Intelligent Minority Game with genetic-crossover strategies

Marko Sysi-Aho,* Anirban Chakraborti,† and Kimmo Kaski‡
Laboratory of Computational Engineering,
Helsinki University of Technology,
P. O. Box 9203, FIN-02015 HUT, Finland.

We develop a game theoretical model of N heterogeneous interacting agents called the intelligent minority game. The "intelligent" agents play the basic minority game and depending on their performances, generate new strategies using the one-point genetic crossover mechanism. The performances change dramatically and the game moves rapidly to an efficient state (fluctuations in the number of agents performing a particular action, characterized by σ^2, reaches a low value). There is no "phase transition" when we vary σ^2/N with $2^M/N$, where M is the "memory" of an agent.

The dynamics of interacting agents competing for scarce resources are believed to underlie the behaviour of complex systems in natural [1, 2, 3] and social [4, 5] sciences. The agents have to be the best in order to survive–similar to the idea of "survival of the fittest" in biology. In studies of market behaviour, tools of statistical physics have been combined with theories of economics [6, 7, 8, 9], like game theory, which deals with decision making of a number of rational opponents under conditions of conflict and competition [10, 11, 12, 13, 14, 15].

In this letter, we present a game theoretical model of a large number of heterogeneous interacting agents called the intelligent minority game, based on the minority game [11]. This provides an alternative to the representative approach of microeconomics, where one has a theory with a single (representative) agent, based on the assumption that all the agents are identical [16]. The minority game model consists of agents having a finite number of strategies and finite amount of public information, interacting through a global quantity (whose value is fixed by all the agents) representing a market mechanism. In the original model the agents choose their strategy through a simple adaptive dynamics based on *inductive reasoning* [5]. Here, we introduce the fact that the agents are *intelligent* and in order to be the best or survive in the market, modify their strategies periodically depending on their performances. For modifying the strategies, we choose the mechanism of *one-point genetic crossover*, following the ideas of genetic algorithms in computer science and operations research. In fact, these algorithms were inspired by the processes observed in natural evolution [17, 18, 19] and it turned out that they solve some extremely complicated problems without knowledge of the decoded world. In nature, one-point crossover occurs when two parents exchange parts of their corresponding chromosomes after a selected point, creating offsprings [19].

The basic minority game consists of an odd number of agents N who can perform only two actions, at a given time t, and an agent wins the game if it is one of the members of the minority group. The two actions, such as "buying" or "selling" commodities, are denoted here by 0 or 1. Further, it is assumed that all the agents have access to finite amount of public information, which is a common bit-string "memory" of the M most recent outcomes. Thus the agents are said to exhibit "bounded rationality" [5]. For example, in case of memory $M = 2$ there are $P = 2^M = 4$ possible "history" bit strings: 00, 01, 10 and 11. A "strategy" consists of a response, i.e., 0 or 1, to each possible history bit strings; therefore, there are $G = 2^P = 2^{2^M} = 16$ possible strategies which constitute the "total strategy space". In our study, we use the reduced strategy space by picking only the uncorrelated strategies (which have Hamming distance $d_H = 1/2$) [20]. At the beginning of the game, each agent randomly picks k strategies, and after a game, assigns one "virtual" point to the strategies which would have predicted the correct outcome; the best strategy is the one which has the highest virtual point. The performance of the player is measured by the number of times the player wins, and the strategy, which the player uses to win, gets a "real" point. We also keep a record of the number of agents who have chosen a particular action, say, "selling" denoted by 1, $N_1(t)$ as a function of time. The fluctuations in the behaviour of $N_1(t)$ indicate the total utility of the system. For example, we may have a situation where only one player is in the minority and thus wins, and all the other players lose. The other extreme case is when $(N-1)/2$ players are in the minority and $(N+1)/2$ players lose. The total utility of the system is highest for the latter case as the total number of the agents who win is maximum. Therefore, the system is more efficient when there are smaller fluctuations around the mean than when the fluctuations are larger. The fluctuations can be characterized by the variance σ^2 so that smaller values of σ^2 would correspond to a more efficient state.

In our model, the players of the basic minority game are assumed to be intelligent and modify their strategies after every time-interval τ depending on their performances. If they find that they are among the fraction n (where $0 < n < 1$) of the worst performing players, they modify any two of their strategies chosen randomly from the pool of k strategies and use one of the new strategies generated. The mechanism by which they modify

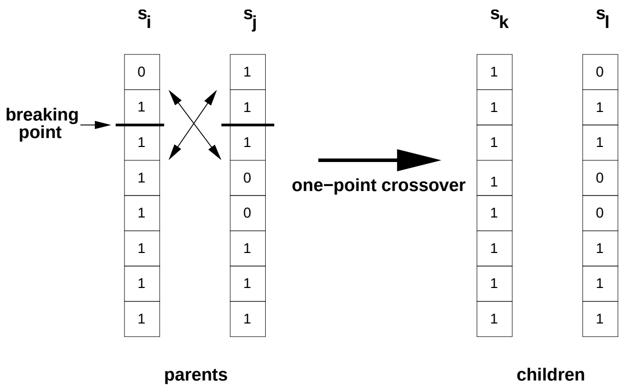

Figure 1: Schematic diagram to show the mechanism of one-point genetic crossover to produce new strategies. The strategies s_i and s_j are the parents. We choose the breaking point randomly and through this one-point genetic crossover, the children s_k and s_l are produced.

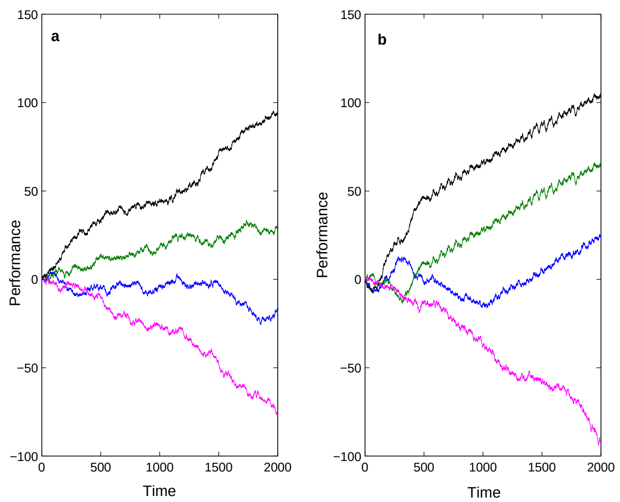

Figure 2: Plots of the performances of the best player (black), the worst player (magenta) and two randomly selected players (green and blue) in (a) the basic minority game, where $N = 1001$, $M = 5$, $k = 10$ and $t = 1999$, and (b) in the intelligent minority game, where $N = 1001$, $M = 5$, $k = 10$, $t = 1999$, $n = 0.3$ and $\tau = 100$.

their strategies is that of one-point genetic crossover illustrated schematically in Figure 1. The strategies s_i and s_j act as the parents and by choosing the breaking point in them randomly, and performing one-point genetic crossover, the children s_k and s_l are produced. We should note that the strategies are changed by the agents themselves and even though the strategy space evolves, it is still of the same size and dimension; thus considerably different from earlier attempts [11, 21, 22].

In Figure 2, the performances of the players in our model are compared with those in the basic minority game. We have scaled the performances of all the players such that the mean is zero for easy comparison of the success of the agents in each case. We find that there are

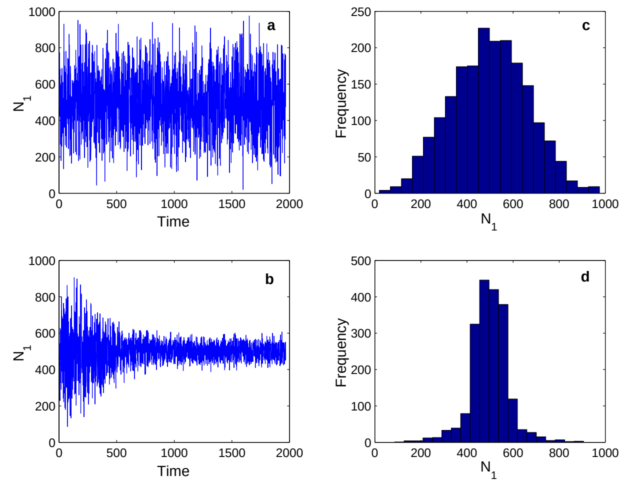

Figure 3: Plots of the (a) time-variation of N_1 for the basic minority game (b) time-variation of N_1 for the intelligent minority game (c) histogram of N_1 for the basic minority game and (d) histogram of N_1 for the intelligent minority game. The simulations for the basic minority game have been made with $N = 1001$, $M = 5$, $k = 10$ and $t = 1999$ and for the intelligent minority game with $N = 1001$, $M = 5$, $k = 10$, $t = 1999$, $n = 0.3$ and $\tau = 100$.

significant differences in the performances of the players. The performance of a player in the basic minority game does not change drastically in the course of the game as shown in Figure 2 (a). However, in our model, the performances of the players may change dramatically even after initial downfalls, and agents often do better after they have produced new strategies with the one-point genetic crossovers, as illustrated in Figure 2 (b).

In order to study the efficiency of the game, we plot the time-variation of N_1 for the basic minority game in comparison to our model in Figures 3 (a) and (b). Also the histograms of N_1 for the basic minority game and our model are plotted in Figures 3 (c) and (d). Clearly evident from these figures is the fact that when we allow one-point genetic crossovers in strategies, the system moves toward a more efficient state since the fluctuations in N_1 decreases and the histogram of N_1 becomes narrower and sharper. We have also studied the effect of increasing the fraction of players n on the distributions of the number of switches and the number of genetic crossovers the players make. The results in Figure 4 illustrate the fact that as n increases, more players have to make large number of switches and crossovers in order to be the best.

Furthermore, we calculate the variance σ^2 of N_1. The variation of σ^2/N against the parameter $2^M/N$ for the basic minority game, have been studied in details in refs. [12, 20, 21, 22]. We show the variation of σ^2/N with the parameter $2^M/N$ for $k = 2$ in Figure 5 (a) for both the games, by varying M and N. Also, we plot the quantity σ^2/N against M (varied from 2 to 12) for $N = 1001$ players and different values of k, in Figure 5 (b). For

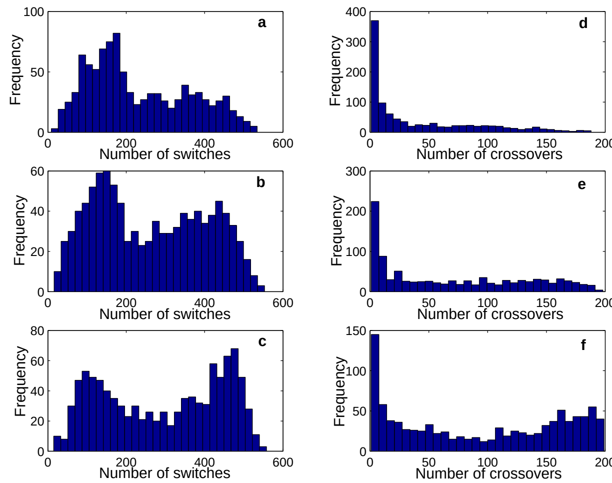

Figure 4: The histograms of the number of switches the players make in the intelligent minority game for (a) $n = 0.3$ (b) $n = 0.4$ (c) $n = 0.5$, and the histograms of the number of genetic crossovers the players make in the intelligent minority game for (d) $n = 0.3$ (e) $n = 0.4$ and (f) $n = 0.5$. The simulations have been made with $N = 1001$, $M = 4$, $k = 10$, $t = 1999$ and $\tau = 10$.

$k = 2$, the quantity σ^2/N is minimum in the basic minority game when $2^M/N \approx 0.5$ and there is a "phase transition" at this value [12, 20, 21, 22]. As we increase the value of k the efficiency decreases and this transition finally smoothens out. However, in the intelligent minority game, we find no such phase transition for any combinations of k, M and N, we have studied. We found that as the value of k is increased, the efficiency decreases, but at a rate much smaller than in the basic minority game. For both games, the values of σ^2/N seem to converge towards a common value for large values of M. If we compare the two games, we find that for large k values and moderate values of M, the differences in σ^2/N is very large.

We have observed that in our model, the worst players were often those who switched strategies most frequently while the best players were those who made the least number of switches after finding a good strategy. Further, we found that the players who do not make any genetic crossovers are unable to compete with those who make genetic crossovers, and their performances were found to fluctuate around the zero mean. Moreover, it was found that as the crossover time-interval τ is increased, the time for the system to reach an efficient state is longer [23].

One advantage of our model is clearly that the dimensionality of the strategy space as well as the number of elements in the strategy space remain the same. It is also appealing that starting from a small number of strategies, many "good" strategies can be generated by the players in the course of the game. Even though the players may not have performed well initially, they often did better when

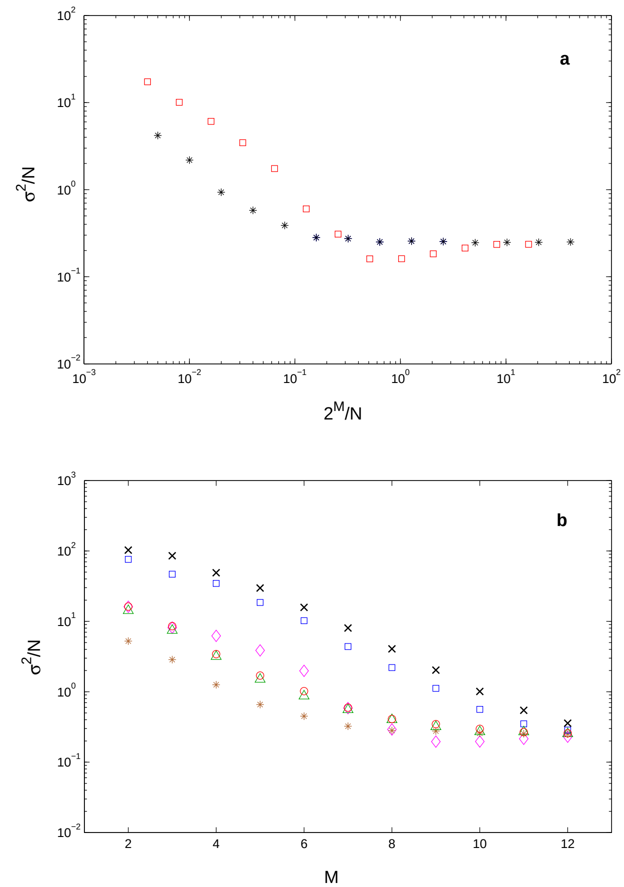

Figure 5: (a) The plot of σ^2/N against the parameter $2^M/N$ for $k = 2$, by varying M from 2 to 11 and N from 25 to 1001 for the basic minority game (red squares) and the intelligent minority game (black asterisk marks). The simulations were made for $t = 5000$ and ten different samples in each case. The parameter values chosen for the intelligent minority game were $\tau = 10$ and $n = 50$. (b) The plot of σ^2/N against M for different values of k for the basic minority game and the intelligent minority game. For the basic minority game, we have studied the cases of $k = 2$ (magenta diamonds), $k = 6$ (blue squares) and $k = 10$ (black cross marks). For the intelligent minority game, we have studied the cases of $k = 2$ (brown asterisk marks), $k = 6$ (green triangles) and $k = 10$ (red circles). The simulations for the basic minority game have been made with $N = 1001$ and $t = 5000$, and for the intelligent minority game have been made with $N = 1001$, $t = 5000$, $n = 50$ and $\tau = 10$, and for five different samples in each case.

they used new strategies generated by the one-point genetic crossovers. Finally, it should be pointed out that even in the framework of genetic algorithms, there are various ways to generate new strategies. One possibility is that we make a one-point genetic crossover between the two worst strategies and replace the parents by the children. Another possibility is to make "hybridized genetic crossover" where we make a one-point genetic crossover between the two best strategies, replace the worst two strategies with the children and retain the parents as well. We defer these modifications and interesting results for a future communication [23].

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* msysiaho@lce.hut.fi
† anirban@lce.hut.fi
‡ Kimmo.Kaski@hut.fi

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