Assumption: $\sqrt{2}$ is rational.

$\sqrt{2} = \frac{p}{q}$, p,q not both even

$\Leftrightarrow \frac{p^2}{q^2} = 2$

$\Leftrightarrow p^2 = 2q^2$

we know that even numbers are divisible by 2
therefore $2q^2$, p^2, and p are even.

we also know that given any even n^2, it is divisible by 4.

$2q^2$ is divisible by 4 and q^2 is divisible by 2.

Therefore p and q are even.

T Cube(c) ∨ Dodec(c)
T Tet(b)

Assume b = c T

T? T T
(b = c) ∧ Tet(b) ∧ [Cube(c) ∨ Dodec(c)]

conjunction elimination: (b = c) ∧ Tet(b) is true

By principle of indiscernibility of identicals, Tet(c) is true.

(b = c) ∧ Tet(b) → Tet(c)

But it is the case that either Cube(c) or Dodec(c)

Assume Cube(c). This contradicts Tet(c).
Assume Dodec(c). " " Tet(c).

Therefore

<=> [(b = c) ∧ Tet(b) ∧ Cube(c)] ∨ [(b = c) ∧ Tet(b) ∧ Dodec(c)]

Both disjuncts are not logically possible. The sentence is always false.

we started

$Small(c) \wedge Dodec(c) \wedge (\neg(Medium(c) \wedge LeftOf(c,b)) \wedge \neg RightOf(c,d))$

$Small(c) \wedge Dodec(c) \wedge (\neg Medium(c) \vee \neg LeftOf(c,b)) \wedge \neg RightOf(c,d)$

$P \wedge (Q \vee (\neg R \wedge Q))$

$\Rightarrow Q \vee (\neg R \wedge Q)$

case 1: Q

case 2: $\neg R \wedge Q \Rightarrow Q$

Therefore Q.

Since P and Q the $P \wedge Q$ by conjunction intro.

$Small(b)$

$Large(d)$

$(b = d) \vee Tet(c)$

$Small(b) \wedge Large(d) \Rightarrow b \neq d$

$b \neq d$

$Tet(c)$

---

1 If he sees blood he can't sleep.

2 He will take test tomorrow.

3 If he does not sleep night before test, he fails test.

4 If he passes test, he will go to medical school.

5 He will go to medical school.

6 Wes Craven movie shows blood

7 Romantic comedy does not show blood.

8 He and girlfriend will see either Craven or romantic comedy.

Assume he goes see the Wes Craven movie.

Then he will see blood, won't sleep, fail the test, can't go to medical school.

This contradicts the last premise. Therefore he will not see the Craven movie.

Therefore he will see the comedy.

All six premises plus the assumption of going to the movie can't all be true because then we obtain a false statement.

If premise 6 is false then the premises are inconsistent.

A proof by cases using premise 8 shows that he can't get into medical school.

1. All doors bolted from inside.
2. No sign of forced entry on doors.
3. Way in through small bathroom window on first floor, left open
4. Way in through unlocked bedroom window, 2nd floor.
5. There are only two ways in.
6. A man weighing 250 lbs does not fit in a small window.
7. A man with arthritis cannot reach the 2nd floor window.
8. Julius weighs 250 lbs.
9. Julius is arthritic.
10. There was a burglary.

3, 4, 5, 10 $\Rightarrow$ The burglar went in through small window | The burglar went in through second floor.

Case 1: The burglar went in through small window

Assume Julius is the burglar. Then Julius went in through the small window.
This contradicts the premise that Julius did not go through because he does not fit.
Therefore, he is not the burglar.

Case 2: The burglar went in through second floor.

Assume Julius is the burglar. Then he went in through 2nd floor.
This contradicts 7.
Therefore he is not the burglar.

5.1

$P \vee Q$

$\neg P$

Q

3 statements
2 premises, 1 conclusion
1 argument

P	Q	$\neg P$	$P \vee Q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

if premises are all true, conclusion is true. Therefore the argument is valid.

Q is logical consequence of ¬P and P∨Q.

$\neg P \wedge (P \vee Q) \Rightarrow Q$

5.2

$P \vee Q$

Q

$\neg P$

Invalid argument

P	Q	$P \vee Q$	$\neg P$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

5.3

$\neg(P \vee Q)$

$\neg P$

Valid

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	T
F	F	F	T	T

5.4

$\neg(P \wedge Q)$

P

$\neg Q$

Valid

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg Q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	F
F	F	F	T	T

5.5

$\neg(P \wedge Q)$

$\neg P$

Invalid

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	T
F	F	F	T	T

5.6

$P \wedge Q$

$\neg P$

R

premises are logically inconsistent : can never be simult. true. Therefore there is no circumstance in which the premises are true and the conclusion false. This is true for any conclusion. Therefore any conclusion is a logical consequence of inconsistent premises.

P	Q	R	$P \wedge Q$	$\neg P$
T	T	T	T	F
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

5.8

BackOf(a,b)	FrontOf(b,a)
T	T
F	F

5.13

P	S
T	T
F	?

Q	S
T	T
F	?

P	Q	P ∨ Q	S
T	T	T	T
T	F	T	
F	T	T	
F	F	F	

P ∨ Q

5.16

(Home(max) ∨ Home(claire)) ∧ (¬Happy(scruffy) ∨ ¬Happy(carl))

([Home(max) | Home(claire)] ∧ ¬Happy(scruffy)) ∨ ([Home(max) | Home(claire)] ∧ ¬Happy(carl))

[[Home(max) ∧ ¬Happy(scruffy)] | [Home(claire) ∧ ¬Happy(scruffy)]] | [[Home(max) ∧ ¬Happy(carl)] | [Home(claire) ∧ ¬Happy(carl)]]