Odd(n+m) → Even(nm)

n	m	n+m	nm
2	4	6	8
3	3	6	9
3	4	7	12

P ↔ Q   ⟺   (P→Q)∧(Q→P)
                (¬P∨Q)∧(¬Q∨P)
                ((¬P∨Q)∧¬Q)∨((¬P∨Q)∧P)
                [(¬Q∧¬P)∨(Q∧¬Q)]∨[(P∧¬P)∨(P∧Q)]
                            F   F

                (P∧Q)∨(¬P∧¬Q)

a∨b
b∨c
a∨c ?

a	b	c	a∨b	b∨c	a∨c
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	F	F	F

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

¬(A∧B)∨C ⟺ (A∧B)→C
¬A∨¬B∨C

B→C   ¬B∨C

A	B	C	B→C	A∧B	(A∧B)→C
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	T	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

¬B

¬A∨¬B∨C   ∨ intro

C

¬A∨¬B∨C   ∨ intro

¬A∨¬B∨C   ∨ elim

A	B	C	B→C	A∨B	(A∨B)→C
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	F	T
F	F	F	T	F	T

¬B∨C

¬B

(A∨B)→C ←→ ¬(A∨B)∨C

(¬A∧¬B)∨C

A∨B
A→C
B→D

  A
  C   → elim

  B
  D   → elim

C∨D   ∨ elim

A←→B
B←→C
A←→C

  A
  B   ←→ elim
  C   ←→ elim

  C
  B   ←→ elim
  A   ←→ elim

A←→C   ←→ intro

8.15 $\quad 1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 1$

6 → 5

n^3 div by 27
$n^3 = 27m = 3 \cdot 9m, \; m \in \mathbb{N}$
n^3 div by 9

5 → 4

n^3 div by 9
$n^3 = 9m = 3 \cdot 3m, \; m \in \mathbb{N}$
n^3 div by 3

4 → 3

n^3 div by 3
3 is prime
$n^3 = 3^3 q^3, \; q \in \mathbb{N}$ Coroll. of Fund. Th. of Arithmetic
$n^2 = 3^2 q^2$
n^2 div by 9

3 → 2

n^2 div by 9
$n^2 = 9m, \; m \in \mathbb{N}$
n^2 div by 3

2 → 1

n^2 div by 3
$n^2 = 3^2 q^2, \; q \in \mathbb{N}$
$n = 3q$
n div by 3

1 → 6

n div by 3
$n = 3m, \; m \in \mathbb{N}$
$n^3 = 27m^3$
n^3 div by 3

R taut con $P_1,...,P_n,Q$ → $Q→R$ taut con $P_1,...,P_n$

$\boxed{\begin{array}{l} R \text{ taut con } P_1,...,P_n,Q \\ \quad \text{Assume } P_1,...,P_n \\ \qquad \text{Assume } Q \\ \qquad \text{Then } R \\ \quad Q→R \\ (P_1∧...∧P_n) → (Q→R) \end{array}}$

$P∨¬P$	Taut Con
$P→Q$	
P	
Q	→ elim
$¬P∨Q$	∨ intro
$¬P$	
$¬P∨Q$	∨ intro
$¬P∨Q$	∨ elim
$(P→Q)→(¬P∨Q)$	→ intro
$¬P∨Q$	
P	
$¬¬P$	¬ elim
$¬Q$	
$¬¬P∧¬Q$	
$¬(¬P∨Q)$	Taut con, DeMorgan
$⊥$	⊥ intro
Q	
$P→Q$	
$(¬P∨Q)→(P→Q)$	→ intro
$(P→Q) <-> (¬P∨Q)$	<-> intro

$P <-> ¬P$

$¬(P∨¬P)$
P
$P∨¬P$
$⊥$
$¬P$

$¬P$
$P∨¬P$
$⊥$
P
$⊥$
$P∨¬P$

$¬P∧¬¬P <=> P∧¬P$

$P∧¬Q <=> ¬(¬P∨Q)$

$¬¬P∧¬Q <=> ¬(¬P∨Q)$

$¬¬P∧¬Q$
$¬¬P$
P
$¬Q$
$\quad ¬P∨Q$
$\quad ¬P$
$\quad ⊥$

$\quad Q$
$\quad ⊥$
$¬(¬P∨Q)$

$P∧¬Q$
$¬P∨Q$

$(P \wedge Q) \to P$

$\neg$

$\neg(P \wedge Q) \vee P$
  $\neg(P \wedge Q)$
  $\neg(P \wedge Q) \vee P$

  P
  $\neg(P \wedge Q) \vee P$

$\neg(\neg(P \wedge Q) \vee P)$
  $\neg(P \wedge Q)$
  $\neg(P \wedge Q) \vee P$
  $\bot$
$P \wedge Q$

        $P \wedge Q$
        P
        $(P \wedge Q) \to P$

$P \vee \neg P$
  P
  $\neg(P \wedge Q) \vee P$

  $\neg P$
    $P \wedge Q$
    P
    $\bot$
  $\neg(P \wedge Q)$
  $\neg(P \wedge Q) \vee P$

$\neg(P \wedge Q) \vee P$