Fine-grained Parameterized Algorithms

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Problem set 8: Fixed-Parameter Intractability

Overview: With this problem set, you can train proving W[1]-hardness.

Instructions: For each skill, select **exactly one** problem below and submit your solution in Moodle; in your submission, make sure to repeat the problem that you are solving. The problems are roughly ordered by difficulty, choose a problem that you find non-trivial and interesting. (You are of course welcome to try the other problems as well and ask us for feedback.)

!! **Skill-8a. Falsify reduction:** *I can explain why a given reduction does not establish* W[1]*-hardness.*

8.1 Vertex-Cover. Let G be a graph and $S \subseteq V(G)$. Prove that S is a maximum independent set of G if and only if $V(G) \setminus S$ is a minimum vertex-cover of G. Use this to give a polynomial-time reduction from Independent Set to Vertex-Cover. Why is this not also a parameterized reduction that establishes that Vertex-Cover is W[1]-hard? Is there a parameterized reduction from Vertex-Cover to Independent Set?

!! **Skill-8b. Design a reduction:** *I can establish* W[1]*-hardness by designing a suitable parameterized reduction from a problem known to be* W[1]*-hard.*

8.2 Dominating Set. Give a parameterized reduction from Set Cover to Dominating Set.

8.3 Multicolored Biclique. In the Multicolored Biclique problem, the input consists of a bipartite graph G with two parts A and B, an integer k, a partition of A into k sets $A_1, A_2, \ldots, A_k$, and a partition of B into k sets $B_1, B_2, \ldots, B_k$; the question is whether there exists a subgraph of G isomorphic to the biclique $K_{k,k}$, with one vertex in each of the sets A_i and B_i. Prove that Multicolored Biclique is W[1]-hard.

8.4 Set Packing. In the Set Packing problem, the input consists of a family $\mathcal{F}$ of subsets of a finite universe U and an integer k, and the question is whether one can find k pairwise disjoint sets in $\mathcal{F}$. Prove that Set Packing is W[1]-hard when parameterized by k.