

MATH.APP.460-2021-2022-1 Numerical Analysis (English).

Sampsa Pursiainen (Juho Kanninen, Jere Mäkinen). **Exercise session 2, Wed 23.3.2022.** Calculate (or try to calculate) the tasks below before the session. Prepare to present orally in the session the solution (or solution trial) of each task marked as done. Alternatively the solutions can be handed in as a single PDF document via a submit link on the course's moodle page. The solutions can be computer assisted and include scripts.

1. Fit data to exponential model by using linearization. Find the 2-norm of the difference between the data points y_i and the best model $c_1 e^{c_2 t_i}$.

t	y
-2	1
0	2
1	2
2	5

1. Apply Newton's Method to find two solutions of the system.

(a)

$$\begin{aligned} u^3 - v^3 + u &= 0 \\ u^2 + v^2 &= 1 \end{aligned}$$

(b)

$$\begin{aligned} 2u^2 - 4u + v^2 + 3w^2 + 6w + 2 &= 0 \\ u^2 + v^2 - 2v + 2w^2 - 5 &= 0 \\ 3u^2 - 12u + v^2 + 3w^2 + 8 &= 0 \end{aligned}$$

2. The bloodstream concentration of a drug, measured hourly after administration, is given in the accompanying table. Fit the model

$$y = c_1 t e^{c_2 t}.$$

Find the estimated maximum and the half-life. Suppose that the therapeutic range for the drug is 4–15 ng/ml. Use the equation solver of your choice to estimate the time the drug concentration stays within therapeutic levels.

hour	concentration (ng/ml)
1	6.2
2	9.5
3	12.3
4	13.9
5	14.6
6	13.5
7	13.3
8	12.7
9	12.4
10	11.9

3. (a) Use Lagrange interpolation to find a polynomial that passes through the points
 - i. $(0, 1), (2, 3), (3, 0)$
 - ii. $(0, -2), (2, 1), (4, 4)$
 (b) Use Newton's divided differences to find the interpolating polynomials of the points in part (a). Verify agreement with the Lagrange interpolating polynomial.
4. How many degree d polynomials pass through the four points $(-1, 3), (1, 1), (2, 3), (3, 7)$? Write one down if possible. (a) $d = 2$ (b) $d = 3$ (c) $d = 6$.
5. Write a program that implements Newton's Divided Difference Interpolation Method. Test your program with the points from Exercise 3(a). Plot the results and report the coefficients returned by the method.