

Coupled enhancer-promoter condensates

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1 Enhancer and promoter

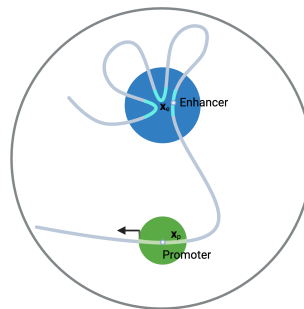


Figure 1: Enhancer and promoter regions on chromatin

2 Free energy functional

$$\begin{aligned}
F(\phi_P(\vec{x}, t), \phi_R(\vec{x}, t)) = & \int d\vec{x} \rho_P (\phi_P - \alpha)^2 (\phi_P - \beta)^2 \\
& + \rho_R \phi_R^2 \\
& - \chi \phi_P \phi_R + c \phi_P^2 \phi_R^2 \\
& + \frac{\kappa}{2} |\nabla \phi_P|^2 \\
& - \chi_{PD} \phi_P \exp \left\{ -\frac{(\vec{x} - \vec{x}_e)^2}{2\sigma^2} \right\} \\
& + F_C
\end{aligned}$$

- Protein-protein double-well potential

$$F_{DW} = \rho_P (\phi_P - \alpha)^2 (\phi_P - \beta)^2$$

- RNA-RNA repulsion

$$\rho_R \phi_R^2$$

- Protein-RNA electrostatic interaction

$$\chi_{PR}(\phi_P, \phi_R) = \chi \phi_P \phi_R + c \phi_P^2 \phi_R^2$$

- Protein-DNA interaction

$$\chi_{PD} \phi_P \phi_D = \chi_{PD} \phi_P \exp \left\{ -\frac{(\vec{x} - \vec{x}_e)^2}{2\sigma^2} \right\}$$

- Interfacial surface-tension

$$\frac{\kappa}{2} |\nabla \phi_P|^2$$

- Chromatin

$$F(\vec{R}) = \frac{3k_B T}{2} \left(\frac{|\vec{R}|^2}{L_C L_P} \right)$$

3 Dynamic equations

- Protein dynamics Model A dynamics. The amount of protein is conserved.

$$\frac{\partial \phi_P}{\partial t} = M_P \nabla^2 \left(\frac{\partial F}{\partial \phi_P} \right) = M_P \nabla^2 \mu_P$$

- RNA dynamics

$$\frac{\partial \phi_R}{\partial t} = M_R \nabla^2 \phi_R + k_p(\vec{x}) \phi_P - k_d \phi_R$$

$$k_p(\vec{x}) = \frac{k_T}{2\pi\sigma^2} \exp \left[\frac{(\vec{x} - \vec{x}_p)^2}{2\sigma^2} \right]$$

- Enhancer region dynamics

– Gradient of free energy functional with respect to the vector \vec{x}_e

$$\frac{\partial \vec{x}_e}{\partial t} = M_D \nabla_{\vec{x}_e} F$$