Università della Svizzera italiana

Facoltà di scienze informatiche

Software Atelier: Differential Equations **Academic Year 2016/2017**

Instructor: Dr. Drosos Kourounis TA: Hardik Kothari

Assignment 2 - Reading assignment

Due date: Monday 10 October 2016

Name: Juraj Kardos
Discussed with: Mr. Nobody

1. section 1

Given the problem

$$-\Delta u + u = f \text{ in } \Omega \tag{1}$$

subject to Dirichlet BC

$$u = u_0 \text{ on } \partial\Omega_D \tag{2}$$

and the Neumann BC

$$\nabla u \cdot n = g \text{ on } \partial\Omega_N \tag{3}$$

we project the problem into the function space of the test function v, that is multiply the equation by the test function and integrate over the domain:

$$\int_\Omega (-\Delta u + u)v \, d\Omega = \int_\Omega fv \, d\Omega. \tag{4}$$

We apply integration by parts on the LHS of the equation (4):

$$\int_\Omega (-\Delta u + u)v \, d\Omega = -\int_\Omega \nabla(v\nabla u) \, d\Omega - \int_\Omega (\nabla v \nabla u) \, d\Omega + \int_\Omega vu \, d\Omega. \tag{5}$$

We further apply Gauss theorem on the first RHS term in equation (5) and use Neumann BC:

$$-\int_\Omega \nabla(v\nabla u) \, d\Omega = -\int_{\partial\Omega} (v\nabla u)n \, d\partial\Omega = -\int_{\partial\Omega} vg \, d\partial\Omega. \tag{6}$$

Substituting (6) back into (5) and (4) we obtain:

$$\int_\Omega (-\nabla v \nabla u + uv) \, d\Omega = \int_\Omega fv \, d\Omega + \int_{\partial\Omega} vg \, d\partial\Omega, \tag{7}$$

which is the weak formulation of the original problem.