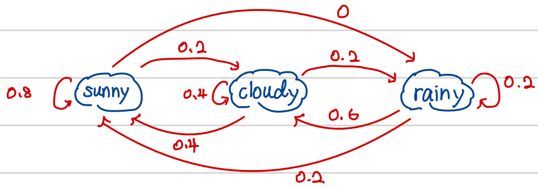
SDC, HW2　　A111137　邱柏諠

Exercise 2.

		tomorrow will be…		
		sunny	cloudy	rainy
today it's…	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2



(a) Day 1 is sunny.

c → c → r

$0.2 \times 0.4 \times 0.2 = 0.016$ #

(b)

```python
#!/usr/bin/env python3

import random

def main():
    print("HW2-1")
    future_day = 10
    weight_list = {"sunny":[8, 2, 0],
                   "cloudy":[4, 4, 2],
                   "rainy":[2, 6, 2]}
    weather = ['sunny', 'cloudy', 'rainy']
    weather_list = ['sunny']

    for day in range(future_day):
        weight = weight_list[weather_list[day]]
        random_result = random.choices(weather,weights = weight)
        weather_list.append(random_result[0])

    print(weather_list)

if __name__ == "__main__":
    main()
```

輸出

```
gogochiou@gogochiouMSI:~/NTHU_course/NYCU_SDC/Homework2$ /bin/python3 /home/gogochiou/NTHU_course/NYCU_SDC/Homework2/HW2-1_simple.py
HW2-1
['sunny', 'cloudy', 'sunny', 'cloudy', 'cloudy', 'cloudy', 'cloudy', 'cloudy', 'rainy', 'cloudy', 'cloudy']
```

(c)

```python
def main():
    print("HW2-2")
    future_day = 1000000
    weight_list = {"sunny":[8, 2, 0],
                   "cloudy":[4, 4, 2],
                   "rainy":[2, 6, 2]}
    weather = ['sunny', 'cloudy', 'rainy']
    weather_list = ['sunny']
    s_count = 0
    c_count = 0
    r_count = 0

    for day in range(future_day):
        weight = weight_list[weather_list[day]]
        random_result = random.choices(weather,weights = weight)
        weather_list.append(random_result[0])
        if random_result[0] == 'sunny':
            s_count+=1
        elif random_result[0] == 'cloudy':
            c_count+=1
        elif random_result[0] == 'rainy':
            r_count+=1

    # print(weather_list)
    print(s_count/future_day)
    print(c_count/future_day)
    print(r_count/future_day)
```

```
0.643924
0.284938
0.071138
gogochiou@gogochiouMSI:~/NTHU_course/NYCU_SDC/Homework2$
6 > 0 > bash *
```

(d) stationary distribution

$$\begin{cases} x_{t+1} = x_t. \\ x_{t+1} = T\,x_t \end{cases}$$

(x_t 是當天 3 种天氣的机率),

where $T = \begin{bmatrix} 0.8 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.6 \\ 0 & 0.2 & 0.2 \end{bmatrix}$

Think as $Ax = \lambda \cdot x$, $\lambda = 1$

$$\begin{cases} 0.8\,x_1 + 0.4x_2 + 0.2\,x_3 = x_1 \\ 0.2\,x_1 + 0.4x_2 + 0.6\,x_3 = x_2 \\ 0.2\,x_2 + 0.2x_3 = x_3 \\ x_1 + x_2 + x_3 = 1 \end{cases} \rightarrow \begin{aligned} x_1 &= \tfrac{9}{14} \doteqdot 0.642857 \\ x_2 &= \tfrac{2}{7} \doteqdot 0.285714 \\ x_3 &= \tfrac{1}{14} \doteqdot 0.071429 \end{aligned}$$ #.

		tomorrow will be…		
		sunny	cloudy	rainy
today it's…	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2

(e) $\sum P_{(x)} \log_2 \frac{1}{P_{(x)}} = 1.1981$ # (by matlab calculate)

(f) $p(x_{t-1} \mid x_t) = \dfrac{P(x_t \mid x_{t-1}) \cdot \underline{P(x_{t-1})}}{P(x_t)}$ ⇝ 假設為 stationary distribution.

$P(x_t) = \sum P(x_t \mid x_{t-1}) \cdot P(x_{t-1})$

$P(x_t = \text{sunny}) = 0.642867$

$P(x_t = \text{cloudy}) = 0.285714$

$P(x_t = \text{rainy}) = 0.071429$.

(昨) 今

sunny → sunny : 0.8

↘ cloudy : $\dfrac{0.2 \times 0.642857}{0.285714} = 0.45$.

↘ rainy : 0.

(昨) 今

cloudy → sunny : $\dfrac{0.4 \times 0.285714}{0.642867} = 0.177775$.

↘ cloudy : 0.4

↘ rainy : $\dfrac{0.2 \times 0.285714}{0.071429} = 0.8$.

(昨) 今

rainy → sunny : $\dfrac{0.2 \times 0.071429}{0.642857} = 0.022$

↘ cloudy : $\dfrac{0.6 \times 0.071429}{0.285714} = 0.15$

↘ rainy : 0.2

		yesterday		
		sunny	cloudy	rainy
today	sunny	0.8	0.177775	0.022
	cloudy	0.45	0.4	0.15
	rainy	0	0.8	0.2

#

(g) Markov assumption 假設只要 x_{t-1} 的狀態，就能知道 x_t，
也就是 $x_{t-2} \sim x_1$ 的 state 並不需要。所以 system still satisfy
Markov assumption，只是 state transition function 在不同時序會
不同，或我們可以把 state 擴展到 不同季節的 sunny, cloudy, rainy,
也就是 4x3 = 12 個 state #

Exercise 3. (呈 ex2.)

		our sensor tells us…		
		sunny	cloudy	rainy
the actual weather is…	sunny	.6	.4	0
	cloudy	.3	.7	0
	rainy	0	0	1

		tomorrow will be…		
		sunny	cloudy	rainy
today it's…	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2

(a) 因 sensor 說 rainy，一定是 rainy，又 by markov assumption.

$P(x_5 = \text{sunny} \mid z_4 = \text{rainy}, z_5 = \text{sunny})$

$= \dfrac{P(z_5 = \text{sunny} \mid x_5 = \text{sunny}) \cdot P(x_5 = \text{sunny} \mid z_4 = \text{rainy})}{P(z_5 = \text{sunny} \mid x_5 = \text{sunny}) P(x_5 = \text{sunny} \mid x_4 = \text{rainy}) + P(z_5 = \text{sunny} \mid x_5 = \text{cloudy}) P(x_5 = \text{cloudy} \mid x_4 = \text{rainy}) + P(z_5 = \text{sunny} \mid x_5 = \text{rainy}) P(x_5 = \text{rainy} \mid x_4 = \text{rainy})}$.

$= \dfrac{0.6 \times 0.2}{0.6 \times 0.2 + 0.3 \times 0.6 + 0} = 0.4$ #

(b) - Way 1

Day 2. $\boxed{x_1 = \text{sunny}}$ $\boxed{z_2 = \text{sunny}}$

$p(x_2 \mid x_1, z_2) = \dfrac{P(z_2 \mid x_2) \; P(x_2 \mid x_1)}{\sum P(z_2 \mid x_2') \, P(x_2' \mid x_1)}$

$\sum P(z_2 \mid x_2') \, P(x_2' \mid x_1) = 0.6 \times 0.8 + 0.3 \times 0.2 = 0.54$.

Day 2 is sunny : $\dfrac{0.6 \times 0.8}{0.54} = \boxed{\dfrac{8}{9}}$

cloudy : $\dfrac{0.3 \times 0.2}{0.54} = \boxed{\dfrac{1}{9}}$

rainy : $\dfrac{0}{0.54} = \boxed{0}$ #

$$\frac{P(z_2, z_3, x_2) \cdot \cancel{P(z_3, x_2)}}{\cancel{P(z_3,} x_2) P(x_2)} \Leftarrow \frac{P(z_2, z_3, x_2)}{P(x_2)}$$

Day 3. $x_2 = \begin{bmatrix} 8/9 \\ 1/9 \\ 0 \end{bmatrix}$ $\boxed{z_3 = \text{sunny}}$

$$p(x_3 \mid x_1, z_{2\cdot 3}) = \frac{P(z_3 \mid x_1, x_3, z_2)\, P(x_3 \mid x_1, z_2)}{\Sigma\, P(z_3 \mid x_3')\cdot P(x_3' \mid x_1, z_2).}$$

$$\Sigma\, P(z_3 \mid x_3')\cdot P(x_3' \mid x_1, z_2) = (0.6 \quad 0.3 \quad 0)\left(\begin{bmatrix} 0.8 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.6 \\ 0 & 0.2 & 0.2 \end{bmatrix}\begin{bmatrix} 8/9 \\ 1/9 \\ 0 \end{bmatrix}\right)$$

$$= (0.6 \quad 0.3 \quad 0)\cdot\begin{pmatrix} 34/45 \\ 2/9 \\ 1/45 \end{pmatrix} = 0.52.$$

Day 3 is sunny : $\frac{0.6 \times 34/45}{0.52} = \boxed{0.8718}$

cloudy : $\frac{0.3 \times 2/9}{0.52} = \boxed{0.1282}$

rainy : $\frac{0}{0.52} = \boxed{0}$ #

Day 4. $\Rightarrow$ sensor tells rainy

$\Rightarrow$ Day 4 is sunny : $\boxed{0}$

cloudy : $\boxed{0}$

rainy : $\boxed{1}$ #

– way 2.

Day 2. with day 3, 4 dat $z_2 = \text{sunny}$, $z_3 = \text{sunny}$, $z_4 = \text{rainy}$.

$$P(x_2 = 1 \mid z_{2:4}) = \eta \cdot \underline{P(z_2 = 1 \mid x_2 = 1, z_{3:4})} \times \underline{P(x_2 = 1 \mid z_{3:4})}$$

$$= \eta \cdot \underline{P(z_2 = 1 \mid x_2 = 1)} \cdot \underline{P(z_3 = 1 \mid x_2 = 1, z_4) \times P(x_2 = 1 \mid z_4)}$$

$$= \eta \cdot P(z_2 = 1 \mid x_2 = 1) \cdot \underline{\left(\Sigma\, P(z_3 = 1 \mid x_3 = i, x_2 = 1, z_4) \times P(x_3 = i \mid x_2 = 1, z_4)\right)} \times \underline{P(x_2 = 1 \mid z_4)}$$

$$= \eta \cdot P(z_2 = 1 \mid x_2 = 1) \cdot \left(\Sigma\, P(z_3 = 1 \mid x_3 = i) \times P(x_3 = i \mid x_2 = 1)\right) \times P(z_4 \mid x_2 = 1) \times P(x_2 = 1)$$

$$= \eta \cdot 0.6 \times (0.6 \times 0.8 + 0.3 \times 0.2) \times \underline{\left(\Sigma\, P(z_4 = 3 \mid x_4 = j, x_2 = 1)\, P(x_4 = j \mid x_2 = 1)\right)} \times P(x_2 = 1)$$

$$= \Sigma\, P(z_4 = 3 \mid x_4 = j) \times \Sigma\, P(x_4 = j \mid x_3 = k) \times P(x_3 = k \mid x_2 = 1)$$

$$= 1 \times \left(\Sigma\, P(x_4 = 3 \mid x_3 = k) \times P(x_3 = k \mid x_2 = 1)\right)$$

$$= 1 \times (0.8 \times 0 + 0.2 \times 0.2 + 0 \times 0.2) = 0.04.$$

$$P(x_2 = 1 \mid z_{2:4}) = \eta \cdot 0.6 \times 0.54 \times 0.04 \times 0.8 = 0.0104\eta.$$

$$P(x_2 = 2 \mid z_{2:4}) = \eta\, P(z_2 = 1 \mid x_2 = 2, z_{3:4})\, P(x_2 = 2 \mid z_{3:4})$$

$$= \eta \cdot P(z_2 = 1 \mid x_2 = 2) \cdot P(z_3 = 1 \mid x_2 = 2, z_4) \times P(x_2 = 2 \mid z_4)$$

$$= \eta\, P(z_2 = 1 \mid x_2 = 2) \cdot \left(\Sigma\, P(z_3 = 1 \mid x_3 = i) \times P(x_3 = i \mid x_2 = 2)\right) \times \left(\Sigma\, P(x_4 = 3 \mid x_3 = k) \times P(x_3 = k \mid x_2 = 2)\right) \cdot P(x_2 = 2)$$

$$= \eta\ 0.3 \times (0.6 \times 0.4 + 0.3 \times 0.4 + 0 \times 0.2) \times (0 \times 0.4 + 0.2 \times 0.4 + 0.2 \times 0.2) \times 0.2$$

$$= 0.0026\eta.$$

$$P(x_2 = 3 \mid z_{2:4}) = \eta\, P(z_2 = 1 \mid x_2 = 3)\, P(z_3 = 1 \mid x_2 = 3) \times P(x_2 = 3 \mid z_4)$$

$$= 0.$$

$$\Rightarrow \eta = \frac{1}{0.0104 + 0.0026} = \frac{1}{0.013}$$

Day 2 is sunny : $\frac{0.0104}{0.013} = \boxed{0.8}$

cloudy : $\frac{0.0026}{0.013} = \boxed{0.2}$

rainy : $\frac{0}{0.013} = \boxed{0}$ #

Day 3 :

$$P(X_3=1 \mid Z_{2:4}) = \frac{P(X_3=1,\ Z_4=3 \mid Z_{2:3})}{P(Z_4=3 \mid Z_{2:3})} = \eta\ P(Z_4=3 \mid X_3=1,\ Z_{2:3}) \times P(X_3=1 \mid Z_{2:3}).$$

$$= \eta\ \left(\Sigma P(Z_4=3 \mid X_4=i)\ P(X_4=i \mid X_3=1)\right) \cdot P(X_3=1 \mid Z_{2:3}).$$

$$= 0.$$

$$P(X_3=2 \mid Z_{2:4}) = \eta\ \left(\Sigma P(Z_4=3 \mid X_4=i)\ (X_4=i \mid X_3=2)\right) \cdot P(X_3=2 \mid Z_{2:3})$$

$$= \eta \cdot (0.2 \times 1) \vee 0.1282$$

$$P(X_3=3 \mid Z_{2:4}) = \eta\ \left(\Sigma P(Z_4=3 \mid X_4=i)\ (X_4=i \mid X_3=3)\right) \times P(X_3=3 \mid Z_{2:3})$$

$$= 0.$$

Day 3 is sunny : $\boxed{0}$

cloudy : $\boxed{1}$

rainy : $\boxed{0}$ #

Day 4 : sunny $\boxed{0}$

cloudy $\boxed{0}$

rainy $\boxed{1}$ #

(c) : By (b) ⇒ the most likely sequence is (hindsight).

D2. → D3 → D4

sunny → cloudy → rainy. #

probability : 0.8 = 80% #