

Forecasting future oil production in Norway and the UK: a general improved methodology

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Abstract

We present a new Monte-Carlo methodology to forecast the crude oil production of Norway and the U.K. based on a two-step process, (i) the nonlinear extrapolation of the current/past performances of individual oil fields and (ii) a stochastic model of the frequency of future oil field discoveries. Compared with the standard methodology that tends to underestimate remaining oil reserves, our method gives a better description of future oil production, as validated by our back-tests starting in 2008. Specifically, we predict remaining reserves extractable until 2030 to be 5.7 ± 0.3 billion barrels for Norway and 3.0 ± 0.3 billion barrels for the UK, which are respectively 45% and 66% above the predictions using an extrapolation of aggregate production.

Forecasting future oil production has been a topic of active interest since the beginning of the past century because of oil central role in our economy. Its importance ranges from energy production, manufacturing to the pharmaceutical industry. As petroleum is a non-renewable and finite resource, it is primordial to be able to forecast future oil production. The fear of a global oil peak, beyond which production will inevitably decline, has been growing due to stagnating supplies and high oil prices since the crisis in

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2008/2009 (Murray & Hansen, 2013). As any industrialized country, Europe is strongly dependent on oil supply to maintain its economic power. In the nowadays difficult geopolitical environment, it is important to know how much of the oil needed in Europe will come from reliable sources. In the past, a big share has been coming from Norway and the U.K., two of Europe's biggest exporters. However, the U.K. already became a net importer in 2005 and Norway's production has been declining rapidly as well (Höök & Aleklett, 2008). This motivates the novel Monte-Carlo methodology to forecast the crude oil production of Norway and the U.K. that we present here.

I. STATE OF THE ART

I.1. Hubbert's Model for peak oil forecasting

The methodology behind forecasting future oil production has not evolved much since M. King Hubbert, who in 1956 famously predicted that the U.S. oil production would peak around 1965-1970 (Hubbert, 1956). That prediction has proven itself to be correct. His main argument was based on the finiteness of oil reserves and to what amounts as the use of the logistic differential equation for the total quantity $P(t)$ of oil extracted up to time t

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K} \right) . \quad (1)$$

The logistic differential equation is characterized by an initial exponential growth, which then decreases to zero as the total oil extracted reaches saturation (no more oil is to be found). The parameter r is commonly referred to as the growth rate, and K as the carrying capacity (total quantity of oil that can be ultimately extracted). If $P(t)$ is the amount of oil extracted up to time t , then $f(t) := \frac{dP}{dt}$ is the oil production rate, the quantity that M. King Hubbert predicted with surprising accuracy to peak. From a methodological point of view, the Hubbert model has enjoyed a longstanding popularity in modeling

future oil production given its simplicity. Various extensions have been studied by Brandt (2007) to account for multi-cycled or asymmetric production curves.

1.2. Back-testing Hubbert's Model

Despite the popularity of the Hubbert model, the literature says little about its out-of-sample performance, apart from the one case that made the prediction famous, namely the prediction by Hubbert of the conventional oil peak that occurred in the US in 1970. There are many ways to address this question and we choose to study the remaining recoverable reserves predicted by the Hubbert Model when going back in time. As the problem of fitting the logistic curve (equation 1) is under-constrained before the peak of $f(t)$, the back-test had to be started when the peak is already well visible in the data. For each time T , a month between the start of the back-test and the date of the latest available date, we fitted the Hubbert model to the data truncated at time T . Based on each fit, we computed the predicted remaining recoverable reserves in 2014.

This back-test allowed us to study how the forecast of Hubbert's model evolves over time. A good model, which captures well the underlying dynamics of the system, should yield a stable forecast over time. An unstable forecast, where the model constantly has to adapt to new data, can be considered as an indicator that the model is missing some aspects (technically, it is called "misspecified"). This is of course assuming that the dynamics of the system is deterministic to a higher degree than the error the model is making in its forecast.

For the studied cases, namely Norway and the UK, a good starting date was chosen to be 01/2000. Fitting Norway's production could be made using the simple logistic equation 1. In contrast, for the UK, the production exhibits two clear peaks, which required the use of a double-logistic function that is the sum of two independent logistic functions with time shifted peaks. The peak of each of the two logistic functions is matched up with one of the two peaks in the UK

production data.

The results of this back-test can be seen in figure 1 and clearly show that the forecast is not stable over time. In 2000, for Norway, the Hubbert model would have predicted 2.08Gb of remaining recoverable reserves by 2014 while, up to the time of writing (July 2014), the predicted amount grew to 2.83Gb (a 36% error) and the growth is not showing sign of slowing down, yet. For the UK, the prediction would have been 0.21Gb but grew all the way to 1.43Gb (an error of 581%), with a recent slow down in growth. These results clearly show that the Hubbert model can be highly unstable and unreliable in its forecast of recoverable reserves. For completeness, let us stress that the relative error between the year 2000 and July 2014 in total production since the beginning is only 17% for Norway and 6.5% for the UK. Nonetheless, the Hubbert model greatly underestimates the fat tail of the production curve.

Our aim is first to understand why the Hubbert model underestimates the tail of the production curve and then to improve upon it. An argument could be the strong increase in the oil price, which widened the amount of economically recoverable reserves. To discuss this hypothesis, figure 1 shows the oil price as well. The forecast for Norway remains fairly stable from 2000 to 2007 during a period when the oil price went from slightly above \$20 per barrel to more than \$80 per barrel. The shift to a larger predicted remaining production does not correlate with the peak, bottom and then rebound of the oil prices since 2007. For the UK, the behaviour is different, with increases in the forecast showing very little delay with respect to the increases in oil prices. One could speculate that Norway had little spare capacities and therefore the increase in remaining reserves was always delayed with respect to oil prices as new capacities had first to be build. On the other hand, UK could have had available or upcoming spare capacities that allowed to respond rapidly to the increase

in the oil price. While it is certain that increases in oil prices will augment the recoverable reserves, it seems difficult to establish a well-defined mathematical link between oil price and the increase in forecasted reserves. Consequently, an extension of the Hubbert model integrating the oil price has not been further explored.

1.3. Beyond Hubbert's Model

The existing forecasts of future oil production all use some version of the Hubbert model (Brecha, 2012; Laherrère, 2002; Lynch, 2002) or some economical model applied to aggregate production (Greiner et al., 2011). However, as just shown in the back-test, the Hubbert model does not provide a forecast that is stable in time, usually underestimating the significant tail in the production curve. Considering that none of these Hubbert-based methodologies go into the details of studying the underlying dynamics, it is not surprising that important features of the dynamics bearing on the total recoverable reserves have been missed. In defense of the existing studies, the main reason for the lack of details is certainly the lack of available data. For this reason, the Hubbert model will likely remain a tool of choice to estimate the total reserves when only the aggregated production of a region (typically a country) is available.

Nonetheless, in this article, a new methodology is introduced to forecast future oil production. Instead of taking the aggregate oil production profile and fitting it with the Hubbert curve or its variants (such as the multi-cyclic Hubbert curve), the production profile of each individual oil field is used. By extending their production into the future and extrapolating the future rate of discovery of new fields, the future oil production is forecasted by means of a Monte Carlo simulation. To demonstrate the generality of the methodology presented here, it is applied to two major oil producing countries with publicly available data at the field level: Norway (NPD) and the U.K. (GOV.UK, 2014).

II. METHODOLOGY

The idea behind the methodology is to model the future aggregate oil production of a country by studying the production dynamics of its individual constituents, the oil fields. The main benefit of this approach, compared to working directly with aggregate production data, is the possibility to forecast non-trivial oil production profiles arising from the combination of all the individual field dynamics. This ability reflects directly the fact that the total production is the sum of the contributions of each individual oil field in production. Thus modelling at the level of each field reflects more closely the reality and is likely to be over-performing and more reliable, as we show below. In order to implement our approach, one must be able 1) to **extend the oil production of each individual field** into the future and 2) to **extrapolate the rate of discoveries** of new oil fields.

II.1. Extending the oil production of individual fields

The first step to predicting the future oil production of a country is to extrapolate the future production of existing fields and to estimate the error on this extrapolation. The data of the fields developed in the past shows a repeating asymmetric pattern. A good example is the Oseberg field shown in figure 2, with a quick ramp up once the field is being developed, and then a peak or plateau before the oil field production starts decaying. The decay can take many different shapes and is governed by a variety of geological and economical factors. The goal of the fitting procedure is to capture as much as possible the impact of these different factors.

II.1.1. Regular, irregular and new fields

To be able to forecast the oil production of each individual field, regularity had to be found in the production's dynamics. Unfortunately, the whole production profile can take a large variety of shapes with often very asymmetric

shapes. Therefore, modeling the whole production profile from the beginning of extraction with the Hubbert model seems elusive. It would require the use of various extensions of the Hubbert model and for each field the most appropriate extension would need to be determined. This is impractical when dealing with hundreds of oil field.

Fortunately, modeling the decay process is sufficient in order to extrapolate future oil production. A preliminary classification is necessary to achieve that goal. Figures 2, 3 and 4 show that, independent of the country, oil fields can be classified into three main categories:

- **Regular fields** - Their decays show some regularity (see figure 2);
- **Irregular fields** - The ones that do not decay in a regular fashion (see figure 3);
- **New fields** - The ones that do not decay yet. As such, there is no easy way to forecast their future oil production based on past data (see figure 4).

All the fields have been fitted using an automated algorithm, but the results have been subsequently checked visually to sort out the irregular fields which could not be fitted. As of January 2014, regular fields make up 85% and 87% of the number of fields and 94% and 71% of the total produced oil volume in Norway and the U.K. respectively. As such, being able to model them is crucial. To capture as many different decay dynamics as possible, the decay part of the oil production rate $f(t) := \frac{dP}{dt}$ has been fitted by the stretched exponential

$$f(t) = f_0 e^{-\left(\frac{t}{\tau}\right)^\beta} . \quad (2)$$

The stretched exponential function has many advantages as it generalises the power law and can therefore capture a broad variety of distributions as shown by Laherrère & Sornette (1998). Moreover, Malevergne et al. (2005) showed that

the power law function can be obtained as an asymptotic case of the stretched exponential family, allowing for asymptotically nested statistical tests. As can be seen in figure 2, the stretched exponential (equation 2) is a good functional form to fit the decay process of regular fields.

For the minority of irregular fields, we assume no difference in the decay of production between giants and dwarfs, which has been modelled as follows: the decay time scale τ has been picked to be the average τ over the regular fields. Then β has been fixed so that the sum of the field production over its lifetime be equal to the official ultimate recovery estimates, when such an estimate is available.

The minority of new fields, which did not yet enter their decay phase, cannot be extrapolated and will therefore be treated as new discoveries. The technical details of how to treat them as new discoveries are discussed in section II.2.3.

II.1.2. Back-testing & Error

For every extrapolation, be it the Hubbert model applied to aggregate production at the country level or the stretched exponential applied to a field's decay, we want to determine how good and stable the predictive power is. To do so, we perform a complete back-testing in every case. This includes a monthly back-test for the extrapolation, using a stretched-exponential, of the decay process of every oil field.

A single back-test is made as follows:

- The production data $\{p_0, \dots, p_N\}$ of the field or country is truncated at a certain date in the past $T \in \{0, \dots, N\}$, where T is the time counted in months since the production start of the field.
- The extrapolation of the oil production rate $f(t) := \frac{dP}{dt}$ is made based on the truncated data $\{p_0, \dots, p_T\}$.

- The future production predicted by the extrapolation function $f(t)$ can be compared to the actual production from the date T in the past up to the present $T_f = N$. The extrapolated total production can be computed as

$$P_e(T) = \int_T^{T_f} f(t)dt \quad (3)$$

and the relative error is given by

$$e(T) = \frac{P_e(T) - \sum_{i=T}^{T_f} p_i}{P_e(T)}, \quad (4)$$

where both $P_e(T)$ and $e(T)$ are functions of the truncation time T .

Computing this back-test, for every month in the past since the production started decaying, yields a plot showing the evolution of the relative error over time defined by expression (5). By construction, the relative error will tend to zero as the truncation time T approaches the present. Nonetheless, it is a useful indicator for the stability of the extrapolation. As can be seen for the Oseberg field in figure 5, the relative error on future production remained fairly stable during the past decade.

From the complete back-test, we compute the average relative error

$$\bar{e} = \frac{1}{N} \sum_{i=0}^N e(i) \quad (5)$$

of the extrapolation made on the future production. Assuming that the relative errors are normally distributed around the average relative error, the standard deviation on the average relative error is given by

$$\sigma_e = \sqrt{\frac{1}{N} \sum_{i=0}^N (e(i) - \bar{e})^2}. \quad (6)$$

As the average relative error is often fairly constant, the extrapolation was corrected by the average relative error, that is, if the extrapolation consistently over-estimated the production by 10% during the back-test, the extrapolation

was reduced by 10%. This results in an extrapolated production $p(t)$, including a 1σ confidence interval, given by

$$p(t) = (1 - \bar{e}) f(t) \pm \sigma_e. \quad (7)$$

An example of such an extrapolation including a one standard deviation range is shown in figure 2 for the Oseberg field.

II.1.3. Aggregate error

Once the individual fields have been extrapolated using formula 7, we compute the extrapolation of the oil production for the whole country. While it is straightforward to sum the extrapolations of the individual fields to obtain the expected production, some care has to be taken with respect to the confidence interval of the production at the country level.

As shown later in section III, the same extrapolation including a complete monthly back-test of total future production has been performed at the country level and the resulting relative error is much smaller than the average error observed on the individual fields. To account for this observation, the assumption made is that the relative error between individual fields is uncorrelated. While one could imagine that some inter-dependence could result from a coordinated response of supply to a sharp increase/decrease of demand, we have not observed this to be the case at a significant level. Therefore, the fields can be considered as a portfolio of assets with a return given by their extrapolation $p(t)$ (eq. 7) and a risk given by σ_{field} (eq. 6). This means that the average standard deviation per field at the country level from the extrapolated production can be computed as

$$\sigma_{country}^2 = \frac{1}{\#fields} \sum_{field \in fields} \sigma_{field}^2. \quad (8)$$

Intuitively, this models well the fact that the uncorrelated errors among fields will mostly cancel out.

II.2. *Discovery rate of new fields*

Knowing the future production rate of existing fields is not enough as new fields will be discovered in the future. The model describing the discovery rate of new fields should satisfy two fundamental observations.

1. The rate of new discoveries should tend to zero as time goes to infinity.
This is a consequence of the finiteness of the number of oil fields.
2. The rate of new discoveries should depend on the size of the oil fields. As of today, giant oil fields are discovered much less frequently than dwarf oil fields.

II.2.1. *Discoveries modeled as logistic growth*

A natural choice for such a model is a non-homogenous Poisson process. The Poisson process is a process that generates independent events at a rate λ . It is nonhomogeneous if the rate is time-dependent, $\lambda \rightarrow \lambda(t)$. The standard way to measure $\lambda(t)$ is to find a functional form for $N(t)$, the statistical average of the cumulative number of events (discoveries) up to time t . Then, $\lambda(t)$ is simply a smoothed estimation of the observed rate $\frac{dN(t)}{dt}$. Figure 6 shows $N(t)$ for Norwegian fields classified according to their size in two classes, dwarfs and giant fields. The logistic curve is a good fit to the data (integral form of equation 1). This implies that after an initial increase, the rate of new discoveries reaches a peak followed by a decrease until no more oil fields are to be found, consistent with our fundamental observations. This same approach has already been successfully applied by Forró et al. (2012) to estimate the number of daily active users on Zynga.

As the discovery and production dynamics are not independent of the field size, the fields have been split into two groups: dwarfs and giants. Unfortunately, the two logistic curves thus obtained are highly sensitive to the splitting size. This results from the major issue, when fitting a logistic curve to data,

that the carrying capacity can not be determined if the data does not already exhibit the slowdown in growth towards the carrying capacity. However, it is mentioned in the literature that often dwarf fields have already been discovered a long time ago, but their production has been postponed for economical reasons (Lynch, 2002, p. 378). Therefore, it is expected that the large oil fields have mostly been found and produced, and that future discoveries will mostly be made up of dwarf fields. Consequently, the splitting size has been picked as small as possible in order to maximize the number of giant fields but large enough to avoid recent discoveries. Our definition is thus:

- **Dwarfs:** Fields which produced less than $50 \cdot 10^6$ barrels.
- **Giants:** Fields which produced more than $50 \cdot 10^6$ barrels.

We note that this definition differs by a factor 10 from the more standard one, for which oil fields with an ultimate recoverable resource of 0.5 billion barrels (Gb) or higher are classified as giants, while oil fields with smaller URR are considered to be dwarfs (Höök & Aleklett, 2008).

The resulting plot shown in figure 6 pictures the dynamics: giant oil fields have mostly been found while the discovery process for dwarf fields is still ongoing. The logistic growth curve fit to the giant discoveries is well constrained, however the fit for the dwarfs is poorly constrained. As can be seen in figure 6, the carrying capacity K of the logistic growth model is not well constrained by the available data. A large spectrum of values for K can lead to an equally good fit of the data. We have not taken into account the possible effect that newly discovered dwarf fields could become smaller and smaller until the new fields have a size that is too small to be economical to drill.

There are in fact two competing effects that are likely to compensate each other. On the one hand, figure 7 shows the complementary cumulative distribution function (CCDF) of known oil field sizes S from Norway and the UK. Two

salient properties can be observed. First, the tails of the distributions are well described by power laws

$$\text{CCDF}(S) \sim \frac{1}{S^{1+\alpha}}, \quad (9)$$

with exponent $\alpha = 1.2 \pm 0.1$ for Norway and $\alpha = 1.4 \pm 0.1$ for the UK. The fact that the estimations of the exponents α are larger than 1 implies that the cumulative oil reserves are asymptotically controlled by the largest fields, and not the small ones (Sornette, 2004). However, the fact that the exponents α are rather close to 1 (which is called “Zipf law”) would make the many small oil field contributing significantly in total. This brings us to the second important feature exhibited by figure 7, namely the roll-overs of the CCDFs for small oil fields, likely due to an under-sampling of the data. Indeed, as for most data sets involving broad distributions of sizes such as oil fields, the distributions are in general incomplete for the small events due to the non exhaustive sampling. This incompleteness raises the spectre that our extrapolations might be grossly underestimating the large potential contributions to the total reserves of the many small yet undiscovered small oil fields. Assuming that the power law (9) would hold for smaller fields down to size of 1 million barrels leads to a number of such fields 10 to 100 times larger than presently known.

But there is another key factor that needs to be considered, namely the fact that small oil fields are not economically viable for exploitation, which leads to an effective truncation in the distribution (9) relevant for the estimation of recoverable oil. In 1980, the threshold for economic viability was as high as 70 million barrels (Smith, 1980, p. 591), but with today’s oil price we expect a much lower threshold. Taking the data from the NPD providing the yearly investments broken by fields in Norway, let us consider the illustrative case of the field GAUPE. The investment spent to develop its exploitation was \$380M,

while its estimated size is $\simeq 1.2\text{Mb}$, which corresponds approximately to a total market value of \$120M at \$100 per barrel. It is thus not a surprise that investment to exploit this field was interrupted in 2013. Figure 8 shows the total estimated revenue divided by investment as a function of the estimated ultimate recovery for a number of small oil fields. It can be inferred that oil fields of sizes smaller than about 10 Mb are not economically viable as long as the market oil price does not grow much higher than \$100 per barrel. This implies that, notwithstanding the large number of unknown small oil fields, economic considerations oblige us to neglect the small oil fields, therefore providing a justification of our procedure. In fact, economic constraints may lead to cap the carrying capacity at a value smaller than the one shown in figure 6.

To address these issues from a more solid angle, the method described in the next section II.2.2 has been used to compute the probability of different carrying capacities.

II.2.2. Likelihood function for the number of discoveries

To overcome the poor constraint on the carrying capacity K obtained from the fitting procedure for dwarf fields, a method already used by Smith (1980) has been implemented. This method makes the following two postulates:

1. "The discovery of reservoirs in a petroleum play can be modeled statistically as sampling without replacement from the underlying population of reservoirs."
2. "The discovery of a particular reservoir from among the existing population is random, with a probability of discovery being dependent on (proportional to) reservoir size."

The fields are split into J size bins denoted S_1, \dots, S_J occurring with frequency n_1, \dots, n_J . Each discovery is considered as a step i at which a field of size $I(i) \in$

$\{S_1, \dots, S_J\}$ is found and m_{ij} denotes the number of fields of size $j \in \{1, \dots, J\}$ discovered before the i^{th} step. Then, the probability that the discovery at step i is of size j can be expressed as

$$P(I(i) = S_j) = \frac{(n_j - m_{ij}) \cdot S_j}{\sum_{k=1}^J (n_k - m_{ik}) \cdot S_k}. \quad (10)$$

The likelihood L for a complete sequence of N discoveries $\{I(1), \dots, I(N)\}$ can then be expressed as

$$L = \prod_{i=1}^N \frac{(n_{I(i)} - m_{iI(i)}) \cdot S_{I(i)}}{\sum_{j=1}^J (n_j - m_{ij}) \cdot S_j}. \quad (11)$$

The unknown parameters are the number of fields n_1, \dots, n_J , whose likelihood can now be estimated based on the existing discoveries. Using a brute force approach, the entire space of plausible values for the variables n_1, \dots, n_J has been sampled. The values n_j have been sampled between the number of existing fields m_{Nj} in the bin j and up to a value n_j^{upper} , such that the scenario with the largest likelihood according to equation (11) has $n_j^{max} = n_j^{upper} / 2$ fields in the bin j . Subsequently, the likelihood of each scenario (value of the tuple n_1, \dots, n_J) has been normalized such that the total likelihood of all generated scenarios equals one.

For the analysis of the discoveries of the North Sea oil fields, the number of size bins has been fixed to $J = 2$ splitting between dwarfs (1) and giants (2) as described in section II.2.1.

The results shown in table 1 are coherent with the intuitive expectation that discovering a new giant field is unlikely and that future discoveries will mostly be made up of dwarf fields. The likelihoods obtained for the carrying capacities of dwarfs and giants have been used to constrain the logistic curve fitted to the discoveries. Figure 6 shows a sample of fitted logistic curves, each curve being weighted by the likelihood of its carrying capacity given by equation (11).

A comparison with the results in table 1 of Smith (1980) show that our results are coherent. Back in 1980, Smith (1980) used $J = 7$, which fortunately maps easily to our splitting using $J = 2$ as his first reservoir size is 0 to 50 Mb as ours, and the sum of the 6 others yields our second reservoir size 50 Mb and above. His estimated total reserves were 43.2 billion barrels with 90% confidence bounds at 38.4 and 65.0, across 203 dwarfs and 117 giants in our definition. Currently, we know that total estimated reserves are reaching 59 Bb, which is still within the 90% confidence interval of Smith. This strongly speaks in favour of his likelihood technique. Given that, in the past 34 years, there have been major technological improvements and a strong increase in the oil price, it is not surprising that the actual reserves are tending towards his upper bound. A linear scaling by $59/43.2$ (actual reserves over Smith expected reserves) of the number of fields Smith predicted, yields 277 dwarfs and 160 giants. This is close to the 296 ± 21 dwarfs and 156 ± 2 giants of our 60% likelihood estimation found for Norway and the UK combined. Such a strong coherence between the two fully independent implementations using different parameters (e.g. $J = 7$ versus $J = 2$) and different data sets (1980 versus 2014) provides a credible validation of the approach.

II.2.3. Future production from discoveries

We now compute an expected oil production coming from future discoveries, which requires to combine the steps described in sections II.2.1 and II.2.2.

The method described in section II.2.2 yields probabilities for the total number of fields (including the not yet discovered fields) in each size bins (called a scenario). However, this likelihood method does not give the time distribution of future discoveries. We propose to use the likelihood function to generate scenarios with their respective occurrence probability.

For a given scenario, the carrying capacity K (= total number of fields) is

given for each size class. This is useful to resolve the instability in fitting the logistic curve to the number of discovered fields. The time distribution of the discoveries is then given for each size class by the fitted logistic curve.

The actual size of a newly discovered field is generated according to the size distribution of the existing fields in its size class. The probability distribution function of field sizes in a given size bin has been fitted by a stretched exponential function.

The production curve is computed based on the average production curve of all existing fields in the same size category. The production curves of the existing fields have all been normalized to a total production of one and then have been averaged. This yields the typical production profile including a one sigma confidence interval. For a new field, this typical production curve is then multiplied by the size of the field.

Superposing the production curves results in the expected production curve from future oil fields for a given scenario.

As the total parameter space is too large to be sampled entirely, a Monte Carlo technique is applied to compute the expected production with confidence interval from future discoveries. In a nutshell, the algorithm works as follows:

1. Draw a scenario (total number of fields in each size bin) based on its probability according to the likelihood function (11). This is done by generating a random number r between 0 and 1, and computing the scenario that is mapped to r by the cumulative distribution function of all scenarios.
2. Compute the time distribution of new discoveries by fitting a logistic curve for each size class.
3. For each discovery, generate a size and the resulting production curve based on the size distribution and production curves of existing fields.

4. Superpose all the production curves.
5. Repeat and average over all drawn scenarios.

The result is the expected production curve of future oil field discoveries. The distribution of generated scenarios yields the confidence interval.

Last but not least, it has to be defined how the expected production from these future oil field discoveries is added to the extrapolated production from existing fields. The start of the simulation of new discoveries does not match up with the date of the latest production data, the reason being that the new fields (defined in section II.1.1), which are already discovered but did not yet enter the decay phase, are not taken into account in the simulation as their final size is not known. The only meaningful way of treating the new fields is to consider them as a discovery. Therefore, the starting point in time of the simulated production resulting from new discoveries has been chosen as the date in the past where it matches the current production from new fields. In other words, the extrapolated production from regular and irregular fields added to the production from simulated future discoveries (which is shifted into the past as to match the production from new fields) must be equal to the current (latest available data) total production from all fields.

III. RESULTS

Based on the methodology described in section II.2, simulating future oil production was straightforward. For each country, the existing oil field productions were extrapolated and the future discoveries were simulated. Figure 9 shows the predicted production resulting from the extrapolation of existing fields and the average of 1000 simulations of future discoveries. For each country, the non-symmetric shape of the production dynamics, which contradicts the prediction based on Hubbert's standard approach, is immediately noticeable.

The results in table 2 show a striking difference between the Hubbert model, the extrapolation of the fit and the Monte-Carlo model forecast. The preliminary study discussed in the introduction already showed that the Hubbert model forecast was not stable over time and therefore unlikely to be accurate. Indeed the results confirm that the Hubbert forecast is smaller than fitting the decaying process and only half of what the Monte-Carlo model predicts. This is not surprising, because the Hubbert curve is symmetric and therefore the fitting procedure will be strongly biased by the rapid increase in production before the peak, therefore predicting a rapid decrease. Only fitting the decay process using a stretched exponential resolves this problem, because the decay is entirely decoupled from the increase before the peak. Consequently, we expect the simple fit of the decay to perform better.

According to the fit (extrapolation of aggregate production), Norway's future oil production would still decay much faster than in the Monte-Carlo case. The remaining reserves estimated with the Monte-Carlo methodology are 45% larger than the estimate from the fit. This difference originates from two different effects:

- The sum of the forecast of the individual existing fields is larger than the extrapolation of the aggregate production.
- The extrapolation of the aggregate production does not capture well the discovery process of dwarf fields.

In the U.K., oil production faced a change of regime during the early nineties due to technological innovation, giving rise to the inverted "w shape" of the oil production profile. However, this has not been an issue as this inverted "w shape" can be fitted well by a double logistic (Hubbert model). Alternatively, we can extrapolate the decay of the aggregate production starting at the second peak. The differences between the three methodologies are very similar to the

Norwegian case. The Hubbert model again predicts the smallest remaining reserves with about half the amount of the Monte-Carlo model. The fit of the aggregate production underestimates the remaining oil reserves by about 66% compared to the Monte-Carlo model.

Which of the models is more trustworthy? Clearly, the implications in adopting one methodology over the other are significant. The only way to answer this question is to back-test them. In other words: "What would each of the models have predicted, had they been used in the past?" The next section addresses this question and presents the validation step of our approach.

IV. VALIDATION

For both countries, namely Norway and the U.K., a back-test using the data truncated in 2008 has been made. Before that date, too many of the giant fields have not entered their decay phase for a sufficiently long time to apply the extrapolation algorithm. Figure 10 shows the resulting production curves for the fit of the aggregate production and of the Monte-Carlo model.

Comparing the forecast of the models with the oil production of the subsequent 6 years shows that the predictive power of the Monte-Carlo model is significantly better than the Hubbert model or than a simple extrapolation of the aggregate past production. Table 3 summarizes the difference between the approaches for the back-testing period. The Hubbert model has the poorest performance, underestimating production by 19% (Norway) and 47% (UK). The Monte-Carlo model is found essentially on target with less than 5% error compared with the actual production, while the extrapolation of past production ("fit") is under-estimating the realised production by 19% and 16% respectively for Norway and the U.K. These results confirm that modelling the production dynamics of the individual fields and the discovery of new fields yields a more accurate prediction than methodologies that just use aggregate production.

The errors of the Hubbert model and of the extrapolation of the past oil production method (“fit”) are not dramatic over the six years from 2008 to 2014. However, the difference between them and the Monte-Carlo approach becomes huge for the time period from 2014 to 2030, as shown in table 4. The Hubbert model and the simple extrapolation decay too fast and entirely miss the fat tails in the decay process of individual fields and the new discoveries. It is interesting to note that the Hubbert model performs similarly then the fit of aggregate production for the UK, but performs much better for Norway. This is due to the fact that the Hubbert model calibrates the decay slope based on the increase before the peak, which in the case of Norway turns out to be beneficial. The simple extrapolation changed massively between the back-test in table 4 and the current fit in table 2 (520% for Norway and 236% for the U.K.). In other words, the simple extrapolation is very unstable in its forecast, while the Monte-Carlo forecasts remains very consistent (less than 10% change). As can be seen in figure 10, the actual production of Norway during the back-testing period remained entirely within the quite narrow 1σ interval of the Monte-Carlo methodology, while totally breaking out of the 1σ interval of the simple extrapolation. For the U.K. the Monte-Carlo methodology only performs slightly better when considering the confidence interval, and the confidence interval is much larger due to the uncertainty on future discoveries and their production profile.

V. CONCLUSION

The detailed discussion of a full back-test of the Hubbert model for Norway and the UK showed that its prediction were unstable in time and therefore unlikely to be reliable. We argue that the Hubbert model is likely to be unable to correctly capture the fat tail in the decay process as it does not model the fat tails in the decay of individual fields nor the discovery of new fields. An

easily quantifiable link with the oil price, which is the driving factor of oil production, could not be found and we therefore did not pursue the exploration of extensions of the Hubbert model to capture properly the tails of the decay production process.

We preferred a two step Monte-Carlo based methodology to forecast future oil production which: (i) extends the oil production of current fields into the future and (ii) models the discovery rate of new fields. This methodology was then applied to forecast the future oil production of Norway and the U.K. These forecasts are significantly different from the ones obtained with an extrapolation of aggregate production. Indeed, our model forecasts 45% to 66% more remaining oil reserves than the standard extrapolation. The back-test performed on the time period between 2008 and 2014 confirmed that the Monte-Carlo based model better captured the production dynamics.

The results suggest that it is highly likely that the decay of Norwegian and U.K. oil production will be much slower than one would expect from a standard extrapolation. Nonetheless, to maintain current levels of oil consumption in the European Union, more of it will have to be imported from outside Europe, as the imports from Norway will vanish (currently accounting for 11% of E.U. oil imports (European Commission)) and the U.K. will need to import more oil.

As shown in table 5, at constant consumption, the Monte-Carlo model predicts that in 2030 the E.U. with Norway will need to increase its daily oil imports by 1.3 million barrels. These imports will most likely have to come from outside Europe, except for non-standard oil sources yet to be developed.

The present methodology can be applied to many other countries and geological areas, as well as updated at the level of the global oil production. Extensions to include the new wave of shale oil and non-standard oil can in principle be considered and constitute an interesting domain of application of

our methodology for the future.

Acknowledgements: This work derives from a Master thesis at ETH Zurich authored by Marc A. Del Degan (July 2012) under the supervision of the authors, entitled “Analysis of peak oil with focus on Norwegian oil production” (http://www.er.ethz.ch/publications/MAS_Thesis_DelDegan_final_July12.pdf), which itself derived from the methodology of pricing social network companies such as Zynga developed by three of the authors Forró et al. (2012).

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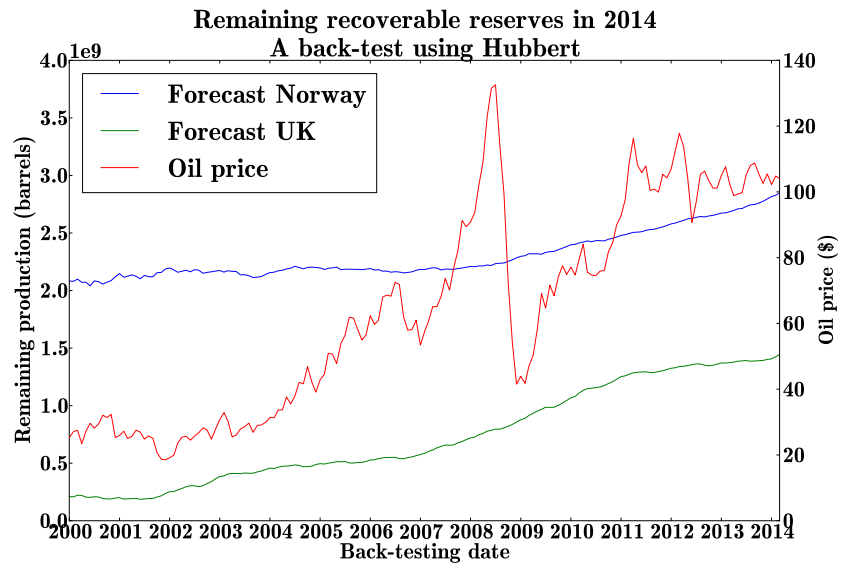


Figure 1: A back-test of the forecast stability of the Hubbert model.

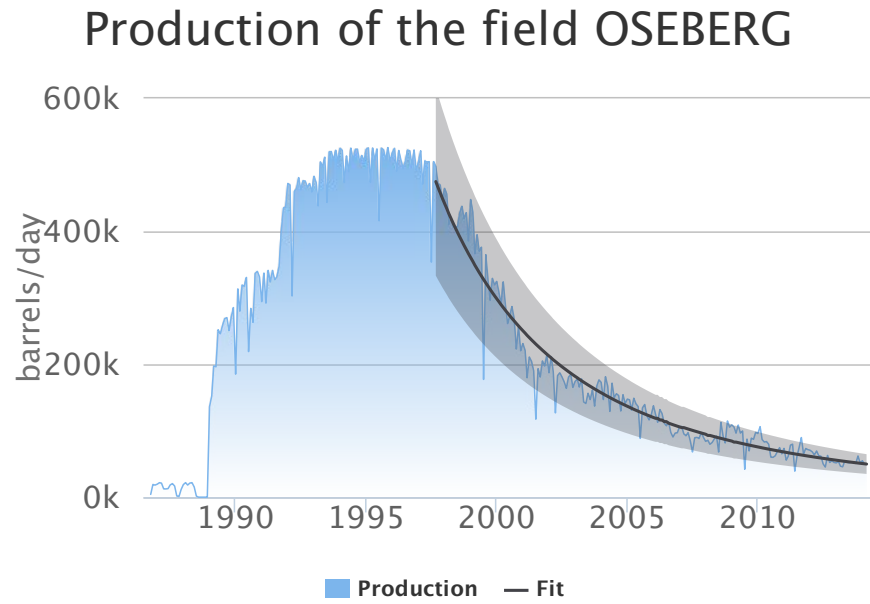


Figure 2: Example of the time evolution of the oil production per day of a regular field, parametrised beyond the peak in the decay regime by the stretched exponential function (2) with $\beta = 0.66 \pm 0.01$ and $\tau = -55 \pm 1$ months, shown with the black line. The one standard deviation given by expression (7) is represented by the grey band.

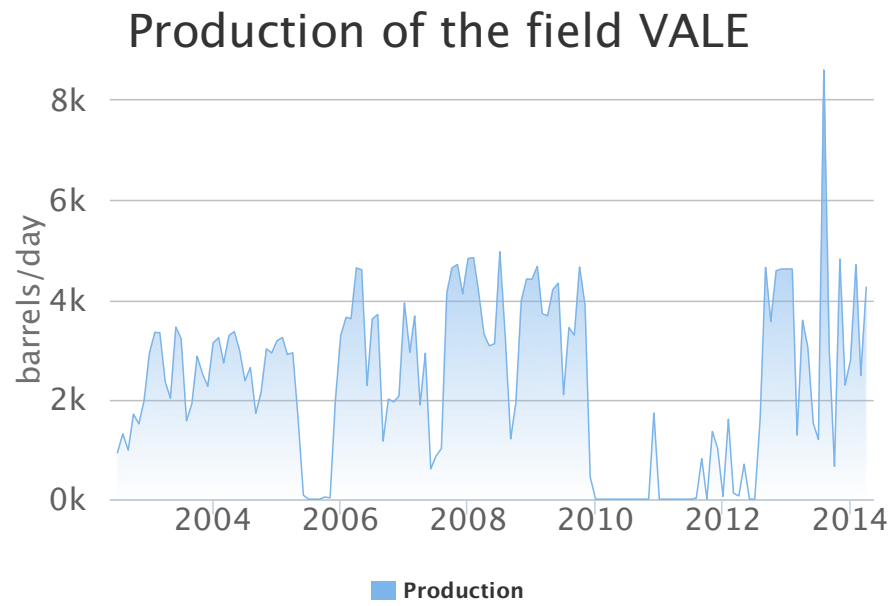


Figure 3: *Example of a irregular field.*

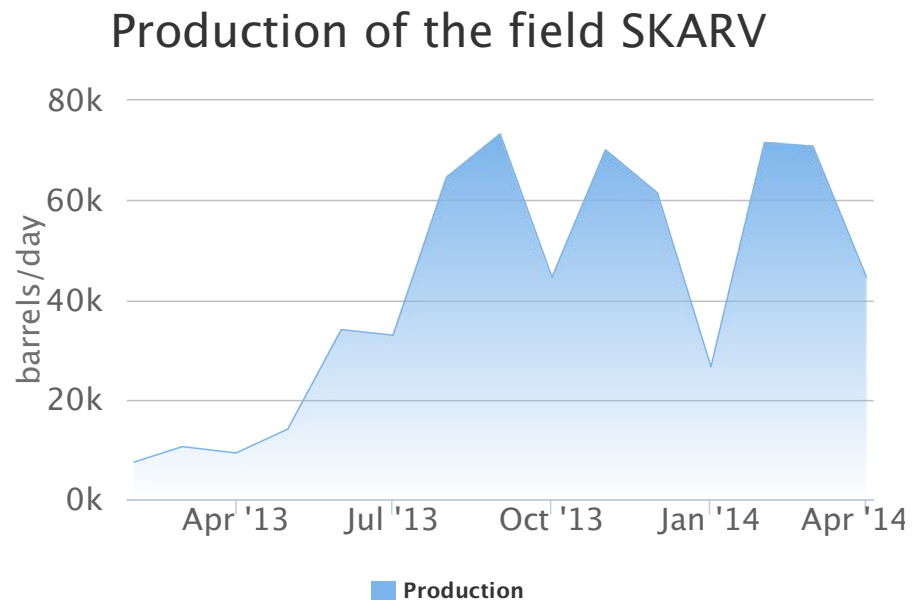


Figure 4: *Example of a new field.*

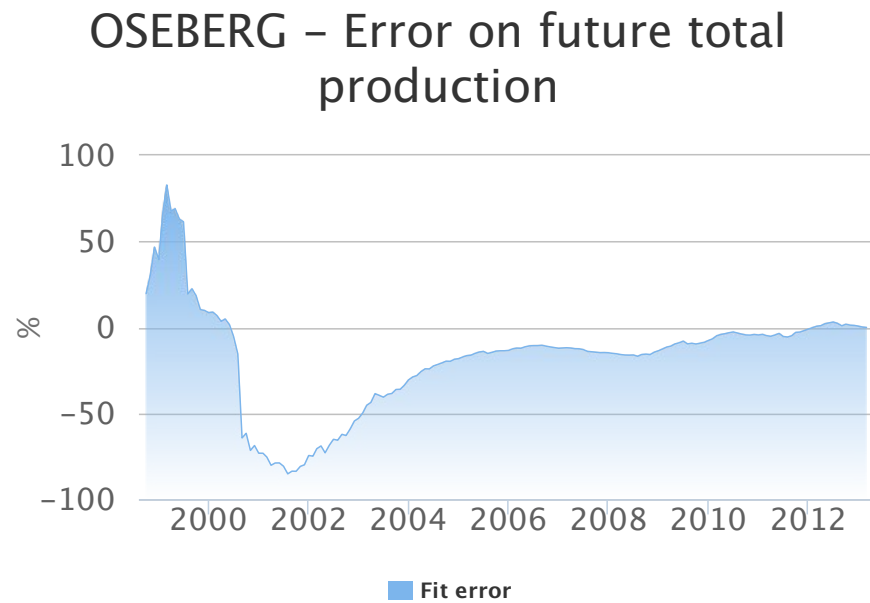


Figure 5: Oseberg field - Relative error defined by expression (4) of the predicted total production from time t indicated in the abscissa until 2014. One can observe that the predicted future total production is over-estimated by as much as 70% in 1999, then under-estimated by the same amount in 2002, while the forecast errors remain smaller than 20% since 2004.

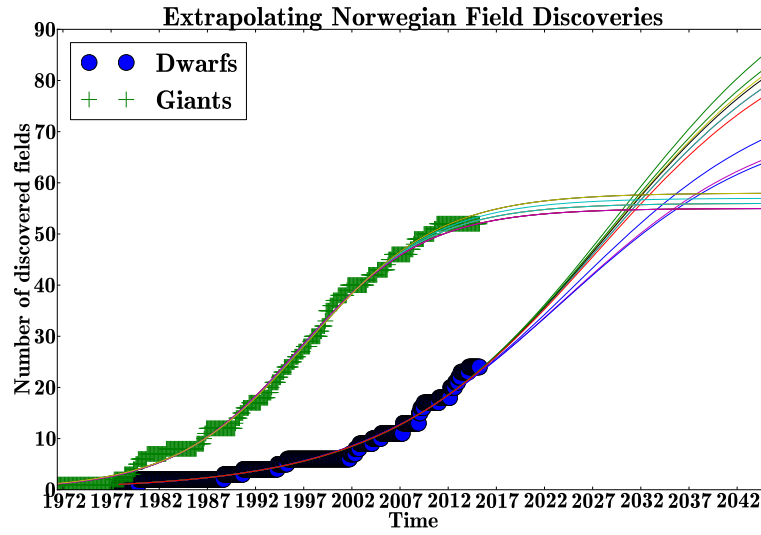


Figure 6: Logistic fit of the function solution of expression (1) to the number of discoveries for Norway. The discovery rate of new oil fields is dependent on their size as explained in the main text.

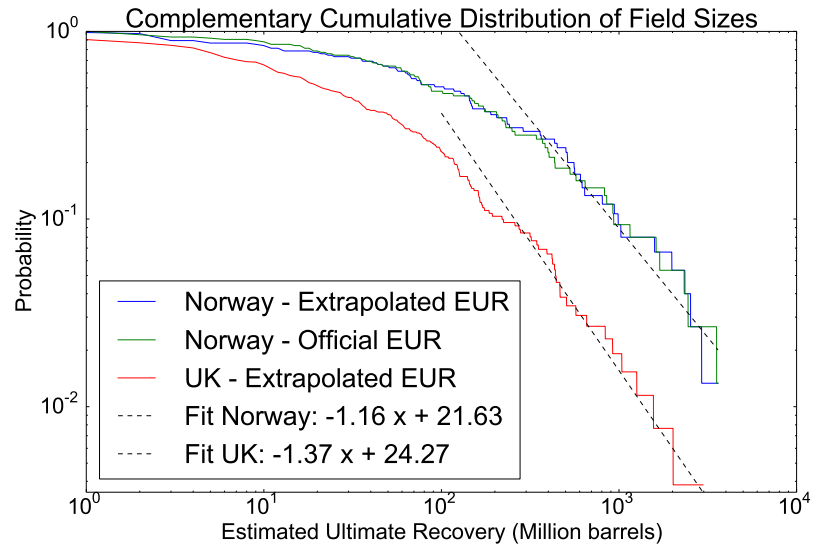


Figure 7: Complementary cumulative distribution function (CCDF) of known oil field sizes S from Norway and the UK. The two dashes lines visualise the power law behaviour (9) of the tail of the distributions.

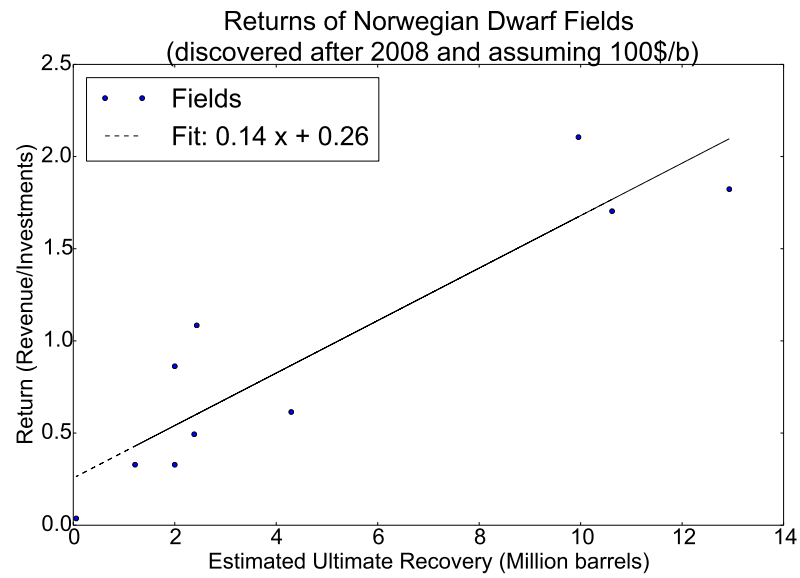
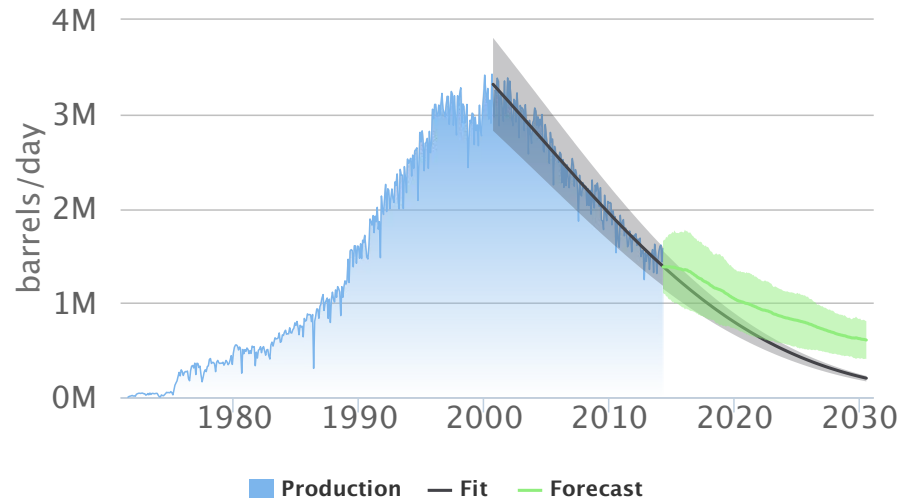


Figure 8: Total estimated revenue divided by investment as a function of the estimated ultimate recovery for a number of small oil fields.

Norwegian oil production and forecast



U.K. oil production and forecast

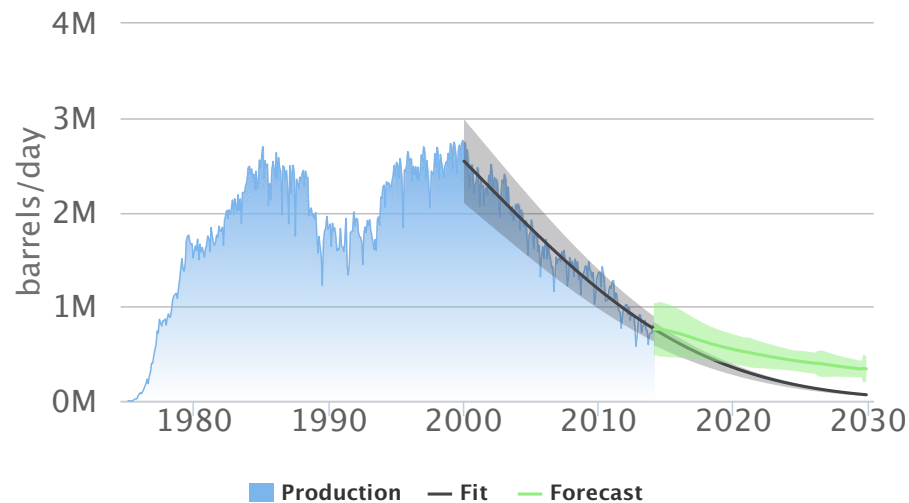
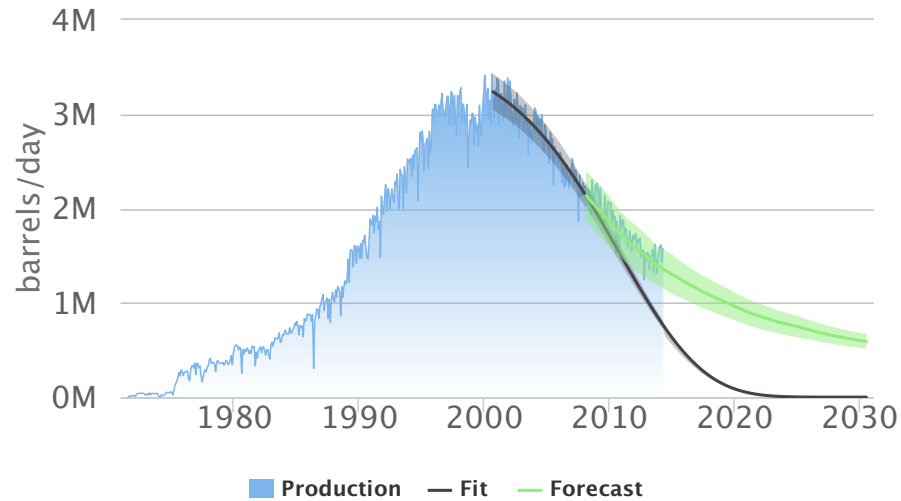


Figure 9: Monte-Carlo (green upper continuous line with standard deviation bank starting in 2014 onward) and fit forecast based on past production data (lower line and grey one standard deviation band) for Norway (top) and the U.K. (bottom). In both cases, the Monte-Carlo model forecasts a significantly slower decay than the fit by taking into account that new fields will come in production.

Norwegian oil production and forecast



U.K. oil production and forecast

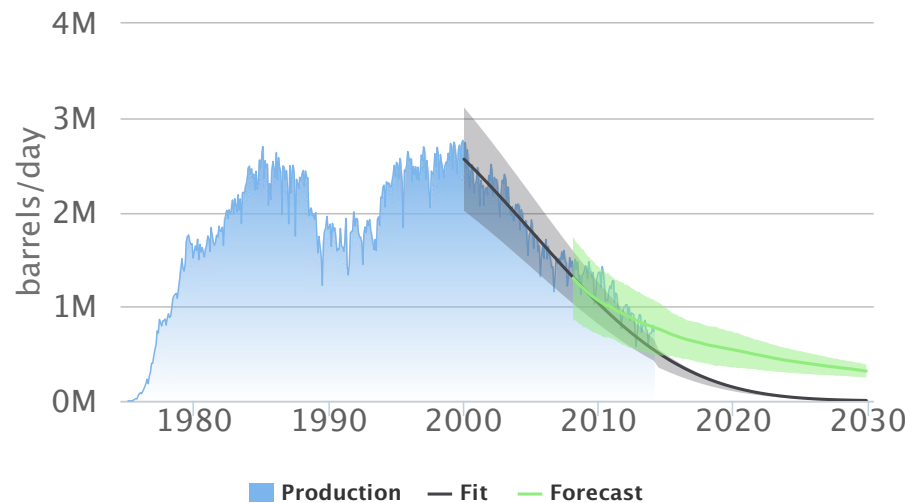


Figure 10: Monte-Carlo (green upper continuous line with standard deviation bank starting in 2014 onward) and Hubbert forecast based on past production data up to 2008 (lower line and grey one standard deviation band) for Norway (top) and the U.K. (bottom). The results can be compared with the subsequent oil production (blue area). In both cases, the Monte-Carlo methodology is more precise.

Table 1: *Likelihood estimation for the number of dwarf (1) and giant (2) fields. For Norway, our logistic fit suggests that up to two new giant fields could be ultimately discovered. For the U.K., the prediction is more bleak, suggesting that the most probable scenario is that one giant field will be found (which is most likely already discovered but classified as a new field).*

	m_{N1}	$n_1 \pm \sigma_1$	m_{N2}	$n_2 \pm \sigma_2$
Norway	24	88.4 ± 10.0	52	56.4 ± 1.6
U.K.	162	208 ± 11	99	100 ± 0.4

Table 2: Remaining oil reserves in billion barrels (Gb) until 2030 predicted by the extrapolation of the **Fit** of the past country production, and predicted by the Monte-Carlo **Model**. The relative difference between these two predictions is defined by $\Delta = \frac{\text{Model}-\text{Fit}}{\text{Fit}}$.

	Hubbert (Gb)	Fit (Gb)	Model (Gb)	Δ
Norway	2.83	3.95	5.72	45%
U.K.	1.43	1.79	2.98	66%

Table 3: *Extrapolation of past oil production (“fit”) and prediction using the Monte-Carlo model are made on the data set truncated in 2008. Their forecast for the period 2008-2014 is compared to the actual realised production. All units in billion barrels (Gb).*

	Actual (Gb)	Hubbert (Gb)	Fit (Gb)	Model (Gb)
Norway	4.05	3.29	3.28	3.95
U.K.	2.40	1.63	2.00	2.28

Table 4: *Remaining oil reserves in billion barrels (Gb) forecasted for the period 2014–2030 when using the data truncated in 2008, according to the extrapolation of past oil production (“fit”) and the Monte-Carlo model. The relative difference between these two predictions is defined by $\Delta = \frac{\text{Model}-\text{Fit}}{\text{Fit}}$.*

	Hubbert (Gb)	Fit (Gb)	Model (Gb)	Δ
Norway 08	2.21	0.76	5.20	–584%
U.K. 08	0.73	0.76	2.77	–264%

Table 5: *Oil import in million barrels per day (Mb/day) at a constant consumption of 1.5 Mb/day for the U.K. and 0.22 Mb/day for Norway. The import for the E.U. and Norway is a lower bound based on the changes in the U.K and Norway. Negative numbers for Norway represent exports.*

	2014	2020	2025	2030
Norway (Mb/day)	−1.23	−0.88	−0.58	−0.43
U.K. (Mb/day)	0.7	0.9	1.0	1.1
E.U. + Norway (Mb/day)	9.8	10.45	10.85	11.1