P	Q	P→Q	(P→Q)→(P→Q)
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

P	Q	P∧Q	(P∧Q)→Q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

P	Q	P∨Q	¬(P∨Q)	¬P	¬Q	¬P∧¬Q	¬(P∨Q)→¬P∧¬Q	¬P∧¬Q → ¬(P∨Q)	¬P∧¬Q ↔ ¬(P∨Q)
T	T	T	F	F	F	F	T	T	T
T	F	T	F	F	T	F	T	T	T
F	T	T	F	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T

logically equivalent
(also tautologically equivalent)

logically necessary

max is home A
claire is at the library B

A	B	¬B	¬B→A	A→¬B
T	T	F	T	F
T	F	T	F	T
F	T	F	T	T
F	F	T	F	T

P	Q	P↓Q	¬P P↓P	¬Q Q↓Q	¬P↓¬Q P∧Q (P↓P)↓(Q↓Q)
T	T	F	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	F

$a \leftrightarrow b$

$(a \land b) \lor (\neg a \land \neg b)$

$\neg(\neg a \lor \neg b) \lor \neg(\neg\neg a \lor \neg\neg b)$

$\neg(\neg a \lor \neg b) \lor \neg(a \lor b)$

a	b	$a \leftrightarrow b$	$\neg a$	$\neg b$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	F
F	F	T	T	T

7.28 $[P \land (Q \lor R)] \lor [\neg P \land \neg (Q \land R)]$

$[P \land (Q \lor R)] \lor \neg [P \lor (Q \land R)]$

7.29

p	q	$p \mid q$ (Scheffer, nand)	$\neg p$	$\neg(p \mid q)$	$(p \mid q) \mid p$	$(p \mid q) \mid q$	$[(p \mid q) \mid p] \mid (p \mid q)$	$\neg[(p \mid q) \mid p] \mid (p \mid q)$
T	T	F	F	T	T	T	T	F
T	F	T	F	F	F	T	T	F
F	T	T	T	F	T	F	F	T
F	F	T	T	F	T	T	F	T

$\neg[(p \mid q) \mid p] \mid (p \mid q) \Leftrightarrow \neg p$

by symmetry, since $p \mid q \Leftrightarrow q \mid p$, we have

$\neg[(p \mid q) \mid q] \mid (p \mid q) \Leftrightarrow \neg q$

p	q	$p \mid q$	$p \land q$	$\neg(p \mid q)$	$p \lor q$	$\neg p$	$\neg q$	$\neg p \mid \neg q$	$\neg(\neg p \mid \neg q)$
T	T	F	T	T	T	F	F	T	F
T	F	T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F	T	F
F	F	T	F	F	F	T	T	F	T

$\neg p \land \neg q \Leftrightarrow \neg(p \lor q)$

$\neg p \land \neg q \Leftrightarrow \neg p \mid \neg q$

$\neg(\neg p \land \neg q) \Leftrightarrow \neg(\neg p \mid \neg q) \Leftrightarrow p \lor q$

7.30

$\rightarrow$

$\perp$

express $\neg P$, $P \wedge Q$, $P \vee Q$

P	$P \rightarrow P$	$\neg P$	$\perp$	$P \rightarrow \perp$
T	T	F	F	F
F	T	T	F	T

7.21 (the monkey principle)

A	B	$A \rightarrow B$	$\perp$	$A \rightarrow \perp$
T	T	T	F	F
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

If $A \rightarrow B$ is true then if B is false, A is false.

P	Q	$P \rightarrow Q$	$\perp$	$Q \rightarrow P$	$P \rightarrow \perp$	$Q \rightarrow \perp$	$\neg(P \rightarrow \perp)$	$Q \rightarrow \perp$	$P \rightarrow (Q \rightarrow \perp)$	$(P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp$
T	T	T	F	T	F	F	T	F	F	T
T	F	F	F	T	F	T	T	T	T	F
F	T	T	F	F	T	F	F	F	T	F
F	F	T	F	T	T	T	F	T	T	F
									$\Leftrightarrow P \vee Q$	$\Leftrightarrow P \wedge Q$

7.31 Max is home (A) whenever Claire is at the library (B).

A	B	$B \rightarrow A$
T	T	T
T	F	T
F	T	F
F	F	T

P	Q	$P \nabla Q$
T	T	F
T	F	T
F	T	T
F	F	F

7.32

∇ exclusive disjunction

$P \nabla Q$

P

$\neg Q$

...

S

Q

$\neg P$

...

T

$S \nabla T$

$(P \wedge \neg Q) \mid (\neg P \wedge Q)$

$P \wedge \neg Q$

...

$S \nabla T$

$\neg P \wedge Q$

...

$S \nabla T$

$S \nabla T$

∇ elimination

$P_1 \nabla P_2$

$\vdots$

$P_1 \wedge \neg P_2$

$\vdots$

$S_1 \nabla S_2$

$\neg P_1 \wedge P_2$

$\vdots$

$S_1 \nabla S_2$

$\triangleright\ S_1 \nabla S_2$

∇ introduction

$(P_1 \wedge \neg P_2) \vee (\neg P_1 \wedge P_2)$

$\vdots$

$\triangleright\ P_1 \nabla P_2$

A B

7.24 Max is home in spite of the fact that Claire is at the library.

In spite of can be interpreted in more than one way.

$A \wedge B$

A and B can be true and S can be false.

Max might be home exactly because Claire is at the library, not in spite of that fact.