4.* Given a commutative ring R and a (2-sided) ideal I. Consider the projection pi : R -> R/I, x -> [x]. Show that there is a one-to-one correspondence between ideals of R/I and ideals containing I. 동일한 과제를 prime ideal, max ideal로 반복. $\to$ correspondence theorem

radical ideal

$\phi : R \to R/I$, ring homomorphism

$A := \{ J \mid \text{ideal of } R \text{ containing } I \}$, $B : \{ K \mid \text{ideal of } R/I \}$

define $\pi : A \to B$

$J \mapsto \{ x + I \mid x \in J \}$. NTS : $\pi(J) \in B$

$\Leftrightarrow \pi(J)$ is an ideal of R/I

① $x+I, y+I \in \pi(J)$, $x-y+I \in \pi(J)$ ⟨subring⟩

② $x+I \in \pi(J)$, $r+I \in R/I$, $(x+I)(r+I) = xr + I \in \pi(J)$ ⟨absorbs product⟩

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1. π is one to one map (injective, 단사)

If $\pi(J_1) = \pi(J_2)$, $J_1 = J_2$

$\pi(J_1) = \{ x+I \mid x \in J_1 \}$, $\pi(J_2) = \{ y+I \mid y \in J_2 \}$

Let $x + I \in \pi(J_1) = \pi(J_2)$. $(x \in J_1, y \in J_2)$

$x + I = y + I \quad \therefore x - y \in I$.

Since $J_2 \in A$ so $I \subseteq J_2$, $x - y \in J_2$

$y \in J_2$

$\Rightarrow x \in J_2$

$\therefore J_1 = J_2$

2. π is onto map (surjective, 전사)

$\forall K \in B$, $\exists J \in A$ s.t. $\pi(J) = K$

$K = S/I = \{ s + I \mid s \in S \}$. NTS: $S \in A$

$\pi(S) = K$

K가 R/I의 아이디얼이므로 K의 원소는 I의 잉여류이다. 임의의 고정된 K에 대해 J가 존재하려면 S는 A의 원소여야 한다. $x \in S$ 이면 $S + I \in K$이다.

$S \in A$ iff ① S is an ideal of R & ② $I \subseteq S$

①-1 Let $x, y \in S$, test $x - y \in S$

$x+I, y+I \in K$, $(x+I)-(y+I) = (x-y)+I \in K$

$\therefore x - y \in S$ ⟨subring⟩

①-2 Let $x \in S$, $r \in R$

Consider $(x+I)(r+I) = xr + I \in K$

$\therefore xr \in S$ ⟨absorbs product⟩

② $x \in I$ then $x \in S$

Consider $x + I = 0 + I \in K$: ideal of R/I

$x + I \in K \Leftrightarrow x \in S$.

$\therefore S \in A$

Thus, π is a bijection from A to B ▨