Homework #2

AMS 507, Fall 2025 **Due September 11**

1. Two pennies, one with $P(\text{head}) = u$ and another one with $P(\text{head}) = w$, are to be tossed together independently. Define

$$p_0 = P(0 \text{ head occurs}), \quad p_1 = P(1 \text{ head occurs}), \quad p_2 = P(2 \text{ heads occur}).$$

   Can u and w be chosen such that $p_0 = p_1 = p_2$? Prove your answer.

2. If A and B are subsets of the sample space, show that

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B).$$

3. A bowl contains 16 chips, of which 6 are white, 7 are red, and 3 are blue. If four chips are taken at random and without replacement, find the probability that

   (a) each of the 4 chips is white.

   (b) none of the 4 chips is white.

   (c) there is at least 1 chip of each color.

4. Prove each of the following statements. Assume that any conditioning event has positive probability.

   (a) If $P(B) = 1$, then $P(A|B) = P(A) \quad \forall A$.

   (b) If $A \subset B$, then $P(B|A) = 1$ and $P(A|B) = P(A)/P(B)$.

   (c) If A and B are disjoint, then

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}.$$

   (d) $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$

5. If $P(B) > 0$ and $\{A_n\}$ is a sequence of mutually exclusive sets, show that

$$P\left(\bigcup_{n=1}^{\infty} A_n \middle| B\right) = \sum_{n=1}^{\infty} P(A_n|B).$$

6. Bowl A contains 6 black chips and 4 white chips. Five of these 10 chips are selected at random and without replacement and put in bowl B, which was originally empty. One chip is then drawn at random from bowl B. Relative to the hypothesis that the chip is white, find the conditional probability that 2 black chips and 3 white chips are transferred from bowl A to bowl B.

7. Urn I contains 3 white and 5 red balls, whereas urn II contains 2 white and 1 red ball. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II.

   (a) What is the probability that the ball selected from urn II is white?

   (b) What is the conditional probability that the transferred ball was white, given that a white ball is selected from urn II?