Abstract

We have developed a Monte-Carlo methodology to forecast the crude oil production of Norway and the U.K. based on the current/past performances of individual oil fields. By extrapolating the future production of these fields and the frequency of new discoveries, we are able to forecast the oil production of these countries. Our results indicate that standard methodologies tend to underestimate remaining oil reserves. We compare our model to those methodologies by making predictions from various points in time the past. It shows that our model gives a better description (hopefully soon) of the evolution of the oil production.

I. Introduction

Forecasting future oil production has been a topic of active interest since the beginning of the past century because of oil's central role in our economy. Its importance ranges from energy production, through manufacturing to pharmaceuticals industry. Petroleum is a non-renewable, finite resource. It is primordial to be able to forecast future oil production since a misestimation of its reserves can have huge consequences on our society.

The methodology behind forecasting future oil production has not evolved much since M. King Hubbert who, in 1956, famously predicted that the U.S. oil production would peak around 1965-1970. That prediction has proven itself to be correct. His main argument was based on the finiteness of oil reserves and use of the logistic differential equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{1}$$

to model oil production. The logistic differential equation is characterized by an initial exponential growth, which then decreases to zero as the total oil extracted reaches saturation (no more oil is to be found). The parameter r is commonly referred to as the growth rate, and K as the carrying capacity (total quantity of oil extracted). If $P(t)$ is the the amount of oil extracted up to time t, then $\frac{dP}{dt}$ is the oil production rate, the quantity that M. King Hubbert predicted with surprising accuracy to peak. From a methodological point of view, the Hubbert model has enjoyed a longstanding popularity in modeling future oil production given its simplicity.

In this paper, we introduce a new methodology to forecast future oil production. Instead of taking the aggregate oil production profile and fitting it with the Hubbert curve or its variants (such as the multi cyclic Hubbert curve), we use the production profile of each individual oil field. By extending their production into the future and extrapolating the future rate of discovery of new fields, we are able to forecast future oil production by the means of a Monte Carlo simulation. To demonstrate the universality of the methodology presented here, we apply it to 2 major oil producing countries with publically available data: Norway and the U.K.

II. Methodology

The idea behind the methodology is to model the future aggregate oil production of a country by studying the production dynamics of its individual constituents, the oil fields. The main benefit of this approach, compared to working directly with aggregate production data, is the possibility to forecast non-trivial oil production profiles arising from the combination of all the individual fields' dynamics. In order to achieve that, one must be able 1) to **extend the oil production of each individual field** into the future and 2) to **extrapolate the rate of discoveries** of new oil fields.

II.1 Extending the oil production of individual fields

The first step to predicting the future oil production of a country is to extrapolate the future production of existing fields and to estimate the error on this extrapolation.

II.1.1 Regular, irregular and new fields

To be able to forecast the oil production of each individual field, regularity needs to be found in the production's dynamics. Modeling the whole production profile from the beginning of extraction seems elusive due to the variety of the forms it can take. Fortunately, modeling the decay process is sufficient in order to extrapolate future oil production. A preliminary classification is necessary to achieve that goal. Figure 1 shows that independent of the country, oil fields can be classified into 3 main categories:

- **Regular fields** - Their decay show some regularity.
- **Irregular fields** - The ones that don't decay in a regular fashion.
- **New fields** - The ones that don't decay yet. As such, there is no easy way to forecast their future oil production based on past data.

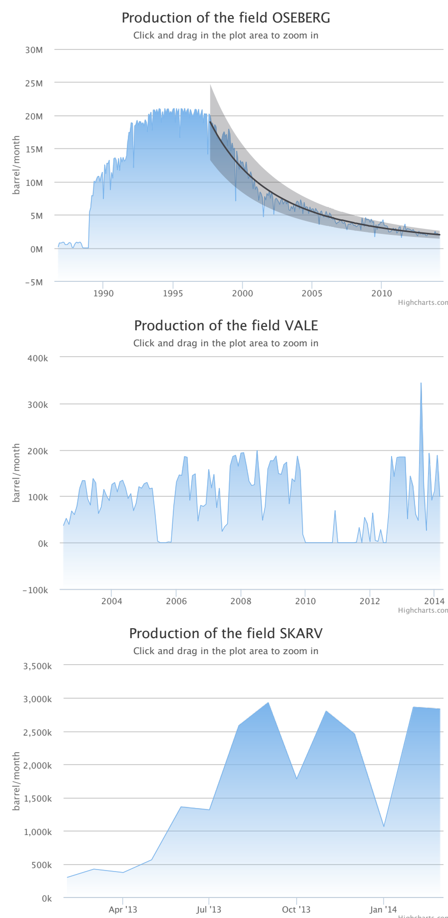

Figure 1: *Example of a regular, irregular and new field.*

All the fields have been fitted using an automated algorithm, afterwards the results have been checked visually to sort out the irregular fields which could not be fitterd. As of January 2013, regular fields make up 85% and 87% of the number of fields and 94% and 71% of the total produced oil volume in the Norway and the U.K. respectively. As such, being able to model them is crucial. As can be seen on figure 1, the stretched exponential (equation 2) is a good functional form to fit the decay process of regular fields.

$$P(t) = P_0 e^{\left(\frac{t}{\tau}\right)^{\beta}} \qquad (2)$$

For the minority of irregular fields, we simply modeled their decay as follows. We picked τ to be the average τ over the regular fields. We then fixed β so that the sum of the fields' production over its lifetime be equal to the official ultimate recovery estimates, when such an estimate is available.

The minority of new fields will be treated as a new discovery, as their final size can simply not be predicted based on the existing production data.

II.1.2 Back-testing & Error

To determine how well the extrapolation based on the stretched exponential predicts the future production, a complete back-testing has been performed on each field. A single back-test is made as follows:

- The production data $\{p_0, \ldots, p_N\}$ of the field is truncated at a certain date in the past $t \in \{0, \ldots, N\}$.
- The extrapolation is made based on the truncated data $\{p_0, \ldots, p_{t_0}\}$.
- The future production predicted by the extrapolation $f(t)$ can be compared to the actual production from the date in the past up to now $t_f = N$. This allows us to compute the resulting error in the extrapolated total production

$$P_e(t) = \int_t^{t_f} f(\tau) d\tau \qquad (3)$$

and the error

$$e(t) = \frac{P_e(t) - \sum_{i=t}^{t_f} p_i}{P_e(t)} \qquad (4)$$

as a function of the truncation time t.

Repeating such a back test for every possible date in the past, allows us to plot the evolution of the error over time.

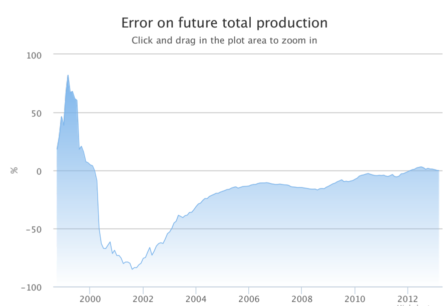

Figure 2: *Oesberg field - Error on predicted total production until 2014*

From the complete back-test, on can compute the average error $\bar{e}$ the extrapolation made on the future production and the standard deviation σ_e from the average error. As the average error is often fairly constant, the extrapolation was corrected by the average error. Meaning that if the extrapolation consistently over-estimated the production by 10% during the back-test, the extrapolation was reduced by 10%. This results in an extrapolated production $p(t)$, including a 1σ confidence interval, as

$$p(t) = (1 - \bar{e})\, f(t) \pm \sigma_e. \tag{5}$$

An example of such an extrapolation including a one standard deviation range is shown on figure 1 for the field Oesberg.

II.1.3 Aggregate error

Once we have extrapolated all the individual fields, the aggregated extrapolation for the country has to be computed. While this is straightforward for the expected production, some care has to be taken with respect to the error on the production at the country level. As shown later, the same extrapolation including a complete back-test has been performed at the country level and the resulting error is much smaller than the average error observed on an individual field. To account for this observation, the necessary assumption is that the error between the individual fields is largely uncorrelated. Therefore, the fields can be considered as a portfolio of stocks with an underlying trend but an overlayed random walk. This means that the error at the country level from the extrapolated production is to be computed as

$$\sigma^2_{country} = \frac{1}{\#fields} \sum_{field \in fields} \sigma^2_{field}. \tag{6}$$

As example, a country with 100 fields having each an error of $\pm 50\%$, will have an expected error of only $\pm 5\%$. Intuitively, this results from the fact that the uncorrelated errors among fields cancel out on average.

II.2 Discovery rate of new fields

Knowing the future production rate of existing fields is not enough as new fields will be discovered in the future. The model describing the discovery rate of new fields should satisfy two fundamental observations.

1. The rate of new discoveries should tend to zero as time goes to infinity. This is a consequence of the finiteness of the number of oil fields.

2. The rate of new discoveries should depend on the size of the oil fields. As of today, giant oil fields are discovered much less frequently than dwarfs.

II.2.1 Discoveries modeled as logistic growth

A natural choice for such a model is a non-homogenous poisson process. The Poisson process is a process that generates independent events at a rate λ. It is inhomogeneous if the rate is time-dependent, $\lambda \to \lambda(t)$. The standard way to measure $\lambda(t)$ is to find a functional form for $N(t)$, the number of events (discoveries) up to time t. Then, $\lambda(t)$ is simply $\frac{dN(t)}{dt}$. Figure 3 shows $N(t)$ for Norwegian fields classified according to their size. The logistic curve is a good fit to the data (integral form of equation 1). This implies that after an initial increase, the rate of new discoveries reaches a peak followed by a decrease until no more oil fields are to be found, consistent with our fundamental observations.

The larger an oil field is, the easier it should be discovered and the more profitable its production will be. Therefore, it is expectable that the large oil fields have all been found, and that future discoveries will mostly be made up of dwarf fields. For the purpose of this model, the fields have been split into two groups:

- **Dwarfs**: Fields which are smaller than $1/10^{th}$ of the largest existing oil field.
- **Giants**: Fields which are larger than $1/10^{th}$ of the largest existing oil field.

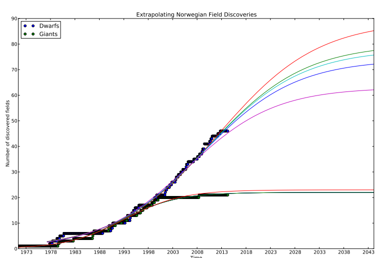

Figure 3: *Logistic fit to the number of discoveries for Norway. The discovery rate of new oil fields is dependent on their size. The logistic model suggests that a large number of dwarf oil fields has not yet been discovered, while the rate of discovery of giant oil fields is almost zero.*

The resulting plot shown in figure 3 confirms our hypothesis. Giant oil fields are mostly all found while the discovery process for dwarf fields is still ongoing. The logistic growth curve fit to the giant discoveries is stable, however the fit for the dwarfs is unstable. As can be seen on figure 3, the carrying capacity K of the logistic growth model is not well constrained by the available data. A large spectrum of values for K leads to a good fit to the data.

II.2.2 Likelihood function for the number of discoveries

To overcome the logistic fitting issue for the dwarf fields, a method already used by Smith 1980 has been implemented. This method makes the following two postulates:

1. "The discovery of reservoirs in a petroleum play can be modeled statistically as sampling without replacement from the underlying population of reservoirs."

2. "The discovery of a particular reservoir from among the existing population is random, with a probability of discovery being dependent on reservoir size."

The fields are splitted into J size bins denoted $S_1, \ldots, S_J$ occuring with frequency $n_1, \ldots, n_J$. Each discovery is considered as a step i at which a field of size $I(i) \in \{S_1, \ldots, S_J\}$ is found and we will denote by m_{ij} the number of fields of size $j \in \{1, \ldots, J\}$ discovered before the i^{th} step. Than the probability that the discovery at step i^{th} is of size j can be expressed as

$$P\left(I(i)=S_j\right)=\frac{\left(n_j-m_{ij}\right)\cdot S_j}{\sum_{k=1}^{J}\left(n_k-m_{ik}\right)\cdot S_k}. \tag{7}$$

The likelihood L for a complete sequence of N discoveries $\{I(1),\ldots,I(N)\}$ can than be expressed as

$$L=\prod_{i=1}^{N}\frac{\left(n_{I(i)}-m_{iI(i)}\right)\cdot S_{I(i)}}{\sum_{j=1}^{J}\left(n_j-m_{ij}\right)\cdot S_j}. \tag{8}$$

The unknown parameters are the number of fields $n_1,\ldots,n_J$, whose likelihood can now be estimated based on the existing discoveries. Using a brute force approach, the entire space of plausible values for the variables $n_1,\ldots,n_J$ has been sampled. The values n_j have been sampled between the number of existing fields m_{Nj} in the bin j and up to a value n_j^{upper} such that the most likely scenario $n_j^{max}=n_j^{upper}/2$. Subsequently, the likelihood of each scenario (value of the tuple $n_1,\ldots,n_J$) has been normalized such that the total likelihood of all generated scenarios equals one.

For the analysis of the north sea oil field discoveries, the number of size bins has been fixed to $J=2$ splitting between dwarfs and giants as described section II.2.1. The results are coherent with the intuitive expectation that discovering a new giant field is unlikely and that the future discoveries of dwarf fields can only be constrainted to a range as presented on figure 3.

II.2.3 Future production from discoveries

The goal is to compute an expected oil production coming from future discoveries, which requires to combine the steps described in sections II.2.1 and II.2.2.

The method described in section II.2.2 yields probabilities for the total number of fields (including the not yet discovered fields) in each size bins (called a scenario). However, this likelihood method does not give the time distribution of future discoveries. The aim is to use the likelihood function to generate scenarios with their respective occurence probability.

For a given scenario, the carrying capactiy K (= total number of fields) is given for each size class. This is useful, because it resolves the instability in fitting the logistic curve to the number of discovered fields. The time distribution of the discoveries is then given for each size class by the fitted logistic curve.

The actual size of a newly discovered field is generated according to the size distribution of the existing fields in its size class. The probability distribution function of field sizes in a given size bin has been fitted by an exponential function.

The production curve is computed based on the average production curve of all existing fields in the same size category. The production curves of the existing fields have all been normalized to a total production of one and then been averaged. This yields an average production curve including a one standard deviation error. For a new field, this average production curve is than multiplied by the size of the field.

Superposing the production curves results in the expected production curve from future oil field for a given scenario.

As the total parameter space is too large to be sampled entirely, a Monte Carlo technique is applied to compute the expected production from future discoveries and standard deviation. In a nutshell, the algorithm works as follows:

1. Draw a scenario based on its probability
2. Compute the time distribution of new discoveries by fitting a logistic curve for each size class.
3. For each discovery generate a size and the resulting production curve based on the size distribution and production curves of existing fields.
4. Superpose all the production curves.
5. Repeat and average over all drawn scenarios.

The result is the expected production curve of future oil field discoveries. The distribution between all the generated scenarios yields the confidence interval.

III. Results

Based on the methodology described in section II.2, simulating future oil production was straightforward. For each country, the existing fields were extrapolated and the future discoveries were simulated. Figure 4 shows the average of 1000 simulations. For each country, we can immediately notice the non-symmetric shape of the production's dynamics contrary to what Hubbert would have predicted.

Table 1: *Remaining oil reserves*

Country	Hubbert	our model	Δ
Norway	$81 \cdot 10^6$	$270 \cdot 10^6$	-70%
U.K.	$275 \cdot 10^6$	$959 \cdot 10^6$	-71%

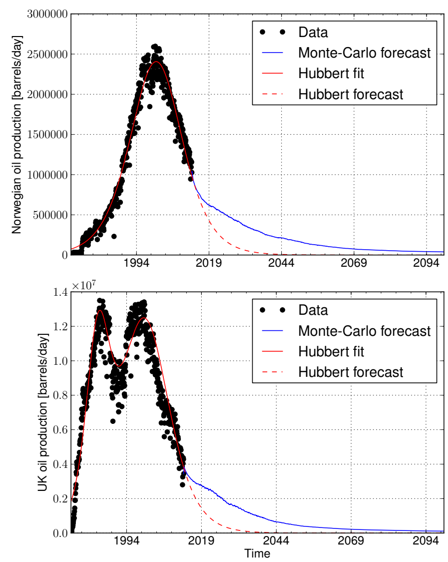

Figure 4: *Monte-Carlo and Hubbert forecast based on past production data for Norway (top) and the U.K. (bottom). In both cases, our model forecasts a significantly slower decay than the Hubbert model.*

The difference between our and the Hubbert-based forecast is striking. In the case, of Norway, the standard Hubbert model was used to fit the production data. According to it, Norway's fu-

ture oil production would decay much faster than in the Monte-Carlo case. As such, the estimated remaining reserves are less than one third of the forecasted value of the Monte-Carlo methodology. In the U.K., oil production faced a change of regime during the early nineties due to technological innovation, giving rise to the inverted "w shape" of the oil production profile. The standard procedure to model oil production in those cases is to use a multi-cyclic Hubbert curve. The multi-cyclic Hubbert model is a generalization of the standard one. Conceptually, it is just a superposition of several standard (single-cyclic) Hubbert curves. Two cycles are commonly used to fit the UK's oil production. The difference between the Hubbert-based methodology and the Monte-Carlo one is very similar to the Norwegian case. The former underestimates the remaining oil reserves by about 70% compared to the latter.
Which of the two models is more thrust worthy? Clearly, the implications in adopting one methodology over the other are significant. The only way to answer this question is to backtest them. In other words we ask: "What would have each of the models predicted had we used them 10, 15, 20 years in the past?"

IV. Validation

For each of the three countries, namely Norway, the U.K. and the U.S.A., we went back in time 10, 15 and 20 years respectively. The reason for chosing an earlier starting point for our backtest in the case of the U.S.A. compared to say, Norway has to do with the number of oil fields available at the time of the backtest. The U.S.A. having the biggest number of oil fields, we could chose an earlier starting point without compromising the quality of our results. Figure 5 shows the outcome of those tests. In the case of Norway, the production data was taken into account until 2003. Comparing the forecast of both models with the oil production of the subsequent 10 years, we find the predictive power of the Hubbert model is impressive. It perfectlty captures the Norwegian oil production's dynamic between 2003 and 2013. The Monte-Carlo method does not perform as well. After underestimating the Norwegian oil production between 2003 and 2009, it overestimates it between 2009 and 2013. For the U.K. the backtesting was performed on data until 1998. The conclusion is not as obvious as for the previous case. None of the two methods captures perfectly the oil production between 1998 and 2013. The following table summarizes the difference between the two models.

Table 2: *Remaining oil reserves*

Start	Country	Hubbert	our model	Δ
2003	Norway	$66 \cdot 10^6$	$251 \cdot 10^6$	-74%
1998	U.K.	$91 \cdot 10^6$	$2.1 \cdot 10^9$	-43%

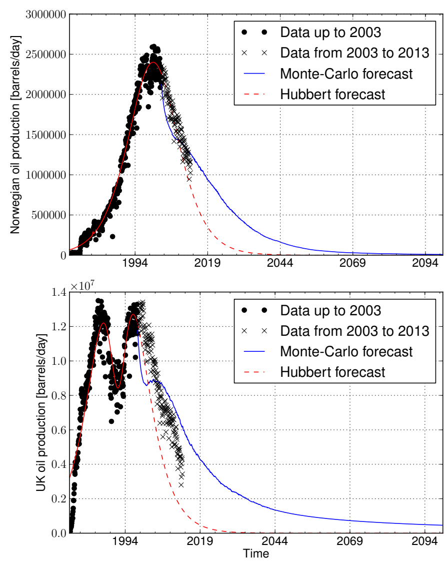

Figure 5: *Monte-Carlo and Hubbert forecast based on past production data up to 2003 for Norway (top) and up to 1998 for the U.K. (bottom). The results can be compared with the subsequent oil production (crosses). In both cases, the Hubbert forecast is more accurate.*

V. Conclusion

We have presented a Monte-Carlo based methodology to forecast future oil production. By extending the oil production of current fields into the future and modeling the discovery rate of new fields, we were able to give forecasts on the future oil production of Norway, the U.K. and the U.S. These forecasts offer significantly scenarios from the ones obtained with standard Hubbert-based methodologies. Indeed, our model gives a 3 times superior estimation of the remaining oil reserves than the standard one. However, our backtesting results seriously questions the credibility of that forecast, since the Hubbert-model clearly outperforms the Monte-Carlo simulation.

References

[1] J. L. Smith A Probablilistic Model of Oil Discovery, The Review of Economics and Statistics, Vol. 62, No. 4 (Nov. 1980), pp. 587-594