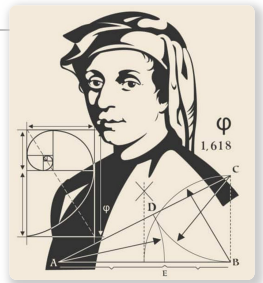
The Fibonacci Sequence



In 1202, the Italian mathematician Leonardo Fibonacci experimented with how a population of rabbits would grow from generation to generation, **give a set of rules**. His rules lead to a **sequence of terms**, which today are called the **Fibonacci sequence**: each term is the sum of the two numbers preceding it.

Expressed as a recurrence relation: $t_n = t_{n-1} + t_{n-2}$

This alone is not sufficient, however; you can define new terms, but **the process has to start somewhere!** You need **at least two terms already available**, which means that the first two terms in the sequence— t_0 and t_1—must be defined explicitly.

Given this qualification, the Fibonacci sequence can be expressed as:

$$t_n = \begin{cases} n & \text{if } n \text{ is 0 or 1} \\ t_{n-1} + t_{n-2} & \text{otherwise} \end{cases}$$

To write a recursive implementation of a *fibonacci(n)* function, you need only plug in the simple cases, plus the recurrence relation, and you're done.

```
int fib(int n)
{
    if (n < 2) { return n; }     // base case
    return fib(n - 1) + fib(n - 2);
}
```

How do you convince yourself that the `fib()` function works? If you begin by tracing through the logic, I guarantee that you'll be confused. Instead, **regard this entire mechanism as irrelevant detail**.

Since the argument values are smaller, **each of these calls represents a simpler case**, and so, applying the recursive leap of faith, **you can assume that the program correctly computes each of these values**. **Case closed.** You don't need the details.