Prospects of Optimal Control for Superconducting Circuits

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Yale Quantum Institute Seminar

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optimum quantum control

steer quantum system in some desired way

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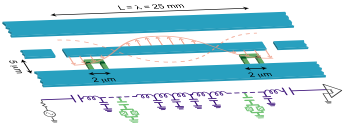
steer quantum system in some desired way

physically:

transmons with shared transmission line:



Blais et al. PRA 75, 032329 (2007)

e.g. CNOT gate:

$$
\begin{aligned}
\vert 00\rangle &\rightarrow \text{CNOT}\,\vert 00\rangle = \vert 00\rangle \\
\vert 01\rangle &\rightarrow \text{CNOT}\,\vert 01\rangle = \vert 01\rangle \\
\vert 10\rangle &\rightarrow \text{CNOT}\,\vert 10\rangle = \vert 11\rangle \\
\vert 11\rangle &\rightarrow \text{CNOT}\,\vert 11\rangle = \vert 10\rangle
\end{aligned}
$$

with *same* control $\epsilon(t)$.

optimum quantum control

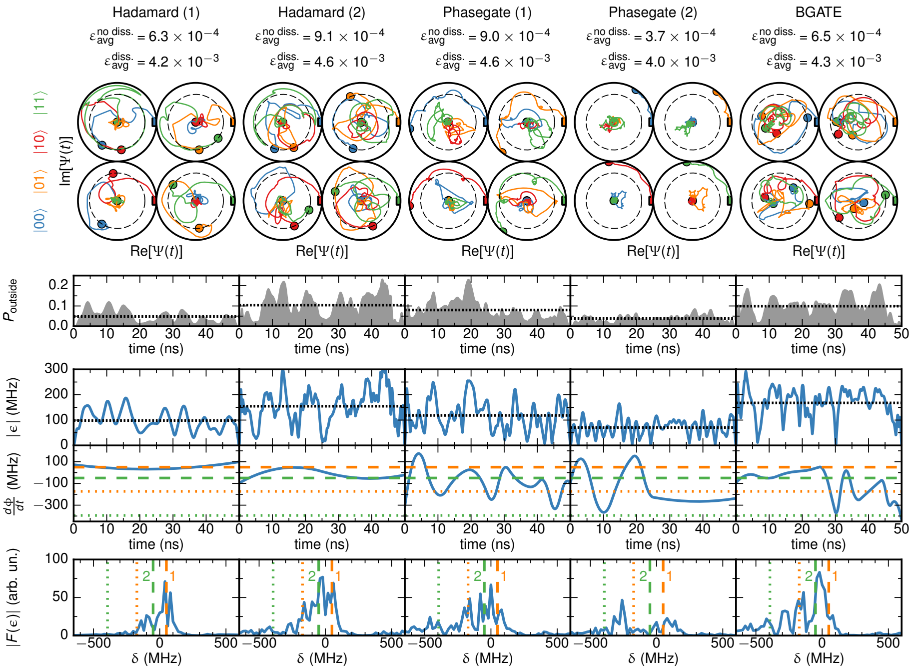
steer quantum system in some desired way

mathematically:

$$\text{minimize functional} \quad J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$$

$$= J_T(\{|\phi_k(T)\rangle\}) + \sum_l \int_0^T g_a(\epsilon_l(t))\,\mathrm{d}t$$

e.g. $J_{T,\mathrm{re}} = 1 - \frac{1}{N}\,\mathrm{Re}\left[\sum_{k=1}^{N} \langle\phi_k^{\mathrm{tgt}}|\phi_k(T)\rangle\right]$



Hadamard (1)
$\varepsilon_{avg}^{no\ diss.} = 6.3 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.2 \times 10^{-3}$
Hadamard (2)
$\varepsilon_{avg}^{no\ diss.} = 9.1 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.6 \times 10^{-3}$
Phasegate (1)
$\varepsilon_{avg}^{no\ diss.} = 9.0 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.6 \times 10^{-3}$
Phasegate (2)
$\varepsilon_{avg}^{no\ diss.} = 3.7 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.0 \times 10^{-3}$
BGATE
$\varepsilon_{avg}^{no\ diss.} = 6.5 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.3 \times 10^{-3}$
$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$
$\mathrm{Im}[\Psi(t)]$
$\mathrm{Re}[\Psi(t)]$
$P_{outside}$
time (ns)
$|\epsilon|$ (MHz)
$\frac{d\phi}{dt}$ (MHz)
$|F(\epsilon)|$ (arb. un.)
δ (MHz)

theory — experiment

general exploration

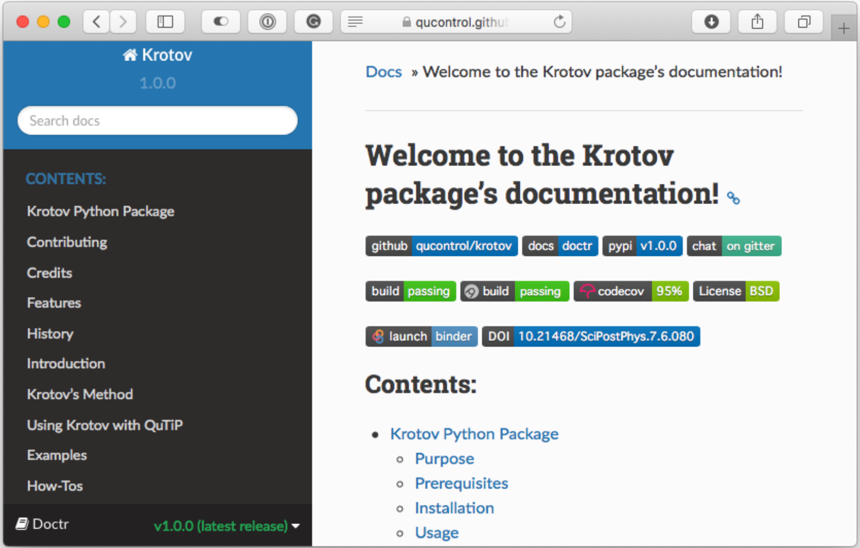
functional/constraint flexibility

- Krotov's method

- GRAPE
- GOAT
- GROUP

- Nelder-Mead
- CRAB
- ...
- Closed loop

https://github.com/qucontrol/krotov



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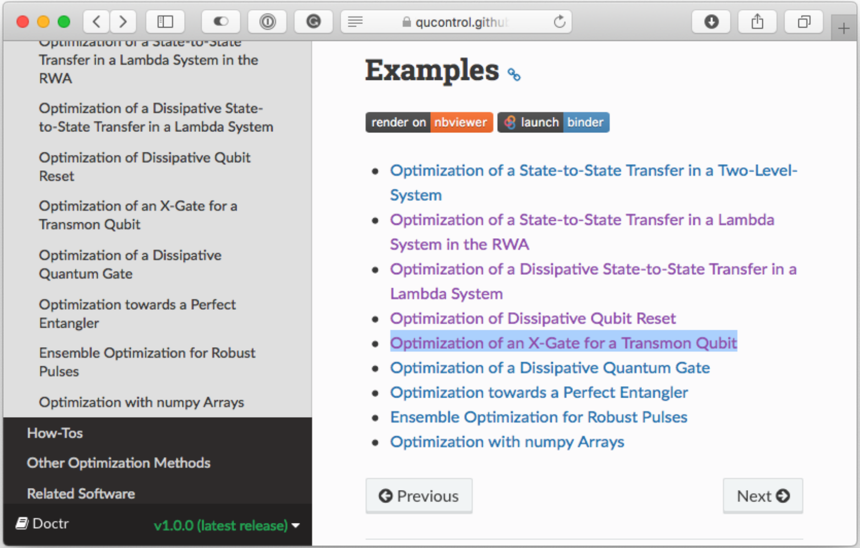
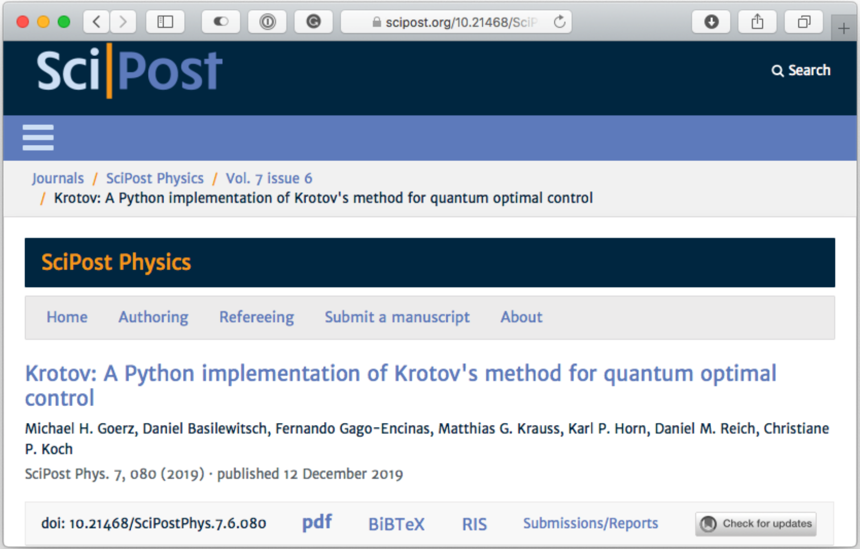
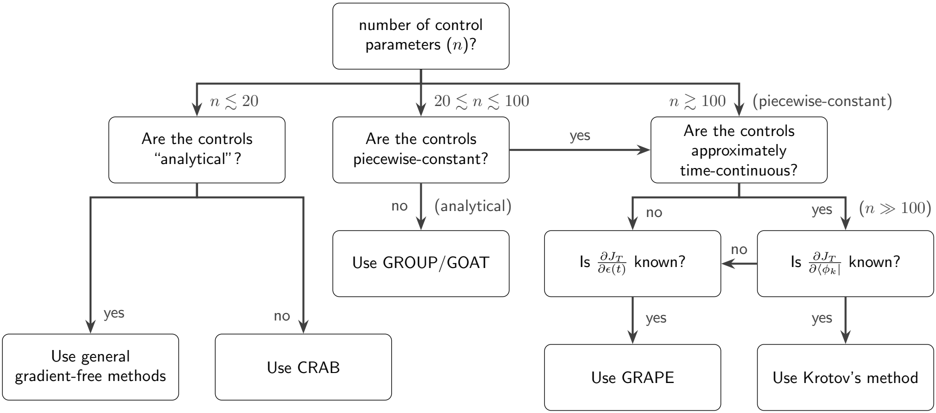
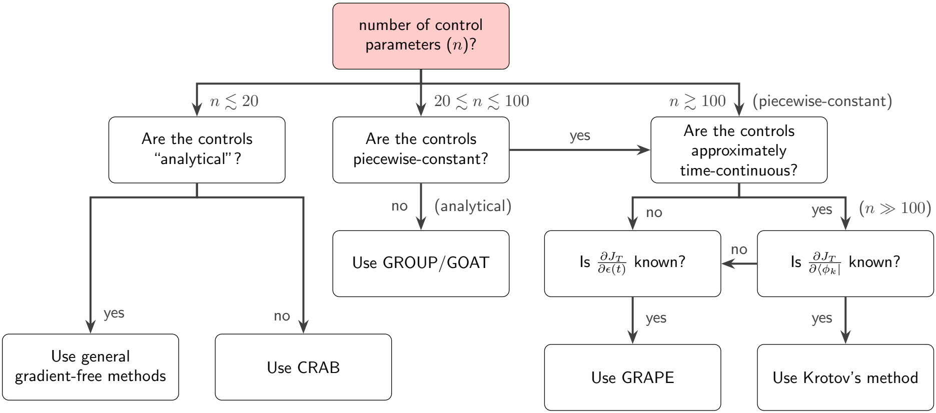


SciPost Phys. 7, 080 (2019)




number of control parameters (n)?
$n \lesssim 20$
$20 \lesssim n \lesssim 100$
$n \gtrsim 100$
(piecewise-constant)
Are the controls "analytical"?
Are the controls piecewise-constant?
yes
Are the controls approximately time-continuous?
no
(analytical)
no
yes
($n \gg 100$)
Use GROUP/GOAT
Is $\frac{\partial J_T}{\partial \epsilon(t)}$ known?
no
Is $\frac{\partial J_T}{\partial \langle \phi_k |}$ known?
yes
no
yes
yes
Use general gradient-free methods
Use CRAB
Use GRAPE
Use Krotov's method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functional $\quad J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$

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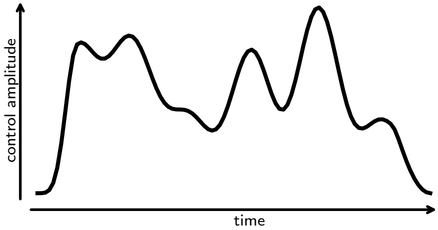


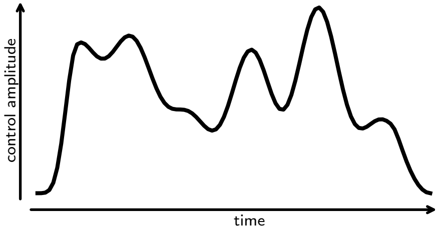
time-continuous fields

$\Downarrow$

functional derivative $\frac{\delta J}{\delta \epsilon_l(t)}$

problem:
interdependence of field and states

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



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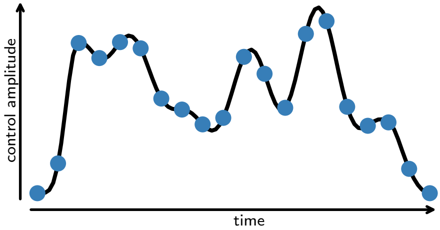
Krotov's method:

- Add "smart zero" to functional to disentangle interdependence of states and field
- Necessary and sufficient conditions for monotonic convergence

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



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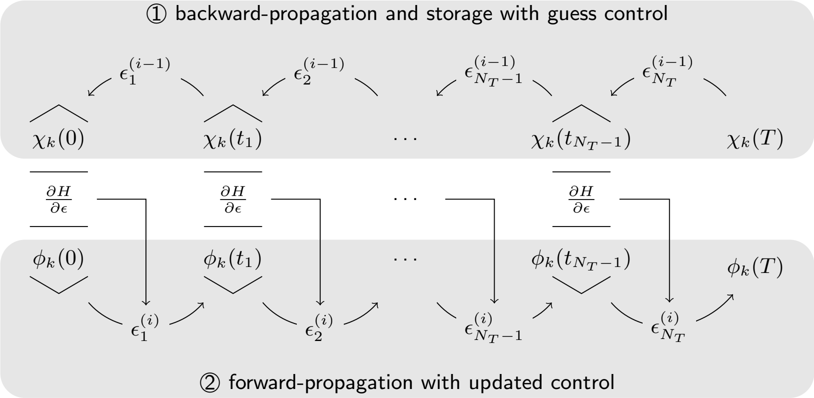
Krotov's method (discretized)

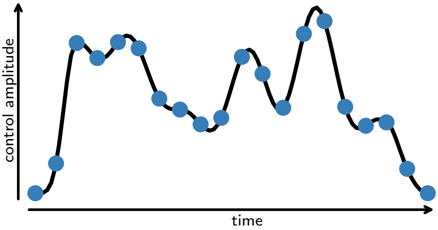
sequential scheme:



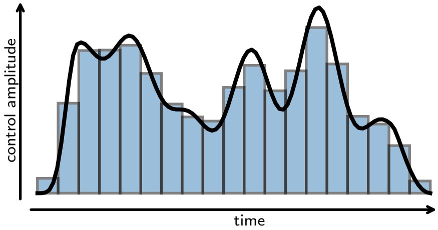
boundary condition:

$$|\chi_k(T)\rangle = -\frac{\partial J_T}{\partial \langle \phi_k(T)|}$$

functional $\quad J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$

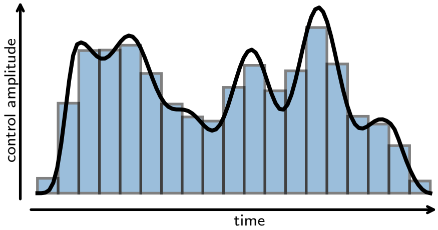


piecewise constant fields

$\Downarrow$

derivative $\frac{\partial J}{\partial \epsilon_l(t_n)}$

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



piecewise constant fields

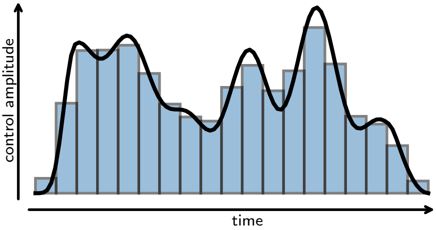
$\Downarrow$

derivative $\frac{\partial J}{\partial \epsilon_l(t_n)}$

Gradient Ascent Pulse Engineering (GRAPE):

- Discretize first! Then calculate the gradient
- L-BFGS-B: Quasi-Hessian for faster convergence

$$\text{functional} \quad J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$$



piecewise constant fields

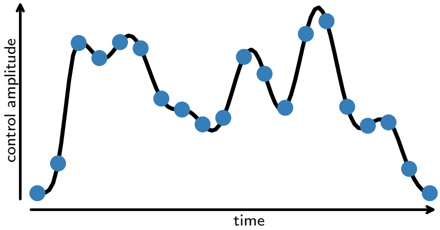
$$\Downarrow$$

derivative $\frac{\partial J}{\partial \epsilon_l(t_n)}$

Gradient Ascent Pulse Engineering (GRAPE):

$$\begin{aligned}
\frac{\partial \tau_k}{\partial \epsilon_n} &= \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle = \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_n \dots \hat{\mathbf{U}}_1 | \phi_k \rangle \\
&= \underbrace{\langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_{n+1}}_{\langle \chi_k(t_{n+1})|} \frac{\partial \hat{\mathbf{U}}_n}{\partial \epsilon_n} \underbrace{\hat{\mathbf{U}}_{n-1} \dots \hat{\mathbf{U}}_1 | \phi_k \rangle}_{|\phi_k(t_n)\rangle}
\end{aligned}$$

functional $\quad J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



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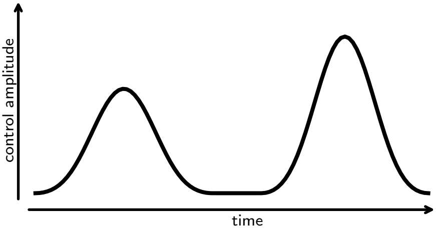
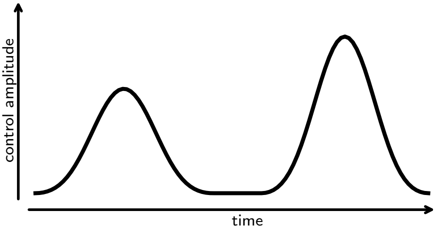


simple parametrized fields

$\Downarrow$

gradient-free methods,
e.g. CRAB

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



simple parametrized fields

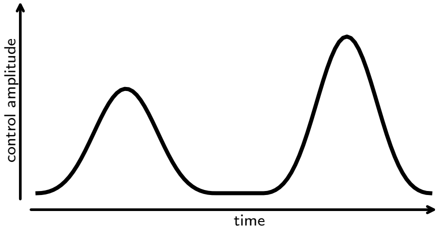
$\Downarrow$

gradient-free methods,
e.g. CRAB

$$\epsilon(t) = E_1 G(t_1, \sigma_1, t) + E_2 G(t_2, \sigma_2, t)$$

$$\epsilon(t) = \sum_{i=1}^{10} (a_n \cos(\omega_n t) + b_n \sin(\omega_n t))$$

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$



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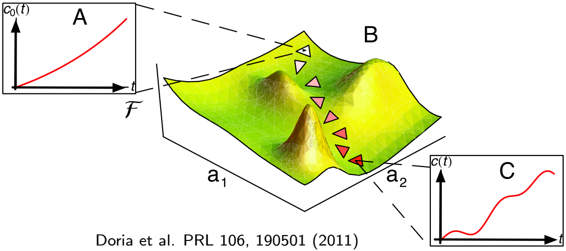
…Filter function of control electronics – non-analytic!

gradient-free optimization



Doria et al. PRL 106, 190501 (2011)

e.g. Nelder-Mead (simplex), genetic algorithms. . .

GROUP & GOAT: apply gradient to parametrized pulses

$$\epsilon(t) = S(t) \sum_n c_n f_n(t) \qquad \text{or generally} \qquad \epsilon(t) = \epsilon(\{c_n\})$$

$$\Rightarrow \frac{\partial J}{\partial c_n} = \sum_i \frac{\partial J}{\partial \epsilon_i} \frac{\partial \epsilon_i}{\partial c_n}$$

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GROUP (collection of methods): chain rule

Sørensen et al., Phys. Rev. A 98, 022119 (2018)

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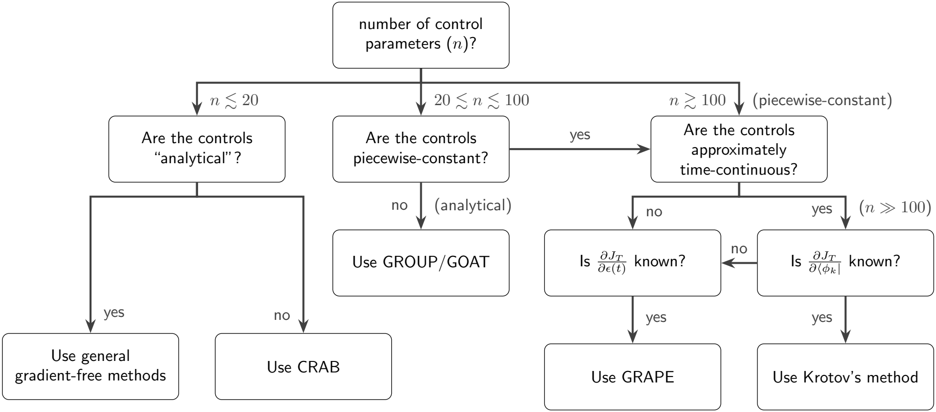
GOAT: evaluate functional and gradient at the same time (once per parameter)

Machnes et al., Phys. Rev. Lett. 120, 150401 (2018)

$$\partial_t \begin{pmatrix} U \\ \partial_{c_n} U \end{pmatrix} = -\frac{i}{\hbar} \begin{pmatrix} H & 0 \\ \partial_{c_n} H & H \end{pmatrix} \begin{pmatrix} U \\ \partial_{c_n} U \end{pmatrix}$$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"hybrid" (multi-stage) methods (example: RIP gate)

1. Start with analytical formula, optimize free parameter with simplex
2. Use simplex-optimized control as starting point for gradient-based method

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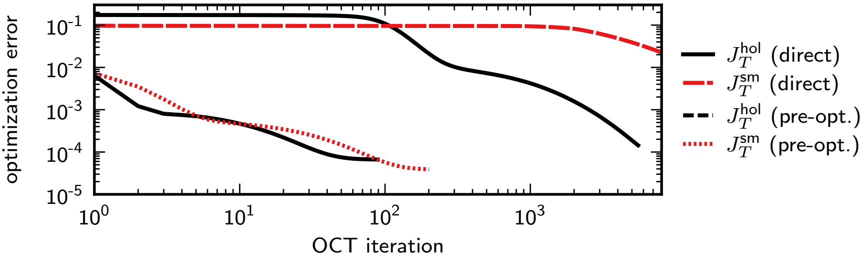
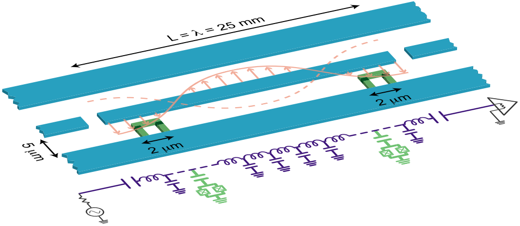


⇒ Goerz et al. EPJ Quantum Tech. 2, 21 (2015)

Charting the circuit QED design landscape using optimal control theory

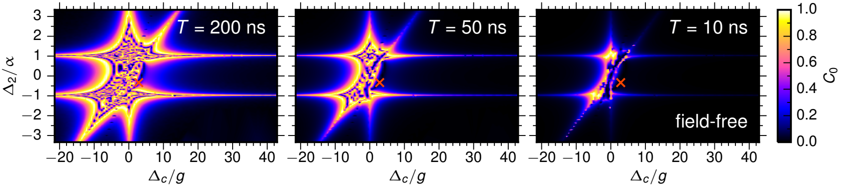
Goerz et al. npj Quantum Information 3, 37 (2017) two transmons with shared transmission line bus:



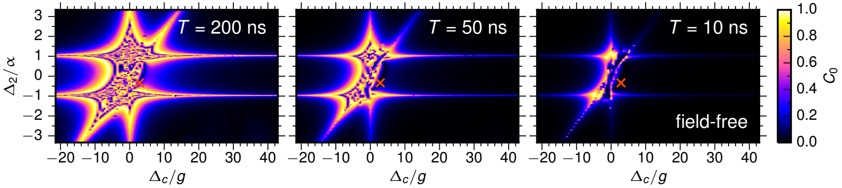
Blais et al. PRA 75, 032329 (2007)

$$\hat{\mathbf{H}} = \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 + \frac{\alpha_1}{2} \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \frac{\alpha_2}{2} \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2$$
$$+ g_1 (\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2 (\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger) + u^*(t) \hat{\mathbf{a}} + u(t) \hat{\mathbf{a}}^\dagger$$

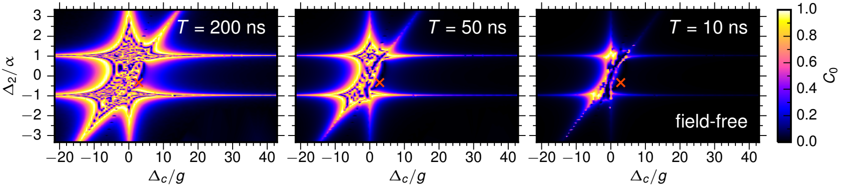
parameter landscape: Δ_c / g, Δ_2 / α



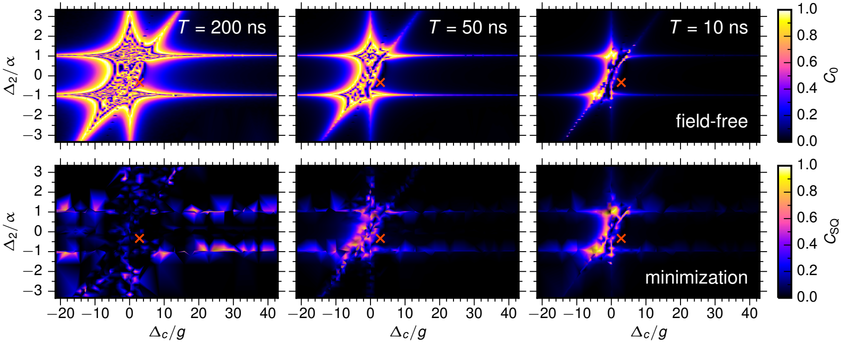
T = 200 ns
T = 50 ns
T = 10 ns
field-free
Δ_2/α
Δ_c/g
C_0



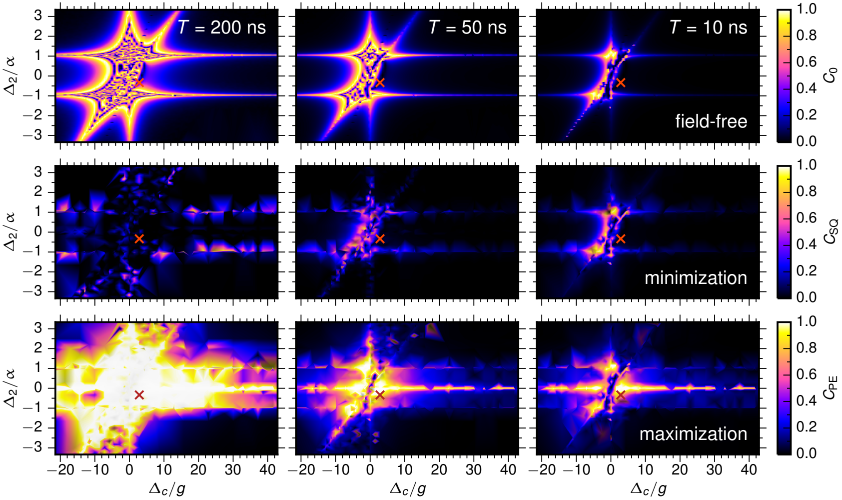
universal quantum computing:
perfect entangler *and* local gates

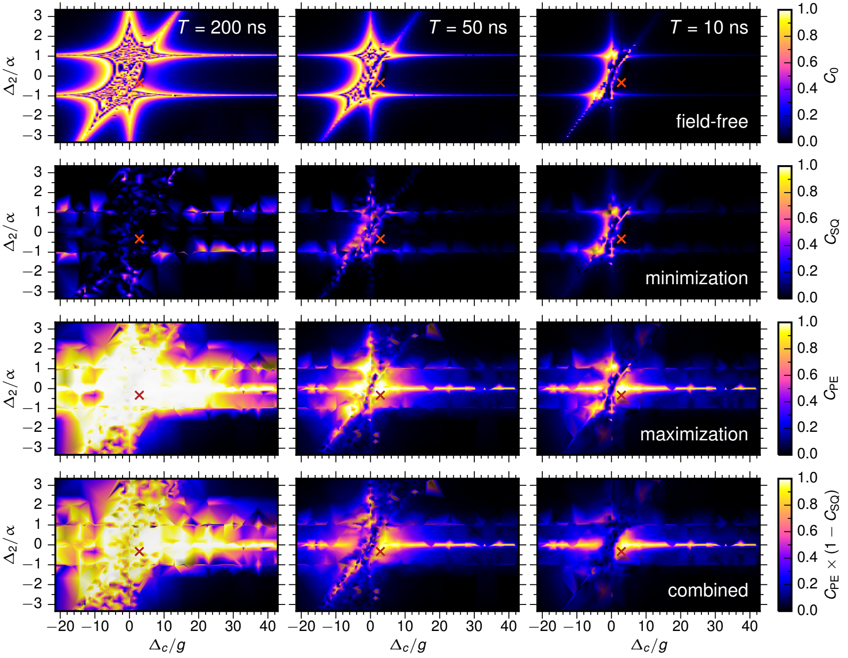


optimize for maximum / minimum entanglement

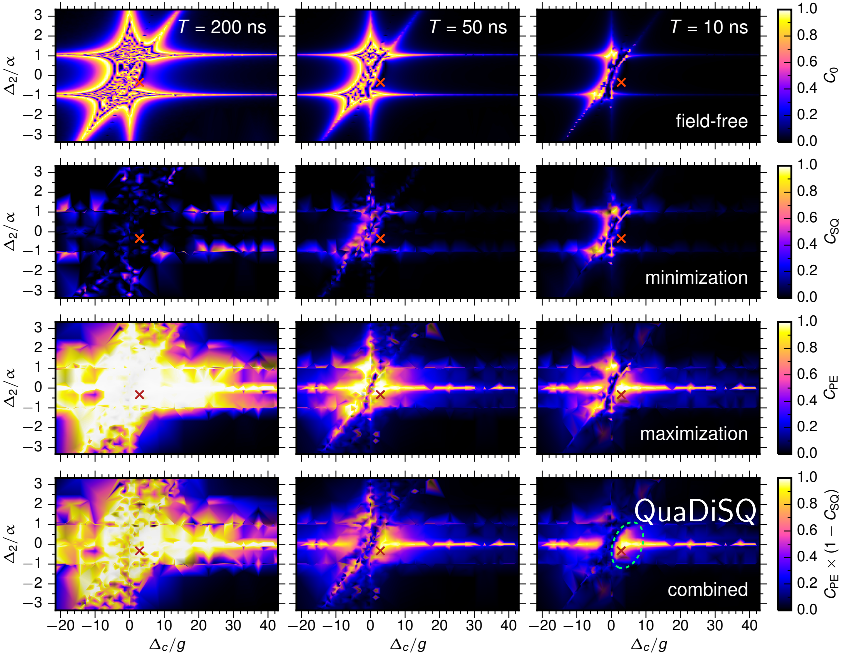


T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
Δ_2/α
Δ_c/g
C_0
C_{SQ}


T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
maximization
Δ_2/α
Δ_c/g
C_0
C_{SQ}
C_{PE}



T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
maximization
combined
Δ2/α
Δc/g
C0
CSQ
CPE
CPE × (1 − CSQ)


T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
maximization
combined
QuaDiSQ
Δ2/α
Δc/g
C0
CSQ
CPE
CPE × (1 − CSQ)

optimize for *specific* set of universal gates:

- Hadamard, Phase single qubit
- BGATE entangler

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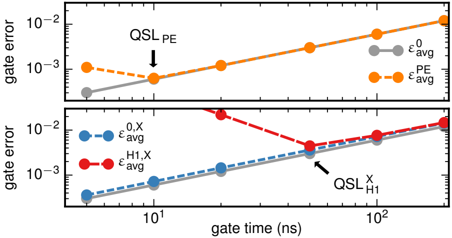
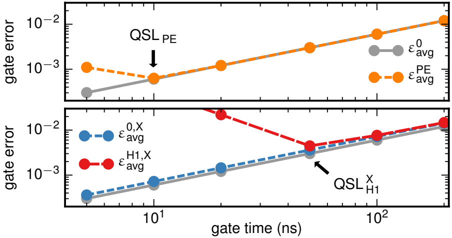


optimize for *specific* set of universal gates:

- Hadamard, Phase single qubit
- BGATE entangler

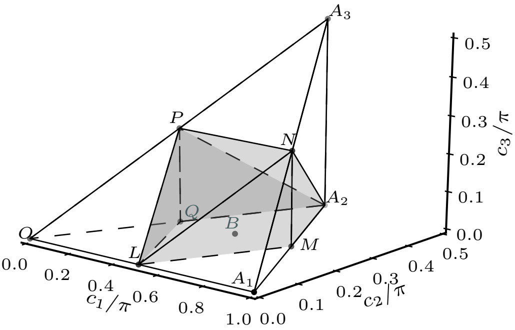


minimum gate duration: 50 ns
gate errors:

- 10^{-4} without dissipation
- 10^{-3} with dissipation

optimizing for entanglement

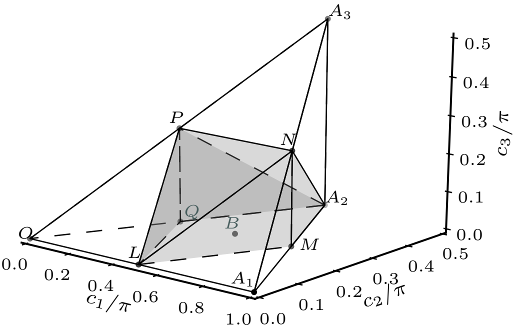
optimizing for entanglement



Cartan decomposition:

$$\hat{\mathbf{U}} = \hat{\mathbf{k}}_1 \exp\left[\frac{\mathrm{i}}{2}\left(c_1 \hat{\boldsymbol{\sigma}}_x \hat{\boldsymbol{\sigma}}_x + c_2 \hat{\boldsymbol{\sigma}}_y \hat{\boldsymbol{\sigma}}_y + c_3 \hat{\boldsymbol{\sigma}}_z \hat{\boldsymbol{\sigma}}_z\right)\right] \hat{\mathbf{k}}_2$$

optimizing for entanglement



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$\Rightarrow$ application: Goerz et al. Phys. Rev. A 91, 062307 (2015)

Outlook: learning from machine learning

Reinforcement learning for quantum control

- Use a neural net (AlphaZero)
  to choose pulse amplitude in each time slice.

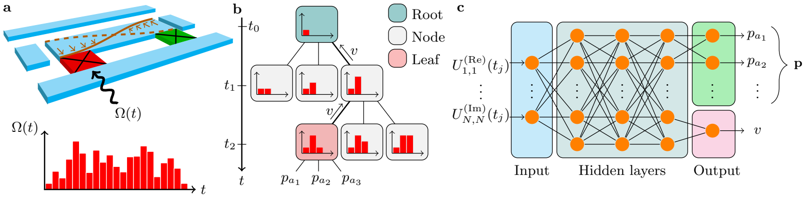

Fig 1 from: Dalgaard et. al, arXiv:1907.05672 (2019)

Automatic Differentiation

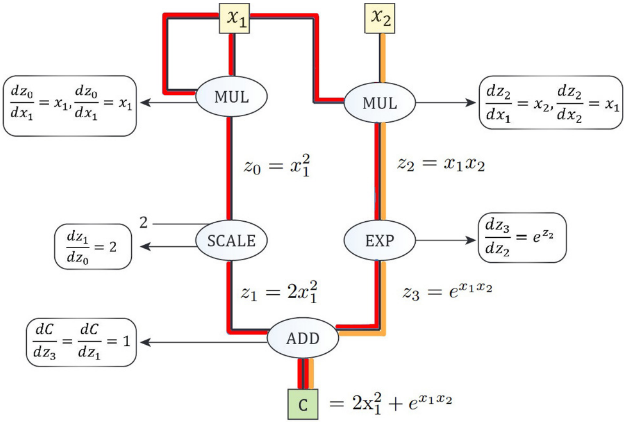

Fig 12 from: Abdelhalfez et. al, PRA 99, 052327 (2019)

Automatic differentiation for MCWF propagation & optimization functional

using Google Tensorflow

PHYSICAL REVIEW A **95**, 042318 (2017)

Speedup for quantum optimal control from automatic differentiation based on graphics processing units

Nelson Leung,[1,*] Mohamed Abdelhafez,[1] Jens Koch,[2] and David Schuster[1]
[1]*The James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA*
[2]*Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*

PHYSICAL REVIEW A **99**, 052327 (2019)

Gradient-based optimal control of open quantum systems using quantum trajectories and automatic differentiation

Mohamed Abdelhafez,[1,*] David I. Schuster,[1] and Jens Koch[2]
[1]*The James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA*
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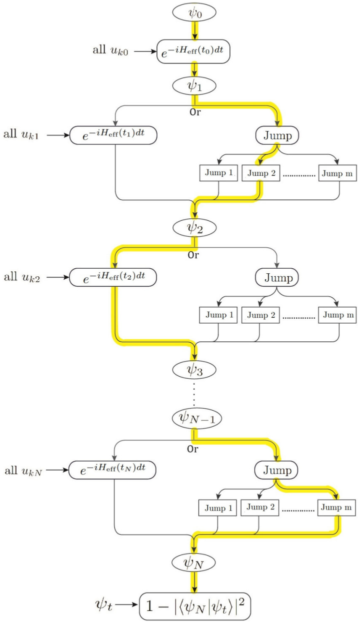

perfect entanglers functional:

$$
\begin{aligned}
F_{PE} = & \left(\frac{1}{\det U_B}\right)\left(\frac{1}{4}(\mathrm{tr}^2[U_B^T U_B] - \mathrm{tr}[U_B^T U_B U_B^T U_B])\right)\left(\frac{1}{16}\,\mathrm{Re}^2[\mathrm{tr}[U_B^T U_B]]\right) + \\
& + \left(\frac{2}{\det U_B}\right)\left(\frac{1}{4}(\mathrm{tr}^2[U_B^T U_B] - \mathrm{tr}[U_B^T U_B U_B^T U_B])\right)\left(\frac{1}{16}\,\mathrm{Im}^2[\mathrm{tr}[U_B^T U_B]]\right) \\
& \left(\frac{1}{16}\,\mathrm{Re}[\mathrm{tr}^2[U_B^T U_B]]\right)
\end{aligned}
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U_B: projection into logical subspace, in Bell basis

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- control constraints
- noise and filter function of control electronics

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"semi-automatic differentiation"

$$\frac{\partial \tau_k}{\partial \epsilon_n} = \frac{\partial}{\partial \epsilon_n}\left\langle \phi_k^{\mathrm{tgt}} \middle| \phi_k(T)\right\rangle = \underbrace{\left\langle \phi_k^{\mathrm{tgt}} \middle| \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_{n+1}\right.}_{\langle \chi_k(t_{n+1})|} \frac{\partial \hat{\mathbf{U}}_n}{\partial \epsilon_n} \underbrace{\left.\hat{\mathbf{U}}_{n-1} \dots \hat{\mathbf{U}}_1 \middle| \phi_k \right\rangle}_{|\phi_k(t_n)\rangle}$$

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$$

express functional in terms of scalar values τ_{ik}

$$
F_{PE} = F_{PE}(\{\tau_{ik}\})
$$

Summary

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- choice of control methods:
  theoretical exploration and experimental constraints
  - Krotov's method best for unconstrained exploration
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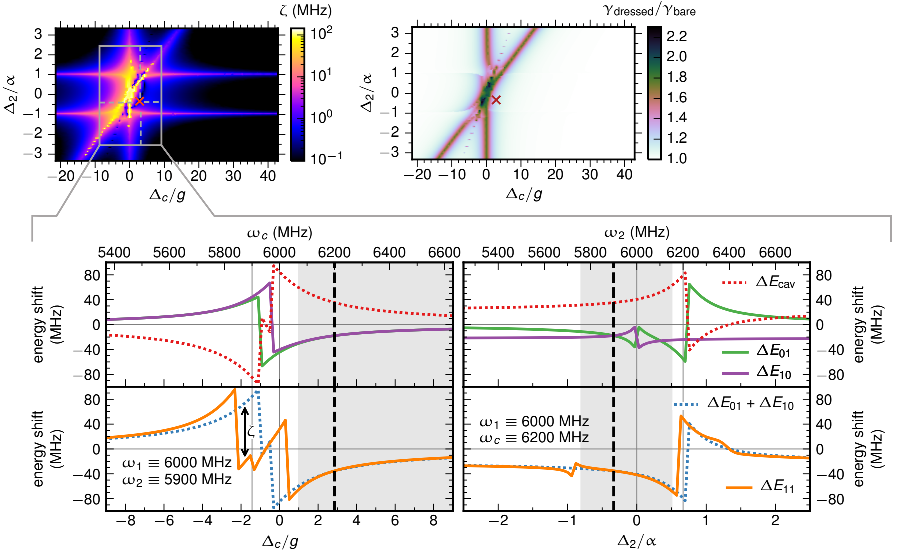
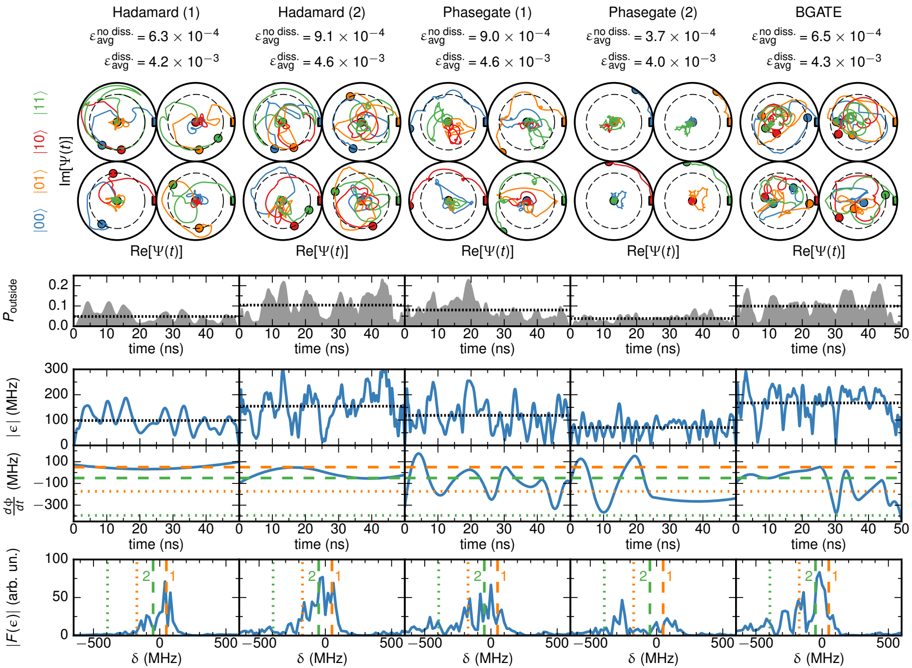
Thank You

properties for field-free Hamiltonian:





Hadamard (1)
$\varepsilon_{avg}^{no\ diss.} = 6.3 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.2 \times 10^{-3}$
Hadamard (2)
$\varepsilon_{avg}^{no\ diss.} = 9.1 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.6 \times 10^{-3}$
Phasegate (1)
$\varepsilon_{avg}^{no\ diss.} = 9.0 \times 10^{-4}$
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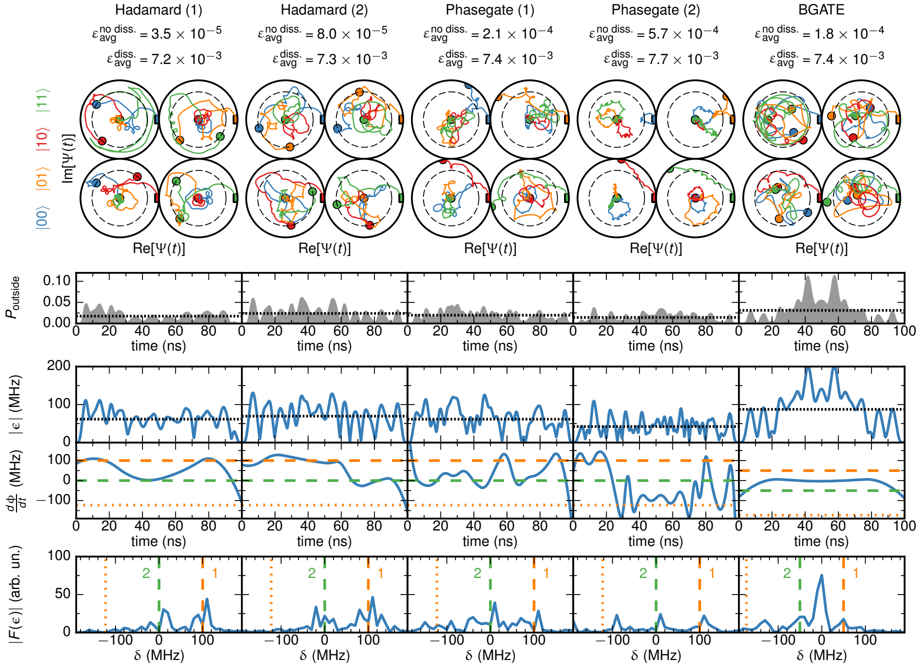
Phasegate (2)
$\varepsilon_{avg}^{no\ diss.} = 3.7 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.0 \times 10^{-3}$
BGATE
$\varepsilon_{avg}^{no\ diss.} = 6.5 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.3 \times 10^{-3}$
$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$
Im[$\Psi(t)$]
Re[$\Psi(t)$]
$P_{outside}$
time (ns)
$|\epsilon|$ (MHz)
$\frac{d\phi}{dt}$ (MHz)
$|F(\epsilon)|$ (arb. un.)
δ (MHz)


Hadamard (1)
$\varepsilon_{avg}^{no\ diss.} = 3.5 \times 10^{-5}$
$\varepsilon_{avg}^{diss.} = 7.2 \times 10^{-3}$
Hadamard (2)
$\varepsilon_{avg}^{no\ diss.} = 8.0 \times 10^{-5}$
$\varepsilon_{avg}^{diss.} = 7.3 \times 10^{-3}$
Phasegate (1)
$\varepsilon_{avg}^{no\ diss.} = 2.1 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 7.4 \times 10^{-3}$
Phasegate (2)
$\varepsilon_{avg}^{no\ diss.} = 5.7 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 7.7 \times 10^{-3}$
BGATE
$\varepsilon_{avg}^{no\ diss.} = 1.8 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 7.4 \times 10^{-3}$
$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$
$\mathrm{Im}[\Psi(t)]$
$\mathrm{Re}[\Psi(t)]$
$P_{outside}$
time (ns)
$|\epsilon|$ (MHz)
$\frac{d\phi}{dt}$ (MHz)
$|F(\epsilon)|$ (arb. un.)
δ (MHz)
1
2

obtained perfect entanglers in the Weyl chamber: