Forecasting future oil production

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Abstract

This paper presents a new Monte-Carlo methodology to forecast the crude oil production of Norway and the U.K. based on the current/past performances of individual oil fields. By extrapolating the future production of these fields and the frequency of new discoveries, the oil production of these countries can be forecasted. The results indicate that standard methodology tends to underestimate remaining oil reserves. A comparison of the model to this methodology is made by making predictions with the full data and a back-test starting in 2008. It shows that the model gives a better description of future oil production.

I. Introduction

Forecasting future oil production has been a topic of active interest since the beginning of the past century because of oil's central role in our economy. Its importance ranges from energy production, through manufacturing to pharmaceuticals industry. As petroleum is a non-renewable and finite resource, it is primordial to be able to forecast future oil production. The fear of a global oil peak, beyond which production will inevitably decline, has been growing due to stagnating supplies and high oil prices since the crisis in 2008/2009 [13]. As any industrialized country, Europe is strongly dependent on oil supply to maintain its economic power. In the nowadays difficult geopolitical environment, it is important to know how much of the oil needed in Europe will come from reliable sources. In the past, a big share has been coming from Norway and the U.K. which have been reliable exporters. However, the U.K. already became a net importer in 2005 and Norway's production has been declining rapidly as well [7].

The methodology behind forecasting future oil production has not evolved much since M. King Hubbert, who in 1956 famously predicted that the U.S. oil production would peak around 1965-1970 [8]. That prediction has proven itself to be correct. His main argument was based on the finiteness of oil reserves and use of the logistic differential equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \quad (1)$$

to model oil production. The logistic differential equation is characterized by an initial exponential growth, which then decreases to zero as the total oil extracted reaches saturation (no more oil is to be found). The parameter r is commonly referred to as the growth rate, and K as the carrying capacity (total quantity of oil extracted). If $P(t)$ is the the amount of oil extracted up to time t, then $\frac{dP}{dt}$ is the oil production rate, the quantity that M. King Hubbert predicted with surprising accuracy to peak. From a methodological point of view, the Hubbert model has enjoyed a longstanding popularity in modeling future oil production given its simplicity. Various extensions have been studied by Brandt [1] to account for multi-cycled or asymmetric production curves.

The existing forecasts of future oil production use some form of the Hubbert model (Brecha, Laherrère or Lynch [2, 9, 12]) or economical model applied to aggregate production (Greiner [6]), but none goes into the details of studying the underlying dynamics. The main reason for the lack of details is certainly the lack of available data. In this paper, a new methodology is introduces to forecast future oil production. Instead of taking the aggregate oil production profile and fitting it with the Hubbert curve or its variants (such as the multi-cyclic Hubbert curve), the production profile of each individual oil field is used. By extending their production into the future and extrapolating the future rate of discovery of new fields, the future oil production is forecasted by the means of a Monte Carlo simulation. To demonstrate the universality of the methodology presented here, it is applied to 2 major oil producing countries with publicly available data: Norway[14] and the U.K. [5].

II. Methodology

The idea behind the methodology is to model the future aggregate oil production of a country by studying the production dynamics of its individual constituents, the oil fields. The main benefit of this approach, compared to working directly with aggregate production data, is the possibility to forecast non-trivial oil production profiles arising from the combination of all the individual field's dynamics. In order to achieve that, one must be able 1) to **extend the oil production of each individual field** into the future and 2) to **extrapolate the rate of discoveries** of new oil fields.

II.1 Extending the oil production of individual fields

The first step to predicting the future oil production of a country is to extrapolate the future production of existing fields and to estimate the error on this extrapolation. The data of the fields developed in the past shows a repeating asymmetric pattern. A good example is the Oseberg field shown in figure 1, with a quick ramp up once the field is being developed, and then a peak or plateau before the field starts decaying. The decay can take many different shapes and is governed by a variety of geological and economical factors which will be captured implicitly by the fitting procedure.

II.1.1 Regular, irregular and new fields

To be able to forecast the oil production of each individual field, regularity had to be found in the production's dynamics. Modeling the whole production profile from the beginning of extraction seems elusive due to the variety of the forms it can take. Fortunately, modeling the decay process is sufficient in order to extrapolate future oil production. A preliminary classification is necessary to achieve that goal. Figures 1, 2 and 3 show that independently of the country, oil fields can be classified into 3 main categories:

- **Regular fields** - Their decay show some regularity.
- **Irregular fields** - The ones that don't decay in a regular fashion.
- **New fields** - The ones that don't decay yet. As such, there is no easy way to forecast their future oil production based on past data.

All the fields have been fitted using an automated algorithm, but the results have been subsequently checked visually to sort out the irregular fields which could not be fitted. As of January 2014, regular fields make up 85% and 87% of the number of fields and 94% and 71% of the total produced oil volume in Norway and the U.K. respectively. As such, being able to model them is crucial. To capture as many different decay dynamics as possible, the stretched exponential

$$P(t) = P_0 e^{sign(\tau)\left(\frac{t}{|\tau|}\right)^{\beta}} \tag{2}$$

has been fitted to the decay of regular fields. The stretched exponential has many advantages as it generalized the power law and can therefore capture a broad variety of distributions as shown by Laherrère and Sornette [10]. As can be seen on figure 1, the stretched exponential (equation 2) is a good functional form to fit the decay process of regular fields.

For the minority of irregular fields, their decay has been modeled as follows: τ has been picked to be the average τ over the regular fields. Then β has been fixed so that the sum of the field's production over its lifetime be equal to the official ultimate recovery estimates, when such an estimate is available.

The minority of new fields, which did not yet enter their decay phase, cannot be extrapolated and will therefore be treated as a new discovery. The technical details of how to treat them as a new discovery are discussed in section II.2.3.

II.1.2 Back-testing & Error

To determine how well the extrapolation based on the stretched exponential predicts the future production, a complete back-testing has been performed on each field. A single back-test is made as follows:

- The production data $\{p_0, \ldots, p_N\}$ of the field is truncated at a certain date in the past $\tau \in \{0, \ldots, N\}$. Where τ is in months since production start of the field.
- The extrapolation$f(t)$ is made based on the truncated data $\{p_0, \ldots, p_\tau\}$.
- The future production predicted by the extrapolation $f(t)$ can be compared to the actual production from the date τ in the past up to now $\tau_f = N$. The extrapolated total production can be computed as

$$P_e(\tau) = \int_{\tau}^{\tau_f} f(t)dt \tag{3}$$

and the relative error is given by

$$e(\tau) = \frac{P_e(\tau) - \sum_{i=\tau}^{\tau_f} p_i}{P_e(\tau)}, \tag{4}$$

where both are a function of the truncation time τ.

Computing this back test for every month in the past since the field starting decaying, yields a plot showing the evolution of the relative error over time 4. By construction, the relative error will tend to zero as the truncation time τ goes to the present. Nonetheless, it is a uself indicator for the stability of the extrapolation. As can been seen for the Oseberg field, the relative error on future production remained fairly stable during the past decade.

From the complete back-test, on can compute the average relative error

$$\bar{e} = \frac{1}{N}\sum_{i=0}^{N} e(i) \tag{5}$$

the extrapolation made on the future production. Assuming that the relative errors are normally distributed around the average relative error, the standard deviation on the average relative error is given by

$$\sigma_e = \sqrt{\frac{1}{N}\sum_{i=0}^{N} (e(i) - \bar{e})^2}. \tag{6}$$

As the average relative error is often fairly constant, the extrapolation was corrected by the average relative error. Meaning that if the extrapolation consistently over-estimated the production by 10% during the back-test, the extrapolation was reduced by 10%. This results in an extrapolated production $p(t)$, including a 1σ confidence interval, as

$$p(t) = (1 - \bar{e})\, f(t) \pm \sigma_e. \tag{7}$$

An example of such an extrapolation including a one standard deviation range is shown on figure 2 for the Oesberg field.

II.1.3 Aggregate error

Once the individual fields have been extrapolated in the sense of formula 7, the aim is to compute the extrapolation for the country. While it is straightforward to sum the extrapolations of the individual fields to obtain the expected production, some care has to be taken with respect to the confidence interval on the production at the country level.

As shown later in section III, the same extrapolation including a complete monthly back-test of total future production has been performed at the country level and the resulting relative error is much smaller than the average error observed on the individual fields. To account for this observation, the assumption made is that the relative error between individual fields is uncorrelated. Therefore, the fields can be considered as a portfolio of assets with a return given by their extrapolation $p(t)$ (eq. 7) and a risk given by σ_{field} (eq. 6). This means that the risk at the country level from the extrapolated production is to be computed as

$$\sigma^2_{country} = \frac{1}{\#fields} \sum_{field \in fields} \sigma^2_{field}. \tag{8}$$

Intuitively, this models well the fact that the uncorrelated errors among fields will mostly cancel out.

II.2 Discovery rate of new fields

Knowing the future production rate of existing fields is not enough as new fields will be discovered in the future. The model describing the discovery rate of new fields should satisfy two fundamental observations.

1. The rate of new discoveries should tend to zero as time goes to infinity. This is a consequence of the finiteness of the number of oil fields.

2. The rate of new discoveries should depend on the size of the oil fields. As of today, giant oil fields are discovered much less frequently than dwarfs.

II.2.1 Discoveries modeled as logistic growth

A natural choice for such a model is a non-homogenous Poisson process. The Poisson process is a process that generates independent events at a rate λ. It is inhomogeneous if the rate is time-dependent, $\lambda \to \lambda(t)$. The standard way to measure $\lambda(t)$ is to find a functional form for $N(t)$, the statistical average of the cumulative number of events (discoveries) up to time t. Then, $\lambda(t)$ is simply $\frac{dN(t)}{dt}$. Figure 5 shows $N(t)$ for Norwegian fields classified according to their size. The logistic curve is a good fit to the data (integral form of equation 1). This implies that after an initial increase, the rate of new discoveries reaches a peak followed by a decrease until no more oil fields are to be found, consistent with our fundamental observations. This same approach has already been successfully applied by Forro [4] to estimate the number of daily active users on Zynga.

As the discovery and production dynamics are not independent of the field size, the fields have been split into two groups: dwarfs and giants. Unfortunately, the two logistic curves thus obtained are highly sensitive to the splitting size. This results from the major issue, when fitting a logistic curve to data, that the carrying capacity can not be determined if the data does not already exhibit the slow down in growth from reaching the carrying capacity. However, it is mentioned in the literature that often dwarf fields have already been discovered a long time ago, but their production has been postponed for economical reasons [12, p. 378]. Therefore, it is expected that the large oil fields have mostly been found and produced, and that future discoveries will mostly be made up of dwarf fields. Consequently, the splitting size has been picked as small as possible in order to maximize the number of giant fields but large enough to avoid recent discoveries:

- **Dwarfs**: Fields which produced less than $50 \cdot 10^6$ barrels.
- **Giants**: Fields which produced more than $50 \cdot 10^6$ barrels.

The resulting plot shown in figure 5 pictures the dynamics: giant oil fields have mostly been found while the discovery process for dwarf fields is still ongoing. The logistic growth curve fit to the giant discoveries is well constrained, however the fit for the dwarfs is poorly constrained. As can be seen on figure 5, the carrying capacity K of the logistic growth model is not well constrained by the available data. A large spectrum of values for K can lead to an equally good fit of the data. Therefore, a method described in section II.2.2 has been used to compute the probability of different carrying capacities.

II.2.2 Likelihood function for the number of discoveries

To overcome the logistic fitting issue for the dwarf fields, a method already used by Smith [15] has been implemented. This method makes the following two postulates:

1. "The discovery of reservoirs in a petroleum play can be modeled statistically as sampling without replacement from the underlying population of reservoirs."

2. "The discovery of a particular reservoir from among the existing population is random, with a probability of discovery being dependent on reservoir size."

The fields are split into J size bins denoted $S_1, \ldots, S_J$ occurring with frequency $n_1, \ldots, n_J$. Each discovery is considered as a step i at which a field of size $I(i) \in \{S_1, \ldots, S_J\}$ is found and m_{ij} denotes the number of fields of size $j \in \{1, \ldots, J\}$ discovered before the i^{th} step. Than the probability that the discovery at step i^{th} is of size j can be expressed as

$$P\left(I(i) = S_j\right) = \frac{\left(n_j - m_{ij}\right) \cdot S_j}{\sum_{k=1}^{J}\left(n_k - m_{ik}\right) \cdot S_k}. \tag{9}$$

The likelihood L for a complete sequence of N discoveries $\{I(1), \ldots, I(N)\}$ can than be expressed as

$$L = \prod_{i=1}^{N} \frac{\left(n_{I(i)} - m_{iI(i)}\right) \cdot S_{I(i)}}{\sum_{j=1}^{J}\left(n_j - m_{ij}\right) \cdot S_j}. \tag{10}$$

The unknown parameters are the number of fields $n_1, \ldots, n_J$, whose likelihood can now be estimated based on the existing discoveries. Using a brute force approach, the entire space of plausible values for the variables $n_1, \ldots, n_J$ has been sampled. The values n_j have been sampled between the number of existing fields m_{Nj} in the bin j and up to a value n_j^{upper}, such that the scenario with the largest likelihood according to equation 10 has $n_j^{max} = n_j^{upper}/2$ fields in the bin j. Subsequently, the likelihood of each scenario (value of the tuple $n_1, \ldots, n_J$) has been normalized such that the total likelihood of all generated scenarios equals one.

For the analysis of the north sea oil field discoveries, the number of size bins has been fixed to $J = 2$ splitting between dwarfs (1) and giants (2) as described in section II.2.1.

Table 1: *Likelihood estimation for the number of dwarf (1) and giant (2) fields.*

	m_{N1}	$n_1 \pm \sigma_1$	m_{N2}	$n_2 \pm \sigma_2$
Norway	24	88.4 ± 10.0	52	56.4 ± 1.6
U.K.	162	208 ± 11	99	100 ± 0.4

The results shown in table 1 are coherent with the intuitive expectation that discovering a new giant field is unlikely and that future discoveries wil mostly be made up of dwarf fields. The likelihoods obtained for the carrying capacities of dwarfs and giants have been used to constrain the logistic curve fitted to the discoveries. Figure 5 shows a sample of fitted logistic curves, each curve being weighted by the likelihood of its carrying capacity given by equation 10.

II.2.3 Future production from discoveries

The goal is to compute an expected oil production coming from future discoveries, which requires to combine the steps described in sections II.2.1 and II.2.2.

The method described in section II.2.2 yields probabilities for the total number of fields (including the not yet discovered fields) in each size bins (called a scenario). However, this likelihood method does not give the time distribution of future discoveries. The aim is to use the likelihood function to generate scenarios with their respective occurrence probability.

For a given scenario, the carrying capacity K (= total number of fields) is given for each size class. This is useful, because it resolves the instability in fitting the logistic curve to the number of discovered fields. The time distribution of the discoveries is then given for each size class by the fitted logistic curve.

The actual size of a newly discovered field is generated according to the size distribution of the existing fields in its size class. The probability distribution function of field sizes in a given size bin has been fitted by a stretched exponential function.

The production curve is computed based on the average production curve of all existing fields in the same size category. The production curves of the existing fields have all been normalized to a total production of one and then have been averaged. This yields the typical production profile including a one sigma confidence interval. For a new field, this typical production curve is than multiplied by the size of the field.

Superposing the production curves results in the expected production curve from future oil field for a given scenario.

As the total parameter space is too large to be sampled entirely, a Monte Carlo technique is applied to compute the expected production with confidence interval from future discoveries. In a nutshell, the algorithm works as follows:

1. Draw a scenario based on its probability.
2. Compute the time distribution of new discoveries by fitting a logistic curve for each size class.
3. For each discovery generate a size and the resulting production curve based on the size distribution and production curves of existing fields.
4. Superpose all the production curves.
5. Repeat and average over all drawn scenarios.

The result is the expected production curve of future oil field discoveries. The distribution of generated scenarios yields the confidence interval.

In order to account for the new fields discussed in section II.1.1, which are already discovered but did not yet enter the decay phase, the simulated production resulting from new discoveries has been shifted to the date in the past where it matches the current production from new fields.

III. RESULTS

Based on the methodology described in section II.2, simulating future oil production was straightforward. For each country, the existing fields were extrapolated and the future discoveries were simulated. Figure 6 shows the average of 1000 simulations. For each country, the non-symmetric shape of the production's dynamics contrary to what Hubbert would have predicted is immediately noticeable.

Table 2: *Remaining oil reserves until 2030 in barrels for the **Fit** of the country's production, the Monte-Carlo **Model** and the relative difference* $\Delta = \frac{Model-Fit}{Model}$ *between the two.*

	Fit (barrels)	Model (barrels)	Δ
Norway	$130 \cdot 10^6$	$188 \cdot 10^6$	31%
U.K.	$59 \cdot 10^6$	$98 \cdot 10^6$	40%

The results in table 2 show a striking difference between the fit and the Monte-Carlo model forecast. According to the fit (extrapolation of aggregate production), Norway's future oil production would decay much faster than in the Monte-Carlo case. The remaining reserves estimated with the Monte-Carlo methodology are 30% larger then the estimate from the fit. This difference originates from two different effects:

- The sum of the forecast of the individual existing fields is higher than the extrapolation of the aggregate production.
- The extrapolation of the aggregate production does not capture well the discovery process of dwarf fields.

In the U.K., oil production faced a change of regime during the early nineties due to technological innovation, giving rise to the inverted "w shape" of the oil production profile. However, this has not been an issue to extrapolate the decay of production starting at the second peak. The difference between the Hubbert-based methodology and the Monte-Carlo one is very similar to the Norwegian case. The former underestimates the remaining oil reserves by about 40% compared to the latter.

Which of the two models is more thrust worthy? Clearly, the implications in adopting one methodology over the other are significant. The only way to answer this question is to back-test them. In other words: "What would have each of the models predicted had they been used in the past?"

IV. Validation

For both countries, namely Norway and the U.K., a back-test using the data truncated in 2008 has been made. Before that date, too many of the giant fields have not entered their decay phase for a sufficiently long time to apply the extrapolation algorithm. Figure 7 shows the outcome of those tests. Comparing the forecast of both models with the oil production of the subsequent 6 years, shows that the predictive power of the Monte-Carlo model is better than a simple extrapolation of aggregate production. The following table 3 summarizes the difference between the two approaches for the back-testing period.

Table 3: *The fit and the Monte-Carlo model are used on the data set truncated in 2008 and their forecast for the period 2008-2014 is compared to the actual production.*

	Actual (barrels)	Fit (barrels)	Model (barrels)
Norway	$133 \cdot 10^6$	$108 \cdot 10^6$	$130 \cdot 10^6$
U.K.	$79 \cdot 10^6$	$66 \cdot 10^6$	$75 \cdot 10^9$

The difference is not huge for the period from 2008 to 2014, but the Monte-Carlo model nevertheless is noticeably closer to the actual value. However, when considering the period after 2014, the difference is huge as shown in table 4.

Table 4: *Remaining oil reserves forecasted for the period 2014-2030 when using the data truncated in 2008.*

	Fit (barrels)	Model (barrels)	Δ
Norway 08	$25 \cdot 10^6$	$171 \cdot 10^6$	-85%
U.K. 08	$25 \cdot 10^6$	$91 \cdot 10^9$	-73%

The simple extrapolation decays too fast and entirely misses the fat tails in the decay process of individual fields and the new discoveries. As well, it must be noted that the simple extrapolation changed massively between the back-test in table 4 and the current fit in table 2 (520% for Norway and 236% for the U.K.). In other words, the simple extrapolation was very unstable in its forecast, while the Monte-Carlo forecasts remained very consistent (less than 10% change).

As can be seen in figure 7, the actual production of Norway during the back-testing period remained entirely within the quite narrow 1σ interval of the Monte-Carlo methodology, while totally breaking out of the 1σ interval of the simple extrapolation. For the U.K. the Monte-Carlo methodology only performs slightly better when considering the confidence interval, and the confidence interval is much larger due to the uncertainty on future discoveries and their production profile.

V. Conclusion

This paper presented a Monte-Carlo based methodology to forecast future oil production. By extending the oil production of current fields into the future and modeling the discovery rate of new fields, the future oil production of Norway and the U.K. could be forecasted. These forecasts are significantly different from the ones obtained with a standard extrapolation. Indeed, our model forecasts 30% to 40% more remaining oil reserves than the standard extrapolation. The back-test performed on the time period between 2008 and 2014 confirmed that the Monte-Carlo based model better captured the production dynamics.

The results are hopeful, as they make it highly likely that the decay of Norwegian and U.K. oil production will be much slower then one would expect from a standard extrapolation. Nonetheless, to maintain current levels of oil consumption in the European Union, more of it will have to be imported from outside Europe, as the imports from Norway will vanish (currently accounting for 11% of E.U. oil imports [3]) and the U.K. will need to import more oil.

Table 5: *Oil import (bbl/day) at a constant consumption of 1.5M bbl/day for the U.K. and 0.22M bbl/day for Norway. The import for the E.U. and Norway is a lower bound based on the changes in the U.K and Norway.*

	2014	2020	2025	2030
Norway	$-1.23 \cdot 10^6$	$-0.88 \cdot 10^6$	$-0.58 \cdot 10^6$	$-0.43 \cdot 10^6$
U.K.	$0.7 \cdot 10^6$	$0.9 \cdot 10^6$	$1.0 \cdot 10^6$	$1.1 \cdot 10^6$
E.U.+Norway (lower bound)	$9.8 \cdot 10^6$	$10.45 \cdot 10^6$	$10.85 \cdot 10^6$	$11.1 \cdot 10^6$

As shown in table 5, at constant consumption, the Monte-Carlo model predicts that in 2030 the E.U. with Norway will need to increase its oil imports by 1.3 million barrels. Imports which will most likely have to come from outside Europe.

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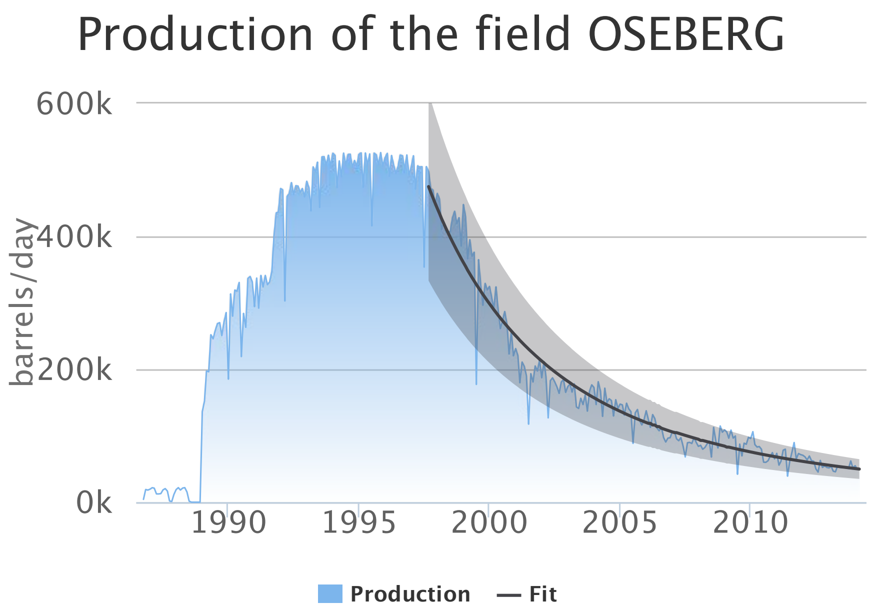

Figure 1: *Example of a regular field with* $\beta = 0.66 \pm 0.01$ *and* $\tau = -55 \pm 1$.

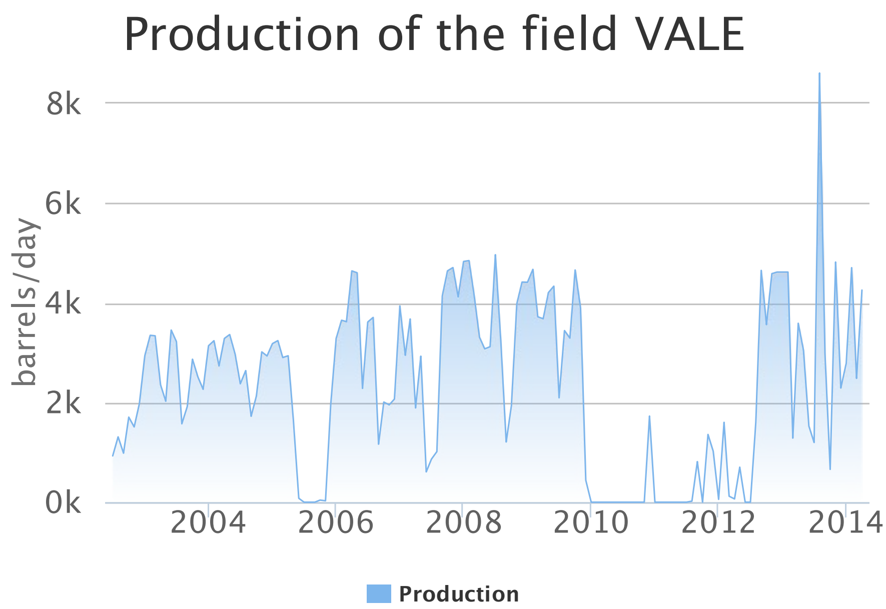

Figure 2: *Example of a irregular field.*

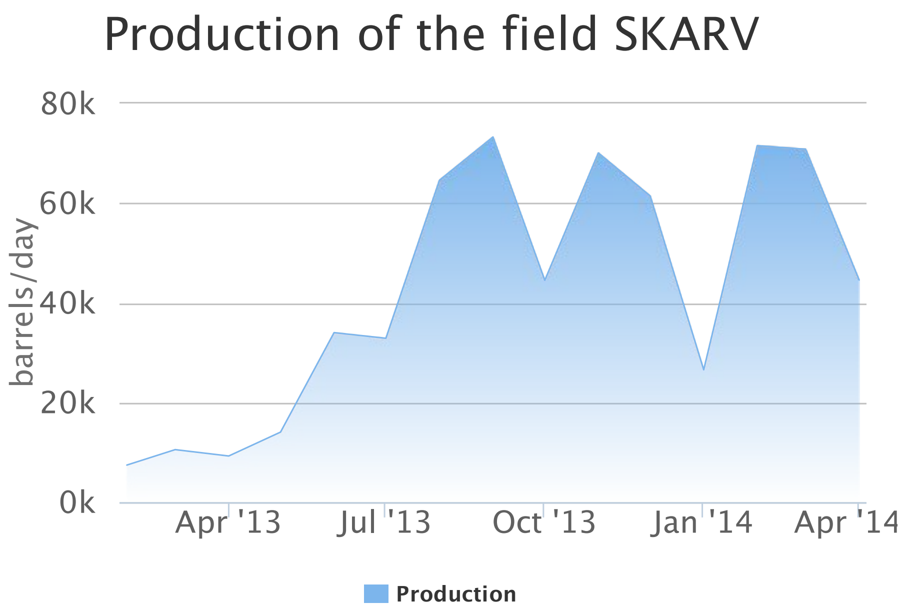

Figure 3: *Example of a new field.*

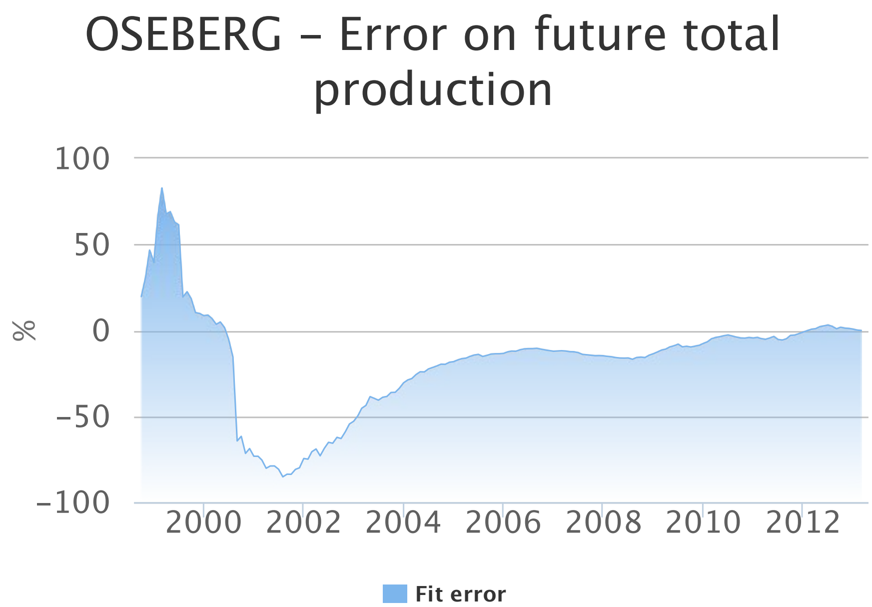

Figure 4: *Oesberg field - Relative error on predicted total production from time t until 2014*

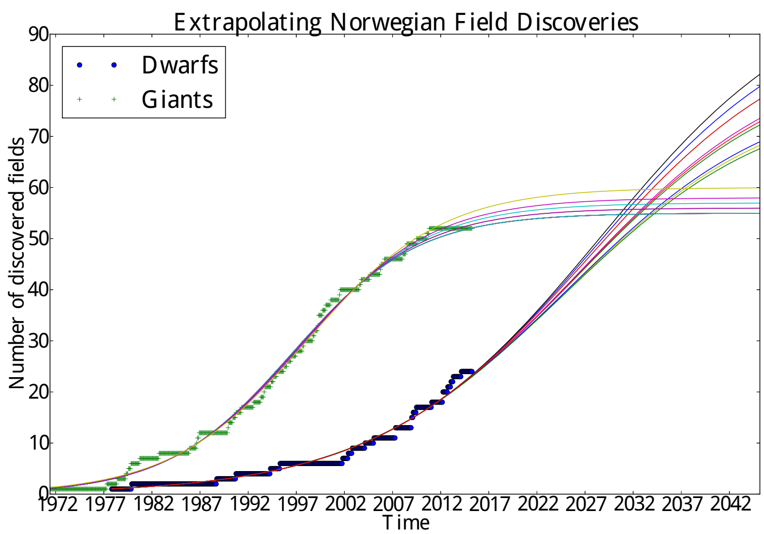

Figure 5: *Logistic fit to the number of discoveries for Norway. The discovery rate of new oil fields is dependent on their size.*

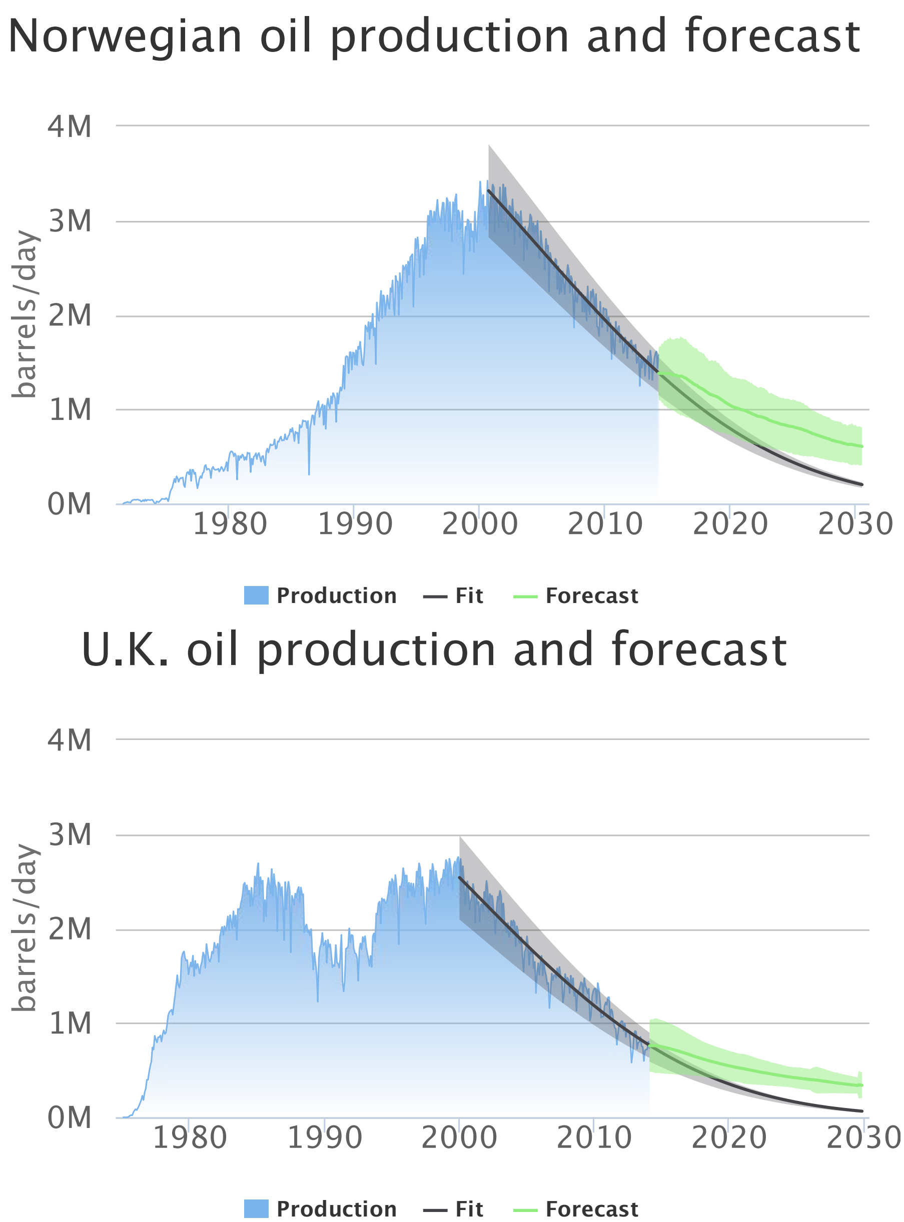

Figure 6: *Monte-Carlo and fit forecast based on past production data for Norway (top) and the U.K. (bottom). In both cases, the Mont-Carlo model forecasts a significantly slower decay than the fit.*

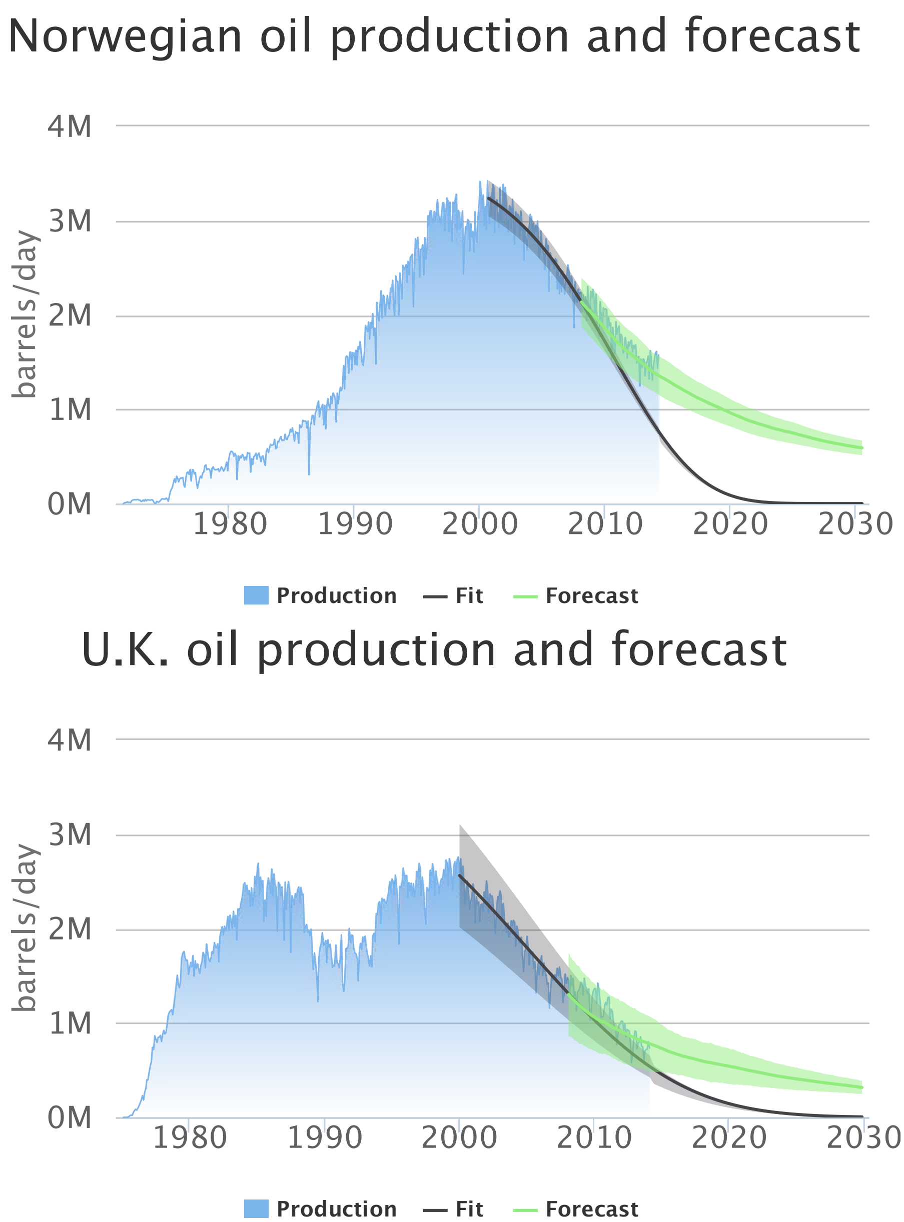

Figure 7: *Monte-Carlo and Hubbert forecast based on past production data up to 2008 for Norway (top) and the U.K. (bottom). The results can be compared with the subsequent oil production (blue area). In both cases, the Monte-Carlo methodology is more precise.*