Coupled enhancer-promoter condensates

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1 Enhancer and promoter

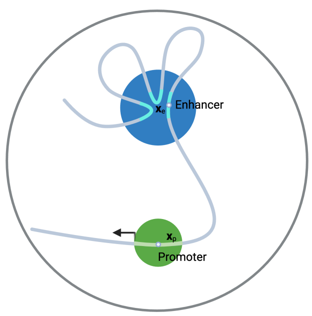

Figure 1: Enhancer and promoter regions on chromatin

2 Free energy functional

$$
\begin{aligned}
F(\phi_{\mathrm{P}}(\vec{x},t),\phi_{\mathrm{R}}(\vec{x},t)) = \int & \mathrm{d}\vec{x}\rho_{\mathrm{P}}(\phi_{\mathrm{P}}-\alpha)^2(\phi_{\mathrm{P}}-\beta)^2 \\
& + \rho_{\mathrm{R}}\phi_{\mathrm{R}}^2 \\
& - \chi\phi_{\mathrm{P}}\phi_{\mathrm{R}} + c\phi_{\mathrm{P}}^2\phi_{\mathrm{R}}^2 \\
& + \frac{\kappa}{2}|\nabla\phi_{\mathrm{P}}|^2 \\
& - \chi_{\mathrm{PD}}\phi_{\mathrm{P}}\exp\left\{-\frac{(\vec{x}-\vec{x}_e)^2}{2\sigma^2}\right\} \\
& + F_C
\end{aligned}
$$

- Protein-protein double-well potential

$$F_{\mathrm{DW}} = \rho_{\mathrm{P}}(\phi_{\mathrm{P}} - \alpha)^2(\phi_{\mathrm{P}} - \beta)^2$$

- RNA-RNA repulsion

$$\rho_{\mathrm{R}}\phi_{\mathrm{R}}^2$$

- Protein-RNA electrostatic interaction

$$\chi_{\mathrm{PR}}(\phi_{\mathrm{P}}, \phi_{\mathrm{R}}) = \chi\phi_{\mathrm{P}}\phi_{\mathrm{R}} + c\phi_{\mathrm{P}}^2\phi_{\mathrm{R}}^2$$

- Protein-DNA interaction

$$\chi_{\mathrm{PD}}\phi_{\mathrm{P}}\phi_{\mathrm{D}} = \chi_{\mathrm{PD}}\phi_{\mathrm{P}} \exp\left\{-\frac{(\vec{x} - \vec{x}_e)^2}{2\sigma^2}\right\}$$

- Interfacial surface-tension

$$\frac{\kappa}{2}|\nabla\phi_{\mathrm{P}}|^2$$

- Chromatin

$$F(\vec{R}) = \frac{3k_BT}{2}\left(\frac{|\vec{R}|^2}{L_C L_P}\right)$$

3 Dynamic equations

- Protein dynamics Model A dynamics. The amount of protein is conserved.

$$\frac{\partial\phi_{\mathrm{P}}}{\partial t} = M_{\mathrm{P}}\nabla^2\left(\frac{\partial F}{\partial\phi_{\mathrm{P}}}\right) = M_{\mathrm{P}}\nabla^2\mu_{\mathrm{P}}$$

- RNA dynamics

$$\frac{\partial\phi_{\mathrm{R}}}{\partial t} = M_{\mathrm{R}}\nabla^2\phi_{\mathrm{R}} + k_p(\vec{x})\phi_{\mathrm{P}} - k_d\phi_{\mathrm{R}}$$

$$k_p(\vec{x}) = \frac{k_T}{2\pi\sigma^2}\exp\left[\frac{(\vec{x} - \vec{x}_p)^2}{2\sigma^2}\right]$$

- Enhancer region dynamics
  - Gradient of free energy functional with respect to the vector $\vec{x}_e$

$$\frac{\partial\vec{x}_{\mathrm{e}}}{\partial t} = M_{\mathrm{D}}\nabla_{\vec{x}_e}F$$