Statistical Modelling W6

P3 $T_i \sim Wei(p, \lambda)$

$$f(t_i) = \frac{p}{\lambda}\left(\frac{t_i}{\lambda}\right)^{p-1} \exp\left[-\left(\frac{t_i}{\lambda}\right)^p\right]$$

(a) $S(x_i) = 1 - F(t_i)$

$$F(t_i) = \int_0^{T_i} f(t_i)\,dt_i = \int_0^{T_i} \frac{p}{\lambda}\left(\frac{t_i}{\lambda}\right)^{p-1} \exp\left[-\left(\frac{t_i}{\lambda}\right)^p\right]$$

$$= \cancel{\;} - \int_0^{T_i} \frac{p}{\lambda^p} t_i^{p-1} \exp\left[-\frac{1}{\lambda^p} t_i^p\right]$$

$$= - \int_0^{T_i} \exp\left(-\left(\frac{t_i}{\lambda}\right)^p\right)$$

$$= 1 - \exp\left[-\left(\frac{T_i}{\lambda}\right)^p\right]$$

$$\Rightarrow S(t_i) = 1 - F(t_i) = \exp\left[-\left(\frac{t_i}{\lambda}\right)^p\right]$$

(b). $h(t_i) = -\dfrac{\partial \log(S(t_i))}{\partial t_i}$

$$= -\frac{\partial \log\left(\exp\left(-\frac{t_i}{\lambda}\right)^p\right)}{\partial t_i}$$

$$= -\frac{\partial \left(\frac{t_i}{\lambda}\right)^p}{\partial t_i}$$

$$h(t_i) = \frac{p}{\lambda}\left(\frac{t_i}{\lambda}\right)^{p-1}$$