0$$

The two conditions

$$F_{\text{air}} + v F'_{\text{air}}$$

$$\frac{\partial F_{\text{air}}(v)}{\partial v} = 0$$

$$\frac{\partial^2 F_{\text{air}}(v)}{\partial v^2} = F''_{\text{air}}$$

$$\frac{\partial (\cancel{v} F_{\text{air}}(v))}{\partial v} = 0$$

$$\frac{\partial^2 (\cancel{v} F_{\text{air}}(v))}{\partial v^2} = F'_{\text{air}} + v F''_{\text{air}} + F'_{\text{air}} = 2F'_{\text{air}} + v F''_{\text{air}}$$