AMS 507, Fall 2025 **Hongshik Ahn**

Sample Exam I

1. Consider a random variable X whose cdf is given below.

$$F(x) = \begin{cases} 0 & \text{if } x < -2 \\ 1/8 & \text{if } -2 \le x < -1 \\ 3/8 & \text{if } -1 \le x < 0 \\ 5/8 & \text{if } 0 \le x < 1 \\ 7/8 & \text{if } 1 \le x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$$

   (a) Find the pmf of X.
   (b) Find EX.
   (c) Find the standard deviation of X.
   (d) Find the 70th percentile.
   (e) Find $P(-1 \le X \le 1)$.
   (f) Compute $P\left(X^2 - 3 \ge 1\right)$.

2. Seventy percent of clay pots are produced by machine 1 and thirty percent by machine 2. Among all the pots produced by machine 1, 4% are defective, and of those produced by machine 2, 8% are defective.

   (a) What percentage of the total production of pots is defective?
   (b) If a pot is found to be defective, what is the probability that it was produced by machine 2?

3. Two of the cards of an ordinary deck of 52 cards are lost. What is the probability that a random card drawn from this defective deck is a diamond?

4. Let P_n denote the probability that n independent Bernoulli trials with success rate p result in an even number of successes (0 being considered as an even number).

   (a) Show that
$$P_n = p(1 - P_{n-1}) + (1 - p)P_{n-1}, \qquad n \ge 1.$$
   (b) Use the result in part (a) to prove that
$$P_n = \frac{1 + (1 - 2p)^n}{2}.$$

5. Let X be a random variable with pmf

$$p(x) = \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \cdots, n,$$

   where $0 < p < 1$. Prove the following equalities.

   (a) $\mathrm{E}(X) = np$
   (b) $\mathrm{Var}(X) = np(1 - p)$

6. A professor has made 30 exams of which 8 are hard, 12 are moderate, and 10 are easy. The exams are mixed up and the professor selects four of them at random to give to four sections of the course he is teaching

   (a) What is the probability that no sections receive a hard test?

   (b) What is the probability that exactly one section receives a hard test?

   (c) Let X be the number of sections that receive a hard test. Compute $\mathrm{E}(X)$.

7. Independent trials, consisting of rolling a pair of fair dice, are performed. What is the probability that an outcome of 5 appears before an outcome of 7, when the outcome of a roll is the sum of the dice?

8. For two events A and B, $P(A) = 4/5$, $P(B|A) = 1/2$, and A and B are not independent. For each statement, determine if it is true, false, or there is not enough information to answer the question.

   (a) A and B are mutually exclusive

   (b) $P(A) = P(A|B)$

   (c) $P(A \cap B) > P(A)P(B)$

   (d) $P(A) \geq P(B)$