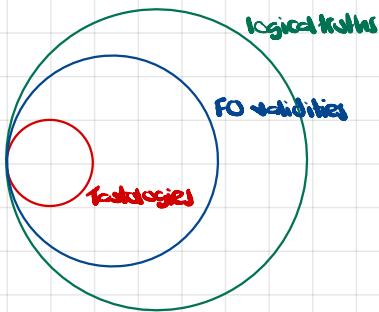


$\forall x \exists y$  FO valid



$\exists x \text{ Cube}(x) \rightarrow \text{Cube}(a)$

$A \rightarrow B$

not taut, not L.T.

$\text{Cube}(a) \rightarrow \exists x \text{ Cube}(x)$  L.T.

$A \rightarrow B$

$\forall x (\text{Cube}(x) \wedge \text{Small}(x)) \rightarrow \forall x (\text{Small}(x) \wedge \text{Cube}(x))$

$A \rightarrow B$

$\forall x (\text{Cube}(x) \leftrightarrow \text{Small}(x)) \leftrightarrow \neg \forall x (\text{Cube}(x) \leftrightarrow \text{Small}(x))$

$A \leftrightarrow \neg A$

$\forall x \text{ Cube}(x) \rightarrow \neg \exists x \neg \text{Cube}(x)$

$A \rightarrow \neg B$

$\forall z (\text{Cube}(z) \rightarrow \text{Large}(z)) \wedge \text{Cube}(b) \rightarrow \text{Large}(b)$

$(A \wedge B) \rightarrow C$

$\exists x \text{ Cube}(x) \rightarrow (\exists x \text{ Cube}(x) \vee \exists y \text{ Dodec}(y))$

$A \rightarrow (A \vee C)$

A	C	$A \wedge C$	$A \rightarrow (A \wedge C)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$(\exists x \text{ Cube}(x) \vee \exists y \text{ Dodec}(y)) \rightarrow \exists x \text{ Cube}(x)$

$(A \vee B) \rightarrow A$

A	B	$A \vee B$	$(A \vee B) \rightarrow A$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

$\frac{(\forall x \text{ Cube}(x) \rightarrow \forall y \text{ Small}(y)) \wedge \neg \forall y \text{ Small}(y)}{A} \rightarrow \neg \forall x \text{ Cube}(x)$

$[(A \rightarrow B) \wedge \neg B] \rightarrow \neg A$

A	B	$A \rightarrow B$	$\neg B$	$(A \rightarrow B) \wedge \neg B$	$\neg A$	$[(A \rightarrow B) \wedge \neg B] \rightarrow \neg A$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

10.6

$$A \rightarrow (B \wedge C)$$

$$D \rightarrow \neg B$$

$$\underline{D}$$

$$E$$

$$A \rightarrow (B \wedge C)$$

$$D \rightarrow \neg B$$

$$\underline{D}$$

$$\neg B$$

 $\rightarrow$  elim

$$B \wedge C$$

$$B$$

$$\perp$$

$$\neg(B \wedge C)$$

$$\neg A$$

$$\neg(B \wedge C) \rightarrow \neg A$$

 $\rightarrow$  intro

Lemma, Contraposition

E can't follow logically.

Now we at this point we have

$$\neg \exists x (cube(x) \wedge large(x))$$

$$\forall x \rightarrow (\neg cube(x) \wedge \neg large(x))$$

The conclusion follows from quantifiers, connectives and predicates here.

But we can also reach the conclusion via predicate knowledge.

$$\exists x A \rightarrow (B \wedge C)$$

$$D \rightarrow \neg B$$

$$\underline{D}$$

$$\forall x \neg A$$

$$\neg B$$

$$\neg(B \wedge C)$$

$$\neg \exists x A$$

$$\forall x \neg A$$

Therefore the argument is first-order valid.

10.7

Takes quantifiers into account

$$A \rightarrow (B \wedge C)$$

$$\neg A \leftrightarrow \neg A$$

$$F \rightarrow \neg B$$

$$F$$

A FO Valid

$$\neg B$$

 $\rightarrow$  elim

$$\neg(B \wedge C)$$

$$A$$

$$A \rightarrow (B \wedge C)$$

$$\neg A \leftrightarrow \neg A$$

$$E \rightarrow \neg B$$

$$E$$

$$\underline{D}$$

$$\neg B$$

$$\neg(B \wedge C)$$

$$\neg A$$

D Taut. Valid



7.  $\forall x (\text{Between}(x, a, d) \vee \neg \text{Between}(x, a, d))$

Everything is between a and d or not between a and d.

TFF: A

Repl.:  $\forall x (A \vee \neg A)$  FO valid ②

logical truth

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8.  $\forall x (\text{Between}(x, a, d) \vee \neg \text{Between}(x, d, a))$

Everything is either between a and d or not between a and d.

TFF: A

Repl.:  $\forall x (A \vee B)$  not FO valid

logical truth

FO counterex:  $\forall x (\text{Tel}(x) \vee \text{Cube}(x))$  in world with one Dodec.

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9.  $\forall x (\text{Dodec}(x) \rightarrow (x = d \vee \text{Small}(x)))$  ③

Every dodec is either d or is small.

not logical truth.

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10.  $\forall x (\text{Cube}(x) \rightarrow \text{LeftOf}(x, e)) \rightarrow \neg \exists y (\text{Cube}(y) \wedge \neg \text{LeftOf}(y, e))$

I) every cube is left of e then no cube is not left of e.

TFF:  $A \rightarrow \neg B$

Repl.:  $\forall x (A \rightarrow B) \rightarrow \neg \exists y (A \wedge \neg B)$  FO valid ③

$$\begin{aligned} \forall x (A \rightarrow B) &\rightarrow \forall x \neg (A \wedge \neg B) \\ &\forall x (\neg A \vee B) \\ &\forall x (A \rightarrow B) \end{aligned}$$

10.14

$$\begin{array}{l} \text{Cube}(a) \\ \text{Dodec}(b) \\ \neg(a=b) \end{array}$$

$$\text{TFF: } \begin{array}{l} A \\ B \\ \neg C \end{array} \quad \text{Rep.: } \begin{array}{l} A \\ B \\ \neg(a=b) \end{array}$$

L.C., not FO valid

FO counterex:

- All moths are mortal.
- Bees make honey.
- Apples are sweeter than avocados.

10.15

$$\begin{array}{l} \text{Cube}(a) \\ \neg(\text{Cube}(b)) \end{array}$$

$$\neg(a=b)$$

$$\text{TFF } \begin{array}{l} A(a) \\ \neg A(b) \\ \neg C(a,b) \end{array} \quad \text{Rep. } \begin{array}{l} A(a) \\ \neg A(b) \\ \neg(a=b) \\ \text{FO valid} \\ \begin{array}{l} a=b \\ A(b) \\ \neg A(b) \\ \perp \\ \neg(a=b) \end{array} \end{array}$$

10.25  $\neg \exists z \text{ Small}(z) \leftrightarrow \exists z \neg \text{Small}(z)$