```
% Numerical analysis - exercise 2
clear
format shortG
% in the exercise, the problems are numbered 1 1 2 3 4 5

% Problem 1.1
% y = c1*exp(c2*t)
% ln(y) = ln(c1) + c2*t
% ln(y) = k + c2*t with c1 = exp(k)
y = [1;2;2;5];
A = [1,-2;1,0;1,1;1,2];
lny = log(y);
kc2 = inv(A'*A)*A'*lny;
c = [exp(kc2(1)); kc2(2)]
```

```
c = 2×1
        1.932
       0.3615
```

```
% Problem 1.2
%a
syms u v
f(u,v) = [(u^3 - v^3 + u), (u^2 + v^2-1)]';
J = jacobian(f, [u,v]);
x0 = [1; 1];
sol1 = newton(f, J, x0, 10^(-9))
```

sol1(u, v) =

$$\begin{pmatrix} 0.50799 \\ 0.86136 \end{pmatrix}$$

```
x0 = [-1; -1];
sol2 = newton(f, J, x0, 10^(-9))
```

sol2(u, v) =

$$\begin{pmatrix} -0.50799 \\ -0.86136 \end{pmatrix}$$

```
%b
syms u v w
f(u,v,w) = [(2*u^2 - 4*u + v^2 + 3*w^2 + 6*w + 2)...
            ,(u^2 + v^2 - 2*v + 2*w^2 - 5)...
            ,(3*u^2 - 12*u + v^2 + 3*w^2 + 8)]';
J = jacobian(f, [u,v, w]);
x0 = [2; 1.5; -1.5];
sol1 = newton(f, J, x0, 10^(-5))
```

```
sol1(u, v, w) =
```

$$\begin{pmatrix} 2.0 \\ 1.0 \\ -1.0 \end{pmatrix}$$

```
x0 = [-1; -1; -1];
sol2 = newton(f, J, x0, 10^(-5))
```

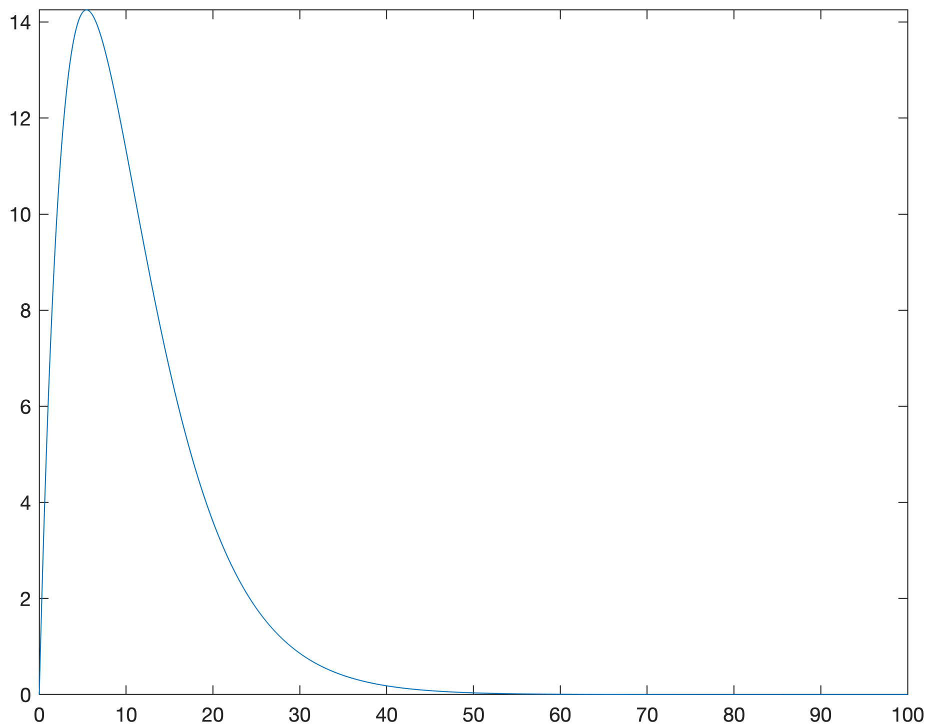
```
sol2(u, v, w) =
```

$$\begin{pmatrix} 1.09602 \\ -1.15925 \\ -0.26115 \end{pmatrix}$$

```
% Problem 2
% y = c1*t*exp(c2*t)
% ln(y) = ln(c1) + ln(t) + c2*t
% ln(y) - ln(t) = k + c2*t (c1 = exp(k))
t = (1:10)';
y = [6.2;9.5;12.3;13.9; 14.6;13.5;13.3;12.7;12.4;11.9];
b = log(y) - log(t);
A = [ones(10,1) t];
kc2 =  inv(A'*A)*A'*b;
c = [exp(kc2(1)); kc2(2)]
```

```
c = 2×1
        7.122
     -0.18385
```

```
x_new = (0:0.001:100);
y_hat = c(1).*x_new.*exp(c(2)*x_new);
plot(x_new,y_hat)
axis tight
```




```
% find max of y = 7.122*t*exp(-0.18385*t)
syms x
f(x) = c(1)*x*exp(c(2)*x);
df = diff(f);
xmax = round(solve(df == 0, x), 5)
```

xmax = 5.43925

```
% max y
round(f(xmax), 5)
```

ans = 14.25108

```
% find x when f = 4
x1 = bisec(f - 4, 0, 1, 10^-8)
```

```
x1 =
      0.63069
```

```
x2 = bisec(f - 4, 10,30,10^-8)
```

```
x2 =
       19.214
```

```
% so, when x1<t<x2, the drug concentration stay within therapeutic level.
```

$\boxed{P3}$

a)

i) $(0,1)\ (2,3)\ (3,0)$

Lagrange interpolation

$$P(x) = 1\times\frac{(\cancel{x}-2)(x-3)}{(0-2)(0-3)} + 3\times\frac{(x-0)(x-3)}{(2-0)(2-3)}$$

$$0\times\frac{(x-0)(x-2)}{(3-0)(3-2)}$$

$$P(x) = \frac{1}{6}(x^2-5x+6) + 3\times\left(-\frac{1}{2}\right)(x^2-3x)$$

$$= -\frac{4}{3}x^2 + \frac{11}{3}x + 1$$

ii) $(0,-2)\ (2,1)\ (4,4)$

$$P(x) = (-2)\frac{(x-2)(x-4)}{(0-2)(0-4)} + (1)\frac{(x-0)(x-4)}{(2-0)(2-4)}$$

$$+ 4\times\frac{(x-0)(x-2)}{(4-0)(4-2)}$$

$$=(-2)\frac{1}{8}(x^2-6x+8)+\frac{(-1)}{4}(x^2-4x)$$

$$+\frac{1}{2}(x^2-2x)$$

$$= 0x^2+\frac{3}{2}x \cancel{-\frac{1}{4}} -2$$

$$= \frac{3}{2}x - \cancel{\frac{11}{4}} - 2$$

b)

i) $(0,1)(2,3)(3,0)$

$$\begin{array}{c|ccc}
0 & 1 & & \\
 & & 1 & \\
2 & 3 & & -\frac{4}{3} \\
 & & -3 & \\
3 & 0 & &
\end{array}
\quad \cancel{\Rightarrow P(x) = 1 + 1(x-0) - 4}$$

$$\Rightarrow P(x) = 1 + 1(x-0) - \frac{4}{3}(x-0)(x-2)$$

$$= 1 + x - \frac{4}{3}(x^2-2x)$$

$$= 1 + \frac{11}{3}x - \frac{4}{3}x^2$$

$\Rightarrow$ Similar to Lagrange interpolation ii)

$$\begin{array}{c|cccc} 0 & -2 & & & \\ & & 3/2 & & \\ 2 & 1 & & 0 & \\ & & 3/2 & & \\ 4 & 4 & & & \end{array}$$

$$\Rightarrow P(x) = -2 + \frac{3}{2}(x-0) + 0$$

$$= -2 + \frac{3}{2}x$$

$\Rightarrow$ Similar to Lagrange Interpolation

$\boxed{P4}$ $(-1,3)$ $(1,1)$, $(2,3)$ $(3,7)$

a) $d = 2$

b) $d = 3$

c) $d = 6$

a and b)

Base on theorem 6.1.6, there is only ~~a~~ 1 polynomials P degree 3 or less that Satisfies $P(x_i) = y_i$ $i = 1, \ldots n$

e

$P(x)$

$\Rightarrow$ only $d = 2$ or $d = 3$.

~~We use Lagrange Interpolation~~

We use Newton divine difference

$$
\begin{array}{c|cccc}
-1 & 3 & & & \\
 & & -1 & & \\
1 & 1 & & 1 & \\
 & & 2 & & 0 \\
2 & 3 & & 1 & \\
 & & 4 & & \\
3 & 7 & & &
\end{array}
$$

$\Rightarrow P(x) = 3 - 1(x+1) + 1(x+1)(x-1)$

$= 3 - x - 1 + x^2 - 1$

$= x^2 - x + 1$

a) $\Rightarrow d = 2$. there exists Polynomial

$P(x) = x^2 - x + 1$

b) Since, theorem 6.1.6 said there is only 1.

$P(x)$ with degree ≤ 3

$\Rightarrow d = 3$ has no polynomial

c) $d = 6$

There are ~~many~~ infinitely polynomials, choose one.

$P(x) = -\frac{11}{240}x^6 + \frac{41}{48}x^5 - \frac{97}{16}x^4 - \frac{3707}{120}x^2 + 19x - 2$

(Matlab part)

```
% Problem 4
% c) d = 6
x0=[0 1 2 3 4 5 6];
y0=[-2 1 1 4 4 3 -2];
c=newtdd(x0,y0,7)
```

```
c = 7×1
          -2
           3
        -1.5
           1
        -0.5
     0.16667
   -0.045833
```

```
syms x
f(x) = -2 + 3*x -1.5*x*(x-1) + x*(x-1)*(x-2) -0.5*x*(x-1)*(x-2)*(x-3)...
    + (1/6)*x*(x-1)*(x-2)*(x-3)*(x-4) -(11/240)*x*(x-1)*(x-2)*(x-3)*(x-4)*(x-5);
f([0 2 4])
```

ans = $\begin{pmatrix} -2 & 1 & 4 \end{pmatrix}$

```
simplify(f)
```

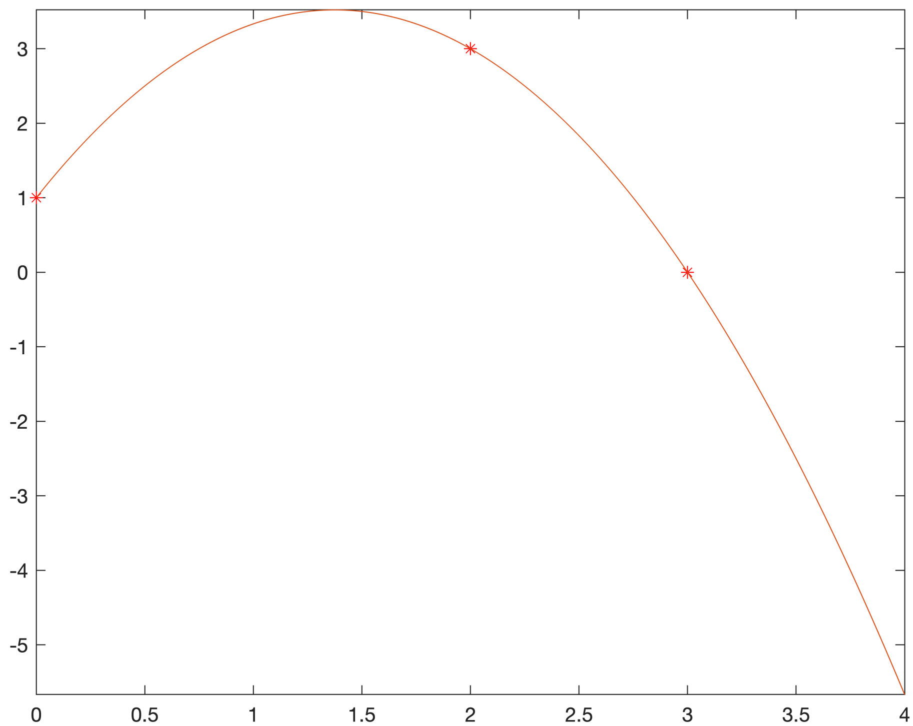
ans(x) =

$$-\frac{11\,x^6}{240}+\frac{41\,x^5}{48}-\frac{97\,x^4}{16}+\frac{967\,x^3}{48}-\frac{3707\,x^2}{120}+19\,x-2$$

```
% Problem 5
%3a i
x0=[0 2 3];
y0=[1 3 0];
c=newtdd(x0,y0,3)
```

```
c = 3×1
           1
           1
     -1.3333
```

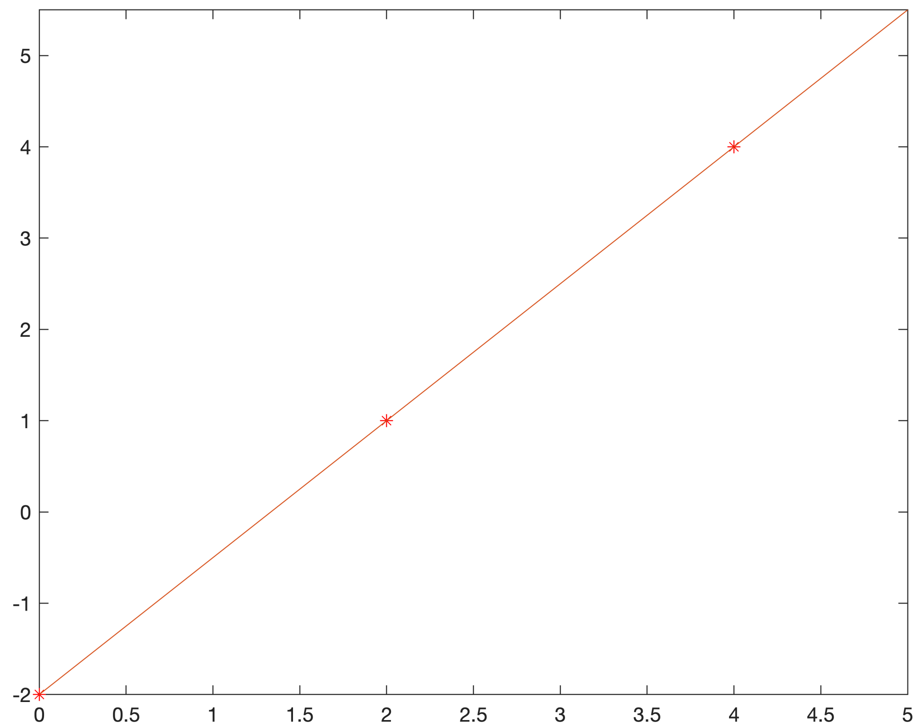
```
x=0:.001:4;
y = c(1) + c(2)*(x - x0(1)) + c(3)*(x - x0(1)).*(x - x0(2));
% y = 1 + x - (4/3)(x)(x - 2)
% y = 1 + x - (4/3)(x^2 -3x +2)
% y = 1 + (11/3)x - (4/3)x^2
% similar to the Lagrange interpolation methods
plot(x0,y0, "r*" ,x,y)
axis tight
```




```
%3a ii
x0=[0 2 4];
y0=[-2 1 4];
c=newtdd(x0,y0,3)
```

```
c = 3×1
       -2
      1.5
        0
```

```
x=0:.001:5;
y = c(1) + c(2)*(x - x0(1)) + c(3)*(x - x0(1)).*(x - x0(2));
% y = -2 + 3/2(x-0) + 0
% y = -2 + (3/2)x
% similar to the Lagrange interpolation methods
plot(x0,y0, "r*" ,x,y)
axis tight
```




```
function [result] = newton(f, J, x0, tol) % Newton method
var = symvar(f)';
result = x0;
Jx = subs(J, var, result);
fxk = subs(f, var, result);
s = linsolve(Jx, -fxk);
x1 = result + s;
e = x1 - x0;

while (norm(e) > tol)
    e = inv(Jx)*fxk;
    Jx = subs(J, var, result);
    fxk = subs(f, var, result);
    s = linsolve(Jx, -fxk);
    result  = result + s;
end

result = round(result, 5);
end

function [result] = bisec(f, a, b, tol)
% a b is the search range [a,b]
% f(x) is the function
% tol is tolerance error
result = Inf;
if (f(a)*f(b) < 0)
    while ((b-a)/2 > tol)
```

```
        c = (a + b)/2;
        if f(c) == 0
            result = c;
            break
        end

        if (f(a) * f(c)<0)
            b = c;
        else
            a = c;
        end
    end
    result = c;
end
end

% Newton's Divided Diferences
function c=newtdd(x,y,n)
v = zeros(n);
c = zeros(n,1);

for j=1:n
    v(j,1)=y(j);
end

for i=2:n
for j=1:n+1-i
v(j,i)=(v(j+1,i-1)-v(j,i-1))/(x(j+i-1)-x(j));
end
end

for i=1:n
c(i)=v(1,i);
end

end
```