1 Point group of triangular lattice

The basis vectors of a triangular lattice are $\vec{a} = \hat{i} + 0\hat{j}$ and $\vec{b} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$.

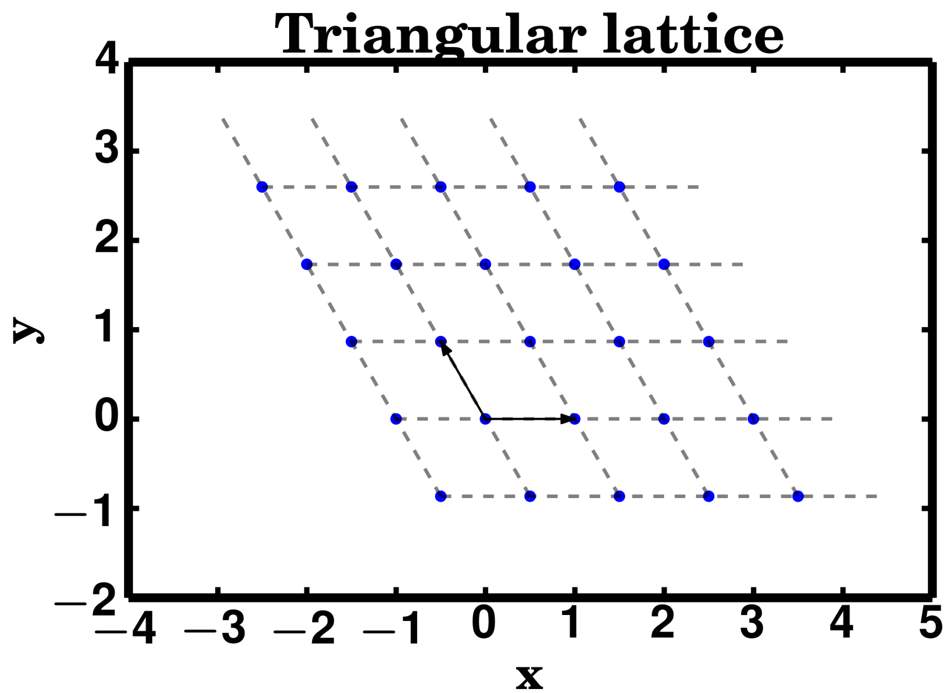

Figure 1: Repeated basis vectors of a triangular lattice

This lattice has a total of 12 symmetry operations:
I0

$$M = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

R60

$$M = \begin{pmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

R120

$$M = \begin{pmatrix} -0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

R180

$$M = \begin{pmatrix} -1.0 & 0.0 \\ 0.0 & -1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

R240

$$M = \begin{pmatrix} -0.5 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

R300

$$M = \begin{pmatrix} 0.5 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{6}$$

M0

$$M = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & -1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{7}$$

M30

$$M = \begin{pmatrix} 0.5 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{8}$$

M60

$$M = \begin{pmatrix} -0.5 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{9}$$

M90

$$M = \begin{pmatrix} -1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{10}$$

M120

$$M = \begin{pmatrix} -0.5 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{11}$$

M150

$$M = \begin{pmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{12}$$

We can verify that it forms a closed group by combining every operation with another and realizing the resulting operation is already part of the group.

I0	R60	R120	R180	R240	R300	M0	M30	M60	M90	M120	M150
R60	R120	R180	R240	R300	I0	M30	M60	M90	M120	M150	M0
R120	R180	R240	R300	I0	R60	M60	M90	M120	M150	M0	M30
R180	R240	R300	I0	R60	R120	M90	M120	M150	M0	M30	M60
R240	R300	I0	R60	R120	R180	M120	M150	M0	M30	M60	M90
R300	I0	R60	R120	R180	R240	M150	M0	M30	M60	M90	M120
M0	M150	M120	M90	M60	M30	I0	R300	R240	R180	R120	R60
M30	M0	M150	M120	M90	M60	R60	I0	R300	R240	R180	R120
M60	M30	M0	M150	M120	M90	R120	R60	I0	R300	R240	R180
M90	M60	M30	M0	M150	M120	R180	R120	R60	I0	R300	R240
M120	M90	M60	M30	M0	M150	R240	R180	R120	R60	I0	R300
M150	M120	M90	M60	M30	M0	R300	R240	R180	R120	R60	I0

Table 1: Multiplication table for the point group of a triangular lattice

2 Factor group of honeycomb lattice

The honeycomb structure has the same lattice vectors as the triangular lattice, but two basis atoms.

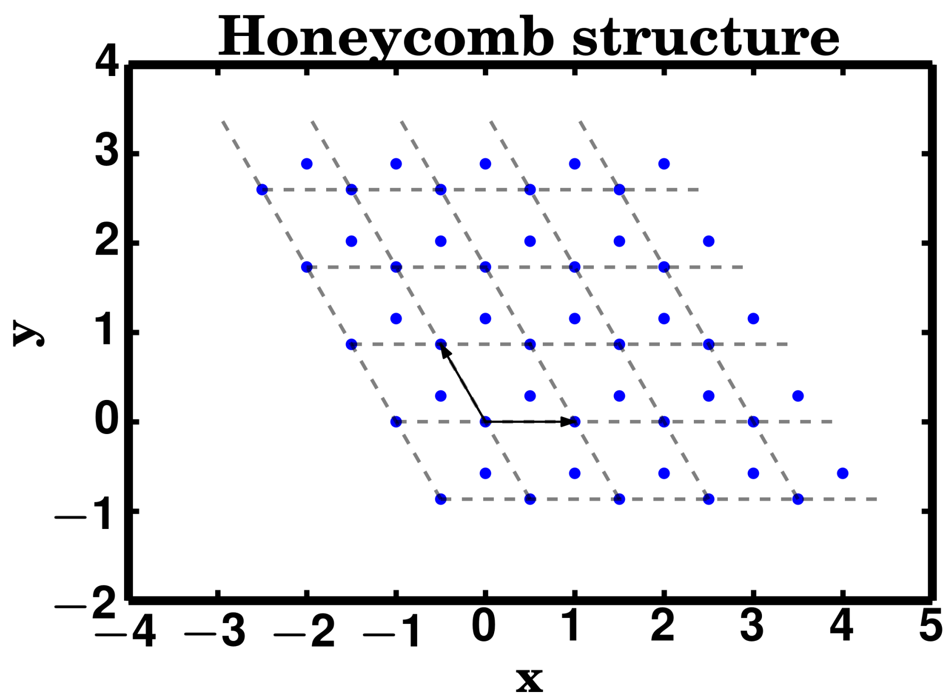

Figure 2: Repeated basis vectors of a triangular lattice

The factor group for this structure is the same as the point group, with some operations involving a translation.

I0

$$M = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{13}$$

R60

$$M = \begin{pmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.5 \\ \frac{\sqrt{3}}{6} \end{pmatrix} \tag{14}$$

R120

$$M = \begin{pmatrix} -0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{15}$$

R180

$$M = \begin{pmatrix} -1.0 & 0.0 \\ 0.0 & -1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.5 \\ \frac{\sqrt{3}}{6} \end{pmatrix} \tag{16}$$

R240

$$M = \begin{pmatrix} -0.5 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{17}$$

R300

$$M = \begin{pmatrix} 0.5 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.5 \\ \frac{\sqrt{3}}{6} \end{pmatrix} \tag{18}$$

M0

$$M = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & -1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.5 \\ \frac{\sqrt{3}}{6} \end{pmatrix} \tag{19}$$

M30

$$M = \begin{pmatrix} 0.5 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{20}$$

M60

$$M = \begin{pmatrix} -0.5 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.5 \\ \frac{\sqrt{3}}{6} \end{pmatrix} \tag{21}$$

M90

$$M = \begin{pmatrix} -1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{22}$$

M120

$$M = \begin{pmatrix} -0.5 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.5 \\ \frac{\sqrt{3}}{6} \end{pmatrix} \tag{23}$$

M150

$$M = \begin{pmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \vec{\tau} = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{24}$$