Fine-grained Parameterized Algorithms

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Problem set 10: The Polynomial Method

!! **10.1 OR has high degree.** Show that representing the OR of n variables $x_1, \ldots, x_n$ exactly with a polynomial over GF(q) where q is prime requires degree exactly n.

!! **10.2 Probabilistic polynomials for bounded-depth circuits.** Let d be a positive integer and let $\delta > 0$. Show that there exists a constant $D > 0$ such that, for every n and every depth-d Boolean circuit with n input variables, there exists a probabilistic polynomial p of degree at most D that satisfies the following for every $x \in \{0,1\}^n$:

$$\Pr_p[C(x) \neq p(x)] \leq \delta\,.$$

Hint: Iteratively convert the gates from the bottom up, with lower gates having a smaller error probability.