Coursework **Fall 2016**
Statistical Pattern Recognition (COMP136)
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Assignment 2

This assignment is due by the start of class on Wednesday, October 19.

1. Find a 3x3 real, symmetric matrix A with $tr(A) = 14$, $det(A) = 36$, satisfying $Av_1 = \lambda_1 v_1$ where $\lambda_1 = 4$, $v_1 = (0; 1; 0)^T$, and where a second eigenvector v_2 has the form $v_2 = (\frac{1}{\sqrt{2}}; a; b)^T$ for some values a, b.

2. Consider a real-valued symmetric matrix S with eigen decomposition $S = V \Lambda V^T$. Now consider the optimization problem:

$$\text{argmax}_{\{x \mid x^T x \leq 1\}} \ \ x^T S x$$

   that is, we seek a vector x of norm at most 1 maximizing the quadratic form $x^T S x$. What is the optimal solution x? First derive your solution in general and then illustrate it in the example of the previous question. (You can assume S is positive semi-definite.)

   Hint: since the columns of V form an orthonormal basis we can write $x = \sum a_k v_k = Va$ for some coefficients a_k. Use this fact to calculate the quadratic form and then analyze the result to identify the optimizing a.

3. Solve problem 3.7 (page 175) in the textbook.

4. Please consult the solution of problem 3.8 from the textbook that is available through the text's web page. Then, solve problem 3.9 (page 175) in the textbook.

5. Solve the second part of problem 3.21 (page 177) in the textbook. That is, you do not need to prove (3.117) but should use it to derive (3.92) from (3.86) by direct differentiation.