MATH.APP.460-2021-2022-1 Numerical Analysis (English).
Sampsa Pursiainen (Juho Kanniainen, Jere Mäkinen). **Exercise session 3, Wed 30.3.2022**. Calculate (or try to calculate) the tasks below before the session. Prepare to present orally in the session the solution (or solution trial) of each task marked as done. Alternatively the solutions can be handed in as a single PDF document via a submit link on the course's moodle page. The solutions can be computer assisted and include scripts.

1. Let $f(x) = e^{-x^2}$. Compare evenly spaced interpolation with Chebyshev interpolation by plotting the degree n polynomials of both types on the interval [-1,1], for $n = 10$ and $n = 20$. For evenly spaced interpolation, the left and right interpolation base points should be -1 and 1. By sampling at a 0.01 step size, create the empirical interpolation errors for each type, and plot a comparison. Can the Runge phenomenon be observed in this problem?

2. Decide whether the equation forms a cubic spline.

   (a)
$$S(x) = \begin{cases} x^3 + x - 1 & \text{on } [0,1] \\ -(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & \text{on } [1,2] \end{cases}$$

   (b)
$$S(x) = \begin{cases} 2x^3 + x^2 + 4x + 5 & \text{on } [0,1] \\ (x-1)^3 + 7(x-1)^2 + 12(x-1) + 12 & \text{on } [1,2] \end{cases}$$

3. Find the equations and plot the natural cubic spline that interpolates the data points (0,3), (1,5), (2,4), (3,1).

   Consider the natural cubic spline through the following world population data points. Estimate the year 1980. Compare with the correct value 4452584592.

   (a)

year	bbl/population
1960	3039585530
1970	3707475887
1990	5281653820
2000	6079603571

4. Apply the composite Trapezoid Rule with $m = 1, 2$ and 4 panels to approximate the integral. Compute the error by comparing with the exact value from calculus

   (a) $\int_0^1 x^2 dx$

   (b) $\int_0^{\pi/2} \cos x dx$

   (c) $\int_0^1 e^x dx$

5. The arc length of the curve defined by $y = f(x)$ from $x = a$ to $x = b$ is given by the integral $\int_a^b \sqrt{1 + f'(x)^2} dx$. Use the composite Simpson's Rule with $m = 32$ panels to approximate the lengths of the curves [**?**](Sauer p. 265)

   (a) $y = x^3$ on $[0, 1]$

   (b) $y = \tan x$ on $[0, \pi/4]$

   (c) $y = \arctan x$ on $[0, 1]$.