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Numerical Evaluation of Multivariate Contingent Claims

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We develop a numerical approximation method for valuing multivariate contingent claims. The approach is based on an n -dimensional extension of the lattice binomial method. Closed-form solutions for the jump probabilities and the jump amplitudes are obtained. The accuracy of the method is illustrated in the case of European options when there are three underlying assets.

The option pricing framework provides a powerful tool for analyzing a range of problems in financial economics. Corporations and their financial advisers have in recent years invented increasingly sophisticated securities with option components. In particular, we find embedded options whose payoffs depend on two or more underlying assets. Stulz (1982) gives a number of examples of options of this nature. Both Schwartz (1982) and Boyle and Kirzner (1985) value optionlike securities linked to the value of an underlying commodity and another risky asset. Carr (1987) considers commodity-linked bonds for which the payoff depends on three risky assets. A number of authors have examined the quality option¹ that is often found in futures con-

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¹ See, for example, Gay and Manaster (1986), Kane and Marcus (1986), Hemler (1988), and Boyle (1988b).

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tracts. In these situations the short has the choice of which of several assets to deliver in order to fulfill the terms of the contract. The analysis of this *cheapest to deliver* option involves the consideration of options on the minimum of several assets.

In most cases involving options on several assets it is not difficult to derive the general partial differential equation that the option price obeys. This differential equation can be solved in some specialized situations. Stulz (1982) derived closed-form solutions for certain types of European options when there were two underlying state variables. A number of authors² have derived closed-form solutions in terms of multivariate normal integrals for European options on the maximum or the minimum of n underlying assets. However, these solutions cannot accommodate the early exercise feature of American options. This can be accomplished by solving the resulting partial differential equation numerically, but when more than two assets are involved the computations become excessively burdensome.

In the case of just one asset, the lattice binomial approach pioneered by Cox, Ross, and Rubinstein (1979) (CRR) has proved to be a powerful and flexible method for valuing American options. Boyle (1988a) recently developed an extension of the CRR procedure for option valuation in the case of two assets, which can be used to handle American options and cash dividends.

The aim of our article is to develop a valuation model for contingent claims involving several underlying assets using a generalized lattice framework. The method is close in spirit to the CRR method. In Section 1 we provide an overview of the approach used. Under certain assumptions the value of a contingent claim satisfies a partial differential equation. We indicate how solutions to this equation may be obtained by modeling the evolution of the underlying asset prices in a discrete-time setting. Section 2 shows how an explicit solution can be obtained in the case of two underlying assets. The key idea is to choose jump sizes and jump probabilities so that the characteristic function of the discrete distribution converges to that of the continuous distribution. Broadly speaking, this can be done in one of two ways. We follow the approach taken by CRR, in which we define the jump sizes as they do and choose the probabilities to match characteristic functions. An alternative approach would be to fix the probabilities (e.g., the probability of an up-jump = .5 = probability of a down-jump in the one-asset case) and then determine jump sizes to ensure convergence. There may be slight advantages of one approach over the other, but the practical differences are probably small. In Section 3, a number of numerical examples are provided for European options involving three underlying assets. The accuracy of the method can be assessed by comparing the option values computed by our procedure with the exact values obtained by an independent approach.

² These include Johnson (1987), Margrabe (1982), Cheng (1987), and Hemler (1988).

1. The Valuation Method

This section provides an overview of the approach. We start with the general valuation equation for a derivative security when there are several underlying assets. We indicate how a discrete-time, lattice approach can be developed to obtain the current price of the derivative asset.

The general valuation equation for a security that depends on n underlying assets was derived by Cox, Ingersoll, and Ross (1985) and also by Garman (1976). Assume that the price of each asset follows a diffusion process.

$$\frac{dV_i}{V_i} = \mu_i dt + \sigma_i dz_i \quad 1 \leq i \leq n \quad (1)$$

where

V_i = the current price of asset i

μ_i = the drift of the process for asset i

σ_i = the volatility of the process for asset i

dz_i = a Gauss-Wiener process

ρ_{ij} = the instantaneous correlation between dz_i and dz_j , $1 \leq i < j \leq n$

The general valuation equation for a security W whose payoff depends on the n assets is

$$\frac{\partial W}{\partial t} + \sum_{i=1}^n V_i \frac{\partial W}{\partial V_i} (\mu_i - \lambda_i \sigma_i) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \sigma_i \sigma_j V_i V_j \frac{\partial^2 W}{\partial V_i \partial V_j} = rW \quad (2)$$

where λ_i is the market price of risk for asset i .

If each of the n assets is a traded security, then Equation (2) is satisfied for $W = V_i$, $1 \leq i \leq n$. This implies

$$\mu_i - \lambda_i \sigma_i = r \quad 1 \leq i \leq n \quad (3)$$

where $r = r(t)$ is the current riskless rate. In this article we assume that r is a constant. We also assume that σ_i and ρ_{ij} are constants.

Both Cox and Ross (1976) and Harrison and Kreps (1979) showed that the no-arbitrage value of a contingent claim can be obtained by replacing the drift terms μ_i by the risk-adjusted drift terms $(\mu_i - \lambda_i \sigma_i)$ and discounting the expected value of the claim at the riskless interest rate. In view of Equation (3), this assumption is equivalent to assuming that each of the underlying assets has an expected rate of return equal to the riskless rate.

Cox, Ross, and Rubinstein (1979) used this technique in the case of options on a single asset to develop a flexible valuation method. They approximated the continuous lognormal distribution with a discrete distribution using a multiplicative binomial lattice. A grid of possible asset prices was computed assuming that the asset earned the riskless rate. The current value of an option was obtained by discounting its expected terminal value at the riskless rate.

To extend the CRR approach to several assets we proceed as follows. First, we construct a discrete probability distribution to approximate the

multivariate lognormal distribution. This discrete distribution converges to the lognormal distribution as the length of the time step tends to zero. The procedure is similar to that used in the CRR method, except that we now have n assets to contend with. We assume that at the end of each time step each asset price can move up or down so that after one time step there will be 2^n states. Second, we value the contingent claim in this discrete setting by discounting its expected terminal value at the riskless rate. Third, we obtain the value of the contingent claim to the desired degree of accuracy by reducing the size of the time interval. We refer to the recent paper by Omberg (1988), which provided the regularity conditions necessary to ensure convergence of the discrete approximation to the true option price.

2. A Lattice Valuation Model for Several Assets

In this section we extend the CRR binomial approach for one asset to the case of several assets. It is convenient to first give the details for two assets and present the results for the n -asset case later. The method used to approximate the bivariate lognormal density by a discrete distribution generalizes readily to the n -dimensional case.

Consider two assets with current prices S_1 and S_2 . Assume these prices have a bivariate lognormal distribution. Over a small time interval we approximate this distribution with a four-jump discrete distribution. It is convenient to use the following notation:

- S_i = current price of asset i ($i = 1, 2$)
- T = time to option maturity (in years)
- X = exercise price of the option
- r = continuously compounded yearly interest rate
- σ_i^2 = variance of the rate of return of the underlying assets (yearly)
- N = number of steps into which the time T is divided
- $h = T/N$; length of one time step
- $S_i u_i$ = asset value after an up-jump ($i = 1, 2$)
- $S_i d_i$ = asset value after a down-jump ($i = 1, 2$)
- ρ_{ij} = correlation coefficient between assets i and j
- $\mu_i = r - \frac{1}{2}\sigma_i^2$ = drift of the continuous lognormal distribution. This is to ensure that both assets earn the riskless rate as required by the assumption of risk neutrality.

The continuous return distribution is approximated by the four-jump process shown in Table 1. Note that we have employed two methods of representing the jump probabilities. Thus p_1 is convenient for writing out the formulas, while p_{uu} , p_{ud} , etc., is more suggestive of the underlying dynamics.

After one time step, the two assets with starting prices (S_1, S_2) can have four possible values represented by (\hat{S}_1, \hat{S}_2) . For convenience we will require that

Table 1
The continuous return distribution approximated by the four-jump process

Nature of jumps	Probability	Asset prices
Up, up	$p_1 = p_{uu}$	$S_1 u_1, S_2 u_2$
Up, down	$p_2 = p_{ud}$	$S_1 u_1, S_2 d_2$
Down, up	$p_3 = p_{du}$	$S_1 d_1, S_2 u_2$
Down, down	$p_4 = p_{dd}$	$S_1 d_1, S_2 d_2$

$$u_i d_i = 1 \qquad i = 1, 2 \tag{4}$$

where

$$u_i = e^{\sigma \sqrt{b}} \qquad i = 1, 2 \tag{5}$$

in analogy with CRR. We will determine the probabilities p_j ($j = 1, \dots, 4$) by requiring that the four-point discrete distribution converge to the corresponding continuous distribution in the limit as b tends to zero.

It is convenient to use the normal distribution rather than the lognormal distribution, because we will be using the moment-generating function, and this function has a simple form in the normal case.³ Hence, we transform the variables of the discrete four-point distribution to new variables using a log transformation. Thus we define

$$(\hat{R}_1, \hat{R}_2) = \left(\ln \left(\frac{\hat{S}_1}{S_1} \right), \ln \left(\frac{\hat{S}_2}{S_2} \right) \right)$$

which has the characteristic function:

$$\Psi_b(\theta_1, \theta_2) = E[\exp(i\theta_1 \hat{R}_1 + i\theta_2 \hat{R}_2)] \tag{6}$$

For the four-jump process this can be written as

$$\begin{aligned} \Psi_b(\theta_1, \theta_2) = & p_1 \exp[i\sqrt{b}(\theta_1 \sigma_1 + \theta_2 \sigma_2)] + p_2 \exp[i\sqrt{b}(\theta_1 \sigma_1 - \theta_2 \sigma_2)] \\ & + p_3 \exp[i\sqrt{b}(-\theta_1 \sigma_1 + \theta_2 \sigma_2)] + p_4 \exp[i\sqrt{b}(-\theta_1 \sigma_1 - \theta_2 \sigma_2)] \end{aligned}$$

Expanding the right-hand side as a Taylor series we obtain

$$\begin{aligned} \psi_b(\theta_1, \theta_2) = & p_1 \left(1 + i\sqrt{b}(\theta_1 \sigma_1 + \theta_2 \sigma_2) - \frac{b}{2}(\theta_1 \sigma_1 + \theta_2 \sigma_2)^2 \right) \\ & + p_2 \left(1 + i\sqrt{b}(\theta_1 \sigma_1 - \theta_2 \sigma_2) - \frac{b}{2}(\theta_1 \sigma_1 - \theta_2 \sigma_2)^2 \right) \\ & + p_3 \left(1 + i\sqrt{b}(-\theta_1 \sigma_1 + \theta_2 \sigma_2) - \frac{b}{2}(-\theta_1 \sigma_1 + \theta_2 \sigma_2)^2 \right) \\ & + p_4 \left(1 + i\sqrt{b}(-\theta_1 \sigma_1 - \theta_2 \sigma_2) - \frac{b}{2}(-\theta_1 \sigma_1 - \theta_2 \sigma_2)^2 \right) + o(b) \end{aligned} \tag{7}$$

By rearranging terms, we can rewrite Equation (7) as

³ The moment-generating function of the lognormal distribution does not have a simple analytical form.

$$\begin{aligned}\psi_b(\theta_1, \theta_2) = & \sum_{j=1}^4 p_j + i\sqrt{b}(\theta_1\sigma_1(p_1 + p_2 - p_3 - p_4) \\ & + \theta_2\sigma_2(p_1 - p_2 + p_3 - p_4)) \\ & - \frac{b}{2}(\theta_1^2\sigma_1^2 + 2\theta_1\theta_2\sigma_1\sigma_2(p_1 - p_2 - p_3 + p_4) \\ & + \theta_2^2\sigma_2^2) + o(b)\end{aligned}\quad (8)$$

To ensure convergence between the discrete distribution and its continuous (bivariate normal) counterpart, we equate the two characteristic functions. The characteristic function for the bivariate normal distribution with a time interval of b is

$$\begin{aligned}\Psi(b)(\theta_1, \theta_2) = & 1 + i\sqrt{b}(\theta_1\mu_1 + \theta_2\mu_2) \\ & - \frac{b}{2}(\theta_1^2\sigma_1^2 + 2\theta_1\theta_2\sigma_1\sigma_2\rho + \theta_2^2\sigma_2^2) + o(b)\end{aligned}\quad (9)$$

By equating like coefficients in Equations (8) and (9), we obtain

$$p_1 + p_2 + p_3 + p_4 = 1 \quad (10a)$$

$$p_1 - p_2 - p_3 + p_4 = \rho \quad (10b)$$

$$p_1 + p_2 - p_3 - p_4 = \sqrt{b}\frac{\mu_1}{\sigma_1} \quad (10c)$$

$$p_1 - p_2 + p_3 - p_4 = \sqrt{b}\frac{\mu_2}{\sigma_2} \quad (10d)$$

This system consists of four equations for the four p 's and can be solved to give

$$p_1 = \frac{1}{4}\left(1 + \rho + \sqrt{b}\left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right)\right) \quad (11a)$$

$$p_2 = \frac{1}{4}\left(1 - \rho + \sqrt{b}\left(\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right)\right) \quad (11b)$$

$$p_3 = \frac{1}{4}\left(1 - \rho + \sqrt{b}\left(-\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right)\right) \quad (11c)$$

$$p_4 = \frac{1}{4}\left(1 + \rho + \sqrt{b}\left(-\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right)\right) \quad (11d)$$

We note that the probabilities given by Equations (11a to 11d) will all be nonnegative if the time step b is sufficiently small. We can ensure that this is the case by increasing N , the number of time steps. As N is increased, the probabilities tend to either $(1/4)(1 + \rho)$ or $(1/4)(1 - \rho)$. Both of these limits are nonnegative.

We now present the corresponding discrete approximation for the n -dimensional multivariate lognormal distribution. It turns out that we also are able to obtain exact expressions for the probabilities, p in this case.

The notation corresponds to that introduced earlier in the section except that we have n assets so that i runs from 1 to n . In analogy with the two-dimensional case we assume that after one time step h each asset can take either an up-jump or a down-jump. This means that there will be 2^n possible values for the n assets after time h . We define u_i and d_i as in Equations (4) and (5). As before, we use the characteristic functions of the discrete distribution and its continuous counterpart to ensure convergence of the two distributions. The development parallels that of the two-dimensional case. After some algebra we obtain a set of linear equations for the jump probabilities.

Corresponding to Equation (10a), we obtain

$$\sum_{j=1}^M p_j = 1 \quad (12)$$

where $M = 2^n$ is the number of distinct states after one time period h .

Corresponding to Equation (10b), we now have a set of $((n^2 - n)/2)$ equations, one for each ρ_{ij} .

$$\sum_{j=1}^M \delta_{km}(j) p_j = \rho_{km} \quad 1 \leq k < m \leq n \quad (13)$$

where $\delta_{km}(j) = 1$ if both asset k and asset m have jumps in the same direction in state j
 $= -1$ if both asset k and asset m have jumps in opposite directions in state j

There are now n equations instead of the two equations (10c) and (10d).

$$\sum_{j=1}^M \delta_k(j) = \sqrt{h} \frac{\mu_k}{\sigma_k} \quad 1 \leq k \leq n \quad (14)$$

where $\delta_k(j) = 1$ (-1) if asset k has an up-jump (down-jump) in state j .

In total, Equations (12), (13), and (14) constitute a set of $((n^2 + n + 2)/2)$ equations for the 2^n unknown p 's. For $n = 1$ and $n = 2$ the number of equations is exactly equal to the number of unknowns. For $n \geq 3$ the number of unknowns exceeds the number of equations so that there is theoretically an infinite number of solutions. However, we can write down an exact solution that is suggested by the symmetry of the expressions obtained in the two-dimensional case.⁴ This solution is

$$p_j = \frac{1}{M} \left(\sum_{k, m=1}^n \delta_{km}(j) \rho_{km} + \sqrt{h} \sum_{k=1}^n \delta_k(j) \frac{\mu_k}{\sigma_k} \right) \quad j = 1, \dots, M \quad (15)$$

⁴ It is helpful to work out the details for the three-dimensional case to see the emerging pattern.

It can be verified by direct substitution that the probabilities given by Equation (15) do indeed satisfy Equations (12), (13), and (14). Our solution technique does not guarantee that these probabilities will be all positive, so this must be checked in each application.

3. Numerical Examples

The numerical procedure is very similar to the CRR algorithm. First, a lattice of future asset prices is generated at each time step until the boundary is attained. In the case of an option whose payoff depends on the values of n assets, we know its value at expiration in terms of the prices of these n assets. The option value one period earlier can be computed as in the CRR framework by discounting the expected option value at the riskless rate. Dividend payments and early exercise can be readily accommodated at each lattice node. The current option value can be obtained by working recursively backward through the lattice.

In this section we present some numerical examples to illustrate the method. We compute the values of European options when there are three underlying assets. For most of our examples accurate option prices can be obtained using numerical integration, and this permits us to examine the speed of convergence. In these examples, we assume

$$\begin{aligned} S_i &= 100 & 1 \leq i \leq 3 \\ \sigma_i &= 0.2 & 1 \leq i \leq 3 \\ \rho_{im} &= 0.5 & 1 \leq i < m \leq 3 \\ r &= 10 - \text{percent per annum} \\ T &= 1 \text{ year} \end{aligned}$$

$$\text{Exercise price} = 100$$

Table 2 shows the results for a range of European options. We evaluate options on the maximum, the minimum, the geometric average, and the arithmetic average of the three underlying assets. Note that as the number of time steps increases the computed option values tend toward the accurate figures. The accurate figures for the options on the maximum and the minimum were obtained by numerical integration of formulas in the literature. For the case of the options on the geometric average an exact closed-form solution can be derived.

As a practical method of approximating the exact values closely without a lot of unnecessary computing, we suggest using Richardson⁵ extrapolation. For example, by solving the price for $n = 20, 40, 60$, and 80 , as in Table 2, we can fit option values as a cubic function of $1/n$. By extrapolating this function, we obtain an approximation to the value corresponding to $n = \infty$. Because computational complexity grows as n^3 , solving the problem for the four values of n suggested above is equivalent to solving a single

⁵ This procedure is described in Dahlquist and Bjorck (1974).

Table 2
European call-and-put option prices on the maximum, minimum, geometric average and arithmetic average of three assets

No. of steps	Maximum	Minimum	Geom. average	Arith. average
European call				
20	22.281	5.226	11.548	12.060
40	22.479	5.237	11.564	12.072
60	22.544	5.241	11.570	12.076
80	22.576	5.243	11.573	12.078
RE ¹	26.673	5.249	11.583	12.084
Accurate value	22.672	5.249	11.581	
European put				
20	0.919	7.240	2.715	2.566
40	0.925	7.323	2.722	2.567
60	0.928	7.350	2.725	2.567
80	0.929	7.364	2.726	2.567
RE	0.936	7.403	2.732	2.566
Accurate value	0.936	7.403	2.729	

Values in this table based on $S_i = 100$, $Y_i = 100$, $r = 0.1$, $\sigma_i = 0.2$, $\sigma_y = 0.2$, $T = 1$ where $1 \leq i < j \leq 3$.

¹ RE denotes Richardson extrapolated value obtained from the figures for $n = 20$, $n = 40$, $n = 60$, and $n = 80$.

problem for $n = 93$. We note that the value obtained by using our approach in conjunction with the Richardson extrapolation produces option prices that agree with the accurate prices to within one penny.

4. Summary

With the increased sophistication of financial instruments, applications involving options on several assets have increased. Such applications include certain types of currency-linked and commodity-linked bonds, the delivery options under some futures contracts, and dynamic asset allocation strategies [see Smith (1988)]. This article provides a method for the valuation of options when there are several underlying assets. The method extends the lattice binomial approach of Cox, Ross, and Rubinstein, which has been so useful in the one-dimensional case. A feature of the method is that it can readily handle the early exercise feature of American options. We illustrate the accuracy of the method in the three-asset case.

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