Homework 2
289A Statistical mechanics of crystalline solids
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Question 1: Pick a nearest neighbor pair on a triangular lattice and determine a symmetry invariant form for its force constant. You can do this by first determining the point group that leaves the pair cluster unchanged. Then express the force constant matrix as a sum over a tensor basis:

$$\begin{pmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{yx} & \Phi_{yy} \end{pmatrix} = \Phi_{xx}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \Phi_{xy}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \Phi_{yy}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \Phi_{yx}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Apply the Reynolds operator to each of the tensor basis elements using the point group that leaves that cluster invariant (note that if you use symmetry operations that exchange the sites of the pair, you need to apply a transpose to your force constant matrix). This will yield a smaller set of symmetry invariant tensor basis elements.

Question 2: Do the same thing as in question 1, but for the nearest neighbor pair of a honeycomb network.