

Prospects of Optimal Control for Superconducting Circuits

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Army Research Lab

Yale Quantum Institute Seminar

January 14, 2020

optimum quantum control

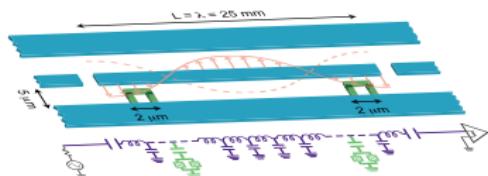
steer quantum system in some desired way

optimum quantum control

steer quantum system in some desired way

physically:

transmons with shared
transmission line:



Blais et al. PRA 75, 032329 (2007)

e.g. CNOT gate:

$$|00\rangle \rightarrow \text{CNOT} |00\rangle = |00\rangle$$

$$|01\rangle \rightarrow \text{CNOT} |01\rangle = |01\rangle$$

$$|10\rangle \rightarrow \text{CNOT} |10\rangle = |11\rangle$$

$$|11\rangle \rightarrow \text{CNOT} |11\rangle = |10\rangle$$

with *same* control $\epsilon(t)$.

optimum quantum control

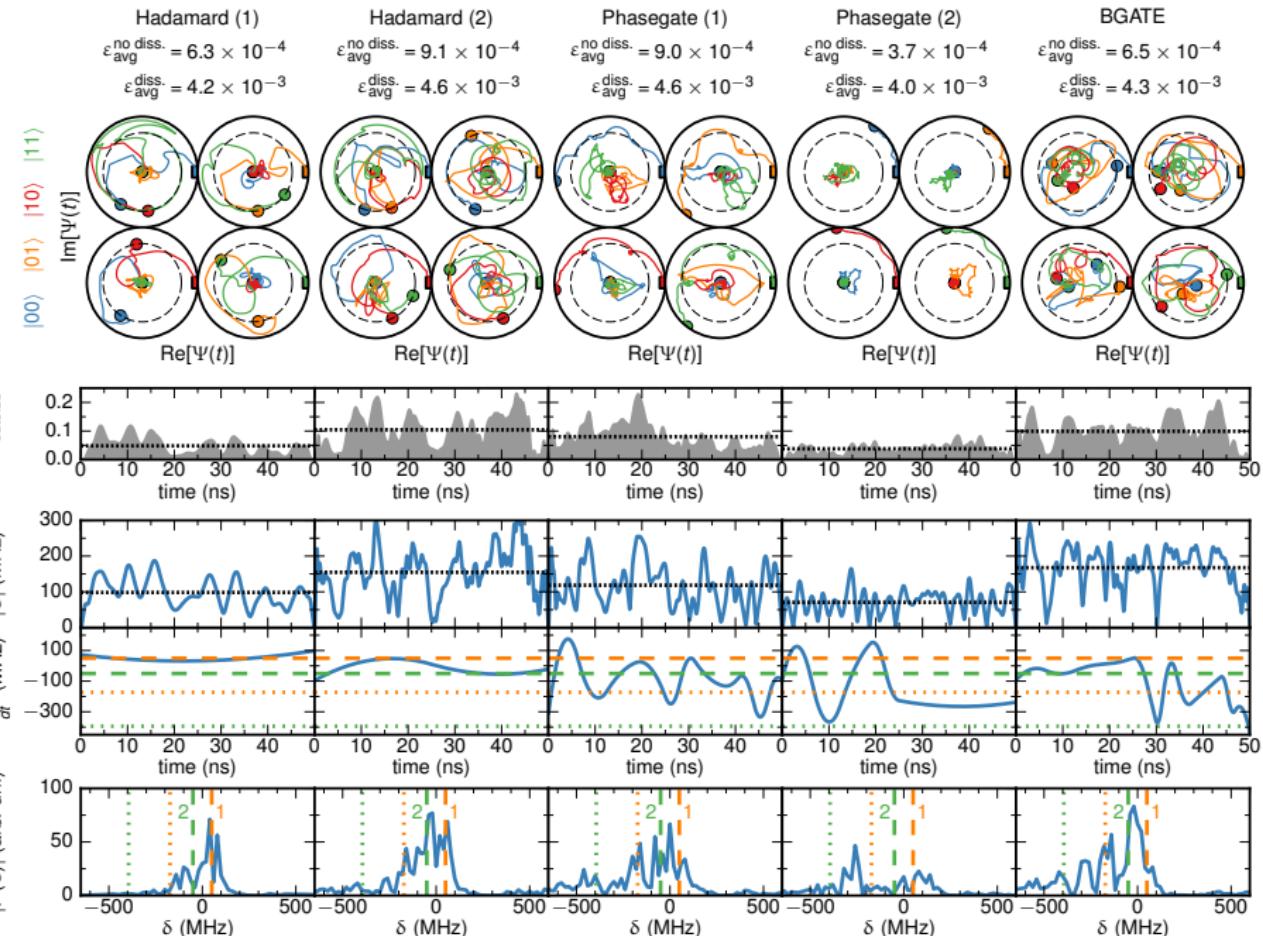
steer quantum system in some desired way

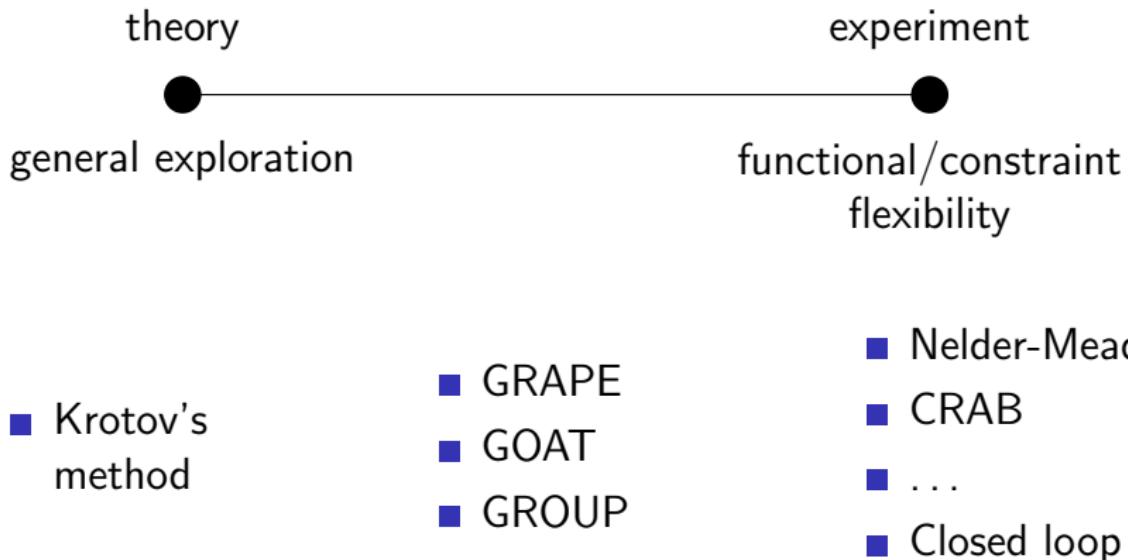
mathematically:

minimize functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$

$$= J_T(\{|\phi_k(T)\rangle\}) + \sum_I \int_0^T g_a(\epsilon_l(t)) dt$$

e.g. $J_{T,\text{re}} = 1 - \frac{1}{N} \operatorname{Re} \left[\sum_{k=1}^N \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle \right]$





<https://github.com/qucontrol/krotov>

The screenshot shows a web browser window with the URL `https://github.com/qucontrol/krotov` in the address bar. The page content is the documentation for the `Krotov` package, version 1.0.0.

Left Sidebar (Contents):

- Krotov Python Package
- Contributing
- Credits
- Features
- History
- Introduction
- Krotov's Method
- Using Krotov with QuTiP
- Examples
- How-Tos

Bottom Left: `Doctr` and `v1.0.0 (latest release)`.

Main Content Area:

Header: Docs » Welcome to the Krotov package's documentation!

Welcome to the Krotov package's documentation! [🔗](#)

[github qucontrol/krotov](#) [docs doctr](#) [pypi v1.0.0](#) [chat on gitter](#)

[build passing](#) [ci build passing](#) [codecov 95%](#) [License BSD](#)

[launch binder](#) [DOI 10.21468/SciPostPhys.7.6.080](#)

Contents:

- [Krotov Python Package](#)
 - [Purpose](#)
 - [Prerequisites](#)
 - [Installation](#)
 - [Usage](#)

<https://github.com/qucontrol/krotov>

The screenshot shows a web browser window with the URL <https://qucontrol.github.io/krotov/> in the address bar. The left sidebar contains a list of optimization examples:

- Optimization of a State-to-State Transfer in a Lambda System in the RWA
- Optimization of a Dissipative State-to-State Transfer in a Lambda System
- Optimization of Dissipative Qubit Reset
- Optimization of an X-Gate for a Transmon Qubit
- Optimization of a Dissipative Quantum Gate
- Optimization towards a Perfect Entangler
- Ensemble Optimization for Robust Pulses
- Optimization with numpy Arrays

The "How-Tos" and "Other Optimization Methods" sections are also listed but are not highlighted.

The main content area features a large heading "Examples" with a percentage symbol. Below it are several buttons: "render on nbviewer", "launch binder", and two others that are partially visible. A list of items follows:

- Optimization of a State-to-State Transfer in a Two-Level-System
- Optimization of a State-to-State Transfer in a Lambda System in the RWA
- Optimization of a Dissipative State-to-State Transfer in a Lambda System
- Optimization of Dissipative Qubit Reset
- Optimization of an X-Gate for a Transmon Qubit
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- Optimization towards a Perfect Entangler
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At the bottom, there are "Previous" and "Next" navigation buttons.

The screenshot shows a web browser window with the SciPost Physics website. The title bar displays the URL scipost.org/10.21468/SciPostPhys.7.080. The main header features the "SciPost" logo with a blue "S" and orange "Post". A search bar is located in the top right corner. Below the header, a blue navigation bar includes a menu icon (three horizontal lines) and links for "Journals", "SciPost Physics", "Vol. 7 issue 6", and the specific article title "Krotov: A Python implementation of Krotov's method for quantum optimal control". The main content area has a dark blue header with the text "SciPost Physics". Below this, a light gray navigation bar contains links for "Home", "Authoring", "Refereeing", "Submit a manuscript", and "About". The main article page features the title "Krotov: A Python implementation of Krotov's method for quantum optimal control" in large blue text. Below the title, the authors are listed as Michael H. Goerz, Daniel Basilewitsch, Fernando Gago-Encinas, Matthias G. Krauss, Karl P. Horn, Daniel M. Reich, and Christiane P. Koch. The publication information "SciPost Phys. 7, 080 (2019) · published 12 December 2019" is also present. At the bottom, there are download links for "doi: 10.21468/SciPostPhys.7.080", "pdf", "BiBTeX", "RIS", and "Submissions/Reports", along with a "Check for updates" button.

SciPost Physics

Journals / SciPost Physics / Vol. 7 issue 6
/ Krotov: A Python implementation of Krotov's method for quantum optimal control

SciPost Physics

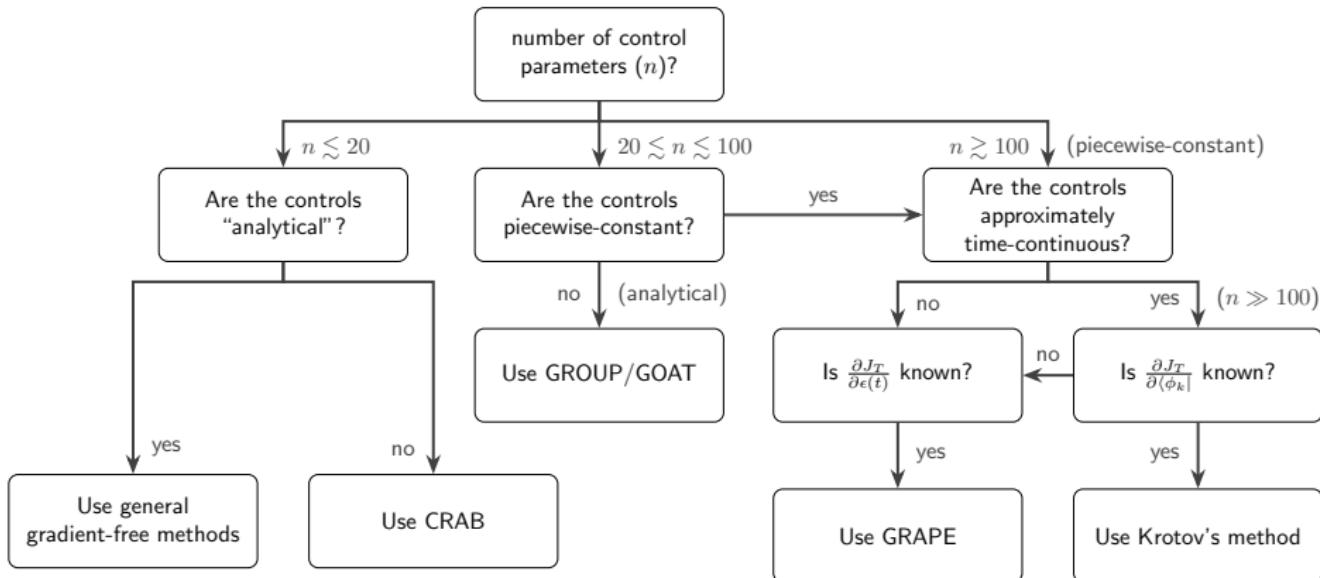
Home Authoring Refereeing Submit a manuscript About

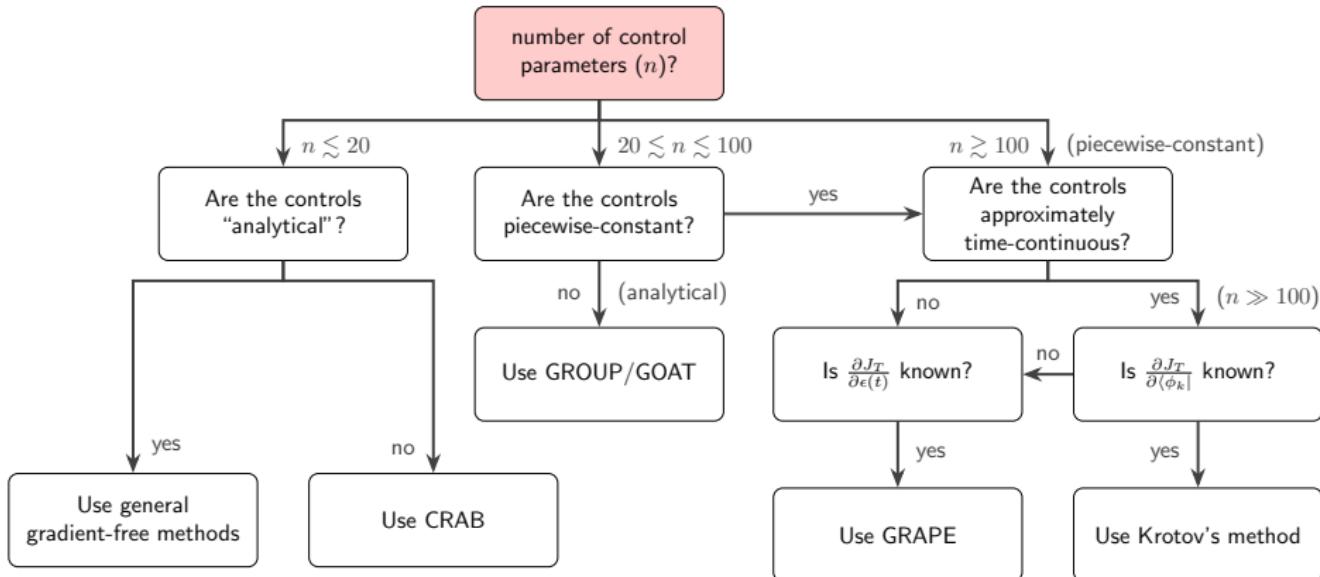
Krotov: A Python implementation of Krotov's method for quantum optimal control

Michael H. Goerz, Daniel Basilewitsch, Fernando Gago-Encinas, Matthias G. Krauss, Karl P. Horn, Daniel M. Reich, Christiane P. Koch

SciPost Phys. 7, 080 (2019) · published 12 December 2019

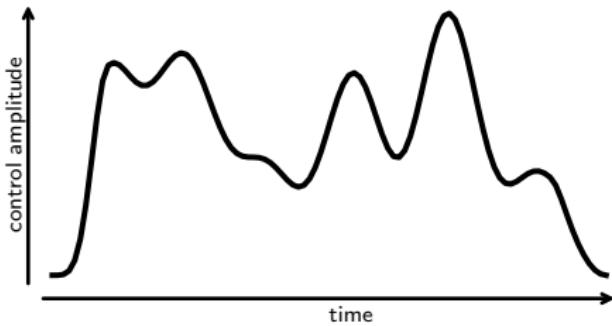
doi: 10.21468/SciPostPhys.7.080 pdf BiBTeX RIS Submissions/Reports Check for updates





functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$

$$\text{functional } J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$

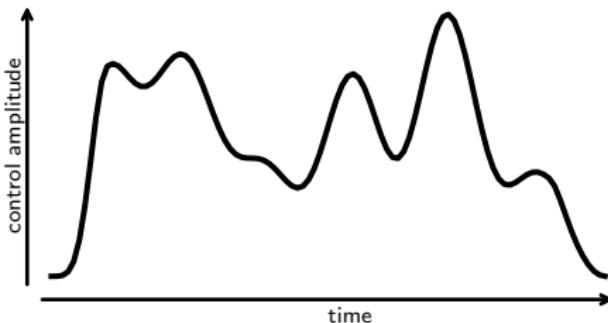


time-continuous fields

↓
functional derivative $\frac{\delta J}{\delta \epsilon_I(t)}$

problem:
interdependence of field
and states

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time-continuous fields

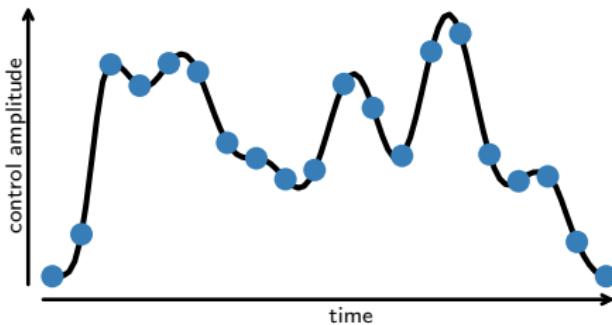
↓
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interdependence of field
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Krotov's method:

- Add “smart zero” to functional to disentangle interdependence of states and field
- Necessary and sufficient conditions for monotonic convergence

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time-continuous fields

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functional derivative $\frac{\delta J}{\delta \epsilon_I(t)}$

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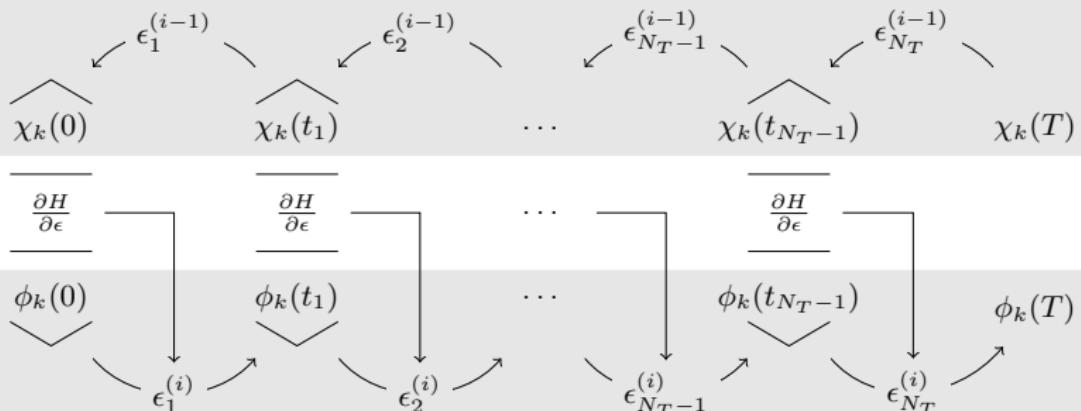
Krotov's method:

- Add “smart zero” to functional to disentangle interdependence of states and field
- Necessary and sufficient conditions for monotonic convergence

Krotov's method (discretized)

sequential scheme:

① backward-propagation and storage with guess control

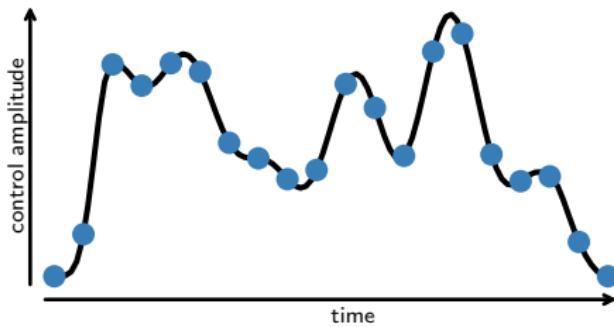


② forward-propagation with updated control

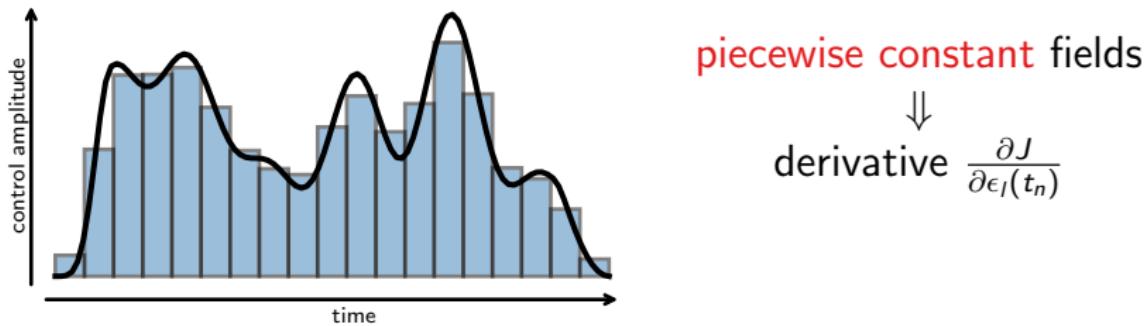
boundary condition:

$$|\chi_k(T)\rangle = -\frac{\partial J_T}{\partial \langle \phi_k(T)|}$$

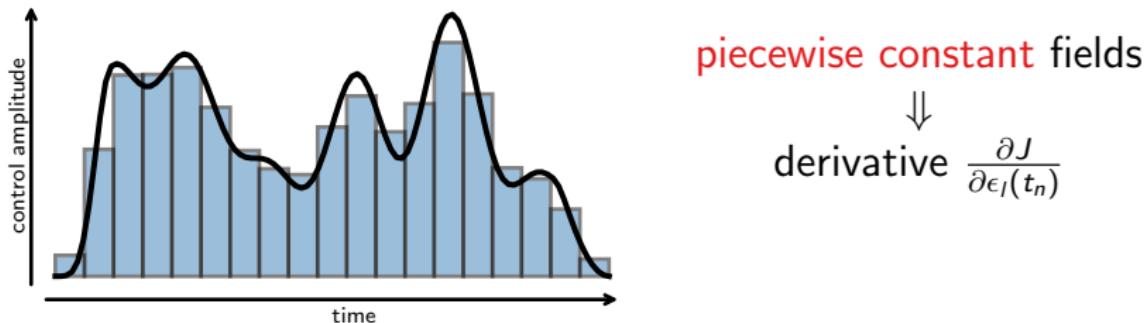
$$\text{functional} \quad J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$



$$\text{functional } J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$



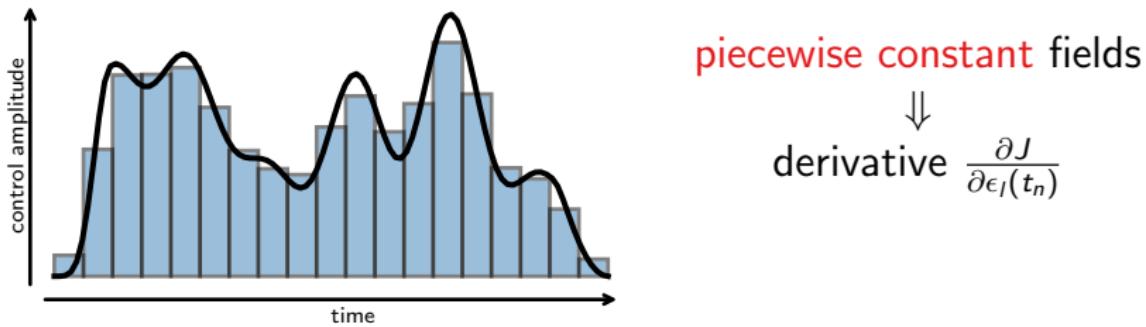
$$\text{functional } J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$



Gradient Ascent Pulse Engineering (GRAPE):

- Discretize first! Then calculate the gradient
- L-BFGS-B: Quasi-Hessian for faster convergence

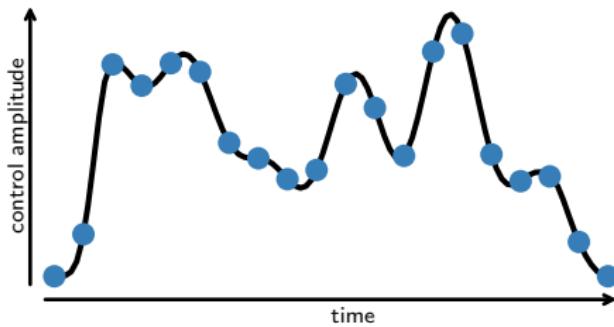
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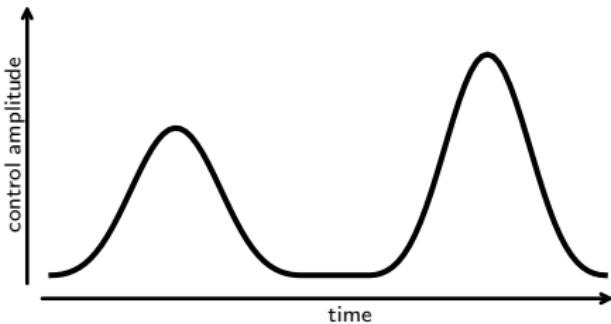
Gradient Ascent Pulse Engineering (GRAPE):

$$\begin{aligned} \frac{\partial \tau_k}{\partial \epsilon_n} &= \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle = \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_n \dots \hat{\mathbf{U}}_1 | \phi_k \rangle \\ &= \underbrace{\langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_{n+1} |}_{\langle \chi_k(t_{n+1}) |} \frac{\partial \hat{\mathbf{U}}_n}{\partial \epsilon_n} \underbrace{\hat{\mathbf{U}}_{n-1} \dots \hat{\mathbf{U}}_1 | \phi_k \rangle}_{| \phi_k(t_n) \rangle} \end{aligned}$$

$$\text{functional } J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$

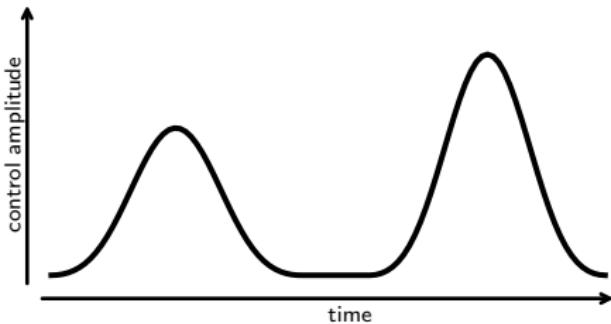


$$\text{functional } J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$



simple parametrized fields
↓
gradient-free methods,
e.g. CRAB

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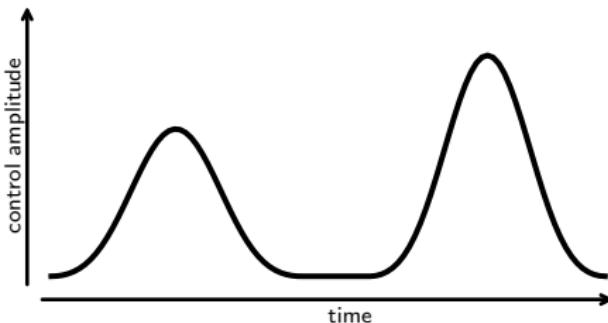


simple parametrized fields
↓
gradient-free methods,
e.g. CRAB

$$\epsilon(t) = E_1 G(t_1, \sigma_1, t) + E_2 G(t_2, \sigma_2, t)$$

$$\epsilon(t) = \sum_{i=1}^{10} (a_n \cos(\omega_n t) + b_n \sin(\omega_n t))$$

$$\text{functional } J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$$



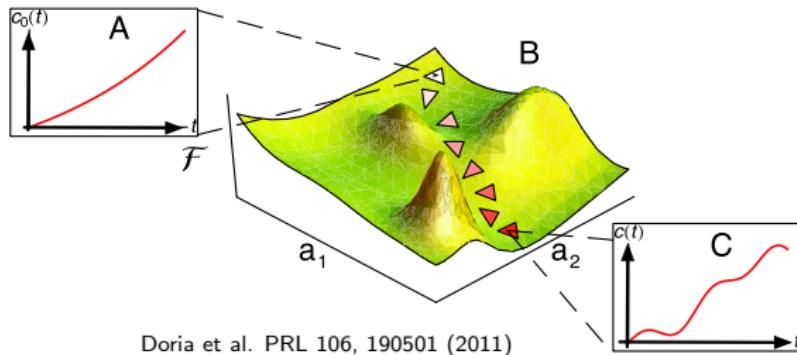
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... Filter function of control electronics – non-analytic!

gradient-free optimization



e.g. Nelder-Mead (simplex), genetic algorithms. . .

GROUP & GOAT: apply gradient to parametrized pulses

$$\epsilon(t) = S(t) \sum_n c_n f_n(t) \quad \text{or generally} \quad \epsilon(t) = \epsilon(\{c_n\})$$
$$\Rightarrow \frac{\partial J}{\partial c_n} = \sum_i \frac{\partial J}{\partial \epsilon_i} \frac{\partial \epsilon_i}{\partial c_n}$$

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GROUP (collection of methods): chain rule

Sørensen et al., Phys. Rev. A 98, 022119 (2018)

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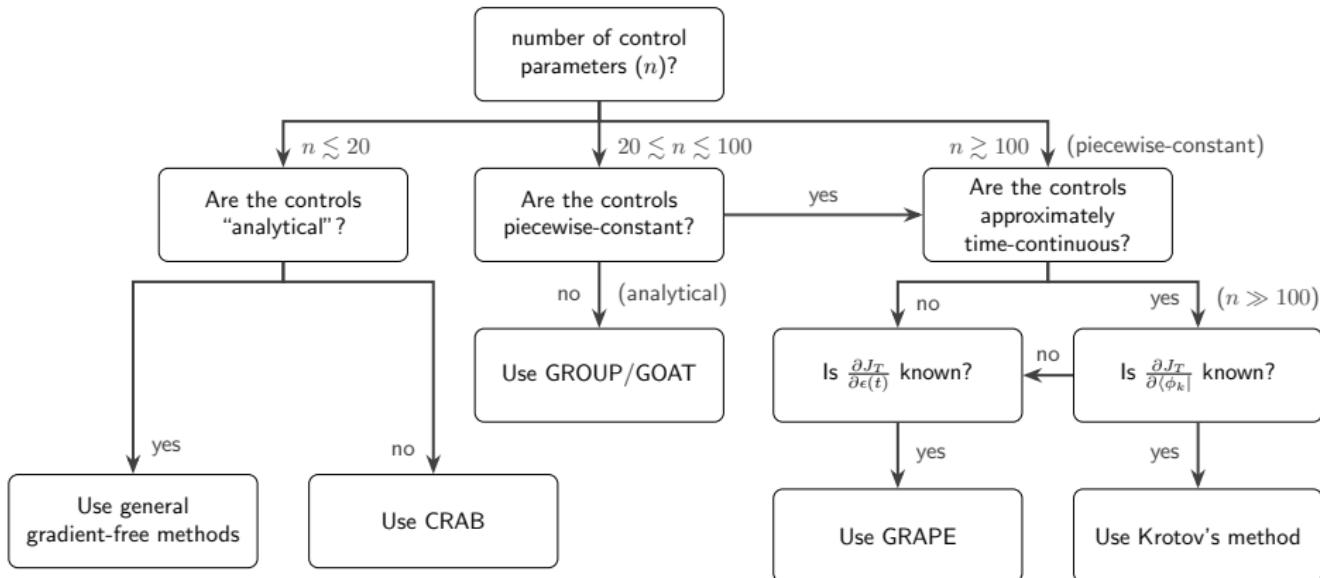
GROUP (collection of methods): chain rule

Sørensen et al., Phys. Rev. A 98, 022119 (2018)

GOAT: evaluate functional and gradient at the same time
(once per parameter)

Machnes et al., Phys. Rev. Lett. 120, 150401 (2018)

$$\partial_t \begin{pmatrix} U \\ \partial_{c_n} U \end{pmatrix} = -\frac{i}{\hbar} \begin{pmatrix} H & 0 \\ \partial_{c_n} H & H \end{pmatrix} \begin{pmatrix} U \\ \partial_{c_n} U \end{pmatrix}$$

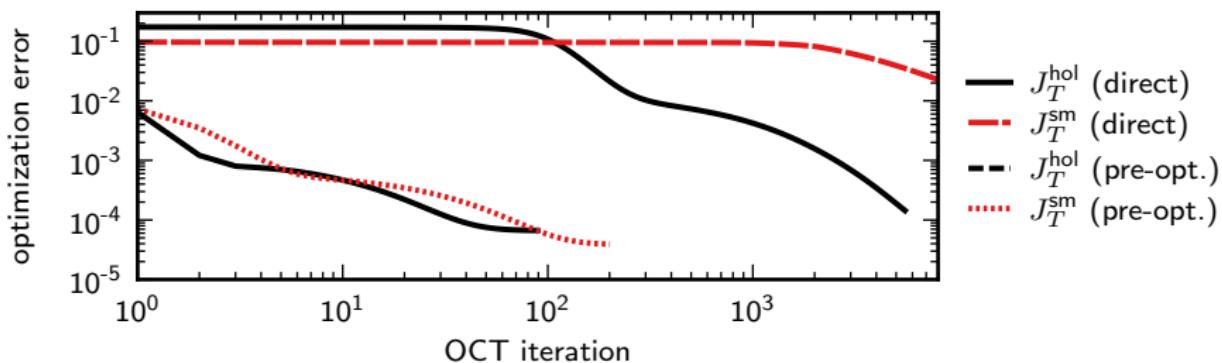


“hybrid” (multi-stage) methods (example: RIP gate)

- 1 Start with analytical formula, optimize free parameter with **simplex**
- 2 Use simplex-optimized control as starting point for **gradient-based** method

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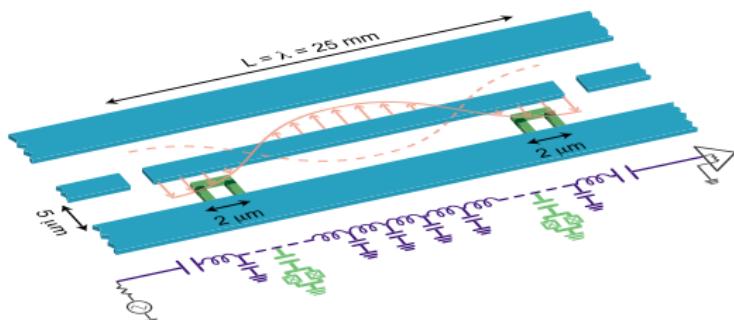


⇒ Goerz et al. EPJ Quantum Tech. 2, 21 (2015)

Charting the circuit QED design landscape using optimal control theory

Goerz et al. npj Quantum Information 3, 37 (2017)

two transmons with shared transmission line bus:



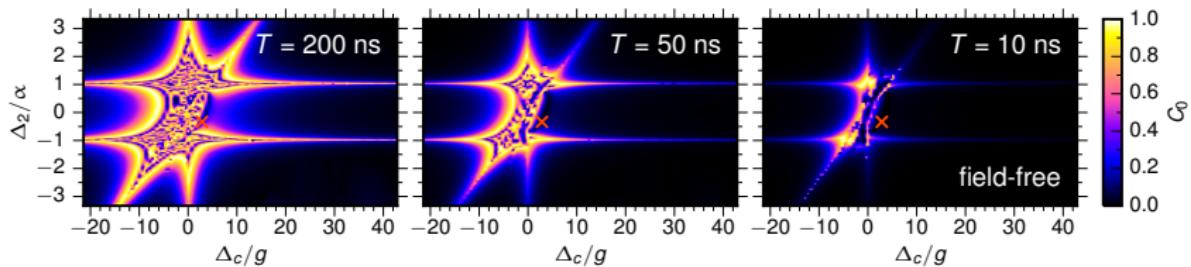
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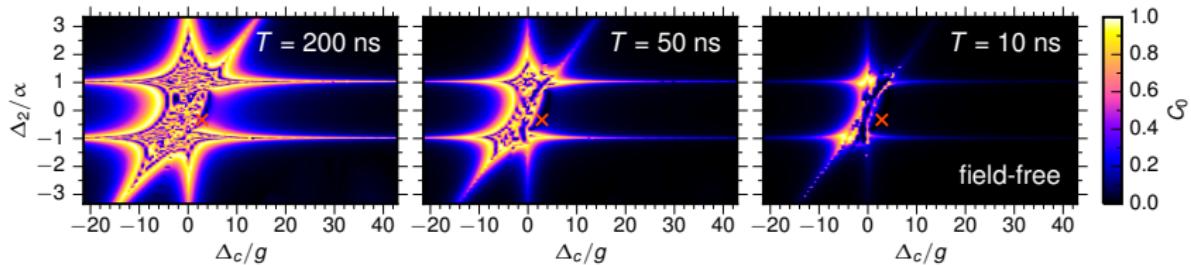
$$\hat{\mathbf{H}} = \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 + \frac{\alpha_1}{2} \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \frac{\alpha_2}{2} \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2 + g_1 (\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2 (\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger) + u^*(t) \hat{\mathbf{a}} + u(t) \hat{\mathbf{a}}^\dagger$$

parameter landscape:

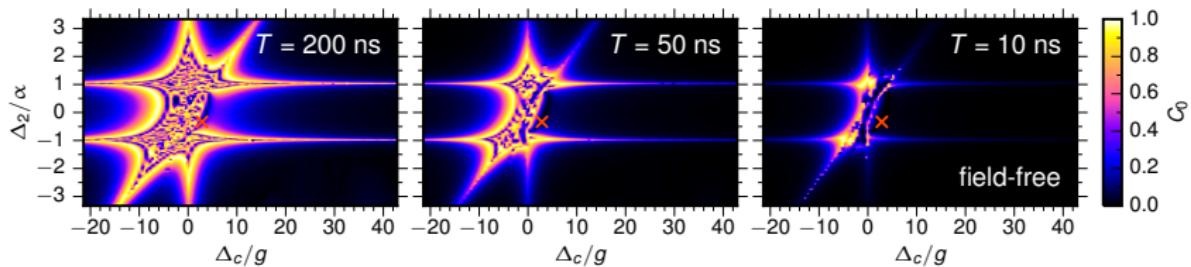
Δ_c/g ,

Δ_2/α

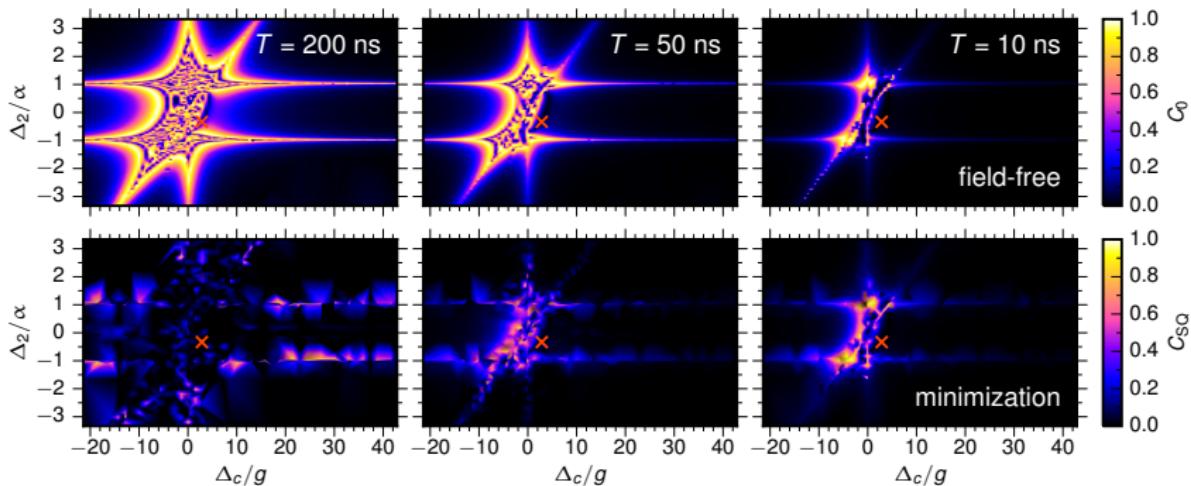


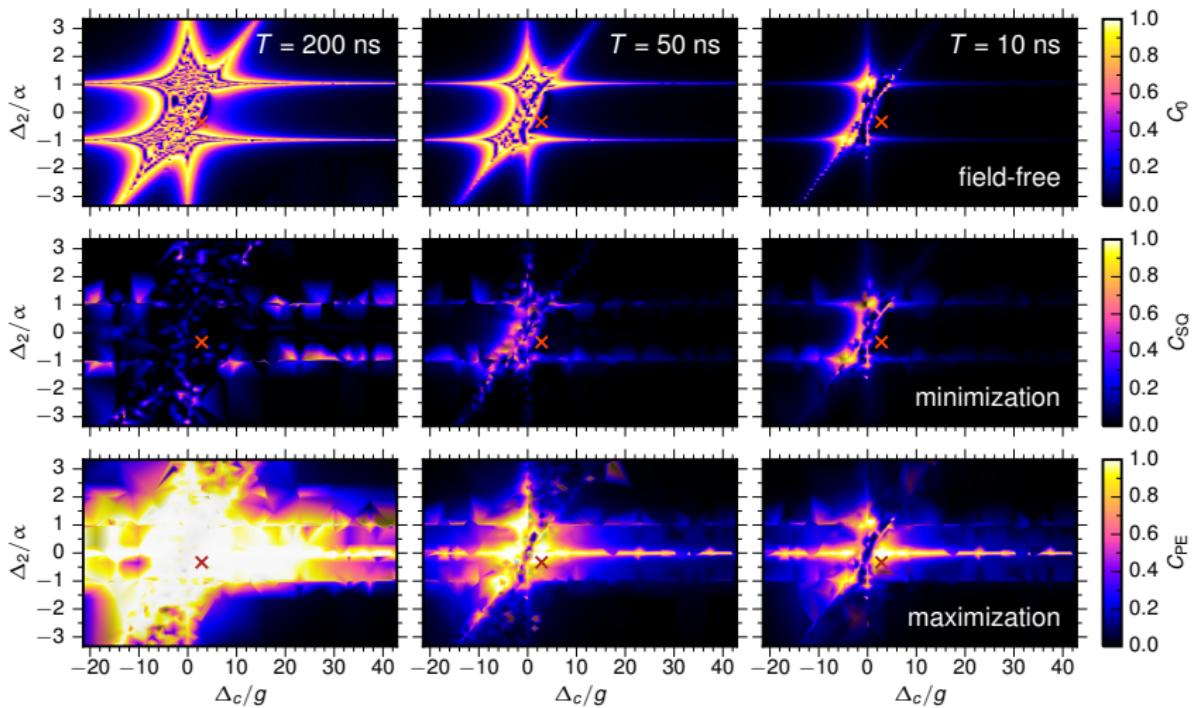


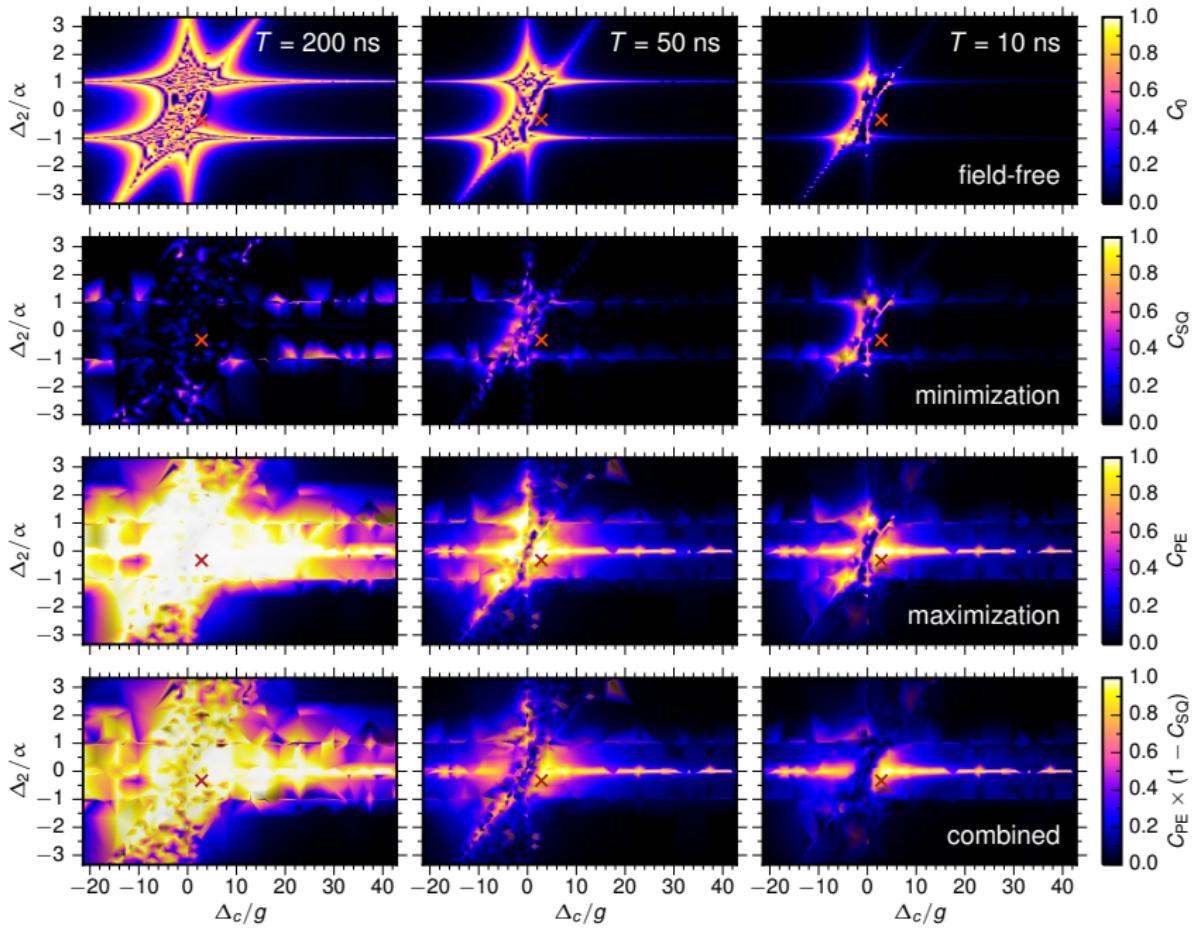
universal quantum computing:
perfect entangler *and* local gates

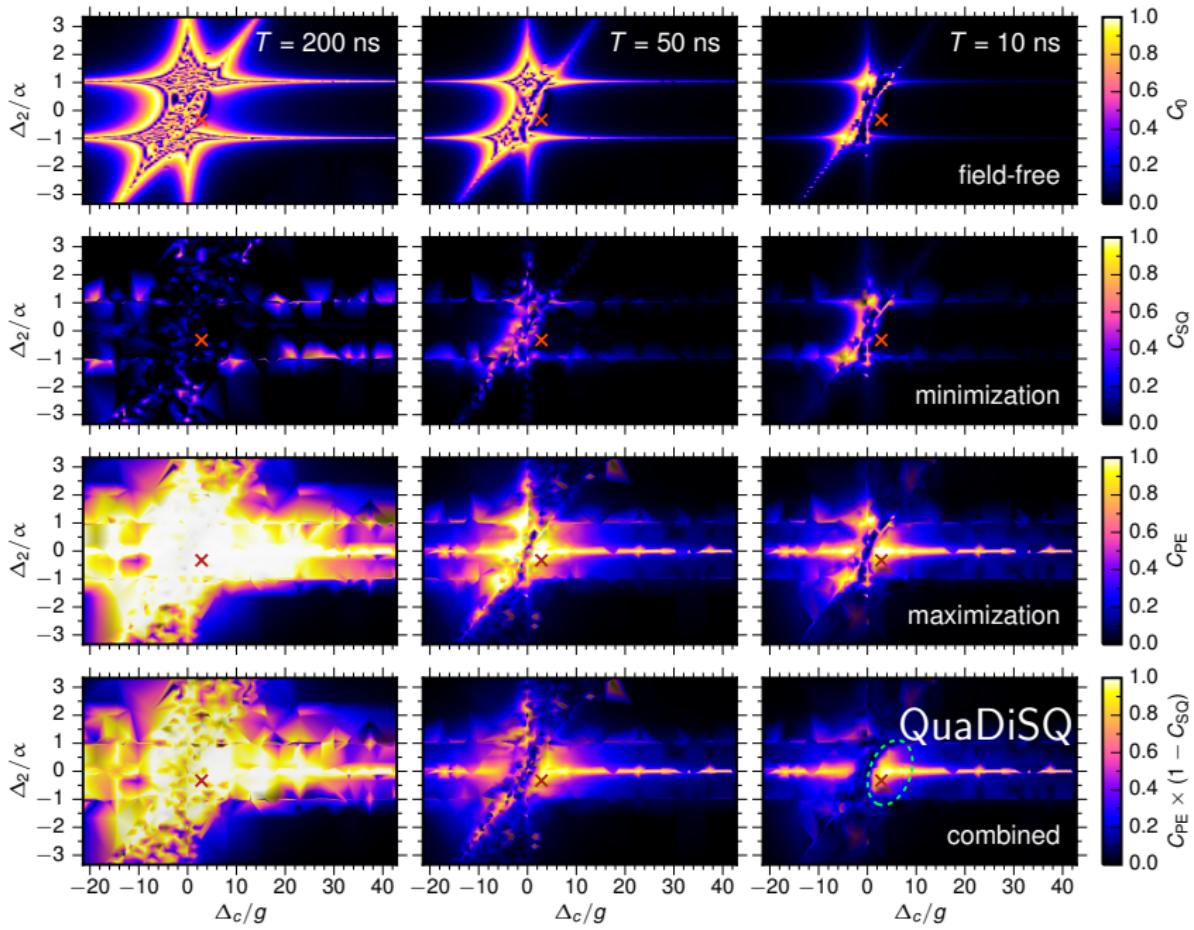


optimize for maximum / minimum entanglement







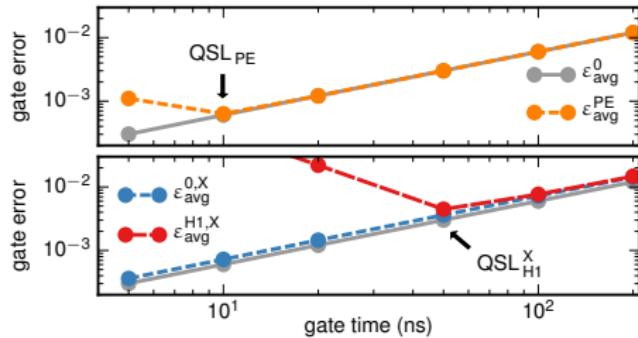


optimize for *specific* set of universal gates:

- Hadamard, Phase single qubit
- BGATE entangler

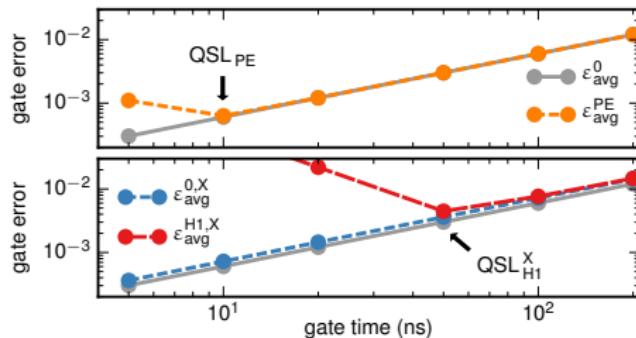
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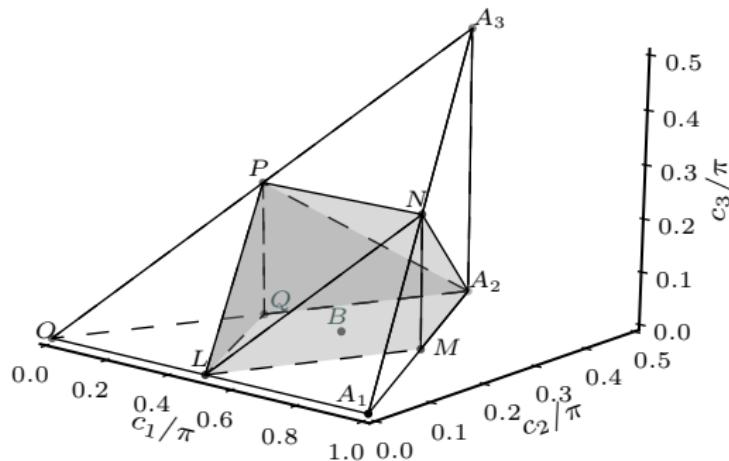
minimum gate duration: 50 ns

gate errors:

- 10^{-4} without dissipation
- 10^{-3} with dissipation

optimizing for entanglement

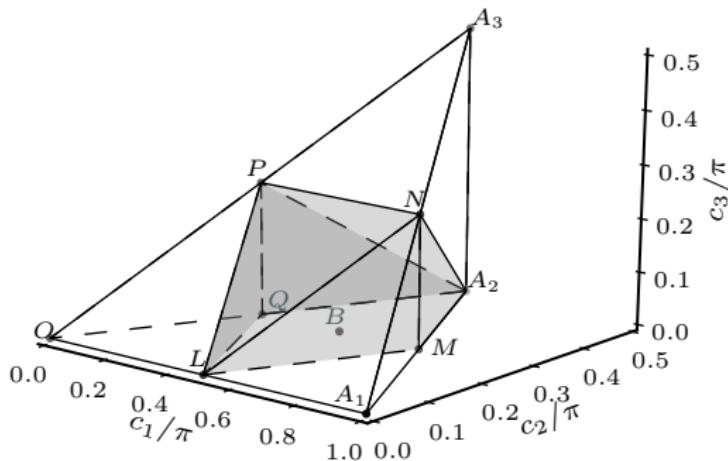
optimizing for entanglement



Cartan decomposition:

$$\hat{\mathbf{U}} = \hat{\mathbf{k}}_1 \exp \left[\frac{i}{2} (c_1 \hat{\sigma}_x \hat{\sigma}_x + c_2 \hat{\sigma}_y \hat{\sigma}_y + c_3 \hat{\sigma}_z \hat{\sigma}_z) \right] \hat{\mathbf{k}}_2$$

optimizing for entanglement



Cartan decomposition:

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⇒ application: Goerz et al. Phys. Rev. A 91, 062307 (2015)

Outlook: learning from machine learning

Reinforcement learning for quantum control

- Use a neural net (AlphaZero) to choose pulse amplitude in each time slice.

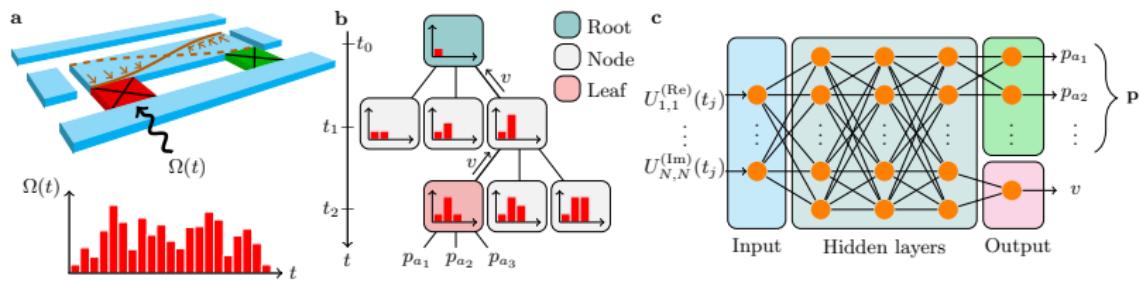


Fig 1 from: Dalgaard et. al, arXiv:1907.05672 (2019)

Automatic Differentiation

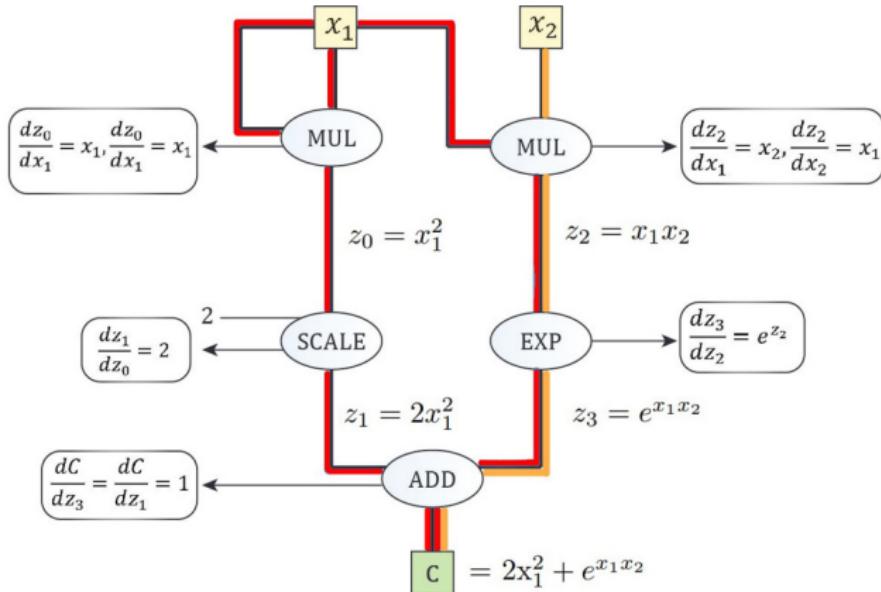


Fig 12 from: Abdelhalfez et. al, PRA 99, 052327 (2019)

Automatic differentiation for MCWF propagation & optimization functional using Google Tensorflow

PHYSICAL REVIEW A **95**, 042318 (2017)

Speedup for quantum optimal control from automatic differentiation based on graphics processing units

Nelson Leung,^{1,*} Mohamed Abdelhafez,¹ Jens Koch,² and David Schuster¹

¹The James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

²Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA

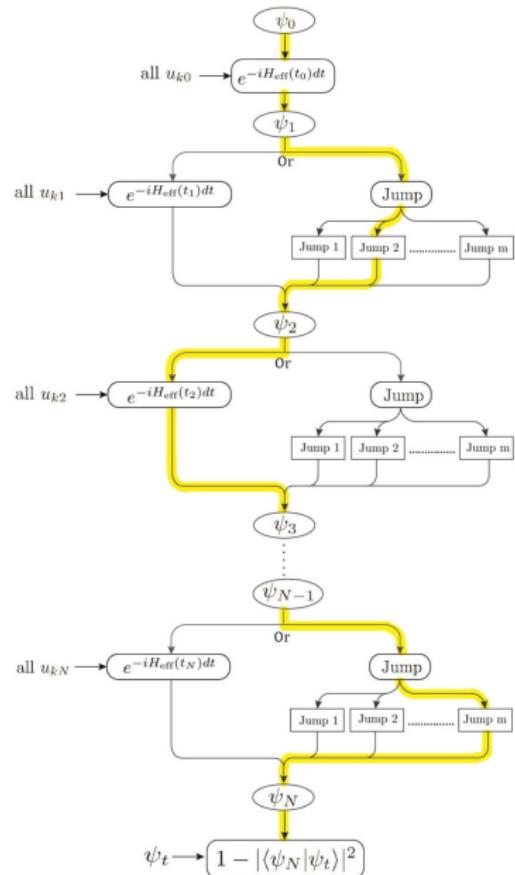
PHYSICAL REVIEW A **99**, 052327 (2019)

Gradient-based optimal control of open quantum systems using quantum trajectories and automatic differentiation

Mohamed Abdelhafez,^{1,*} David I. Schuster,¹ and Jens Koch²

¹The James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

²Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA



perfect entanglers functional:

$$\begin{aligned} F_{PE} = & \left(\frac{1}{\det U_B} \right) \left(\frac{1}{4} (\text{tr}^2[U_B^T U_B] - \text{tr}[U_B^T U_B U_B^T U_B]) \right) \left(\frac{1}{16} \text{Re}^2[\text{tr}[U_B^T U_B]] \right) + \\ & + \left(\frac{2}{\det U_B} \right) \left(\frac{1}{4} (\text{tr}^2[U_B^T U_B] - \text{tr}[U_B^T U_B U_B^T U_B]) \right) \left(\frac{1}{16} \text{Im}^2[\text{tr}[U_B^T U_B]] \right) \\ & \left(\frac{1}{16} \text{Re}[\text{tr}^2[U_B^T U_B]] \right) \end{aligned}$$

U_B : projection into logical subspace, in Bell basis

perfect entanglers functional:

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U_B : projection into logical subspace, in Bell basis

- control constraints
- noise and filter function of control electronics

perfect entanglers functional:

$$\begin{aligned} F_{PE} = & \left(\frac{1}{\det U_B} \right) \left(\frac{1}{4} (\text{tr}^2[U_B^T U_B] - \text{tr}[U_B^T U_B U_B^T U_B]) \right) \left(\frac{1}{16} \text{Re}^2[\text{tr}[U_B^T U_B]] \right) + \\ & + \left(\frac{2}{\det U_B} \right) \left(\frac{1}{4} (\text{tr}^2[U_B^T U_B] - \text{tr}[U_B^T U_B U_B^T U_B]) \right) \left(\frac{1}{16} \text{Im}^2[\text{tr}[U_B^T U_B]] \right) \\ & \left(\frac{1}{16} \text{Re}[\text{tr}^2[U_B^T U_B]] \right) \end{aligned}$$

U_B : projection into logical subspace, in Bell basis

“semi-automatic differentiation”

$$\frac{\partial \tau_k}{\partial \epsilon_n} = \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle = \underbrace{\langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_{n+1}}_{\langle \chi_k(t_{n+1}) \rangle} \underbrace{\frac{\partial \hat{\mathbf{U}}_n}{\partial \epsilon_n} \hat{\mathbf{U}}_{n-1} \dots \hat{\mathbf{U}}_1 | \phi_k \rangle}_{|\phi_k(t_n)\rangle}$$

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express functional in terms of scalar values τ_{ik}

$$F_{PE} = F_{PE}(\{\tau_{ik}\})$$

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theoretical exploration and experimental constraints
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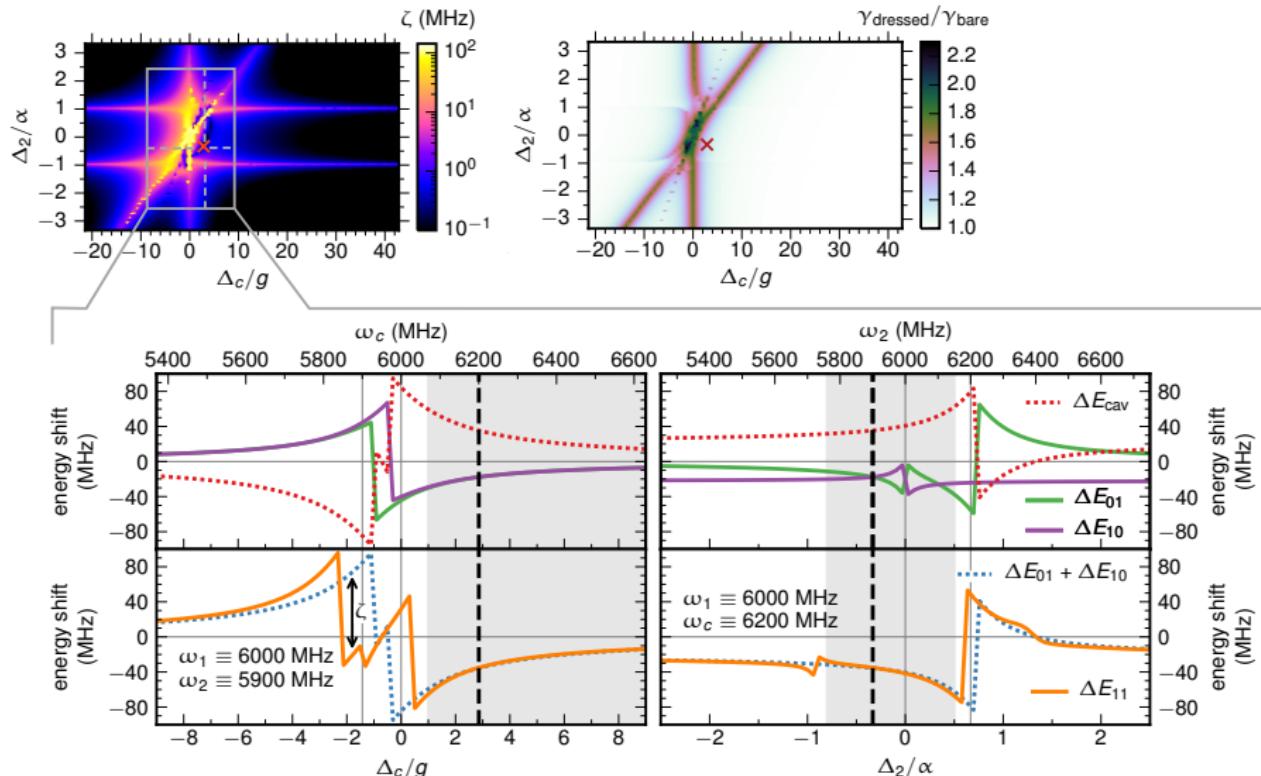
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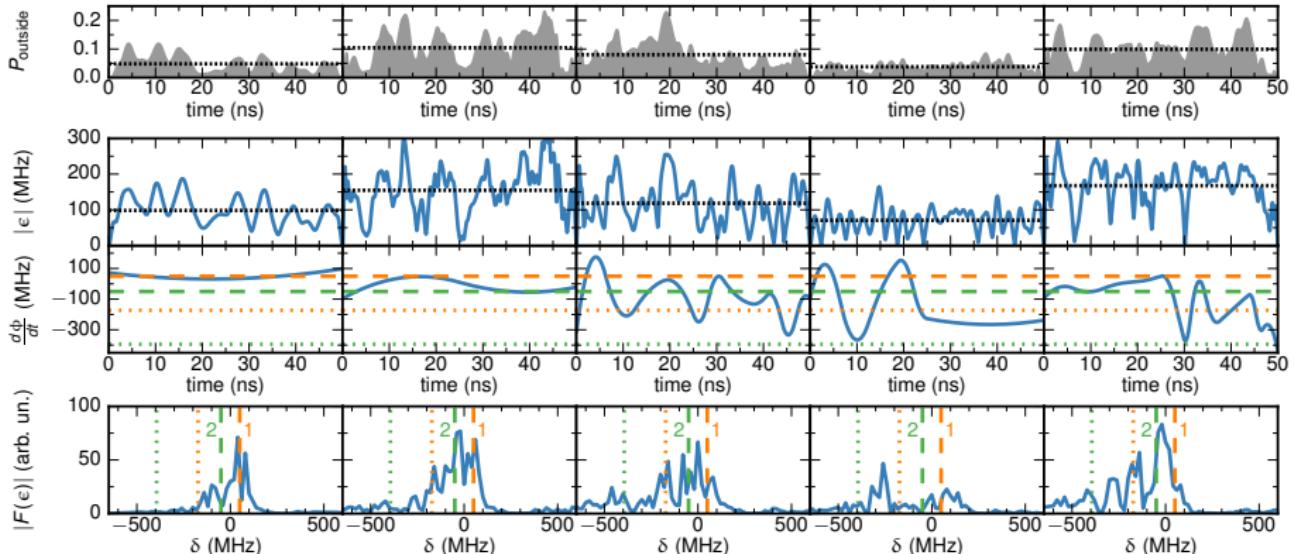
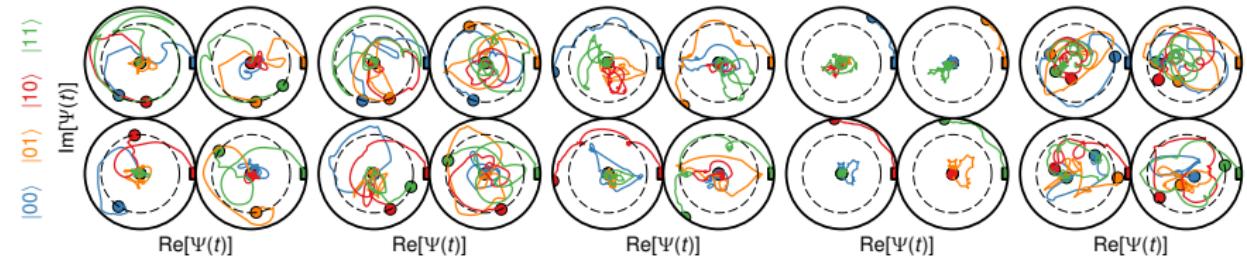
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Thank You

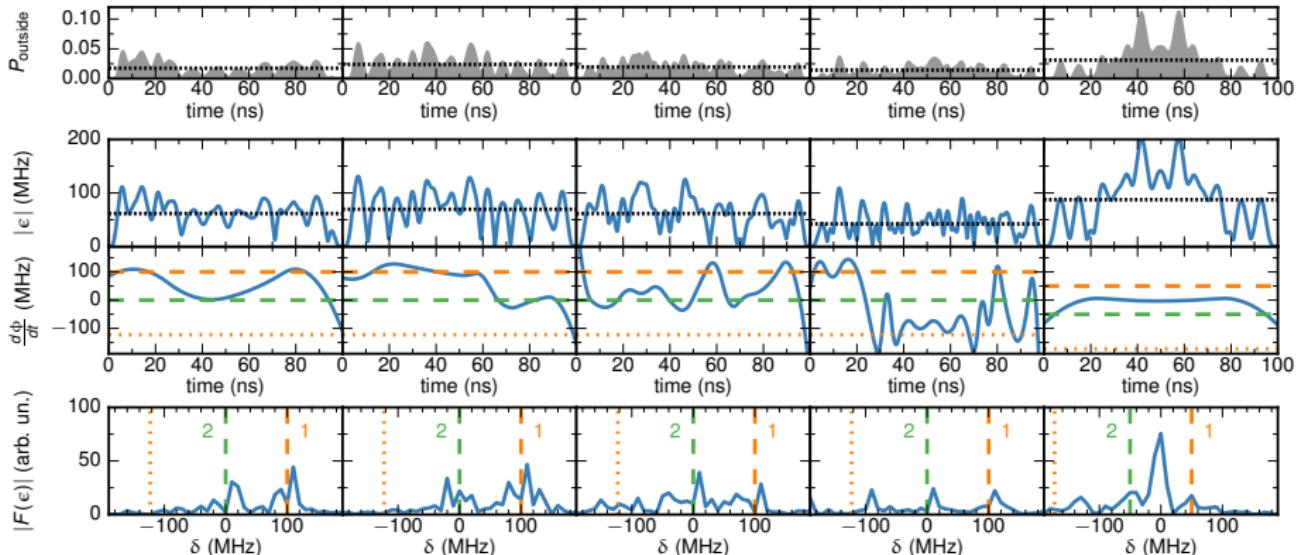
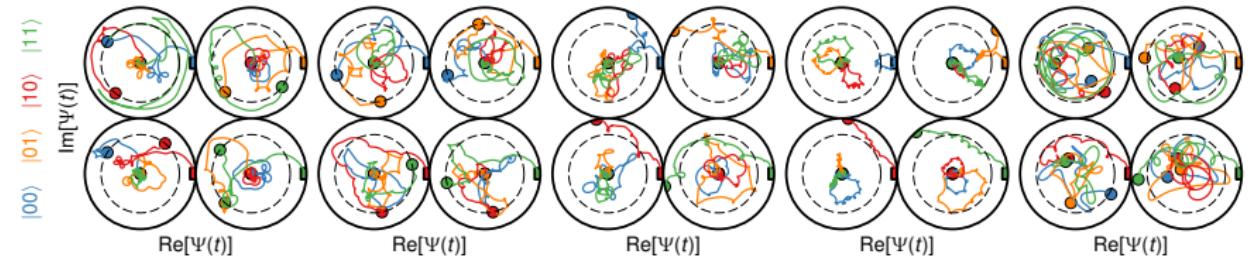
properties for field-free Hamiltonian:



Hadamard (1)	Hadamard (2)	Phasegate (1)	Phasegate (2)	BGATE
$\varepsilon_{\text{avg}}^{\text{no diss.}} = 6.3 \times 10^{-4}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 9.1 \times 10^{-4}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 9.0 \times 10^{-4}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 3.7 \times 10^{-4}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 6.5 \times 10^{-4}$
$\varepsilon_{\text{avg}}^{\text{diss.}} = 4.2 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 4.6 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 4.6 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 4.0 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 4.3 \times 10^{-3}$



Hadamard (1)	Hadamard (2)	Phasegate (1)	Phasegate (2)	BGATE
$\varepsilon_{\text{avg}}^{\text{no diss.}} = 3.5 \times 10^{-5}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 8.0 \times 10^{-5}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 2.1 \times 10^{-4}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 5.7 \times 10^{-4}$	$\varepsilon_{\text{avg}}^{\text{no diss.}} = 1.8 \times 10^{-4}$
$\varepsilon_{\text{avg}}^{\text{diss.}} = 7.2 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 7.3 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 7.4 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 7.7 \times 10^{-3}$	$\varepsilon_{\text{avg}}^{\text{diss.}} = 7.4 \times 10^{-3}$



obtained perfect entanglers in the Weyl chamber:

