7 Week 7: The triple scalar product

The *triple scalar product* $(\vec{a}, \vec{b}, \vec{c})$ of the vectors $\vec{a}, \vec{b}, \vec{c}$ is the real number $(\vec{a} \times \vec{b}) \cdot \vec{c}$.

Proposition 7.1. *If* $[\vec{i}, \vec{j}, \vec{k}]$ *is a direct orthonormal basis and*

$$\begin{aligned} \vec{a} &= a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k} \\ \vec{b} &= b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k} \\ \vec{c} &= c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k} \end{aligned}$$

then

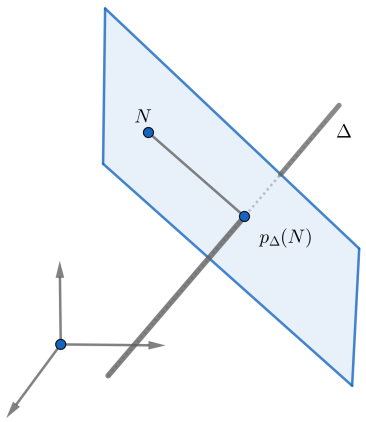
$$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \tag{7.1}$$

Proof. Indeed, we have successively:

$$\begin{aligned} (\vec{a}, \vec{b}, \vec{c}) &= (\vec{a} \times \vec{b}) \cdot \vec{c} \\ &= \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k} \right) \cdot (c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k}) \\ &= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} c_1 - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} c_2 + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} c_3 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}. \end{aligned}$$

□

Remark 7.1. Taking into account the formula (7.2) for the distance $\delta(N, \Delta)$ from the point $N(x_N, y_N, z_N)$ to the straight line $\Delta : \dfrac{x - x_0}{p} = \dfrac{y - y_0}{q} = \dfrac{z - z_0}{r}$ as well as Proposition 6.3 we deduce that



$$\begin{aligned} \delta(N, \Delta) &= \| \overrightarrow{Np_{_\Delta}(N)} \| \\ &= \| \overrightarrow{NO} + \overrightarrow{Op_{_\Delta}(N)} \| = \left\| \overrightarrow{NA_0} - \frac{\vec{d}_{_\Delta} \cdot \overrightarrow{NA_0}}{\| \vec{d}_{_\Delta} \|^2} \vec{d}_{_\Delta} \right\| \end{aligned} \tag{7.2}$$

$$= \frac{\| \left(\vec{d}_\Delta \cdot \vec{d}_\Delta\right) \overrightarrow{NA_0} - \left(\vec{d}_\Delta \cdot \overrightarrow{NA_0}\right) \vec{d}_\Delta \|}{\| \vec{d}_\Delta \|^2}$$
$$= \frac{\| \vec{d}_\Delta \times (\overrightarrow{NA_0} \times \vec{d}_\Delta)\|}{\| \vec{d}_\Delta \|^2} = \frac{\| \overrightarrow{NA_0} \times \vec{d}_\Delta \|}{\| \vec{d}_\Delta \|}.$$

Thus, we recovered the distance formula from one point to one straight line (see formula 6.4) by using different arguments.

Corollary 7.2. *1. The free vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent (collinear) iff $(\vec{a}, \vec{b}, \vec{c}) = 0$*

2. The free vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly independent (noncollinear) if and only if $(\vec{a}, \vec{b}, \vec{c}) \neq 0$

3. The free vectors $\vec{a}, \vec{b}, \vec{c}$ form a basis of the space $\mathcal{V}$ if and only if $(\vec{a}, \vec{b}, \vec{c}) \neq 0$.

4. The correspondence $F : \mathcal{V} \times \mathcal{V} \times \mathcal{V} \to \mathbb{R}$, $F(\vec{a}, \vec{b}, \vec{c}) = (\vec{a}, \vec{b}, \vec{c})$ is trilinear and skew-symmetric, i.e.

$$\begin{array}{c} (\alpha \vec{a} + \alpha' \vec{a}\,', \vec{b}, \vec{c}) = \alpha(\vec{a}, \vec{b}, \vec{c}) + \alpha'(\vec{a}\,', \vec{b}, \vec{c}) \\ (\vec{a}, \beta \vec{b} + \beta' \vec{b}\,', \vec{c}) = \beta(\vec{a}, \vec{b}, \vec{c}) + \beta'(\vec{a}, \vec{b}\,', \vec{c}) \\ (\vec{a}, \vec{b}, \gamma \vec{c} + \gamma' \vec{c}\,') = \gamma(\vec{a}, \vec{b}, \vec{c}) + \gamma'(\vec{a}, \vec{b}, \vec{c}\,') \end{array} \tag{7.3}$$

$\forall\, \alpha, \beta, \gamma, \alpha', \beta', \gamma' \in \mathbb{R}$, $\forall\ \vec{a}, \vec{b}, \vec{c}, \vec{a}\,', \vec{b}\,', \vec{c}\,' \in \mathcal{V}$ *şi*

$$(\vec{a}_1, \vec{a}_2, \vec{a}_3) = sgn(\sigma)(\vec{a}_{\sigma(1)}, \vec{a}_{\sigma(2)}, \vec{a}_{\sigma(3)}),\ \forall\ \vec{a}_1, \vec{a}_2, \vec{a}_3 \in \mathcal{V} \text{ şi } \forall \sigma \in S_3 \tag{7.4}$$

Remark 7.2. *One can rewrite the relations (7.4) as follows:*

$$\begin{aligned} (\vec{a}_1, \vec{a}_2, \vec{a}_3) &= (\vec{a}_2, \vec{a}_3, \vec{a}_1) = (\vec{a}_3, \vec{a}_1, \vec{a}_2) \\ &= -(\vec{a}_2, \vec{a}_1, \vec{a}_3) = -(\vec{a}_1, \vec{a}_3, \vec{a}_2) = -(\vec{a}_3, \vec{a}_2, \vec{a}_1), \end{aligned}$$

$\forall\ \vec{a}_1, \vec{a}_2, \vec{a}_3 \in \mathcal{V}$

Corollary 7.3. *1. $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c}) \forall\ \vec{a}, \vec{b}, \vec{c} \in \mathcal{V}$.*

2. For every $\vec{a}, \vec{b}, \vec{c}, \vec{d} \in \mathcal{V}$ the Laplace formula holds:

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}.$$

Proof. While the first identity is obvious, for the Laplace formula we have successively:

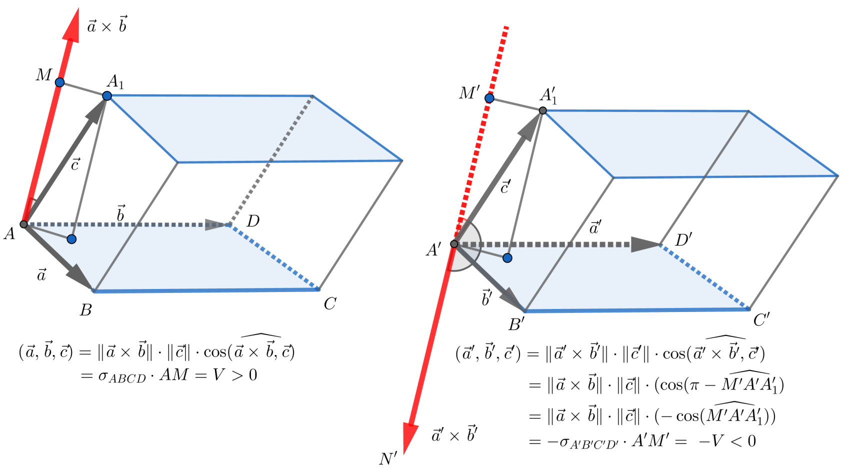
$$\begin{aligned} (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) &= \left(\vec{a}, \vec{b}, \vec{c} \times \vec{d}\right) = \left(\vec{c} \times \vec{d}, \vec{a}, \vec{b}\right) \\ &= \left[(\vec{c} \times \vec{d}) \times \vec{a}\right] \cdot \vec{b} = -\left[(\vec{a} \cdot \vec{d})\, \vec{c} - (\vec{a} \cdot \vec{c})\, \vec{d}\right] \cdot \vec{b} \\ &= -(\vec{a} \cdot \vec{d})(\vec{c} \cdot \vec{b}) + (\vec{a} \cdot \vec{c})(\vec{d} \cdot \vec{b}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}. \end{aligned}$$

□

Definition 7.1. The basis $[\vec{a}, \vec{b}, \vec{c}]$ of the space $\mathcal{V}$ is said to be *directe* if $(\vec{a}, \vec{b}, \vec{c}) > 0$. If, on the contrary, $(\vec{a}, \vec{b}, \vec{c}) < 0$, we say that the basis $[\vec{a}, \vec{b}, \vec{c}]$ is *inverse*

Definition 7.2. The *oriented* volume of the parallelepiped constructed on the noncoplanar vectors $\vec{a}, \vec{b}, \vec{c}$ is $\varepsilon \cdot V$, where V is the volume of this parallelepiped and $\varepsilon = +1$ or -1 insomuch as the basis $[\vec{a}, \vec{b}, \vec{c}]$ is directe or inverse respectively.

Proposition 7.4. *The triple scalar product* $(\vec{a}, \vec{b}, \vec{c})$ *of the noncoplanar vectors* $\vec{a}, \vec{b}, \vec{c}$ *is equal with the oriented volume of the parallelepiped constructed on these vectors.*



7.1 Applications of the triple scalar product

7.1.1 The distance between two straight lines

If $d_1,\ d_2$ are two straight lines, then the distance between them, denoted by $\delta(d_1, d_2)$, is being defined as

$$\min\{||\ \overrightarrow{M_1M_2}\ ||\ |\ M_1 \in d_1,\ M_2 \in d_2\}.$$

1. If $d_1 \cap d_2 \neq \emptyset$, then $\delta(d_1, d_2) = 0$.

2. If $d_1 || d_2$, then $\delta(d_1, d_2) = ||\ \overrightarrow{MN}\ ||$ where $\{M\} = d \cap d_1$, $\{N\} = d \cap d_2$ and d is a straight line perpendicular to the lines d_1 and d_2. Obviously $||\ \overrightarrow{MN}\ ||$ is independent on the choice of the line d.

3. We now assume that the straight lines $d_1,\ d_2$ are noncoplanar (skew lines). In this case there exits a unique straight line d such that $d \perp d_1, d_2$ and $d \cap d_1 = \{M_1\},\ d \cap d_2 = \{M_2\}$. The straight line d is called the *common perpendicular* of the lines $d_1,\ d_2$ and obviously $\delta(d_1, d_2) = ||\ \overrightarrow{M_1M_2}\ ||$.

Assume that the straight lines $d_1,\ d_2$ are given by their points $A_1(x_1, y_1, z_1)$, $A_2(x_2, y_2, z_2)$ and their vectors şi au vectorii directori $\vec{d}_1\ (p_1, q_1, r_1)\ \vec{d}_2\ (p_2, q_2, r_2)$, that is, thei equations are

$$d_1 : \frac{x - x_1}{p_1} = \frac{y - y_1}{q_1} = \frac{z - z_1}{r_1}$$

$$d_2 : \frac{x - x_2}{p_2} = \frac{y - y_2}{q_2} = \frac{z - z_2}{r_2}.$$

The common perpendicular of the lines d_1, d_2 is the intersection line between the plane containing the line d_1 which is parallel to the vector $\vec{d}_1 \times \vec{d}_2$, and the plane containing the line d_2 which is parallel to $\vec{d}_1 \times \vec{d}_2$. Since

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} = \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} \vec{i} + \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix} \vec{j} + \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \vec{k}$$

it follows that the equations of the common perpendicular are

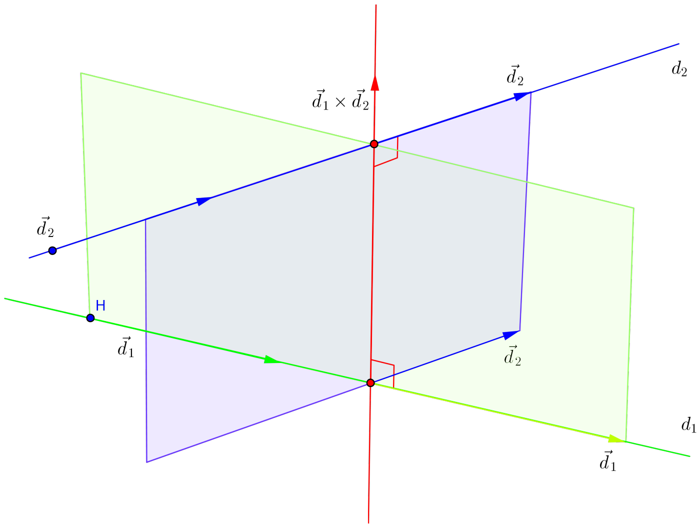


Figure 8: Perpendiculara comună a dreptelor d_1 şi d_2

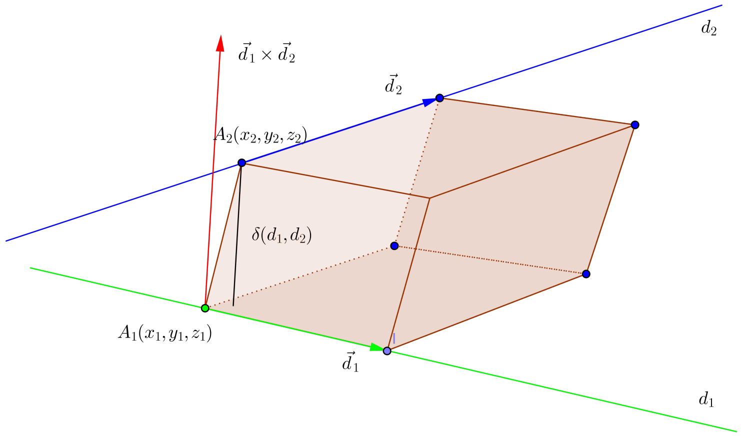
$$\begin{cases} \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ p_1 & q_1 & r_1 \\ \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} & \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix} & \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \end{vmatrix} = 0 \\ \begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ p_2 & q_2 & r_2 \\ \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} & \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix} & \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \end{vmatrix} = 0. \end{cases} \tag{7.5}$$

The distance between the straight lines d_1, d_2 can be also regarded as the height of the parallelogram constructed on the vectors $\vec{d}_1, \vec{d}_2, \vec{d}_1 \times \vec{d}_2$. Thus

$$\delta(d_1, d_2) = \frac{|(\overrightarrow{A_1A_2}, \vec{d}_1, \vec{d}_2)|}{\| \vec{d}_1 \times \vec{d}_2 \|}. \tag{7.6}$$

Therefore we obtain

$$\delta(d_1, d_2) = \frac{\left| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} \right|}{\sqrt{\begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix}^2 + \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix}^2 + \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix}^2}} \tag{7.7}$$



7.1.2 The coplanarity condition of two straight lines

Using the notations of the previous section, observe that the straight lines d_1, d_2 are coplanar if and only if the vectors $\overrightarrow{A_1A_2}, \vec{d}_1, \vec{d}_2$ are linearly dependent (coplanar), or equivalently $(\overrightarrow{A_1A_2}, \vec{d}_1, \vec{d}_2) = 0$. Consequently the stright lines d_1, d_2 are coplanar if and only if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} = 0 \tag{7.8}$$

7.2 Problems

1. **(2p)** Show that

   (a) $|(\vec{a}, \vec{b}, \vec{c})| \leq \| \vec{a} \| \cdot \| \vec{b} \| \cdot \| \vec{c} \|$;
   Solution.

(b) **(2p)** $(\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}) = 2(\vec{a}, \vec{b}, \vec{c})$.
Solution.

2. **(3p)** Prove the following identity:

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = (\vec{a}, \vec{c}, \vec{d})\, \vec{b} - (\vec{b}, \vec{c}, \vec{d})\, \vec{a} = (\vec{a}, \vec{b}, \vec{d})\, \vec{c} - (\vec{a}, \vec{b}, \vec{c})\, \vec{d}\,.$$

Solution. By using the identity $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\, \vec{v} - (\vec{u} \cdot \vec{v})\, \vec{v}$ for $\vec{u} = \vec{a} \times \vec{b}$, $\vec{v} = \vec{c}$ and $\vec{w} = \vec{d}$ we obtain

$$\begin{aligned}(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) &= \vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\, \vec{v} - (\vec{u} \cdot \vec{v})\, \vec{v} \\ &= \left[(\vec{a} \times \vec{b}) \cdot \vec{d}\right] \vec{c} - \left[(\vec{a} \times \vec{b}) \cdot \vec{c}\right] \vec{d} \\ &= (\vec{a}, \vec{b}, \vec{d})\, \vec{c} - (\vec{a}, \vec{b}, \vec{c})\, \vec{d}\,.\end{aligned}$$

By using the identity $(\vec{u} \times \vec{v}) \times \vec{w} = (\vec{u} \cdot \vec{w})\, \vec{v} - (\vec{v} \cdot \vec{w})\, \vec{u}$ for $\vec{u} = \vec{a}$, $\vec{v} = \vec{b}$ and $\vec{w} = \vec{c} \times \vec{d}$ we obtain

$$\begin{aligned}(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) &= (\vec{u} \times \vec{v}) \times \vec{w} = (\vec{u} \cdot \vec{w})\, \vec{v} - (\vec{v} \cdot \vec{w})\, \vec{u} \\ &= \left[\vec{a} \cdot (\vec{c} \times \vec{d})\right] \vec{b} - \left[\vec{b} \cdot (\vec{c} \times \vec{d})\right] \vec{a} \\ &= (\vec{a}, \vec{c}, \vec{d})\, \vec{b} - (\vec{b}, \vec{c}, \vec{d})\, \vec{a}\,.\end{aligned}$$

3. **(3p)** Prove the following identity: $(\vec{u} \times \vec{v}, \vec{v} \times \vec{w}, \vec{w} \times \vec{u}) = (\vec{u}, \vec{v}, \vec{w})^2$.

Solution. We have successively:

$$\begin{aligned}(\vec{u} \times \vec{v}, \vec{v} \times \vec{w}, \vec{w} \times \vec{u}) &= \left[(\vec{u} \times \vec{v}) \times (\vec{v} \times \vec{w})\right] \cdot (\vec{w} \times \vec{u}) \\ &= \left[(\vec{u}, \vec{v}, \vec{w})\, \vec{v} - (\vec{u}, \vec{v}, \vec{v})\, \vec{w}\right] \cdot (\vec{w} \times \vec{u}) \\ &= (\vec{u}, \vec{v}, \vec{w}) \left[\vec{v} \cdot (\vec{w} \times \vec{u})\right] = (\vec{u}, \vec{v}, \vec{w})(\vec{v}, \vec{w}, \vec{u}) = (\vec{u}, \vec{v}, \vec{w})^2.\end{aligned}$$

4. **(3p)** The *reciprocal vectors* of the noncoplanar vectors $\vec{u}, \vec{v}, \vec{w}$ are defined by

$$\vec{u}' = \frac{\vec{v} \times \vec{w}}{(\vec{u}, \vec{v}, \vec{w})},\quad \vec{v}' = \frac{\vec{w} \times \vec{u}}{(\vec{u}, \vec{v}, \vec{w})},\quad \vec{w}' = \frac{\vec{u} \times \vec{v}}{(\vec{u}, \vec{v}, \vec{w})}.$$

Show that:

(a)

$$\begin{aligned}\vec{a} &= (\vec{a} \cdot \vec{u}')\, \vec{u} + (\vec{a} \cdot \vec{v}')\, \vec{v} + (\vec{a} \cdot \vec{w}')\, \vec{w} \\ &= \frac{(\vec{a}, \vec{v}, \vec{w})}{(\vec{u}, \vec{v}, \vec{w})}\, \vec{u} + \frac{(\vec{u}, \vec{a}, \vec{w})}{(\vec{u}, \vec{v}, \vec{w})}\, \vec{v} + \frac{(\vec{u}, \vec{v}, \vec{a})}{(\vec{u}, \vec{v}, \vec{w})}\, \vec{w}\,.\end{aligned}$$

(b) the reciprocal vectors of $\vec{u}', \vec{v}', \vec{w}'$ are the vectors $\vec{u}, \vec{v}, \vec{w}$.

Solution. (4a) Obviously $\vec{a} = \alpha\, \vec{u} + \beta\, \vec{v} + \gamma\, \vec{c}$, as $\vec{u}, \vec{v}, \vec{w}$ are three linearly independent vectors of the three dimensional vector space $\mathcal{V}$, i.e. $\vec{u}, \vec{v}, \vec{w}$ form a basis of $\mathcal{V}$. Moreover we have

$$\begin{aligned}\vec{a} \cdot \vec{u}' &= \frac{\vec{a} \cdot (\vec{v} \times \vec{w})}{(\vec{u}, \vec{v}, \vec{w})} = \frac{(\vec{a}, \vec{v}, \vec{w})}{(\vec{u}, \vec{v}, \vec{w})} = \frac{(\alpha\, \vec{u} + \beta\, \vec{v} + \gamma\, \vec{c}, \vec{v}, \vec{w})}{(\vec{u}, \vec{v}, \vec{w})} \\ &= \frac{\alpha(\vec{u}, \vec{v}, \vec{w}) + \beta(\vec{v}, \vec{v}, \vec{w}) + \gamma(\vec{w}, \vec{v}, \vec{w})}{(\vec{u}, \vec{v}, \vec{w})} = \alpha.\end{aligned}$$

One can similalry show that

$$\vec{a} \cdot \vec{v}' = \frac{(\vec{u}, \vec{a}, \vec{w})}{(\vec{u}, \vec{v}, \vec{w})} = \beta \text{ and } \vec{a} \cdot \vec{w}' = \frac{(\vec{u}, \vec{v}, \vec{a})}{(\vec{u}, \vec{v}, \vec{w})} = \gamma.$$

(4b) Let us first observe that

$$(\vec{u}', \vec{v}', \vec{w}') = (\vec{w}', \vec{u}', \vec{v}') = \frac{(\vec{u} \times \vec{v}, \vec{v} \times \vec{w}, \vec{w} \times \vec{u})}{(\vec{u}, \vec{v}, \vec{w})^3} = \frac{(\vec{u}, \vec{v}, \vec{w})^2}{(\vec{u}, \vec{v}, \vec{w})^3} = \frac{1}{(\vec{u}, \vec{v}, \vec{w})}.$$

On the other hand we have:

$$\frac{\vec{v}' \times \vec{w}'}{(\vec{u}', \vec{v}', \vec{w}')} = (\vec{u}, \vec{v}, \vec{w})(\vec{v}' \times \vec{w}') = (\vec{u}, \vec{v}, \vec{w}) \frac{(\vec{w} \times \vec{u}) \times (\vec{u} \times \vec{v})}{(\vec{u}, \vec{v}, \vec{w})^2} = \frac{(\vec{w}, \vec{u}, \vec{v})\, \vec{u} - (\vec{w}, \vec{u}, \vec{u})\, \vec{v}}{(\vec{u}, \vec{v}, \vec{w})} = \vec{u}\,.$$

One can similarly show that

$$\frac{\vec{w}' \times \vec{u}'}{(\vec{u}', \vec{v}', \vec{w}')} = \vec{v} \text{ and } \frac{\vec{u}' \times \vec{v}'}{(\vec{u}', \vec{v}', \vec{w}')} = \vec{w}\,.$$

5. **(2p)** Find the value of the parameter α for which the pencil of planes through the straight line AB has a common plane with the pencil of planes through the straight line CD, where $A(1, 2\alpha, \alpha)$, $B(3, 2, 1)$, $C(-\alpha, 0, \alpha)$ and $D(-1, 3, -3)$.

   Solution.

6. **(2p)** Find the value of the parameter λ for which the straight lines

$$(d_1)\ \frac{x-1}{3}=\frac{y+2}{-2}=\frac{z}{1},\ (d_2)\ \frac{x+1}{4}=\frac{y-3}{1}=\frac{z}{\lambda}$$

are coplanar. Find the coordinates of their intersection point in that case.

Solution.

7. **(2p)** Find the distance between the straight lines

$$(d_1)\ \frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{1},\ (d_2)\ \frac{x+1}{3}=\frac{y}{4}=\frac{z-1}{3}$$

as well as the equations of the common perpendicular.

Solution.

8. **(2p)** Find the distance between the straight lines M_1M_2 and d, where $M_1(-1,0,1)$, $M_2(-2,1,0)$ and

$$(d)\left\{\begin{array}{rcrcrcl} x & + & y & + & z & = & 1 \\ 2x & - & y & - & 5z & = & 0. \end{array}\right.$$

as well as the equations of the common perpendicular.

Solution.

