$\boxed{Ex3}$ $\hat{\beta}_0 = 1$, $\hat{\beta}_1 = 0.5$ $\quad x_i = 5$

(i) $Y_i \sim Poi(\mu_i)$

$$\log(\mu_i) = \beta_0 + \beta_1 x_i$$

$$\Rightarrow \mu_i = e^{\hat{\beta}_0 + \hat{\beta}_1 x_i} = e^{1+0.5\times 5} \simeq 33.1155$$

(ii) $Y_i \sim Poi(\mu_i)$

$$\sqrt{\mu_i} = \beta_0 + \beta_1 x_i$$

$$\Rightarrow \mu_i = (\hat{\beta}_0 + \hat{\beta}_1 x_i)^2 = (1 + 0.5\times 5)^2 = 12.25$$

(iii) $Y_i \sim Poi(\mu_i)$

$$\log\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 x_i \qquad t_i = 10$$

$$\log(\mu_i) = \log(t_i) + \beta_0 + \beta_1 x_1$$

$$\mu_i = t_i e^{\hat{\beta}_0 + \hat{\beta}_1 x_i} = 331.155$$

$$\cancel{= 1 + 0.5\times 5} = 3.5$$

(iv) $\mu_i = \theta_i \cdot 0 + (1-\theta_i)\, e^{\beta_0 + \beta_1 x_i}$

$$= 0.75 \times e^{1+0.5\times 5}$$

$$= 24.8366$$

b) $Y_i \sim Poi(\mu_i)$

$$f(y_i|\mu_i) = \frac{e^{-\mu_i}\mu_i^{y_i}}{y_i!}$$

$$= \exp\left(\frac{y_i \log(\mu_i) - e^{\log(\mu_i)}}{1} - \log(y_i!)\right)$$

$\cancel{y_i \theta_i} = y_i \log(\mu_i)$

$b(\theta_i) = e^{\log(\mu_i)}$

$a(\phi) = 1.$

$c(y_i, \phi) = -\log(y_i!)$

$\theta_i = \log(\mu_i)$

$b = e.$

$\Rightarrow Poi(\mu_i)$ belog to the Exponential Family of Distribution.

c) $Y_i \sim Poi(\mu_i)$

$$\mu_i = \eta_i = \beta_0$$

$$Var(Y_i) = \mu_i = \beta_0$$

$$\frac{\partial \ell(\beta, \phi)}{\partial \beta_j} = \sum_{i=1}^{n} \frac{y_i - \mu_i}{Var(Y_i)} \underbrace{x_{ij}}_{1} \underbrace{\left(\frac{\partial \mu_i}{\partial \eta_i}\right)}_{1} = 0$$

$$= \sum_{i=1}^{n} \frac{y_i - \beta_0}{\beta_0}$$

$$= \frac{1}{\beta_0}\left(\sum y_i - n\beta_0\right)$$

$$\frac{\partial \ell(\beta, \phi)}{\partial \beta_0} = 0$$

$$\Leftrightarrow \frac{1}{\beta_0}\left(\sum y_i - n\beta_0\right) = 0$$

$$\Leftrightarrow \hat{\beta}_0 = \bar{y}$$