Ring homo $\phi : R \to R/I$

① ϕ : 전사함수 $\because \forall r+I \in R/I$, $\exists r \in R$

② R/I 의 임의의 아이디얼 J에 대해, $I \subseteq \phi^{-1}(J)$ $\because I = \phi^{-1}(0) \subseteq \phi^{-1}(J)$

③ $\pi : \{J : \text{ideal of } R\} \longrightarrow \{K : \text{ideal of } R/I\}$ is 일대일대응

$\because \forall K$ in R, $\phi(K)$ is ideal in R/I ($\because \phi$: 전사함수)

$\forall J$: ideal in R/I, $\phi^{-1}(J)$ is ideal in R.

($\because$ $\phi^{-1}(j_1) - \phi^{-1}(j_2) = \phi^{-1}(j_1 - j_2) \in \phi^{-1}(J)$: Additive subgroup. —— (1)

$r \cdot \phi^{-1}(j) \in R \Rightarrow \phi(r) \cdot j \in J \Rightarrow r \cdot \phi^{-1}(j) \in \phi^{-1}(J)$ —— (2)

Hence, $\phi^{-1}(J)$ is ideal.)

$\therefore$ π : 전단사 ■

④ $\phi^{-1}(\text{prime}) = \text{prime}$. $\because \phi^{-1}(p_1) \cdot \phi^{-1}(p_2) = \phi^{-1}(p_1 p_2) \in \phi^{-1}(P)$

$\Rightarrow p_1$ or $p_2 \in P \Rightarrow \phi^{-1}(p_1)$ or $\phi^{-1}(p_2) \in \phi^{-1}(P)$

⑤ $\phi^{-1}(\max) = \max$.

$\because$ 귀류법가정 : $\phi^{-1}(\max) \neq \max \Rightarrow \phi^{-1}(\max) \subsetneq J \subsetneq R$ ($\exists$ ideal J)

$\Rightarrow \max \subseteq \phi(J) \subseteq \phi(R) = R/I$ ($\exists$ ideal $\phi(J)$)

이때 $\{\text{ideal in } R\} \overset{1\text{-}1}{\longleftrightarrow} \{\text{ideal in } R/I\}$ 이므로,

$\phi(\phi^{-1}(\max)) \neq \phi(J) \neq \phi(R)$

$\max \subsetneq \phi(J) \subsetneq R/I$ $\Rightarrow$ max에 모순. 米

($\because$ max보다 $\phi(J)$가 더 큰 진부분 아이디얼)