**Homework #3**

**AMS 507, Fall 2025** **Due September 18**

1. Let the three independent events $A_1,\ A_2$ and $A_3$ be such that $P(A_1) = P(A_2) = P(A_3) = 1/4$. Find
$$P\left[(A_1^c \cap A_2^c) \cup A_3\right].$$

2. Prove that if $P(B|A) = P(B|A^c)$, then $A$ and $B$ are independent events, and give an intuitive statement as to why this result seems reasonable.

3. Bowl 1 contains 3 red chips and 7 blue chips. Bowl 2 contains 6 red chips and 4 blue chips. A single chip is drawn from a randomly selected bowl.
   (a) What is the probability that this chip is red?
   (b) Given that the chip is red, what is the probability that it came from bowl 2?
   (c) Let $A = \{$the chip is red $\}$ and $B = \{$the chip came from bowl 2$\}$. Are they independent?

4. For each of the following, find the constant $c$ so that $p(x)$ satisfies the condition of being a pmf of $X$.
   (a)
   $$p(x) = c\left(\frac{2}{3}\right)^x,\ x = 1, 2, \cdots$$
   (b)
   $$p(x) = cx,\ x = 1, 2, 3, 4, 5, 6$$

5. Let $X$ have the pmf $p(x) = x/5050$, $x = 1, 2, \cdots, 100$.
   (a) Find $P(X \leq 50)$.
   (b) Show that the cdf of $X$ is $F(x) = [x]([x] + 1)/10100$ for $1 \leq x \leq 100$, where $[x]$ is the greatest integer which is less than or equal to $x$.

6. Let $p(x) = 1/2^x$, $x = 1, 2, \cdots$ be the pmf of $X$. Let $Y = X^3$.
   (a) Find the pmf of $Y$.
   (b) Find the cdf of $Y$.

7. Two distinct integers are chosen at random and without replacement from the set of integers $\{1, 2, 3, 4, 5, 6\}$. Let $X$ be the absolute value of the difference of these two numbers. Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

8. Suppose $X$ and $Y$ are independent random variables such that $\mathrm{E}(X) = 5$, $\mathrm{Var}(X) = 9$, $\mathrm{E}(Y) = 3$ and $\mathrm{Var}(Y) = 25$. Find $\mathrm{E}(U)$ and $\mathrm{Var}(U)$ where $U = 2X - Y + 1$.

9. Let $X$ be a discrete random variable whose range is the nonnegative integers. Show that

$$\mathrm{E}(X) = \sum_{k=0}^{\infty} [1 - F(k)]$$

10. Suppose that $p(x) = 1/5$, $x = 1, 2, 3, 4, 5$ is the pmf of $X$. Find

    (a) $\mathrm{E}(X)$

    (b) $\mathrm{Var}(X)$

    (c) $\mathrm{E}\left[(X+2)^2\right]$