1. Consider the data set in the file tirerealiability.txt:

```
> data<-read.table("tirereliability.txt", sep="\t", dec=".", header=TRUE)
> head(data)
  tireAge wedge interBelt EB2B peelForce crbnBlk survival complete
1    1.22  0.81      0.88 1.07      0.63    1.02     1.02        0
2    1.19  0.69      0.77 0.92      0.68    1.02     1.05        1
3    0.93  0.77      1.01 1.11      0.72    0.99     1.22        0
4    0.85  0.80      0.57 0.98      0.75    1.00     1.17        1
5    0.85  0.85      1.26 1.03      0.70    1.02     1.09        0
6    0.91  0.89      0.94 1.00      0.77    1.03     1.09        1

Source: V.V. Kristov, D.E. Tanako, T.P. Davis (2002). "Regression Approach
to Tire Reliability Analysis," Reliability Engineering and System Safety,
Vol. 78, pp. 267-273.

Description: Study to compare tire survival as a function of:
Tire Age  (tireAge)
Wedge gauge   (wedge)
Interbelt Gauge  (interBelt)
EB2B
Peel Force    (peelForce)
Carbon Black %   (crbnBlk)
Survival    (survival)
Censoring indicator w/ 1=Complete, 0=censored    (complete)
```

Denote variables as following

$$Y = \text{complete},\ T = \text{survival}.$$

(a) Let $X = \text{wedge}$. Consider the Cox proportional hazards regression model

$$h_i(t|x_i) = h_0(t)e^{\beta x_i}$$

Calculate the estimate for the parameter β.

(2 points)

(b) Continue with the model

$$h_i(t|x_i) = h_0(t)e^{\beta x_i},$$

where $X = \text{wedge}$. Estimate the value of the survival function $S(t|x_i) = P(T \geq t|x_i)$ at the time point $t = 1.00$ when $x_i = 0.6$. Also present graphically how the estimate of the survival function $S(t|x_i)$ is behaving when $x_i = 0.6$.

(1 point)

(c) Furthermore, continue with the model

$$h_i(t|x_i) = h_0(t)e^{\beta x_i},$$

where $X =$ wedge. Estimate the hazard ratio

$$\frac{h_i(t|x_i = 0.6)}{h_i(t|x_{i_*} = 1.6)}.$$

(1 point)

(d) Consider the following Cox proportional hazards regression model

$$h_i(t|\mathbf{x}_i) = h_0(t) \cdot \exp(\beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i1} x_{i2}),$$

where $X_1 =$ wedge, $X_2 =$ peelForce and $X_3 =$ interBelt. Test at 5% significance level, is the explanatory variable $X_1 =$ wedge statistically significant variable.

(1 point)

(e) Continue using the model

$$h_i(t|\mathbf{x}_i) = h_0(t) \cdot \exp(\beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i1} x_{i2}),$$

where $X_1 =$ wedge, $X_2 =$ peelForce and $X_3 =$ interBelt. Create 95% confidence interval for the survival function $S(t|\mathbf{x}_i) = P(T \geq t|\mathbf{x}_i)$ at the time point $t = 1.00$ when $x_{i1} = 0.6$, $x_{i2} = 0.8$, and $x_{i3} = 0.7$.

(1 point)

2. Let us continue with the data set tirerealiability.txt:

   (a) Let X = wedge. Consider the Weibull proportional hazards regression model
   $$h_i(t|x_i) = \frac{p}{\lambda}\left(\frac{t}{\lambda}\right)^{p-1} e^{\beta x_i}.$$
   Estimate the hazard ratio
   $$\frac{h_i(t|x_i = 0.6)}{h_i(t|x_{i_*} = 1.6)}.$$
   (2 points)

   (b) Continue with the model
   $$h_i(t|x_i) = \frac{p}{\lambda}\left(\frac{t}{\lambda}\right)^{p-1} e^{\beta x_i},$$
   where X = wedge. Find the estimate for the expected value $\mathrm{E}(T_{i_*})$, when $x_{i_*} = 1.6$.
   (1 point)

   (c) Furthermore, continue with the model
   $$h_i(t|x_i) = \frac{p}{\lambda}\left(\frac{t}{\lambda}\right)^{p-1} e^{\beta x_i},$$
   where X = wedge. Create 80% prediction interval for new observation T_f, when $x_f = 1.6$.
   (2 points)

   (d) Consider the Weibull proportional hazards regression model
   $$h_i(t|\mathbf{x}_i) = \frac{p}{\lambda}\left(\frac{t}{\lambda}\right)^{p-1} \cdot \exp(\beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i1} x_{i2}),$$
   where X_1 = wedge, X_2 = peelForce and X_3 = interBelt. where $S_0(t) = e^{-t}$ and explanatory variables are as X_1 = wedge, X_2 = peelForce and X_3 = interBelt. Estimate the value of the survival function $S(t|x_i) = P(T \geq t|\mathbf{x}_i)$ at the time point $t = 1.00$ when $x_{i1} = 0.6$, $x_{i2} = 0.8$, and $x_{i3} = 0.7$.
   (1 point)

Extra assignment! If you do not return this assignment, you will not loose any points. But you can gain some extra 6 points if you can do this assignment.

Suppose that the random variable T_i follows the Weibull distribution $T_i \sim Wei(p, \lambda)$. Then the random variable T_i has the density function

$$f(t_i) = \frac{p}{\lambda} \left(\frac{t_i}{\lambda} \right)^{p-1} \cdot \exp \left[- \left(\frac{t_i}{\lambda} \right)^{p} \right].$$

(a) Derive the survival function $S(t_i)$ from the density function $f(t_i)$.

(3 points)

(b) Derive the hazard function $h(t_i)$ from the survival function $S(t_i)$.

(3 points)