1.

Euclid - Geometry — 5-A

Russell - the, both, neither — 6-B

Grice - Implicature — 4-C

Gödel - no complete axiomatization of arithmetic in FOL — 2-F

2.

A: All FOL sentences are wffs.

B: All wffs are expressions.

C: All FOL sentences are expressions.

A: $\forall x\,(\text{Sentence}(x) \to \text{Wff}(x))$

B: $\forall x\,(\text{Wff}(x) \to \text{Expr}(x))$

C: $\forall x\,(S(x) \to E(x))$

$\boxed{a}$ $S(a)$

$S(a) \to W(a)$

$W(a) \to E(a)$

$W(a)$

$E(a)$

$\forall x\,(S(x) \to E(x))$

3.

$\exists x\,(CNF(x) \land NNF(x))$

$\forall x\,(DNF(x) \to \neg NNF(x))$

$\exists x\,(CNF(x) \land \neg DNF(x))$

$\boxed{a}$ $CNF(a) \land NNF(a)$

$NNF(a)$

$\neg(\neg NNF(a))$

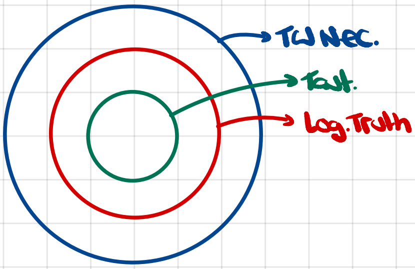
$\neg DNF(a)$

$CNF(a)$

$CNF(a) \land \neg DNF(a)$

$\forall x\,(CNF(x) \land \neg DNF(x))$

4.



$\exists x\,(T(x) \land L(x))$ T

$\forall x\,(L(x) \to N(x))$ T

$\exists x\,(T(x) \land N(x))$

5.

All axioms are necessary truths. F

No theorem is a necessary truth. F

No theorems are axioms

$\forall x\,(A(x) \to N(x))$

$\forall x\,(T(x) \to \neg N(x))$

$\forall x\,(T(x) \to \neg A(x))$

$\boxed{a}$ $T(a)$

$T(a) \to \neg N(a)$

$A(a) \to N(a)$

$\neg N(a)$

$\neg A(a)$

$\forall x\,(T(x) \to \neg A(x))$

6.

$\forall x\,(AI(x) \to \neg S(x))$ T

$\exists x\,(S(x) \wedge \neg V(x))$ F

$\forall x\,(AI(x) \to \neg V(x))$

counterexample:

y is an unsound argument that is valid.

y has inconsistent premises.

ie $\neg S(y) \wedge V(y)$

$AI(y)$

Therefore $AI(y) \wedge V(y)$

$\exists x\,(AI(x) \wedge V(x))$

$\exists x\,\neg(\neg AI(x) \vee \neg V(x))$

$\exists x\,\neg(AI(x) \to \neg V(x))$

$\neg \forall x\; AI(x) \to \neg V(x)$

7.

Abraham can fool all of the people all of the time.

$\forall x \forall y\,((Time(y) \wedge Person(x)) \to CanFool(abraham, x, y))$

Abraham can't fool ...

$\neg \forall x \forall y\,((Time(y) \wedge Person(x)) \to CanFool(abraham, x, y))$

$\neg \forall x \forall y\,(\neg(Time(y) \wedge Person(x)) \vee CanFool(abraham, x, y))$

$\exists x \exists y\; \neg(\neg(Time(y) \wedge Person(x)) \vee CanFool(abraham, x, y))$

$\exists x \exists y\,((Time(y) \wedge Person(x)) \wedge \neg CanFool(abraham, x, y))$

8. There is a sucker born every minute

11. One man in his time plays many parts.

many y
satisfying Part(y)
satisfy $Q(y)$

many y
satisfying $y = y$
satisfy $Q(y)$

Invariant under expansion (P) $\leftrightarrow$ $(P_u \rightarrow P_{u^+})$

" " contraction(P) $\leftrightarrow$ $(P_u \rightarrow P_{u^-})$

$\exists x (\forall y (x \neq y \rightarrow \text{Smaller}(x,y)) \rightarrow \exists z\, \text{Tet}(z))$

$\exists x (\neg(\forall y (x \neq y \rightarrow \text{Smaller}(x,y)) \vee \exists z\, \text{Tet}(z))$

$\exists x (\neg(\forall y (x = y \vee \text{Smaller}(x,y))) \vee \exists z\, \text{Tet}(z))$

$\exists x (\exists y (x \neq y \wedge \neg \text{Smaller}(x,y)) \vee \exists z\, \text{Tet}(z))$

$\exists x \exists y \exists z ((x \neq y \wedge \neg \text{Smaller}(x,y)) \vee \exists z\, \text{Tet}(z))$

$\exists x \exists y \exists z (\neg (x = y \vee \text{Smaller}(x,y)) \vee \exists z\, \text{Tet}(z))$

$\exists x \exists y \exists z ((x \neq y \rightarrow \text{Smaller}(x,y)) \rightarrow \exists z\, \text{Tet}(z))$

True

$\exists x (\forall y (x \neq y \rightarrow \text{Smaller}(x,y))$ false

$\exists x (\exists y \neg (x \neq y \rightarrow \text{Smaller}(x,y))$ true
$\neg (x = y \vee \text{Smaller}(x,y))$
$(x \neq y \wedge \neg \text{Smaller}(x,y)$

inv exp.
$\neg$ inv con.

$\exists x \neg (\forall y (x \neq y \rightarrow \neg \text{Tet}(y)) \rightarrow \neg \exists z\, \text{Smaller}(x,z))$

$\exists x (\forall y (x \neq y \rightarrow \text{Smaller}(x,y)) \rightarrow \exists z\, \text{Tet}(z))$

$\exists x (\forall y (A(x,y) \rightarrow B(x,y)) \rightarrow \exists z\, C(z))$

$\exists x (\neg(\forall y (A(x,y) \rightarrow B(x,y))) \vee \exists z\, C(z))$

$\exists x (\neg(\forall y (\neg A(x,y) \vee B(x,y))) \vee \exists z\, C(z))$

$\exists x (\exists y \neg (\neg A(x,y) \vee B(x,y)) \vee \exists z\, C(z))$

$\exists x \exists y \exists z ((A(x,y) \rightarrow B(x,y)) \rightarrow \exists z\, C(z))$

$\forall x \forall y \ x = y$

$\exists x \forall y \ x = y \mid x \neq y$

$\forall x (\exists y (x \neq y \to \text{SameSize}(x,y)) \to \neg \exists z \, \text{Tet}(z))$

$\forall x (\neg \exists y (x \neq y \to \text{SameSize}(x,y)) \vee \neg \exists z \, \text{Tet}(z))$

$\forall x (\forall y \ \neg (x \neq y \to \text{SameSize}(x,y)) \vee \neg \exists z \, \text{Tet}(z))$

$\forall x (\forall y \ \neg (x = y \vee \text{SameSize}(x,y)) \vee \ \forall z \ \neg \text{Tet}(z)$

$\forall x \forall y \forall z \ ((x \neq y \to \text{SameSize}(x,y)) \to \neg \text{Tet}(z))$

when is $\forall x (\exists y (x \neq y \to \text{SameSize}(x,y)) \to \neg \exists z \, \text{Tet}(z))$ true?

- $\forall x, \exists y (x \neq y \to \text{SameSize}(x,y))$ false.

$\forall x (\neg \exists y (x \neq y \to \text{SameSize}(x,y)))$ true

$\forall x (\forall y \ \neg (x = y \vee \text{SameSize}(x,y)))$

$\forall x \forall y \ (x \neq y \wedge \neg \text{SameSize}(x,y))$

↳ Inv Exp.

- $\neg \exists z \, \text{Tet}(z)$ true

Inv Con.

when is $\forall x \forall y \forall z \ ((x \neq y \to \text{SameSize}(x,y)) \to \neg \text{Tet}(z))$ true?

- $\forall x \forall y \forall z \ (x \neq y \to \text{SameSize}(x,y))$ false

$\forall x \forall y \forall z \ \neg (x \neq y \to \text{SameSize}(x,y))$

$\forall x \forall y \forall z \ \neg (x = y \vee \text{SameSize}(x,y))$

- $\forall x \forall y \forall z \ \ \neg \text{Tet}(z)$ true

$\exists x \neg (\forall y (x \neq y \to \neg Tet(y)) \to \neg \exists z\, Smaller(x,z))$

$\exists x \neg (\neg \forall y (x \neq y \to \neg Tet(y)) \vee \neg \exists z\, Smaller(x,z))$

$\exists x \neg (\exists y \neg (x \neq y \to \neg Tet(y)) \vee \neg \exists z\, Smaller(x,z))$

$\exists x (\neg \exists y \neg (x \neq y \to \neg Tet(y)) \wedge \exists z\, Smaller(x,z))$

$\exists x (\forall y (x \neq y \to \neg Tet(y)) \wedge \exists z\, Smaller(x,z))$

$\exists x \forall y \exists z ((x \neq y \to \neg Tet(y)) \wedge Smaller(x,z))$

$\exists x \forall y \exists z \neg (\neg (x \neq y \to \neg Tet(y)) \vee \neg Smaller(x,z))$

$\exists x \forall y \exists z \neg ((x \neq y \to \neg Tet(y)) \to \neg Smaller(x,z))$

True if

$x \neq y \to \neg Tet(y)$ and $Smaller(x,z)$
$x = y \vee \neg Tet(y)$ and $Smaller(x,z)$

There is one object and it isn't a tet. There is another object and the sole object is smaller than it.

Always false. Inconsistent.