Laboratory 1

0. Read carefully and try the commands written in the file Lab1-Sage-tutorial.pdf or in the first 3 pages of the file Lab-Maple8-tutorial.pdf

1. Evaluate the number using floating point arithmetic: $\frac{1}{2}$ (1./2) ; e=exp(1) ; $\sqrt{3}$ (sqrt(3.));

π (evalf(Pi) in Maple, numerical_approx(pi) in Sage) ; $\sin(0.1)$

2. Assign the following expression to a variable and then expand it:

a) $(x^2 + 2 \cdot x - 1)^3 \cdot (x^2 - 2)$ b*) $(x + n)^5$. Unassign the used variables (a:='a' in Maple, a=var('a') in Sage).

3. Factorize: $x^8 - 1$

4. Add the following rational expressions by applying the factor command: $\frac{2 \cdot x^2}{x^3 - 1} + \frac{3 \cdot x}{x^2 - 1}$

5. Simplify the trigonometric expression (In Maple using simplify(expression, trig) and in Sage using expr.trig_simplify()): $\sin(x)^2 + \cos(x)^2$

6. Evaluate using both subs and eval the expression $e^x + \ln(x)$ in x=1.

7. Solve: a) the equation $x^2 - 4 \cdot x + 3 = 0$ where x is the unknown;
b) the equation $x^2 \cdot y + 2 \cdot y - x = 0$ where x is a parameter and y is the unknown;
c*) the equation $x^2 \cdot y + 2 \cdot y - x = 0$ where y is a parameter and x is the unknown;
d) the equation $x - \cos(x) = 0$ where x is the unknown; (for this equation one can find only approximate roots. So, use fsolve in Maple, and find_root in Sage. Hint: it has 1 real root, and it belongs to the interval [-1,1])
e*) the equation $x^5 - 3 \cdot x^3 - 1 = 0$ where x is the unknown; (for this equation one can find only approximate roots. So, use fsolve in Maple, and find_root in Sage. Hint: it has 3 real roots, all in the interval [-2,2])
f) the system of two equations $4x + 3y = 10$, $3x - y = 1$ where x and y are the unknowns.

8* (only in Maple). Assign to a variable f the function (not the expression) $f: R \rightarrow R$, $f(x) = e^x - \sin(x)$. Evaluate $f(0), f(-1), f'(0), f'(1)$. Calculate the first and second order derivatives of f (using both D and diff) and a primitive of f. Evaluate $\int_{-1}^{1} f(x) \, dx$. Unassign f. Do not forget that for f'(0) (the derivative of a function in a given point you just have to write D(f)(0)).

9. Assign to a variable g the expression $e^x - \sin(x)$. Evaluate this expression in x=0. Compute its first order derivative and then evaluate it in x=0. Find a primitive. Evaluate $\int_{-1}^{1} g \, dx$. Assign to a variable f the second order derivative of g and evaluate it in x=0.

10. Find $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x}$ and $\lim\limits_{x \to \pi} \dfrac{\cos(x) + 1}{x - \pi}$.

11. First plot the graph of $f(x) = \sin(x)$ by hand on paper. Then plot it using Sage in the interval $[0, 2 \cdot \pi]$, or using Maple, in each of the intervals: $[0, 2 \cdot \pi]$, $[-4\pi, 4 \cdot \pi]$, $[-100, 100]$, $(-\infty, \infty)$.

12. First plot the graph of $f(x) = \dfrac{1}{x}$ by hand on paper. Then plot it using Maple or Sage in the interval [-1,1]. In Sage use the option detect_poles=True and specify a range for the values of this function.

13. Plot the planar curve of parametric equations $x = 2 - t^2$, $y = t - t^3$, $t \in [-2, 2]$.

14. First plot by hand on paper the planar curve of parametric equations $x = \cos(t)$, $y = \sin(t)$ in each of the intervals: $\left[0, \dfrac{\pi}{6}\right]$, $\left[0, \dfrac{\pi}{3}\right]$, $\left[0, \dfrac{\pi}{2}\right]$, $[0, \pi]$, $\left[0, \dfrac{3 \cdot \pi}{2}\right]$, $[0, 2 \cdot \pi]$, $[0, 4 \cdot \pi]$.
Then do the same using Maple or Sage. What do you notice?

15*. Plot the planar curves of parametric equations a) $x = 2 \cdot \cos\left(\dfrac{t}{3}\right)$, $y = 2 \cdot \sin\left(\dfrac{t}{3}\right)$
b) $x = \cos(4 \cdot t)$, $y = \sin(4 \cdot t)$ in different intervals at your choice. What do you notice?

16. Write by hand on paper the implicit equations of a circle centered in (9,10), an ellipse and a parabola. Then plot using Maple or Sage at leats a circle, an ellipse and a parabola.

17. Plot the planar curves of implicit equations a) $x^2 - 2 \cdot x \cdot y - y^2 = 1$ b*) $y^3 + y^2 - 5 \cdot y - x^2 = -4$. You have to choose properly a rectangle where to see the curve.

18. First draw by hand on paper the graph of the function $H(x, y) = x^2 + y^2$. Then plot it in 3d using Maple or Sage (using the command plot3d). For the variable (x,y) choose a rectangle centered at (0,0). What remarkable planar curves are the level curves of H?