$\forall x \forall y [(Tet(y) \rightarrow LeftOf(x,y)) \rightarrow \forall z (Dodec(z) \rightarrow LeftOf(x,z))]$

Rewrite the conditional as a disjunction:

$\forall x \forall y [\neg(Tet(y) \rightarrow LeftOf(x,y)) \vee \forall z (Dodec(z) \rightarrow LeftOf(x,z))]$

Use null quantification to place quantifier $\forall y$ into the first disjunct

$\forall x [\forall y \neg(Tet(y) \rightarrow LeftOf(x,y)) \vee \forall z (Dodec(z) \rightarrow LeftOf(x,z))]$

Use DeMorgan Laws for quantifiers

$\forall x [\neg \exists y (Tet(y) \rightarrow LeftOf(x,y)) \vee \forall z (Dodec(z) \rightarrow LeftOf(x,z))]$

$\exists x P(x) \rightarrow \forall x Q(x)$

$\neg \exists x P(x) \vee \forall x Q(x)$
$\forall x \neg P(x) \vee \forall x Q(x)$
$\forall x (\neg P(x) \vee \forall y Q(y))$
$\forall x \forall y (\neg P(x) \vee Q(y))$
$\forall x \forall y (P(x) \rightarrow Q(y))$

$\forall z \forall y \; [(\exists x \, (\text{Tet}(x) \wedge \text{Between}(x,z,y))) \rightarrow (\text{Small}(y) \wedge \text{Small}(z))]$

$\forall z \forall y \; [\neg \exists x \, (\text{Tet}(x) \wedge \text{Between}(x,z,y)) \vee (\text{Small}(y) \wedge \text{Small}(z))]$

$\forall z \forall y \; [\forall x \, \neg(\text{Tet}(x) \wedge \text{Between}(x,z,y)) \vee (\text{Small}(y) \wedge \text{Small}(z))]$

$\forall z \forall y \forall x \; [\neg(\text{Tet}(x) \wedge \text{Between}(x,z,y)) \vee (\text{Small}(y) \wedge \text{Small}(z))]$

11.20

Anything to the left of a is smaller than something that is in the back of every cube to the right of b.

$\forall x \; [\text{LeftOf}(x,a) \rightarrow \exists z \, \forall y \; [((\text{Cube}(y) \wedge \text{RightOf}(y,b)) \rightarrow \text{BackOf}(z,y)) \wedge \text{Smaller}(x,z)]]$

Every cube is smaller than some dodec but no cube is smaller than every dodec

Every cube has the property that it is smaller than some dodec and there is no cube w/ the property that it is smaller than every dodec.

$\forall x \; [\text{Cube}(x) \rightarrow \exists y \, (\text{Dodec}(y) \wedge \text{Smaller}(x,y))] \wedge \neg \exists x \; [\text{Cube}(x) \wedge \forall y \, (\text{Dodec}(y) \rightarrow \text{Smaller}(x,y))]$

If a is larger than some cube then it is smaller than every tetrahedron.

If some cube has the property that a is larger than it, then every tetrahedron has the property that the cube is smaller than it.

$\exists x \; ((\text{Cube}(x) \wedge \text{Larger}(a,x)) \rightarrow \forall y \, (\text{Tet}(y) \rightarrow \text{Smaller}(x,y))$

Only dodecs are larger than everything else.

$\forall x \, (\forall y \; \text{Larger}(x,y) \rightarrow \text{Dodec}(x))$

All objects w/ nothing in front of them are tetrahedra.

$\forall x \, (\forall y \; \neg \text{FrontOf}(y,x) \rightarrow \text{Tet}(x))$

Nothing is between two objects of same shape.

$\forall x \forall y \, (\text{SameShape}(x,y) \rightarrow \neg \exists z \; \text{Between}(z,x,y))$

Nothing but a cube is between two other objects.

$\forall x \forall y \, ((\exists z \; \text{Cube}(z) \wedge \text{Between}(z,x,y)) \vee (\neg \exists z \; \text{Between}(z,x,y)))$

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b has something behind it which has at least two objects behind it.

something has prop. that it is behind b and has prop. that

$\exists x (BehindOf(x,b) \wedge \exists y \exists z (BehindOf(y,x) \wedge BehindOf(z,x)))$

More than one thing is smaller than something larger than b.

there is one object with the property (...) and it is not the case that for all objects different than the first they don't have the prop.

$\exists x ((\exists z\, Larger(z,b) \wedge Smaller(x,z)) \wedge \exists y\, y \neq x\, (\exists z\, Larger(z,b) \wedge Smaller(x,z))$

11.26

1. Every cube is between a pair of dodecahedra.

strong: $\exists y \exists z \forall x\, (Cube(x) \rightarrow Between(x,y,z))$

weak: $\forall x (Cube(x) \rightarrow \exists y \exists z\, Between(x,y,z))$

2. Every cube to the right of a dodec is smaller than it is.

strong: $\exists y (\forall x ((Cube(x) \wedge Dodec(y) \wedge RightOf(x,y)) \rightarrow Smaller(x,y)))$

weak: $\forall x \forall y ((Cube(x) \wedge Dodec(y) \wedge RightOf(x,y)) \rightarrow Smaller(x,y))$

3. Cube a is not larger than every dodec.

$\forall x (Dodec(x) \rightarrow \neg Larger(a,x))$

$\neg \forall x (Dodec(x) \rightarrow Larger(a,x))$

$\exists x \neg (\neg Dodec(x) \vee Larger(a,x))$

$\exists x (Dodec(x) \wedge \neg Larger(a,x))$

4. No cube is to the left of some dodec.

strong: $\exists x (Dodec(x) \wedge (\forall y (Cube(y) \rightarrow \neg LeftOf(x,y))))$

weak: $\forall x (Cube(x) \rightarrow \neg \exists y (Dodec(y) \wedge LeftOf(x,y)))$

5. (At least) two cubes are between (at least) two dodecs.

Everything to the left of every tetrahedron is also to the left of every dodec.

$$\forall x \; \left(\forall y \, (\text{Tet}(y) \rightarrow \text{LeftOf}(x,y)) \rightarrow \forall z \, (\text{Dodec}(z) \rightarrow \text{LeftOf}(x,z)) \right)$$