Krotov's Method

$$J = J_T\left(\{|\psi_k(T)\rangle\}\right) + \int g_a\left(\{\varepsilon_e(t)\}\right) dt + \int g_b\left(\{|\psi(t)\rangle\}\right) dt$$

- given: guess $\varepsilon_e^{(0)}(t)$
- necessary and sufficient conditions for new field $\varepsilon_e^{(1)}(t)$

so that $J\left(\{\varepsilon_e^{(1)}(t)\}\right) \leq J\left(\{\varepsilon_e^{(0)}(t)\}\right)$

$$\frac{\partial g_a}{\partial \varepsilon^{(1)}} = 2 \,\mathrm{Im} \sum_k \langle \chi_k^{(0)}(t) | \frac{\partial H}{\partial \varepsilon} | \psi_k^{(1)}(t) \rangle$$

$$|\chi_k^{(0)}(t)\rangle = \frac{\partial J_T}{\partial \langle \psi_k^{(0)}(t)|}$$

$$g_a = \frac{\lambda_a}{S(t)} \int \left(\Delta \varepsilon_e(t)\right)^2 dt \quad ; \quad \Delta\varepsilon_e(t) = \varepsilon_e^{(1)}(t) - \varepsilon_e^{(0)}(t)$$

$$\Rightarrow \Delta\varepsilon = \frac{S(t)}{\lambda_a} \sum_k \langle \chi_k^{(0)}(t) | \frac{\partial H}{\partial \varepsilon} | \psi_k^{(1)}(t) \rangle$$