5. Transport properties

5.1. Semi-classical approximation

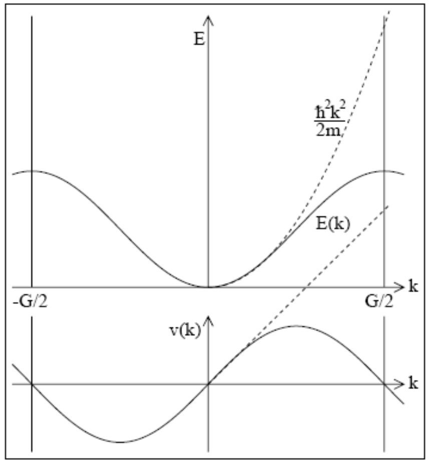
Electron is a particle moving with group velocity:

$$\vec{v} = \frac{1}{\hbar}\nabla_{\vec{k}} E(\vec{k})$$

Wave-function changes according to:

$$\hbar\dot{\vec{k}} = -e\vec{\mathcal{E}}$$

i.a. $\hbar\vec{k} \neq m\vec{v}$

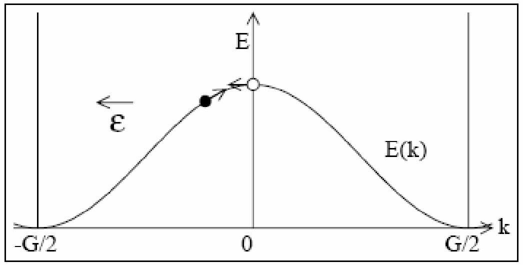


Bloch oszillation

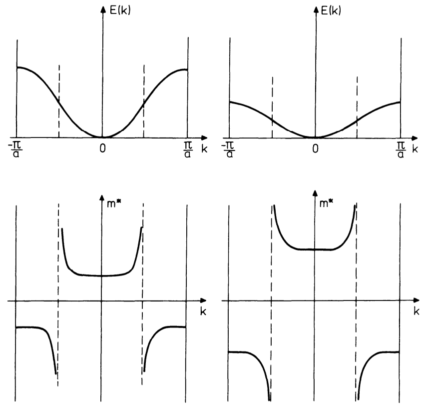
5.2 Electrical conductivity

$$\vec{j} = \frac{-2e}{(2\pi)^3}\int_{BZ,occ.} d^3k\ \vec{v}(\vec{k}) = \frac{-2e}{(2\pi)^3}[\int_{BZ} d^3k\ \vec{v}(\vec{k}) - \int_{BZ,unocc.} d^3k\ \vec{v}(\vec{k})] = \frac{2e}{(2\pi)^3}\int_{BZ,unocc.} d^3k\ \vec{v}(\vec{k})$$

electrons　　　　holes



5.2 Electrical conductivity

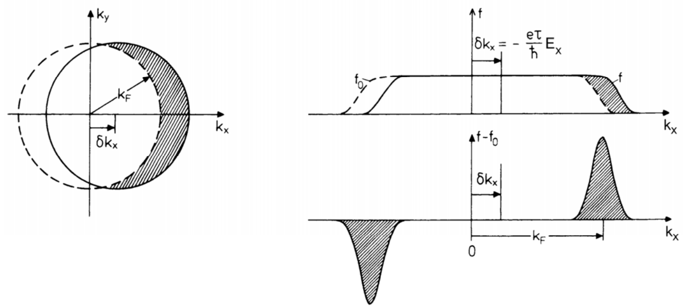


Effective mass tensor

$$\frac{1}{m^*_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E(\vec{k})}{\partial k_i \partial k_j}$$

flat band $\Rightarrow$ heavy electrons or holes
dispersing band $\Rightarrow$ light electrons or holes

5.2 Boltzmann equation



$$f(\vec{k},t) = f_0(\vec{k}) + \frac{e\mathcal{E}}{\hbar}\tau(\vec{k})\nabla_{\vec{k}} f(\vec{k},t)$$

$$\simeq f_0(\vec{k} + \frac{e\mathcal{E}}{\hbar}\tau(\vec{k}))$$

5.2 Electrical conductivity

$$\sigma_{ij} = \frac{2e^2}{(2\pi)^3} \int_{BZ} d^3k \; v(\vec{k})_i \; v(\vec{k})_j \; \tau(\vec{k}) \delta(E(\vec{k}) - E_F)$$

for τ independent of $\vec{k}$:

$$\sigma = \frac{e^2}{3} v_F^2 \; \tau_F D(E_F)$$

for free electron gas $D(E_F) = \frac{3}{2}\frac{n}{E_F}$:

$$\sigma = \frac{e^2 \tau_F}{m} \cdot n = \mu \cdot n$$

with carrier mobility

$$\mu = \frac{e^2 \tau}{m}$$

5.2 Electrical conductivity of metals

Table 3 Electrical conductivity and resistivity of metals at 295 K

(Resistivity values as given by G. T. Meaden, *Electrical resistance of metals*, Plenum, 1965; residual resistivities have been subtracted.)

Conductivity in units of 10^5 (ohm-cm)$^{-1}$.
Resistivity in units of 10^{-6} ohm-cm.

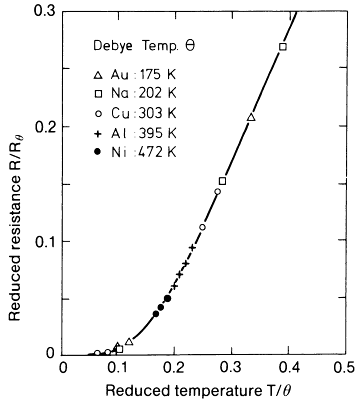
Li	Be											B	C	N	O	F	Ne
1.07	3.08																
9.32	3.25																
Na	Mg											Al	Si	P	S	Cl	Ar
2.11	2.33											3.65					
4.75	4.30											2.74					
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
1.39	2.78	0.21	0.23	0.50	0.78	0.072	1.02	1.72	1.43	5.88	1.69	0.67					
7.19	3.6	46.8	43.1	19.9	12.9	139.	9.8	5.8	7.0	1.70	5.92	14.85					
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn (w)	Sb	Te	I	Xe
0.80	0.47	0.17	0.24	0.69	1.89	~0.7	1.35	2.08	0.95	6.21	1.38	1.14	0.91	0.24			
12.5	21.5	58.5	42.4	14.5	5.3	~14.	7.4	4.8	10.5	1.61	7.27	8.75	11.0	41.3			
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg liq.	Tl	Pb	Bi	Po	At	Rn
0.50	0.26	0.13	0.33	0.76	1.89	0.54	1.10	1.96	0.96	4.55	0.10	0.61	0.48	0.086	0.22		
20.0	39.	79.	30.6	13.1	5.3	18.6	9.1	5.1	10.4	2.20	95.9	16.4	21.0	116.	46.		
Fr	Ra	Ac															

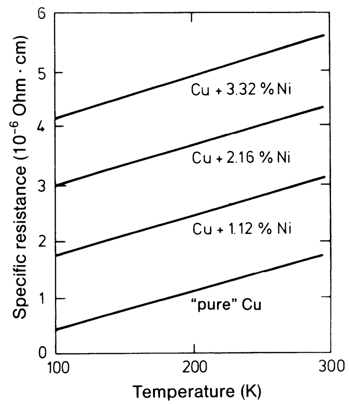
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
0.12	0.15	0.17		0.10	0.11	0.070	0.090	0.11	0.13	0.12	0.16	0.38	0.19
81.	67.	59.		99.	89.	134.	111.	90.0	77.7	81.	62.	26.4	53.
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
0.66		0.39	0.085	0.070									
15.2		25.7	118.	143.									

5.2 Electrical conductivity of metals

Temperature dependence (Matthiesen rule)

$$\rho = 1/\sigma = \rho_0(\alpha + \beta \cdot T)$$





At low temperatures:

$$\rho \propto (\frac{T}{\Theta_D})^5$$

5.3. Thermal conductivity

$$\vec{j}_Q = -\lambda \cdot \nabla T$$

Elektronen:

$$\lambda_{el} = \frac{1}{3} c_v^{el} \cdot v_F^2 \cdot \tau_{el}$$

Leitfhigkeit:

$$\sigma = n e^2 \tau_\sigma / m$$

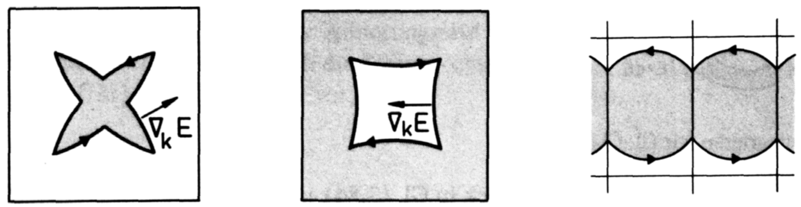
$$c_v^{el} = (\pi^2/3) k_B T^2 D(E_F) = (\pi^2/2) k_B T^2 (n/E_F)$$

$$\boxed{\frac{\lambda_{el}}{\sigma} = \frac{1}{3}(\frac{\pi k_B}{e})^2 T}$$

Wiedemann – Franz´sches Gesetz

5.4. Electrons in magnetic fields

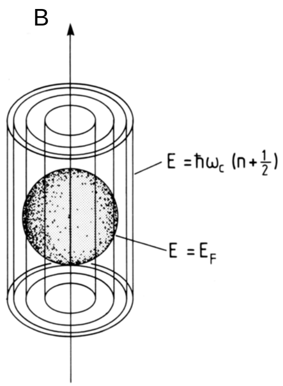
$$\hbar \dot{\vec{k}} = -\frac{e}{\hbar}(\nabla_{\vec{k}} E(\vec{k}) \times B)$$



electron — hole — open orbit

$$\dot{E}(\vec{k}) = \nabla_{\vec{k}} E(\vec{k}) \cdot \dot{\vec{k}} = 0$$
$$E(\vec{k}) = const.$$

5.4. Electrons in magnetic fields



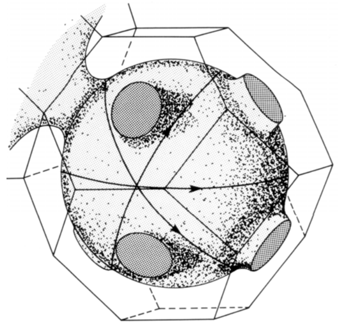
condensation on <u>Landau levels,</u>
Landau – cylinders

$$E_\nu = E_0(k_z) + (\nu + 1/2)\hbar\omega_c$$

$$\omega_c = \frac{e \cdot B}{m_c^*}$$

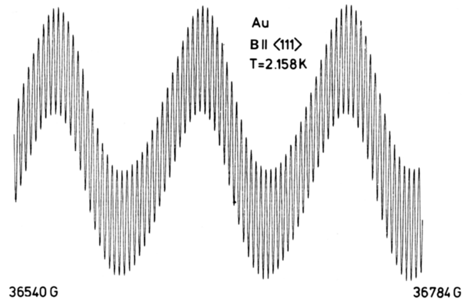
$$m_c^* = \frac{\hbar^2}{2\pi}\frac{\partial A(E,k_z)}{\partial E}$$

5.4. de Haas – van Alphen effect



Oscillation of the magnetic susceptibility
With external magnetic field

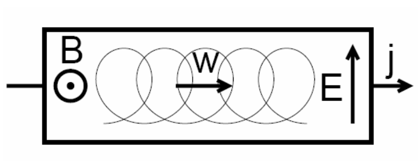
$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar}\frac{1}{A_F}$$



5.5. E ⊥ B – Hall Effect

$$\hbar\dot{\vec{k}} = -e(\vec{\mathcal{E}} + \vec{v} \times \vec{B}) = -e(\vec{v} - \vec{w}) \times \vec{B}$$

Drift velocity: $\vec{w} = \frac{\vec{\mathcal{E}} \times \vec{B}}{B^2}$ $\qquad \vec{j} = -ne\ \vec{w}$



$$\boxed{\begin{array}{l} \text{Hall field: } E_y = R_H\ j_x \cdot B,\ j_y = 0 \\ \text{Hall coefficient: } R_H = -1/ne \end{array}}$$

Non-closed orbits or carriers with different mobility or concentration
⇒ R_H depends on magnetic field B