As this year's implementation is quite special because the electronic exam is not organized, I was thinking that it would be good if I give an extra problem and if you solve it, you get max three (3) extra points added to the exam result. Importantly, this is not obligatory, and you can get the full 12/12 points from the paper and pen exam.

Below is the problem. So, if you solve this by the 6th of March, 23:55, then I'll assess your solution and 0-3 points will be given. The solutions will be returned in Moodle.

Suppose that under the risk-neutral probability measure the stock prices follow

$dS(t) = rS(t)dt + \sqrt{v(t)}S(t)dW_1^Q(t)$, and volatility follows according to Heston model,

$dv(t) = \kappa^Q(\theta^Q - v(t))dt + \eta\sqrt{v(t)}dW_2^Q(t)$.

All the parameters are non-negative constants and $S(0) = S_0 > 0$ and $v(0) = v_0 > 0$. Moreover, W_1 and W_2 are Wiener process with correlation $dW_1(t)dW_2(t) = \rho dt, \rho \in [-1, 1]$.

The initial stock price and volatility are $S_0 = 1$ and $v_0 = 0.04$, respectively. Price a European up-and-in call option with strike $K = 0.98$, barrier level $H = 1.1$ and maturity time $T = 3$. That is, the option pays $\max(S_T - K, 0)$ if $S_t \geq H$ for some $0 \leq t \leq T$.

Use Heston model with **Monte Carlo simulation with antithetic variates** with the following parameter values:
$r = 0.01, \kappa^Q = 6, \theta^Q = 0.05, \eta = 0.5, \rho = -0.7, \Delta t = 1/252$. Use Monte Carlo with 10,000 iterations. To generate correlated random data with correlation coefficient ρ, $\varepsilon_2 = \rho\varepsilon_1 + \sqrt{1 - \rho^2}e_{12}$, where e_{12} is independent of ε_1, and all random variables are i.i.d. Gaussian.

In the simulation of the volatility process, use the Milstein Scheme with the truncation scheme. That is,

$$S(t + \Delta t) = S(t)\exp\left(\left(r - \tfrac{1}{2}v(t)\right)\Delta t + \sqrt{v(t)\Delta t}\varepsilon_1(t)\right),$$
$$v(t + \Delta t) = \max\left[0, v(t) + \kappa^Q(\theta^Q - v(t))\Delta t + \eta\sqrt{v(t)\Delta t}\varepsilon_2(t) + \tfrac{1}{4}\eta^2\Delta t(\varepsilon_2(t)^2 - 1)\right].$$

Express the option price in terms of **implied volatility**.

Please provide the Matlab/Python/R code that solves the problem.