AMS 572: Data Analysis I
Fall 2025
Homework 1: Due 09/21/2025 at 11:59 PM
Total Points: 50

1. Prove that the sample mean is an unbiased estimator of the population mean. How to interpret the unbiasedness when we have an estimate of the population mean μ=4 from a set of random samples {2, 4, 6, 6, 4, 2}?
2. In a Bernoulli trial with unknown probability of success p, in the process of estimating $\eta = \frac{1}{p}$, prove that there is no unbiased estimator (for η) based on the sample mean of Bernoulli observations.
3. If $X \sim U(0, 1)$ and $Y = X^n$, $n > 0$, find the distribution of Y using

   (a) the c.d.f. method.

   (b) the Jacobian method.
4. Let X1, . . . Xn be a random sample from $U(0, \theta)$. Find the MLE and MME of θ.
5. Let $X \sim$ Poisson(λ). Compute P(X is even). (Hint: $e^y = 1 + \frac{y}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \ldots\ldots$,

   $e^{-y} = 1 - \frac{y}{1!} + \frac{y^2}{2!} - \frac{y^3}{3!} + \ldots\ldots$, use these sum of the two series to simplify the summation in this question. Even numbers include zero.)