EE 152: Digital Image Processing (Winter 2018)
Homework 1

Due date: January 18, 2018

Description: The questions in this assignment are primarily aimed at reviewing 1D signal processing, linear algebra, and our discussion in the first 2 to 3 lectures.

Optional reading: Gonzalez and Woods Ch. 1,2; Szeliski Ch. 1,2; Bovik Ch. 1

Homework and lab assignment submission policy:
All homework and lab assignments must be submitted online via https://iLearn.ucr.edu.
Homework solutions should be written and submitted individually, but discussions among students are encouraged.
All assignments should be submitted by the due date. There will be 25% penalty per day for late assignments. No grade will be given to homework submitted 3 days after the due date.

H1.1 Suppose we record 100 discrete samples of a signal in an interval of 1 second at regular intervals (i.e., uniform sampling at 100 samples/second). Answer the following: **(4 points)**

(a) Explain what is the range of frequencies that we can uniquely distinguish?

(b) Explain what is the maximum number of unique frequencies we can distinguish?

(c) Design two continuous-time signals that would yield identical discrete-time samples.

(d) Plot these signals and their Fourier transform (use Matlab if needed).

H1.2 Consider a sinusoidal signal of length $N = 16$ as

$$x(n) = \sum_{k=1,5,9} a_k \cos(2\pi kn/N),$$

where the a_k are scalar coefficients. Calculate the discrete Fourier transform of x? **(3 points)**

H1.3 Compute the convolution of the following N-length discrete-time signal with itself (assume N is even): (2 points)

$$x(n) = \begin{cases} 1 & 1 \leq n \leq N/2 \\ 0 & \text{elsewhere.} \end{cases} \tag{1}$$

H1.4 Compute the convolution of the following signal with $x(n)$ in (1) **(3 points)**

$$y(n) = \begin{cases} 0 & 1 \leq n \leq N/2 \\ 1 & N/2 < n \leq N. \end{cases} \tag{2}$$

H1.5 Select a vector of length 4 (e.g., $\mathbf{a} = [1;\ 2;\ 3;\ 4]$) **(5 points)**

(a) Compute its inner product with following four vectors:

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{w}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \ \mathbf{w}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

(b) Write a matrix expression for these inner products ($\mathbf{u} = \mathbf{W}\mathbf{a}$):

$$\underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}_{\mathbf{u}} = \underbrace{\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}_{\mathbf{a}}$$

(c) How can we compute $\mathbf{a}$ from $\mathbf{u}$?

(d) Is there any special property of $\mathbf{W}$?

(e) What are the eigenvalues of $\boldsymbol{W}^T\boldsymbol{W}$?

H1.6 Select a 4×4 image $\mathbf{A}$ **(4 points)**

(a) Compute another 4×4 image $\mathbf{U}$ whose (k, l)th entry is defined as

$$U_{k,l} = \mathbf{w}_k^T \mathbf{A} \mathbf{w}_l \quad \text{for all} \quad k, l = 1, 2, 3, 4$$

where the $\mathbf{w_k}$ are defined in H1.5.

(b) How can we compute $\mathbf{A}$ from $\mathbf{U}$?

H1.7 Show that $\langle \mathbf{a}\mathbf{b}^T, \mathbf{C} \rangle = \mathbf{a}^T\mathbf{C}\mathbf{b}$, where $\mathbf{a}$ is a vector of length M, $\mathbf{b}$ is a vector of length N, and $\mathbf{C}$ is a matrix of size $M \times N$. Build on this to show that we can compute $\mathbf{A}^T\mathbf{C}\mathbf{B}$ using $\mathbf{a}_i^T\mathbf{C}\mathbf{b}_j$ for all i, j. $\mathbf{A} = [\mathbf{a}_1\ \mathbf{a}_2 \ldots \mathbf{a}_M]$ is an $M \times M$ matrix and $\mathbf{B} = [\mathbf{b}_1\ \mathbf{b}_2 \ldots \mathbf{b}_N]$ is an $N \times N$. **(4 points)**

H1.8 *Problem 2.5 in Gonzalez and Woods textbook (requires reading of Section 2.1.2)*
A CCD camera chip of dimension 7×7 mm, and having 1024×1024 elements, is focused on a square, flat area, located 0.5 m away. How many line pairs per mm will this camera be able to resolve? The camera is equipped with a 35-mm lens. **(2 points)**

H1.9 Apple's Retina display has roughly 300 pixels per inch. According to Apple marketing, humans cannot see more dense pixels if the device is held at approximately 1 foot away. You may assume that human retina has 150,000 elements per mm^2. **(3 points)**
(You may find some relevant information in these links: link 1, link 2, link 3.)

(a) Discuss how would you justify this claim (maximum of 5 lines).

(b) Assuming that Apple's claim is true, what would be the minimum resolution for a TV display that is 10 ft away? Justify your answer by drawing a picture.

H1.10 Tell us about one of your most favorite optical illusion and explain the reason behind that illusion. You can find some good examples here: distractify illusions link **(2 points)**

Maximum points: 20