| B | C |
|---|---|
| T | T |
| T | F |
| F | T |
| F | F |

$P_1, \dots, P_n \vdash_T S \rightarrow S$ taut con — Soundness

$S$ taut con $P_1, \dots, P_n \rightarrow P_1, \dots, P_n \vdash_T S$ — Completeness

B is not taut. con of C.
Therefore, by contrapositive of completeness theorem, there is no proof in $F_T$ of B given C

| | | |
|---|---|---|
| 1 | $\neg(A \wedge (B \vee C))$ | |
| 2 | $(B \rightarrow C) \rightarrow B$ | |
| 3 | $(C \rightarrow B) \rightarrow A$ | |
| 4 | $\quad$ \| $B$ | |
| 5 | $\quad$ \| \| $C$ | |
| 6 | $\quad$ \| \| $B$ | reit 4 |
| 7 | $\quad$ \| $C \rightarrow B$ | $\rightarrow$ intro, 5-6 |
| 8 | $\quad$ \| $A$ | $\rightarrow$ elim, 3,7 |
| 9 | $\quad$ \| $B \vee C$ | $\vee$ intro 4 |
| 10 | $\quad$ \| $A \wedge (B \vee C)$ | $\wedge$ intro 8,9 |
| 11 | $\quad$ \| $\bot$ | $\bot$ intro 10,1 |
| 12 | $\neg B$ | $\neg$ intro 4-11 |

| B | C | $B \rightarrow C$ | B | $(B \rightarrow C) \rightarrow B$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | F | F |

| | | |
|---|---|---|
| 1 | $A \to (B \vee C)$ | |
| 2 | $\neg(\neg A \vee C)$ | |
| 3 | $\neg B$ | |
| 4 | $\neg C$ | |
| 5 | $A$ | |
| 6 | $B \vee C$ | $\to$ elim 1,5 |
| 7 | $B$ | |
| 8 | $\neg C$ | |
| 9 | $B$ | reit 7 |
| 10 | $\neg B$ | reit 3 |
| 11 | $\perp$ | $\perp$ intro 9,10 |
| 12 | $\neg\neg C$ | $\neg$ intro 8-11 |
| 13 | $C$ | $\neg$ elim 12 |
| 14 | $C$ | |
| 15 | $C$ | reit 14 |
| 16 | $C$ | $\vee$ elim 6,7-13,14-15 |
| 17 | $\neg C$ | reit 4 |
| 18 | $\perp$ | $\perp$ intro 16,17 |
| 19 | $\neg A$ | $\neg$ intro 5-18 |
| 20 | $\neg A \vee C$ | $\vee$ intro 19 |
| 21 | $\neg(\neg A \vee C)$ | reit 2 |
| 22 | $\perp$ | $\perp$ intro 20,21 |
| 23 | $\neg\neg C$ | $\neg$ intro 4-22 |
| 24 | $C$ | $\neg$ elim 23 |
| 25 | | |

$A \to B$

$B \to (C \to D)$

$\neg A \to (E \wedge F)$

---

$C \to (F \vee D)$

$C$

$A \vee \neg A$

$A$

$B$

$C \to D$

$D$

$F \vee D$

$\neg A$

$E \wedge F$

$F$

$F \vee D$

$F \vee D$

$C \to (F \vee D)$

$(P \to Q) \to ((P \to \neg Q) \to \neg P)$

$P \to Q$

$P \to \neg Q$

$P$

$Q$

$\neg Q$

$\perp$

$\neg P$

$(P \to \neg Q) \to \neg P$

$\land \; \lor \; \neg \; \bot \; \rightarrow \; \leftrightarrow$

$\land$ intro

one prev. sentence cited that is a conjunct

example bug:

(...)

5 A

6 $A \land B$ $\land$ intro 5

$A \land B$ is not a taut. consequence of A

The system allows proofs of statements that are not taut. con.

Conversely, if a statement is not a taut. con., a proof can still exist.

Soundness: $P_1, \ldots, P_n \vdash_T S \rightarrow S$ taut con

$\sim(P_1, \ldots, P_n \vdash_T S) \lor (S$ taut con$)$

But now, $\sim(\sim(S$ taut con$) \rightarrow \sim(P_1, \ldots, P_n \vdash_T S))$

$\sim(S$ taut con $\lor \sim(P_1, \ldots, P_n \vdash_T S)$

$\sim(S$ taut con$) \land (P_1, \ldots, P_n \vdash_T S)$

completeness: $S$ taut con $\rightarrow P_1, \ldots, P_n \vdash_T S$

$\sim(S$ taut con$) \lor (P_1, \ldots, P_n \vdash_T S)$

still true. But now we also have $\sim(S$ taut con$) \land (P_1, \ldots, P_n \vdash_T S)$

Bug gets corrected.

second bug: $\land$ intro absent.

These are taut con. whose proofs depend on $\land$ intro.

$$\begin{tikzcd} \text{Proof} \arrow[r, bend left, "\text{Soundness}"] \arrow[r, phantom, "\Leftrightarrow"] & \text{Taut Con} \arrow[l, bend left, "\text{completeness}"] \end{tikzcd}$$

Proof $\rightarrow$ Taut Con

$\sim$(Taut Con) $\rightarrow \sim$ Proof

Taut Con $\rightarrow$ Proof

$\sim$Proof $\rightarrow \sim$(Taut Con)

| A(d) | D(d) | A(d) $\oplus$ D(d) | $A \vee D$ | $\neg(A \wedge D)$ | $(A \vee D) \wedge \neg(A \wedge D)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | T | T | T | T |
| F | T | T | T | T | T |
| F | F | F | F | T | F |

$(A(d) \wedge \neg D(d)) \mid (\neg A(d) \wedge D(d))$

| A | E | $A \vee E$ | $A \oplus E$ | $A \wedge E$ | $(A \oplus E) \oplus (A \wedge E)$ |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | T | T | F | T |
| F | T | T | T | F | T |
| F | F | F | F | F | F |

| A | B | $A \vee B$ | $B \vee A$ | $A \oplus B$ | $B \oplus A$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | T | T | T | T |
| F | T | T | T | T | T |
| F | F | F | F | F | F |

| A | B | C | $A \oplus B$ | $(A \oplus B) \oplus C$ | $B \oplus C$ | $A \oplus (B \oplus C)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | T | F | T |
| T | T | F | F | F | T | F |
| T | F | T | T | F | T | F |
| T | F | F | T | T | F | T |
| F | T | T | T | F | F | F |
| F | T | F | T | T | T | T |
| F | F | T | F | T | T | T |
| F | F | F | F | F | F | F |

| A | B | C | $A \oplus B$ | $B \oplus C$ | $A \oplus C$ |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | T | F | F | T | T |
| T | F | T | T | T | F |
| T | F | F | T | F | T |
| F | T | T | T | F | T |
| F | T | F | T | T | F |
| F | F | T | F | T | T |
| F | F | F | F | F | F |

| A | $A \oplus A$ | $A \vee A$ |
|---|---|---|
| T | F | T |
| F | F | F |

$\neg(A \oplus B) \Leftrightarrow (\neg A \wedge \neg B)$

$\neg(A \oplus B)$

$(A \wedge B) \vee (\neg A \wedge \neg B)$

| A | B | $A \oplus B$ | $\neg(A \oplus B)$ | $\neg A \wedge \neg B$ |
|---|---|---|---|---|
| T | T | F | T | F |
| T | F | T | F | F |
| F | T | T | F | F |
| F | F | F | T | T |

$\neg(A \wedge B) \Leftrightarrow (\neg A \oplus \neg B)$

| A | B | $A \wedge B$ | $\neg(A \wedge B)$ | $\neg A$ | $\neg B$ | $\neg A \oplus \neg B$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | F | T | F | T | T |
| F | T | F | T | T | F | T |
| F | F | F | T | T | T | F |

$(A \oplus B) \wedge C \Leftrightarrow (A \wedge C) \oplus (B \wedge C)$

| A | B | C | $A \oplus B$ | $(A \oplus B) \wedge C$ | $A \wedge C$ | $B \wedge C$ | $(A \wedge C) \oplus (B \wedge C)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T | F |
| T | T | F | F | F | F | F | F |
| T | F | T | T | T | T | F | T |
| T | F | F | T | F | F | F | F |
| F | T | T | T | T | F | T | T |
| F | T | F | T | F | F | F | F |
| F | F | T | F | F | F | F | F |
| F | F | F | F | F | F | F | F |

$(A \wedge B) \oplus C \Leftrightarrow (A \oplus C) \wedge (B \oplus C)$

| A | B | C | $A \wedge B$ | $(A \wedge B) \oplus C$ | $A \oplus C$ | $B \oplus C$ | $(A \oplus C) \wedge (B \oplus C)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | F | F | F |
| T | T | F | T | T | T | T | T |
| T | F | T | F | T | F | T | F |
| T | F | F | F | F | T | F | F |
| F | T | T | F | T | T | F | F |
| F | T | F | F | F | F | T | F |
| F | F | T | F | T | T | T | T |
| F | F | F | F | F | F | F | F |

| A | B | C | Δ | Δ→Δ | ¬(Δ→Δ) |
|---|---|---|---|---|---|
| T | T | T | T | T | F |
| T | T | F | F | T | F |
| T | F | T | F | T | F |
| T | F | F | F | T | F |
| F | T | T | F | T | F |
| F | T | F | F | T | F |
| F | F | T | T | T | F |
| F | F | F | F | T | F |

$(A \wedge B \wedge C) \vee (\neg A \wedge \neg B \wedge C)$

$C \wedge [(A \wedge B) \vee (\neg A \wedge \neg B)]$

$\Leftrightarrow C \wedge [(A \wedge B) \vee \neg (A \vee B)]$

$\Delta(A, B, C) \Leftrightarrow C \wedge [(A \wedge B) \vee \neg (A \vee B)]$

$\Delta(A, B, A \vee B) \Leftrightarrow (A \vee B) \wedge [(A \wedge B) \vee \neg (A \vee B)]$

$\Leftrightarrow [A \wedge B \wedge (A \vee B)] \vee [\neg A \wedge \neg B \wedge (A \vee B)]$

$\Leftrightarrow [(A \wedge B \wedge A) \vee (A \wedge B \wedge B)] \vee [\underbrace{(\neg A \wedge \neg B \wedge A)}_{F} \vee \underbrace{(\neg A \wedge \neg B \wedge B)}_{F}]$

$\Leftrightarrow (A \wedge B) \vee (A \wedge B)$

$\Leftrightarrow A \wedge B$

A: it is sunny tomorrow

B: we will go to the park

C: we will go to the movies

$(A \to B) \wedge (\neg A \to C)$

A then B, otherwise C

| A | B | C | A→B | ¬A | ¬A→C | (A→B)∧(¬A→C) | (A→B)∨C | (A→B)∨(¬A→C) |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | F | T | T | T | |
| T | T | F | T | F | T | T | T | |
| T | F | T | F | F | T | F | T | |
| T | F | F | F | F | T | F | F | |
| F | T | T | T | T | T | T | T | |
| F | T | F | T | T | F | F | T | |
| F | F | T | T | T | T | T | T | |
| F | F | F | F | T | F | F | F | |

$((A \to B) \wedge \neg C) \vee ((\neg A \to C) \wedge \neg B)$

$(A \wedge B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge C) \vee (\neg A \wedge B \wedge \neg C)$

$(A \wedge B) \wedge \underbrace{(C \vee \neg C)}_{T}$ $\qquad$ $(\neg A \wedge B) \wedge \underbrace{(C \vee \neg C)}_{T}$

$(A \wedge B) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B)$

$(A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge C) \vee (\neg A \wedge \neg B \wedge C)$

$(A \wedge C \wedge B) \vee (A \wedge C \wedge \neg B)$ $\qquad$ $(\neg A \wedge C \wedge B) \vee (\neg A \wedge C \wedge \neg B)$

$(A \wedge C) \quad \vee \quad (\neg A \wedge C)$

$C \wedge (A \vee \neg A)$

$C$

---

| A | B | ¬A | A\|B | A\|A | A∧B | (A\|B)\|(A\|B) |
|---|---|---|---|---|---|---|
| T | T | F | F | F | T | T |
| T | F | F | T | F | F | F |
| F | T | T | T | T | F | F |
| F | F | T | T | T | F | F |

| A | B | A\|B | ⊥ | A\|A | A\|(A\|A) | A→B | A\|(A\|B) |
|---|---|---|---|---|---|---|---|
| T | T | F | F | F | T | T | T |
| T | F | T | F | F | T | F | F |
| F | T | T | F | T | T | T | T |
| F | F | T | F | T | T | T | T |