

$\text{Cube}(a) \wedge \neg \text{Cube}(a)$

Always false.

TT-possible

$a=b \wedge \text{Larger}(a,b) \wedge \text{Larger}(b,a)$

logically possible

$\text{Larger}(a,b)$

$\wedge \text{Larger}(b,a)$

TW-possible

$\text{Large}(a)$

$\text{Tet}(a) \vee \neg \text{Tet}(a)$

$\text{Small}(b) \vee \text{Medium}(b) \vee \text{Large}(b)$

$\text{Tet}(a) \vee \text{Cube}(a) \vee \text{Dodec}(a)$

$a=a \wedge b=b$

$\neg (\text{Larger}(a,b) \wedge \text{Larger}(b,a))$

$\text{Cube}(a) \wedge \text{Larger}(a,b)$

$$\neg \neg (A \vee \neg B) \wedge \neg (\neg \neg A \wedge \neg B)$$

$$(A \vee \neg B) \wedge \neg (A \wedge \neg B)$$

$$(A \vee \neg B) \wedge (\neg A \vee B) \quad \text{(NNF)}$$


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$$A \wedge (B \wedge A)$$

$$(B \wedge A) \wedge A \quad \text{Comm.}$$

$$B \wedge (A \wedge A) \quad \text{Assoc.}$$

$$B \wedge A \quad \text{Idempotence of } \wedge$$

$$A \wedge B$$


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$$(A \vee B) \wedge C \wedge (\neg (\neg B \wedge \neg A) \vee B)$$

$$(A \vee B) \wedge C \wedge ((B \vee A) \vee B)$$

$$(A \vee B) \wedge C \wedge (A \vee B)$$

$$(A \vee B) \wedge C \quad \text{NNF, CNF}$$

$$(C \wedge A) \vee (C \wedge B) \quad \text{DNF}$$


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$$A \quad \text{NNF, CNF, DNF}$$


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$$(L(a,b) \vee B(a,b)) \wedge Cube(a)$$

$$(Cube(a) \wedge L(a,b)) \vee (Cube(a) \wedge B(a,b))$$

$$L(a,b) \wedge (Cube(a) \vee Tet(a) \vee a=b)$$

$$(L(a,b) \wedge Cube(a)) \vee (L(a,b) \wedge Tet(a)) \vee (L(a,b) \wedge a=b)$$

$$(Between(a,b,c) \vee Tet(a) \vee \neg Tet(b)) \wedge Dodec(c)$$

$$C(a) \wedge C(b) \wedge (\neg S(a) \vee \neg S(b))$$

$$(C(a) \wedge C(b) \wedge \neg S(a)) \vee (C(a) \wedge C(b) \wedge \neg S(b))$$

$$(S(a) \vee H(a)) \wedge (C(a) \vee \neg D(a))$$

$$[(S(a) \vee H(a)) \wedge C(a)] \vee [(S(a) \vee H(a)) \wedge \neg D(a)]$$

$$(C(a) \wedge S(a)) \vee (C(a) \wedge H(a)) \vee (\neg D(a) \wedge S(a)) \vee (\neg D(a) \wedge H(a))$$


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$$4.33 \quad (A \wedge B) \wedge A \stackrel{\text{commut.}}{\Leftrightarrow} A \wedge (A \wedge B) \stackrel{\text{assoc.}}{\Leftrightarrow} (A \wedge A) \wedge B \stackrel{\text{idemp.}}{\Leftrightarrow} A \wedge B$$

$$4.34 \quad (B \wedge (A \wedge B \wedge C)) \stackrel{\text{commut.}}{\Leftrightarrow} B \wedge (B \wedge A \wedge C) \stackrel{\text{assoc.}}{\Leftrightarrow} (B \wedge B) \wedge A \wedge C \stackrel{\text{idemp.}}{\Leftrightarrow} B \wedge A \wedge C \stackrel{\text{commut.}}{\Leftrightarrow} A \wedge B \wedge C$$

$$4.37 \quad (A \wedge B) \vee C \vee (B \wedge A) \vee A$$

$$\Leftrightarrow (A \wedge B) \vee (B \wedge A) \vee C \vee A$$

$$\Leftrightarrow (A \wedge B) \vee (A \wedge B) \vee C \vee A$$

$$\Leftrightarrow ((A \wedge B) \vee (A \wedge B)) \vee C \vee A$$

$$\Leftrightarrow (A \wedge B) \vee C \vee A \quad \text{DNF}$$

4.40

$$1. \neg [(C(a) \wedge \neg S(a)) \vee (\neg C(a) \wedge S(a))] \quad \text{NNF}$$

$$4.43 \quad A \wedge (A \wedge (B \vee (A \wedge C)))$$

$$\Leftrightarrow A \wedge (A \wedge ((B \vee A) \wedge (B \vee C)))$$

$$\Leftrightarrow (A \wedge A) \wedge ((B \vee A) \wedge (B \vee C))$$

$$\Leftrightarrow A \wedge (B \vee A) \wedge (B \vee C) \quad \text{CNF}$$

$$A \wedge (A \wedge (B \vee (A \wedge C)))$$

$$\Leftrightarrow (A \wedge A) \wedge (B \vee (A \wedge C))$$

$$\Leftrightarrow A \wedge (B \vee (A \wedge C))$$

$$\Leftrightarrow (A \wedge B) \vee (A \wedge (A \wedge C))$$

$$\Leftrightarrow (A \wedge B) \vee ((A \wedge A) \wedge C)$$

$$\Leftrightarrow (A \wedge B) \vee (A \wedge C)$$