# Gradient of State Overlap

$$\frac{\partial}{\partial \varepsilon_{ml}} \tau_n = \frac{\partial}{\partial \varepsilon_{ml}} \langle \tau_n(T) | \tau_n^{tgt} \rangle$$

$$| \tau_n(T) \rangle = \hat{U}_N \ldots \hat{U}_2 \hat{U}_1 | \tau_n(0) \rangle$$

$$\frac{\partial \tau_n}{\partial \varepsilon_{ml}} = \frac{\partial}{\partial \varepsilon_{ml}} \langle \tau_n(0) | \hat{U}_1^+ \hat{U}_2^+ \ldots \hat{U}_n^+ \ldots \hat{U}_N^+ | \tau_n^{tgt} \rangle$$

$$= \underbrace{\langle \tau_n(0) | \hat{U}_1^+ \ldots \hat{U}_{n-1}^+}_{\langle \tau_n(t_m) |} \frac{\partial \hat{U}_n^+}{\partial \varepsilon_{ml}} \underbrace{\hat{U}_{n+1}^+ \ldots \hat{U}_N^+ | \tau_n^{tgt} \rangle}_{| \chi_n(t_{m+1}) \rangle}$$

forward $-$ prop          bw $-$ prop