Friedrich-Alexander-Universität
Technische Fakultät

Lecture on the MATLAB practical course

Experiment 3 Measurement data analysis

Felix Binder

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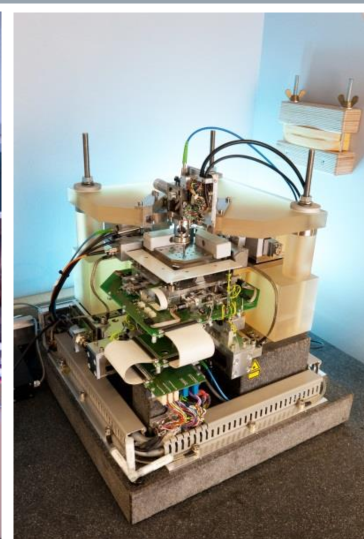



Overview of lecture content

I. Measurement technology

a. Fourier analysis

b. Evaluation of measurement results

c. Determination of measurement uncertainty according to GUM

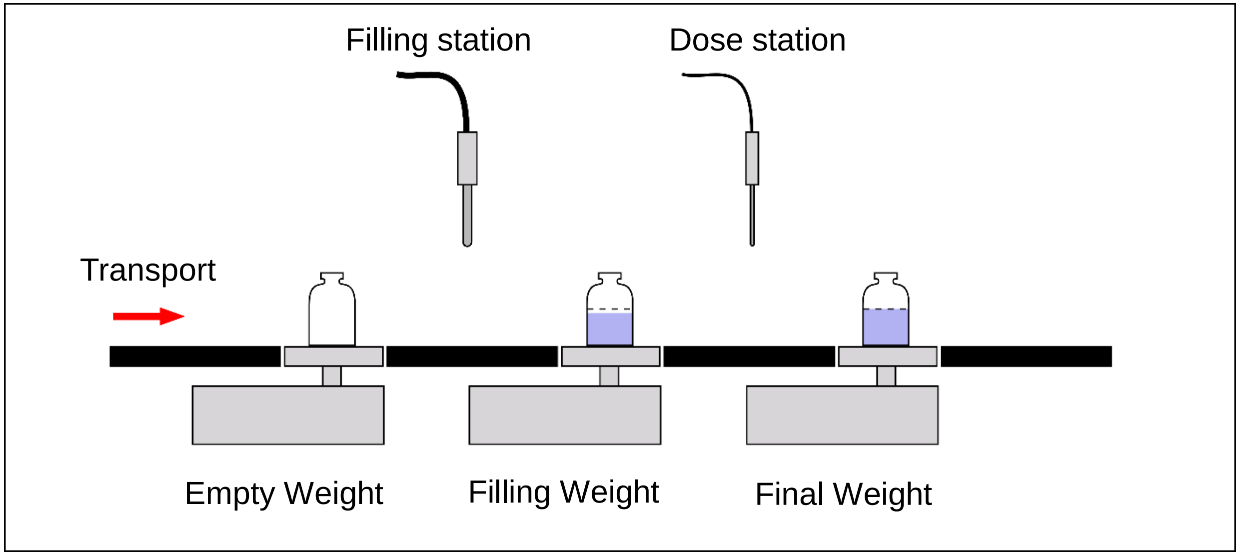
II. MATLAB

a. Basics of programming with MATLAB

b. Visualize data

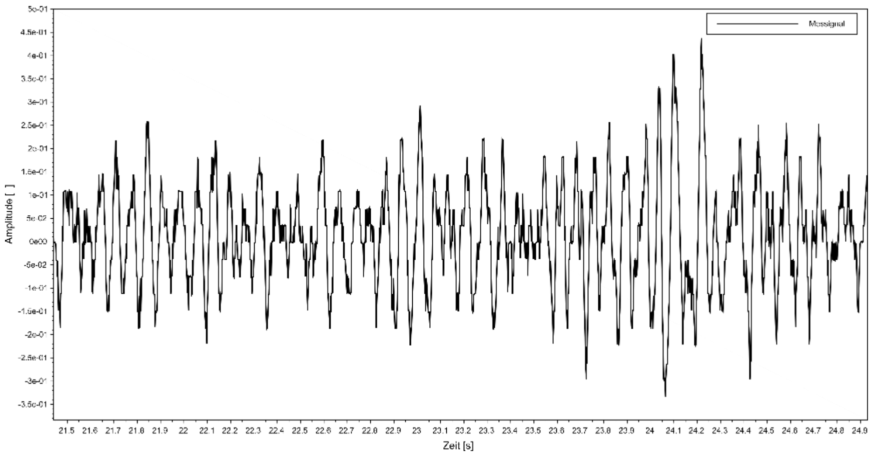
c. Symbolic Math Toolbox and useful functions

I.a: Example application - level measurement

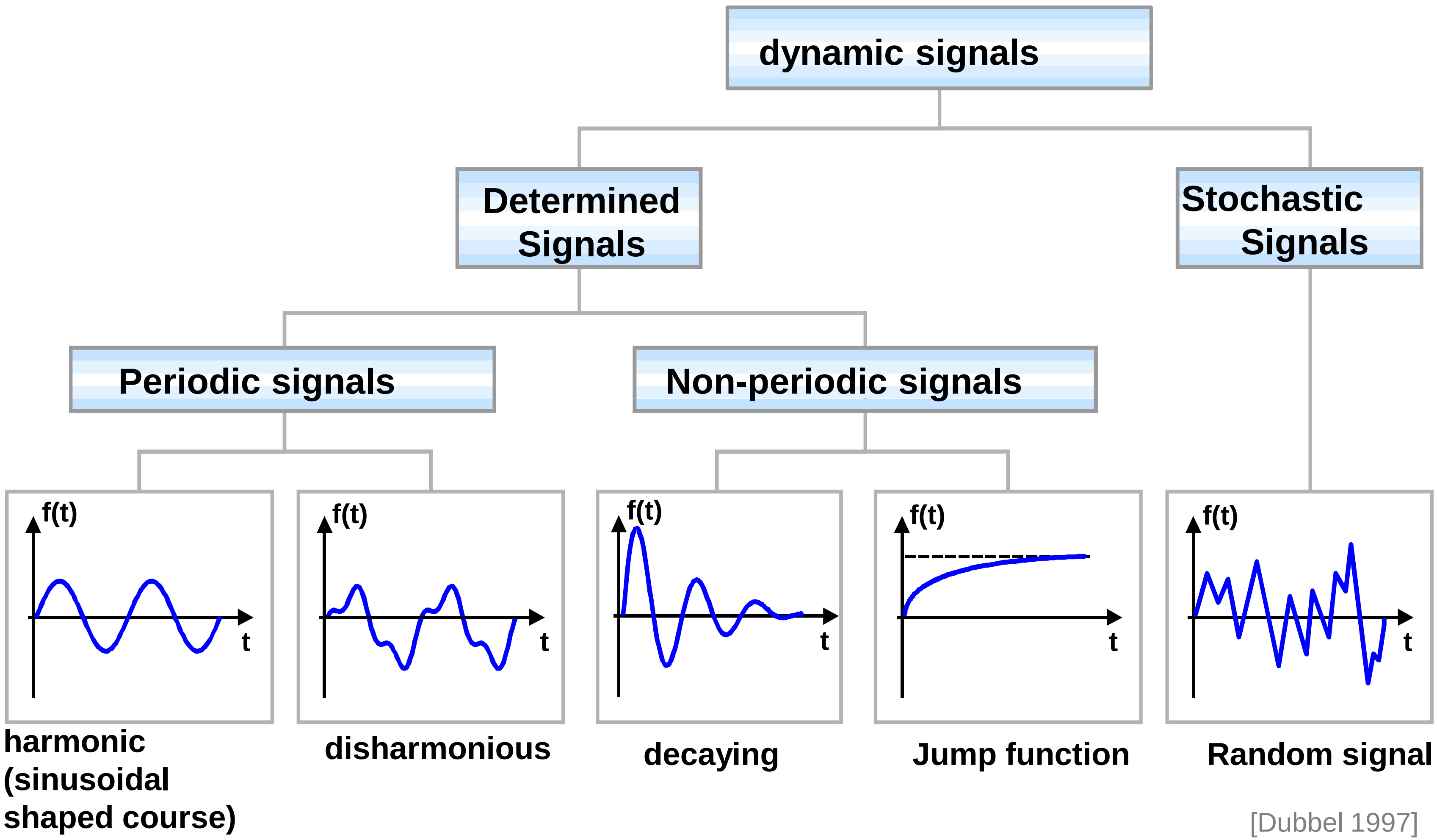


Possible main influencing factors:

- **"laminar flow" ventilation**
- **Sorting pot**
- **Transported liquid**



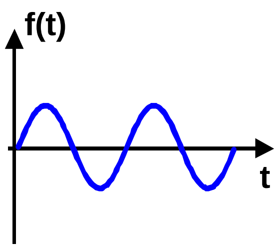
I.a.: Classification of dynamic signals



[Dubbel 1997]

I.a: Signal description

- Harmonic signals (periodic sinusoidal curve)

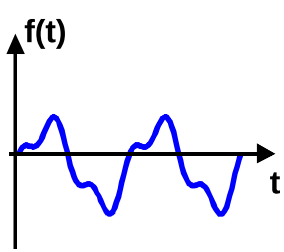


Parameters for the description:
- Offset
- Amplitude
- Frequency
- Phase

$$f(t) = A_0 + A \sin(\omega t + \varphi)$$

$$f(t) = A_0 + a \sin(\omega t) + b \cos(\omega t)$$

- disharmonic signals (periodic non-sinusoidal course)

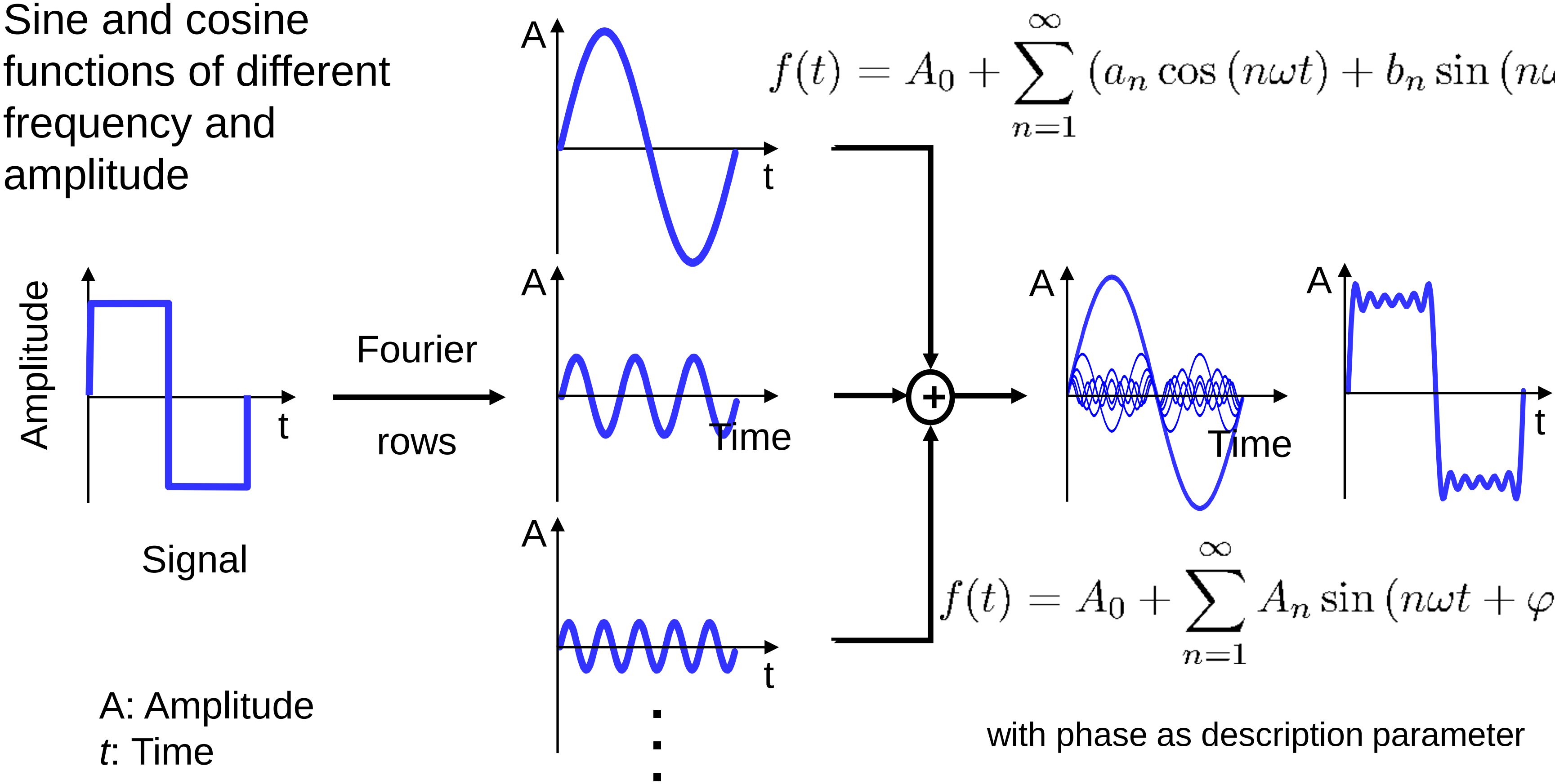


How to describe?

I.a: Signal description with Fourier series I

- Harmonic signals can be described very well
- Idea: describe a disharmonic signal as a composition of several harmonic signals

Sine and cosine functions of different frequency and amplitude



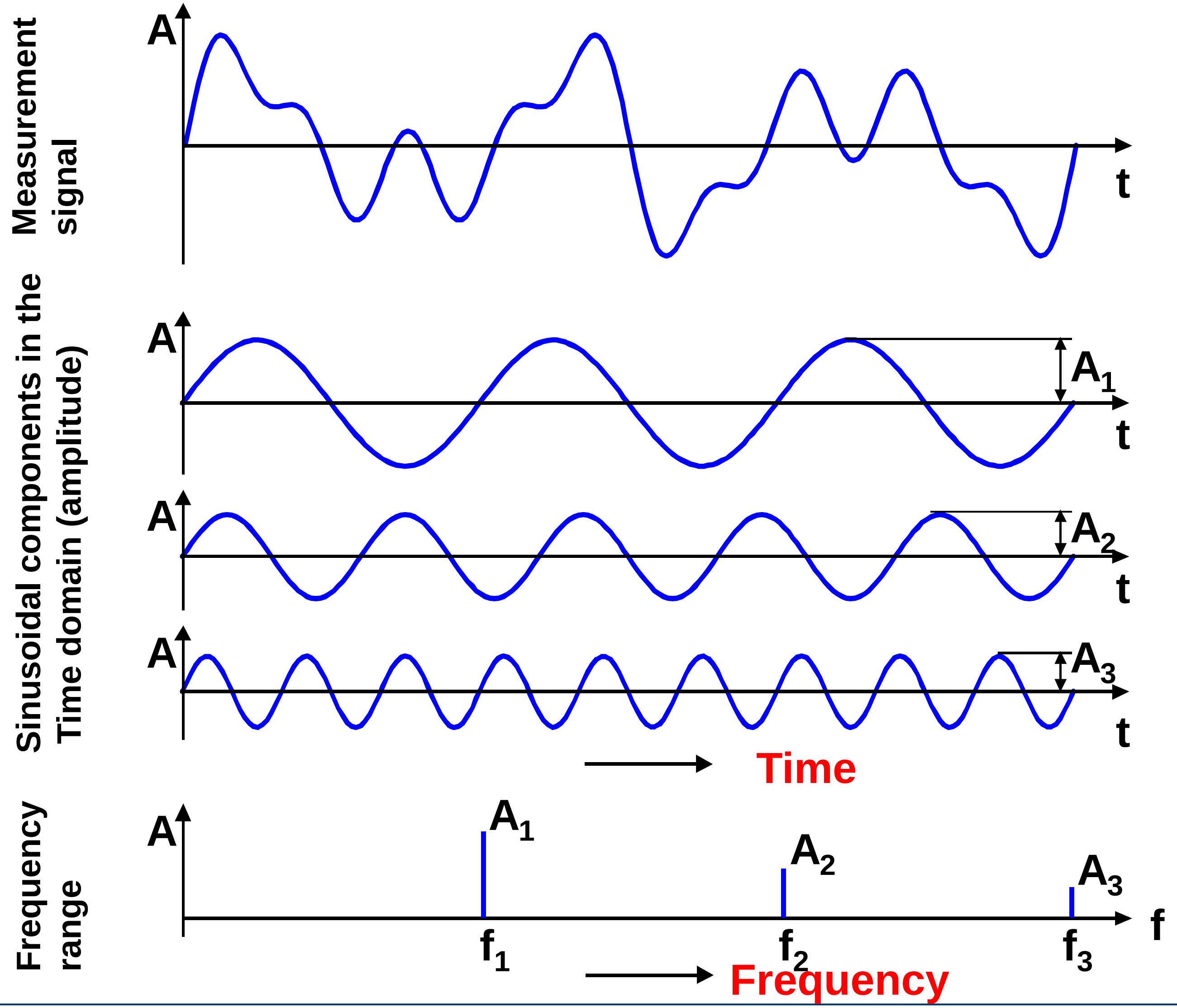
$$f(t) = A_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

with phase as description parameter

A: Amplitude
t: Time

I.a: Signal description with Fourier series II



[Profos 1992]

I.a.: Fourier series and transformation I

Jean Baptiste Fourier
(1768 - 1830)

Fourier proved in 1822 that periodic functions can be split into a sum of angle functions

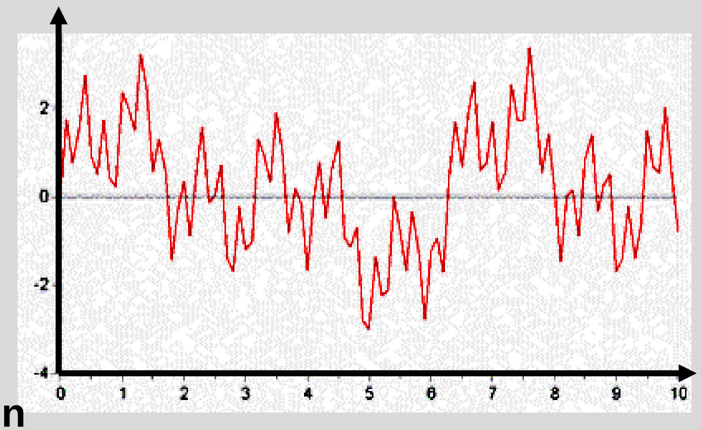
$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin\left(n\omega t + \varphi_n\right)$$

Possibility of breaking down a measured signal into individual frequency

$$\mathfrak{F}\{f(t)\} = F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-j\omega t} f(t)\, dt$$

Tension

Signal in Time range

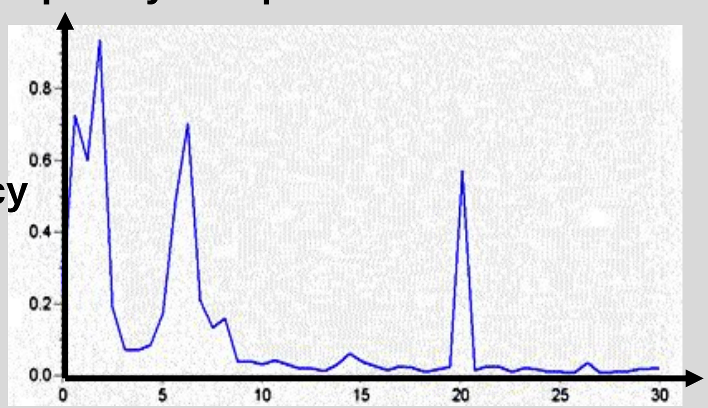


Time

suitable Transformation

Frequency component

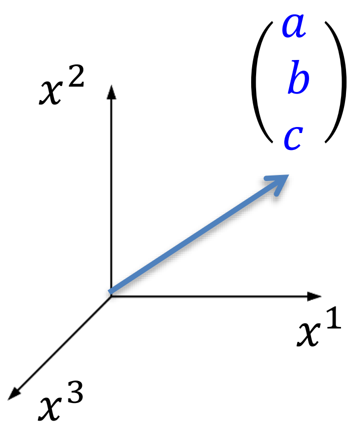
Spectrum in the frequency range



Frequency

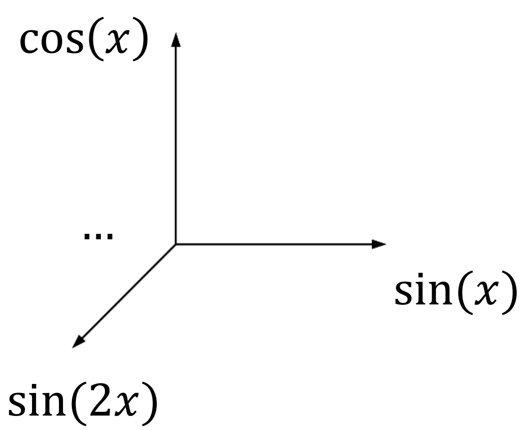
Fourier transformation

I.a.: Fourier series and transformation II



$$f(x) = ax^1 + bx^2 + cx^3$$

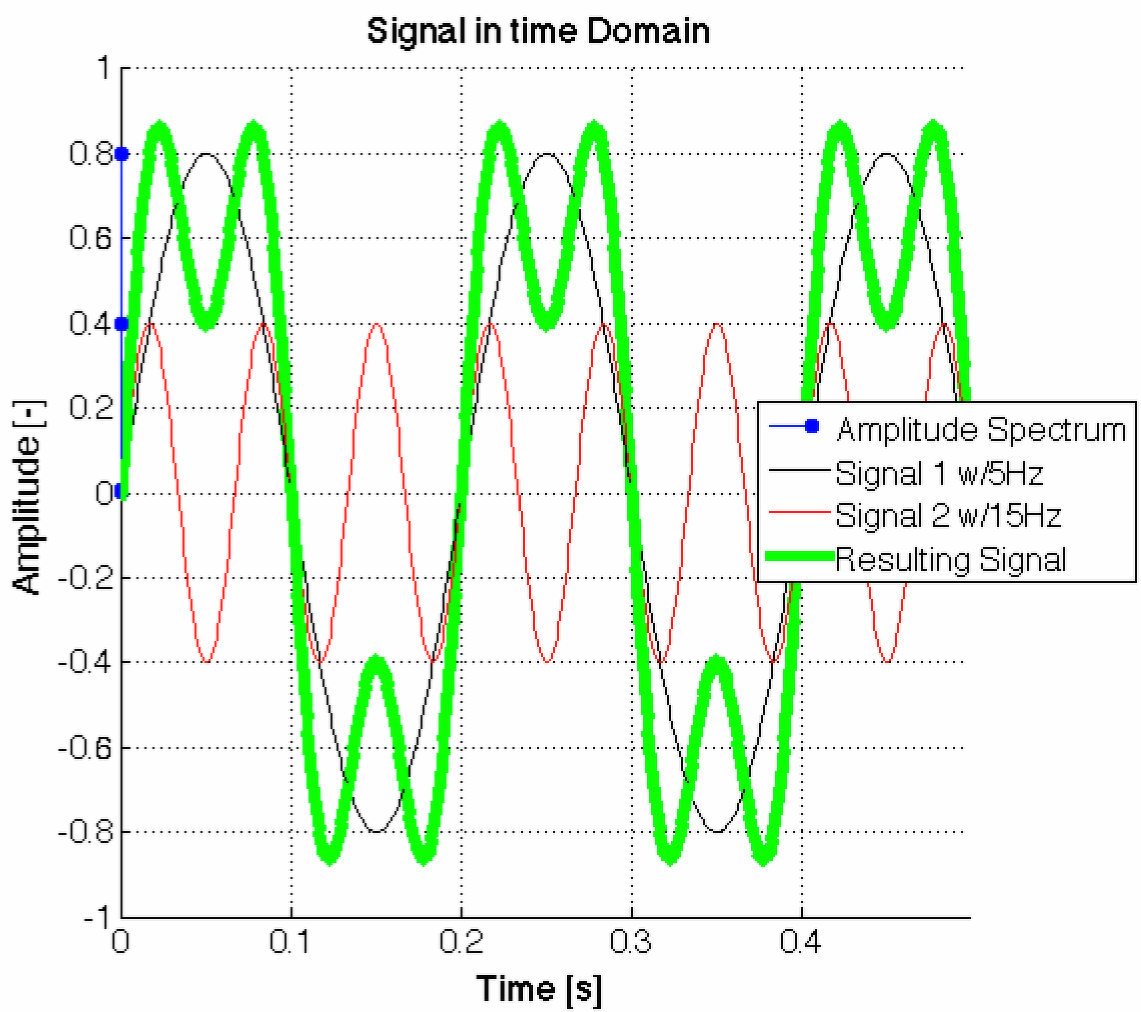
for 3-dimensions



$$f(x) = a\sin(x) + b\cos(x) + c\sin(2x) + \ \dots$$

for N-dimensions

I.a.: Fourier series and transformation III



I.a: Determining the parameters of the Fourier series

Mathematical description of the signal curve

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

Circular frequency of the fundamental frequency:

$$\omega_0 = \frac{2\pi}{T_0}$$

Basic parameters:

T_0 : Period duration of the fundamental oscillation

Fourier coefficients:

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt$$

$$a_k = \frac{2}{T_0} \int_0^{T_0} x(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0} \int_0^{T_0} x(t) \sin(k\omega_0 t)\, dt$$

[Lerchner 1990]

I.a.: Practical realization of the Fourier transform

- analytically solvable for given functions $x(t)$
- Mathematical description of the signal curve usually not available

  $\rightarrow$ Signal is recorded discretely in terms of time and value (e.g. A/D converter)
- Calculation of Fourier coefficients on the basis of time-discretized values

  $\rightarrow$ discrete Fourier transform (DFT) or fast Fourier transform (FFT)

Basic parameters:

T_0 : Period duration of the fundamental oscillation

f_{max} : maximum frequency

k_{max} : maximum number of coefficients

T_A : Sampling time

M: Exponent of base 2

Practice-oriented equation set

$$T_A \leq \frac{1}{5} \cdot \frac{1}{f_{\max}}$$ **modified Nyquist criterion**

$$N = \frac{T_0}{T_A} \qquad N = 2^M$$

$$k_{\max} = f_{\max} \cdot T_0$$

[Lerchner 1990]

I.a.: Practical realization of the Fourier transform

Fourier transform

$$\mathfrak{F}\{f(t)\} = F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-j\omega t} f(t)\, dt \quad \bullet\!\!-\!\!\circ \quad \mathfrak{F}^{-1}\{F(\omega)\} = f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\, d\omega$$

Discrete Fourier transformation

$$X(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{-j2\pi \frac{nk}{N}}$$

Inverse discrete Fourier transform

$$x(k) = \sum_{n=0}^{N-1} X(n) e^{j2\pi \frac{kn}{N}}$$

- DFT = implementation of the Fourier transform for discrete signals
- Discrete-time equivalent of the Fourier series ↷
  - Line spectrum from 0 to f_{max} =0.5/T_A (from 0.5/T_A to 1/T_a mirrored, sampling theorem) with line spacing Δf=1/(NT)$_A$
  - Spectrum of the DFT is periodic in contrast to the spectrum of the Fourier series
- FFT = numerically efficient implementation of the DFT for signals with length L=2 ^{N}

I.a: Prerequisites and calculation steps of the DFT

Prerequisites

- Signal must be sampled periodically
- Scanning row must be finite and form a window
- the cut-out time window continues periodically

Spectrum is completely described if the finite number of spectral lines of a period is known

DFT calculation steps

- Defining the parameters of the Fourier series
- Algorithmic calculation of the Fourier coefficients

```
public void dft(double[] in_real, double[] in_imag, double[] out_real, double[]
    out_imag){
    int N = in_real.length;

    for(int n = 0; n < N; n++){
        double sum_real = 0;
        double sum_imag = 0;

        for(int m = 0; m < N; m++){
            double angle = 2*Math.PI*n*m/N;

            sum_real += in_real[n]*Math.cos(angle)+in_imag[n]*Math.sin(angle);
            sum_imag += -in_real[n]*Math.sin(angle)+in_imag[n]*Math.cos(angle);
        }
        out_real[n] = sum_real;
        out_imag[n] = sum_imag;
    }
}
```

Implementierung der DFT als Java Funktion

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I.a.: "Fast Fourier Transformation" I

FFT according to J. W. Cooley and J. W. Tukey ("An Algorithm for the Machine Calculation of Complex Fourier Series", 1964) reduces the complexity of to by utilizing the symmetry properties of the Fourier transform.

The data must therefore be reduced to or expanded with 0 ("zero padding").

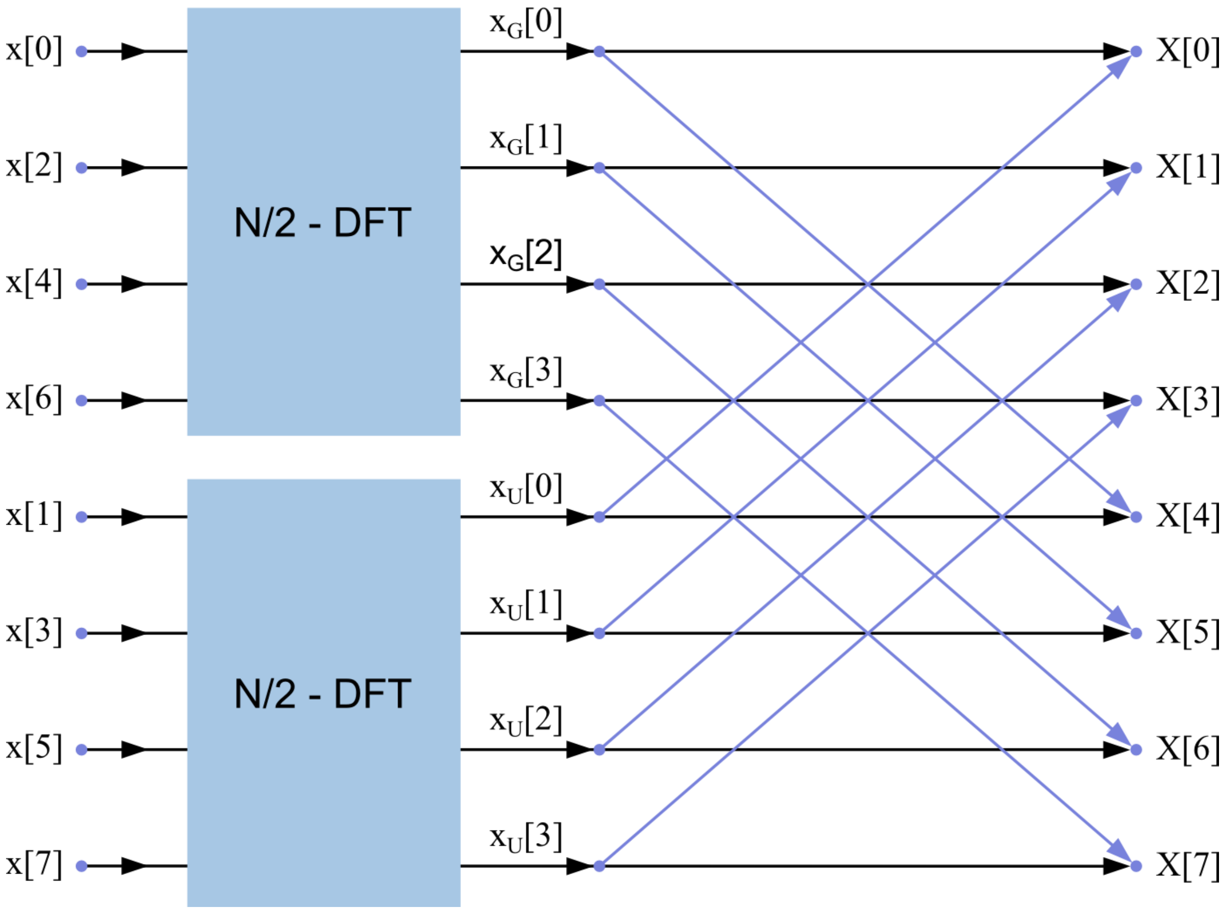
The indices can be divided according to the following scheme.

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2} mk}}_{G_k\text{: DFT of even index}} + e^{-\frac{2\pi i}{N} k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2} mk}}_{U_k\text{: DFT of uneven index}}$$

$$\begin{aligned} X_k &= G_k + e^{-\frac{2\pi i}{N} k} \cdot U_k \\ X_{k+\frac{N}{2}} &= G_k - e^{-\frac{2\pi i}{N} k} \cdot U_k \end{aligned}$$

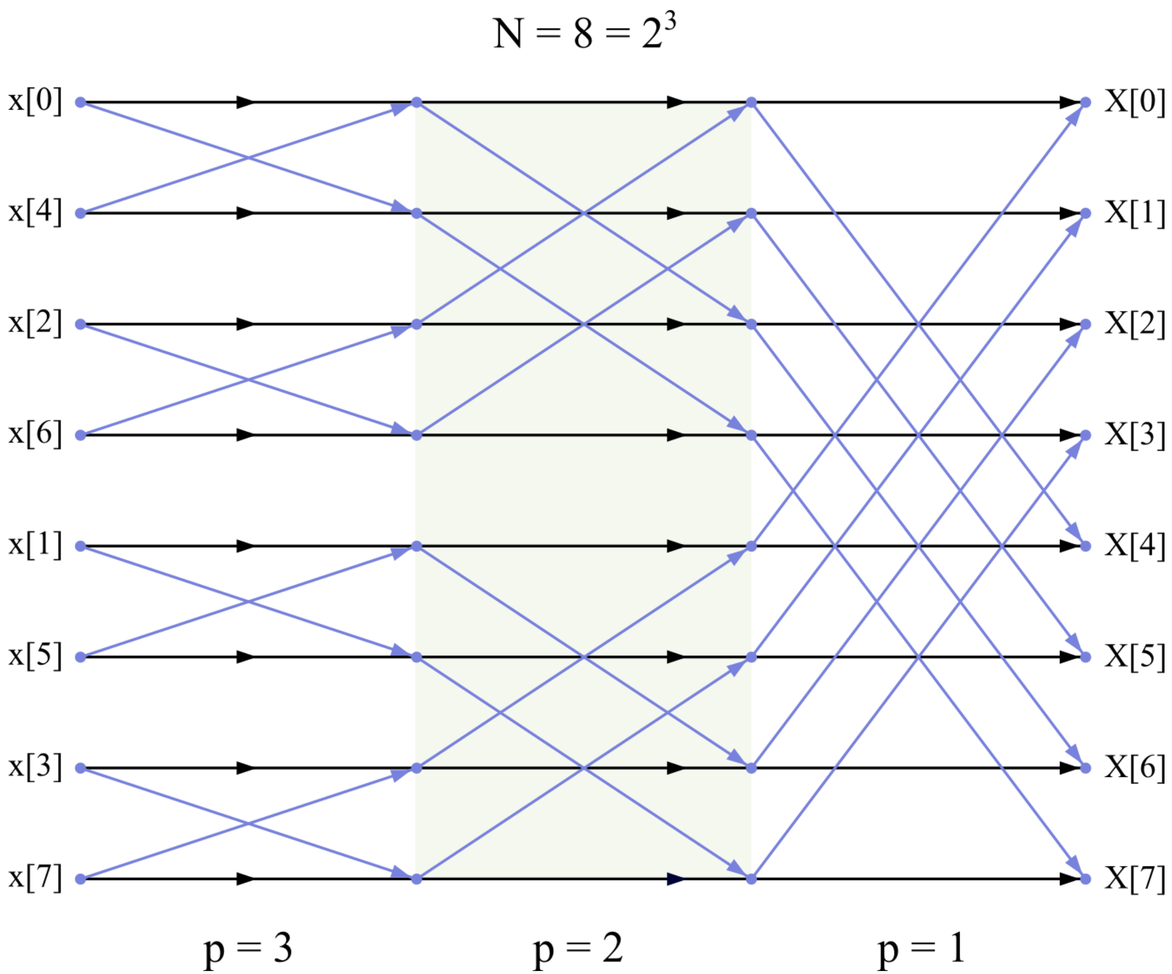
I.a: "Fast Fourier Transformation" II

Signal flow diagram of the FFT for with halving of the DFT input data.



I.a.: "Fast Fourier Transformation" III

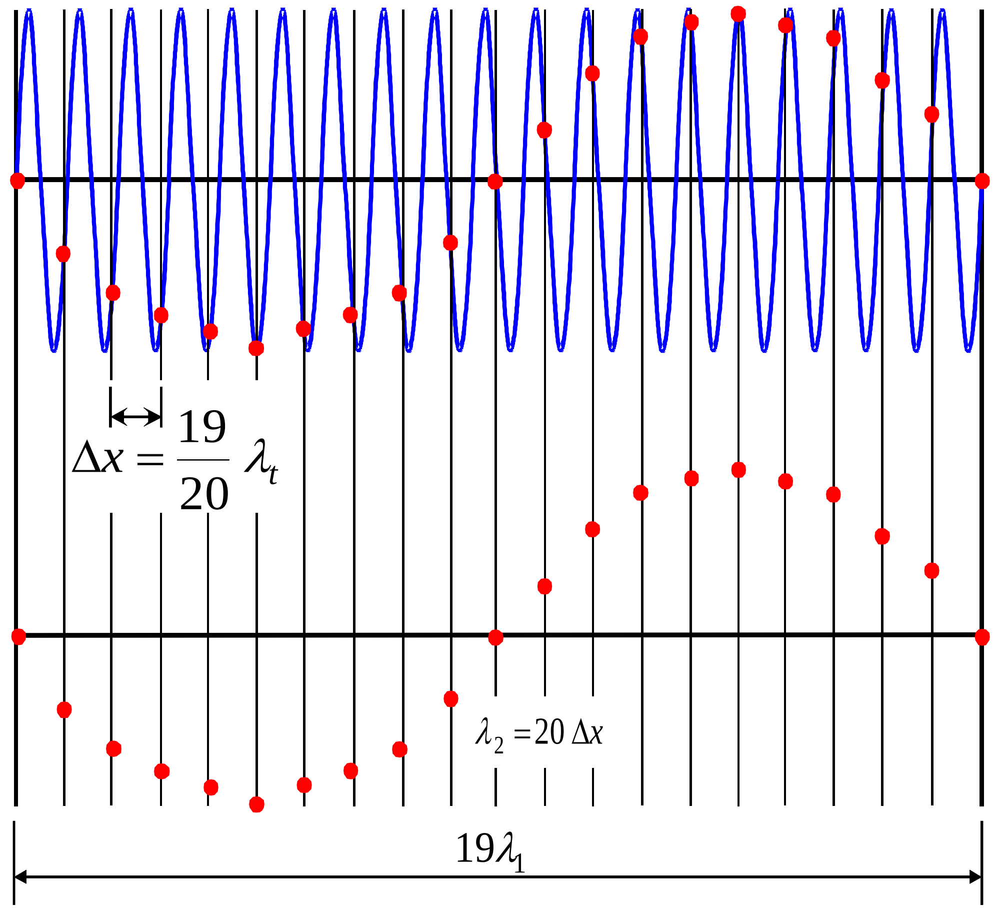
Further reduction of the DFT input data for by swapping the indices in *p-steps*:



I.a: Sensing disorder I

- Error in signal acquisition due to non-compliance with the sampling theorem (sampling frequency too low)

- lead to digitized signal sequences that are not contained in the original signal



[Jähne 1991]

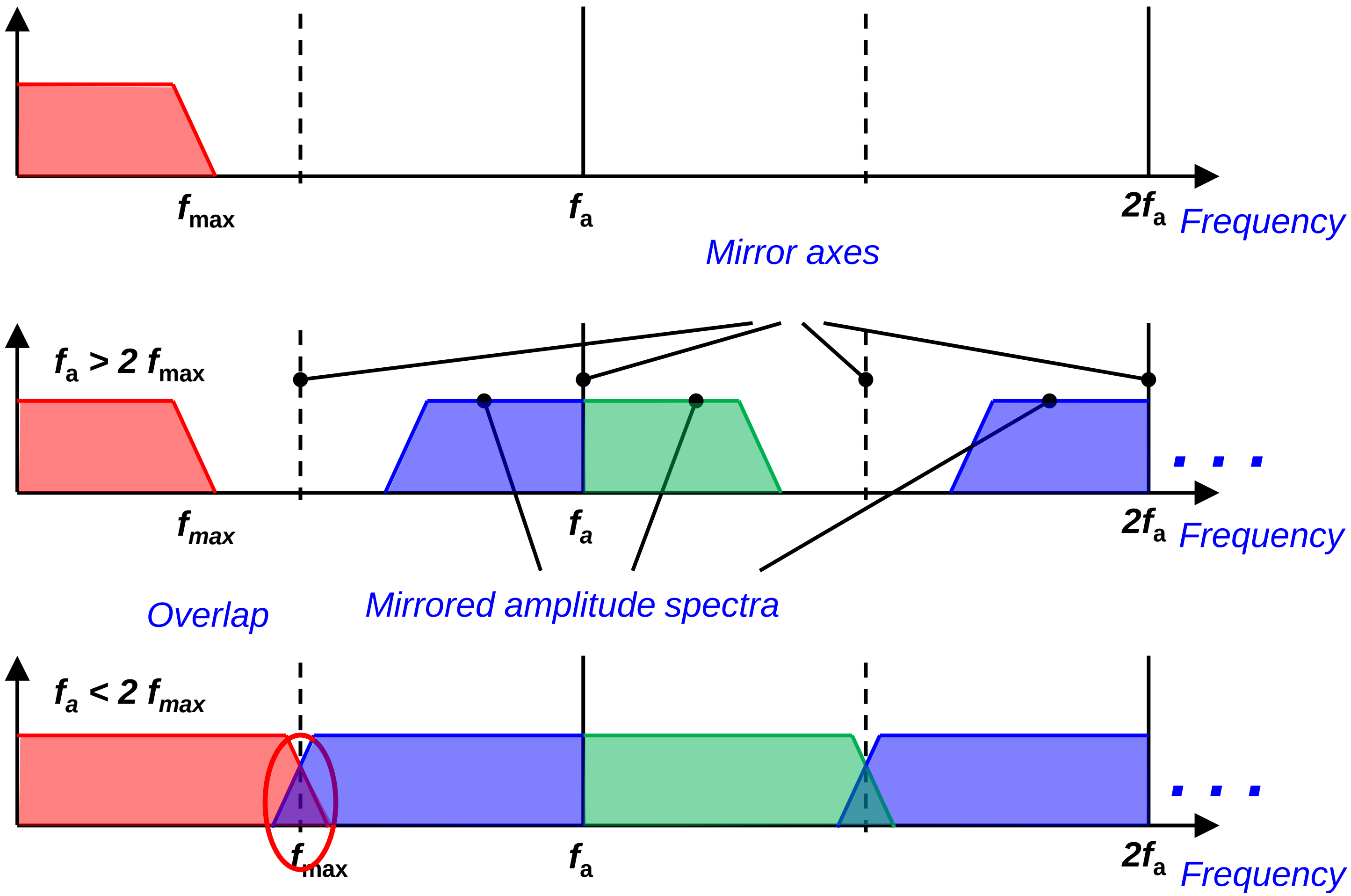
I.a: Sensing disorder II

Introduction of errors in the frequency spectrum of a sampled signal when components with frequencies too high to be analyzed at the sampling interval used contribute to the amplitudes of the components with lower frequencies [DIN IEC 60050-351:2013-07]

I.a.: Shannon's sampling theorem

Amplitude spectrum of an analog signal

Amplitude spectrum of a digital signal



$\boxed{f_a \geq 2 f_{max}}$

$f_a = 1/T_a$: ***Sampling frequency***

f_{max} : ***maximum signal frequency***

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c. Determination of measurement uncertainty according to GUM

II. MATLAB

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I.b.: Definition of measurement deviation according to VIM 2012

No measurement is completely controllable, nothing is completely known

Measurement deviation *= measured value - reference value*

[VIM 2012 2.16]

Systematic measurement deviation:

*"Component of the **measurement error** that remains **constant** or **changes** in a **predictable manner** with **repeated measurements"***

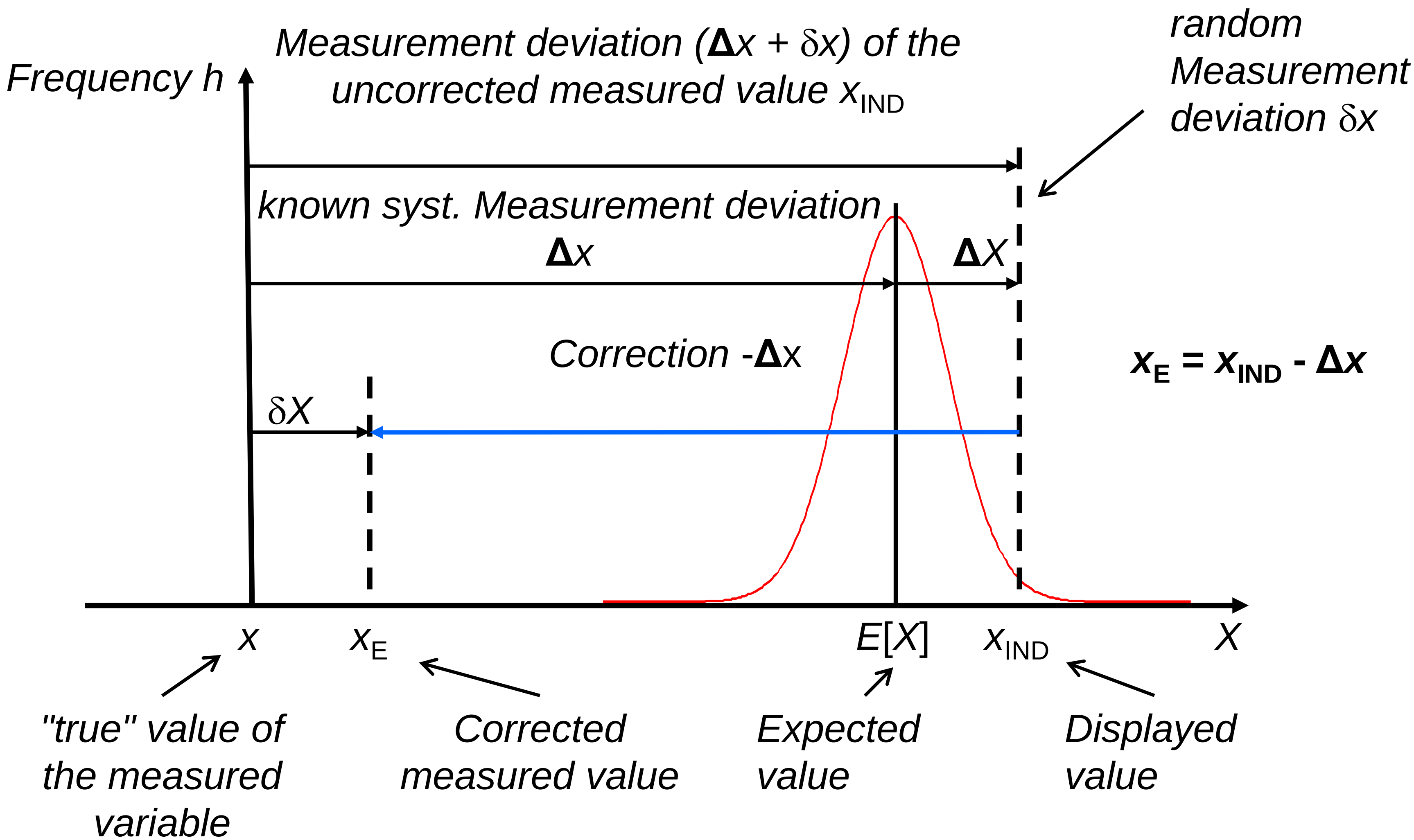
[VIM 2012, 2.17]

Random measurement deviation:

"Component of the measurement deviation that occurs with repeated measurements fluctuates in an unpredictable manner"

[VIM 2012, 2.19]

I.b.: Dealing with measurement deviations



I.b.: How good is a measurement?

random and systematic deviations
lead to uncertain measurement results

How good is the measurement?

How reliable are the measurement results?

How can the measurement result be quantitatively evaluated?

I.b.: Measuring precision

***Measurement precision*:**
*"Extent of agreement of **indications** or **measured values** obtained by **repeated measurements** on the same or similar objects under specified conditions"*
[VIM 2012, 2.15]

***Parameters for* measurement *precision*:**
Standard deviation s, variance s^2 or coefficient of variation c under specified measurement conditions

$$c = \frac{s}{\bar{x}}$$

I.b.: Measuring accuracy

***Measuring accuracy*:**
"Extent of approximation ***of a*** *measured* ***value***
to a ***true value*** *of a* ***measurand****"*
[VIM 2012, 2.13]

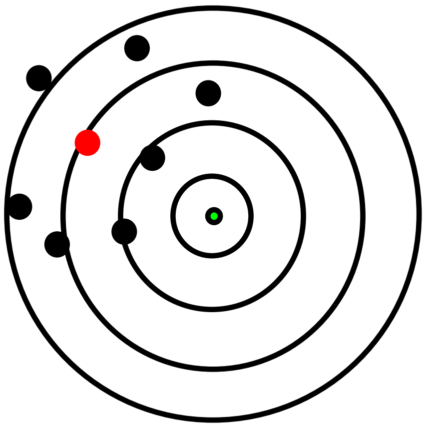
- *is* ***not a variable*** *and is* ***not expressed quantitatively***
- *It is said that a measurement is more accurate if it has a smaller measurement deviation*

I.b.: Measurement accuracy

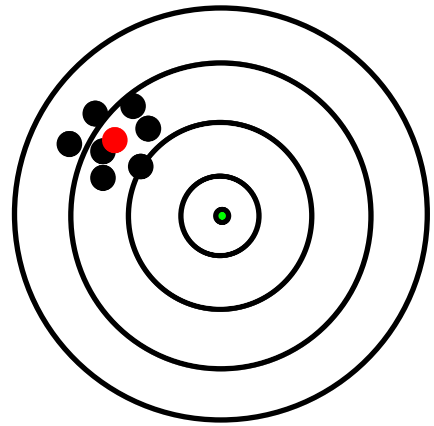
Measurement accuracy**:**
"Extent of ***approximation of*** *the* ***mean value*** *of a*
infinite number of repeated measured values
a ***reference value****"*
[VIM 2012, 2.14]

- *is* ***not a variable*** *and is* ***not expressed quantitatively***
- *is inversely related to the systematic error of measurement*

I.b.: Example: Shooting at a target

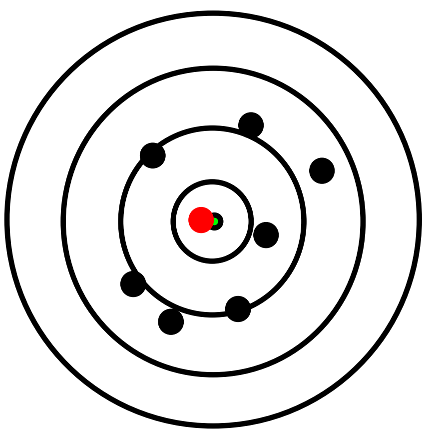


- − *Precision*
- − *Measurement accuracy*
- − *Accuracies*

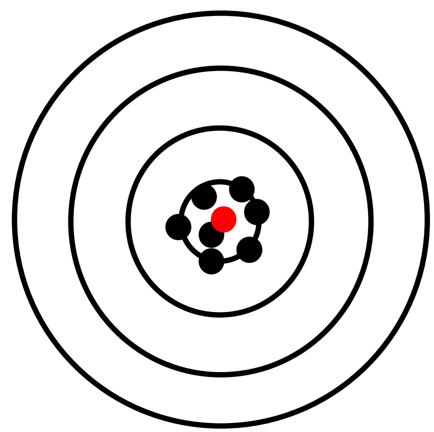


- + *Precision*
- − *Measurement accuracy*
- − *Accuracies*

● *single value mean value true value*



- − *Precision*
- + *Measurement accuracy*
- − *Accuracies*



- + *Precision*
- + *Measurement accuracy*
- + *Accuracies*

Overview of lecture content

I. **Measurement technology**
   a. Fourier analysis
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   c. Determination of measurement uncertainty according to GUM

II. **MATLAB**
   a. Basics of programming with MATLAB
   b. Visualize data
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I.c.: Error propagation law (old concept)

The Gaussian error propagation law is used to determine how large the expected error of a measurement result calculated from deviating measured values is. according to [Hoffmann 2000]

$$y = f(x_1, \ldots, x_N)$$

***for random deviations*:**

$$s_\mathrm{y} = \sqrt{\sum_{j=1}^{N} \left(\frac{\partial f}{\partial x_j} s_j \right)^2}$$

s_y *Standard deviation of the overall result*

x_j *Individual sizes*

s_j *Standard deviation of the individual variables*

$\frac{\partial f}{\partial x_j}$ *partial derivatives*

N *Number of input variables*

I.c.: Error propagation law (old concept)

for systematic deviations:

$$\Delta y = \sum_{j=1}^{N} \left(\frac{\partial f}{\partial x_j} \Delta x_j \right)$$

Δy *Systematic deviation of the overall result*

x_j *Individual sizes*

Δx_j *Average deviation of the individual variables*

$\frac{\partial f}{\partial x_j}$ *partial derivatives*

N *Number of input variables*

I.c.: Disadvantages of classic error analysis

***Disadvantages*:**
- *Systematic deviations are usually not taken into account*
- *Correlations not taken into account*

Other reasons:

1. *Quantification of measurement deviations **ultimately requires knowledge of a** "(single) **true value**"*
2. ***Overemphasis on statistically determined deviations***
3. ***Separate determination of measurement result and uncertainty***
4. *Measurement model ("measurement equation") plays practically no role*
5. *There are neither completely known systematic measurement deviations nor completely unknown*
6. *No generally accepted definition of uncertainty*

I.c.: Measurement uncertainty

Measurement uncertainty:

*"non-negative parameter **characterizing** the dispersion of **values** associated with **the measurand on the** basis of the information used"*

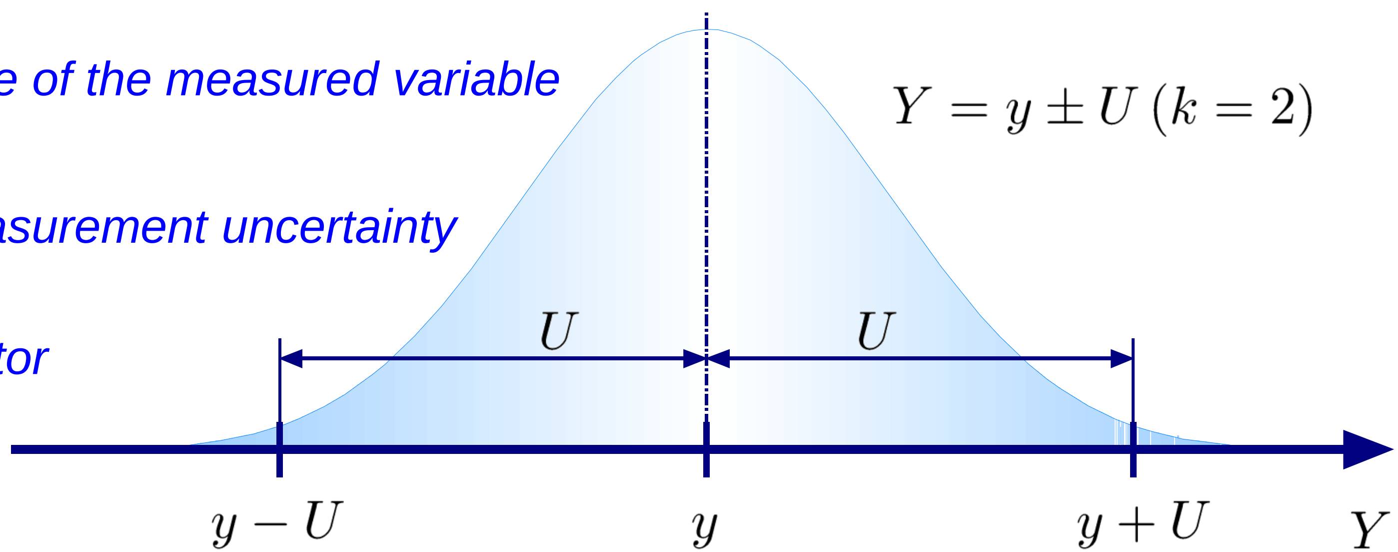
[VIM 2012, 2.26]

Y *Measured variable*

y *Expected value of the measured variable*

U *expanded measurement uncertainty*

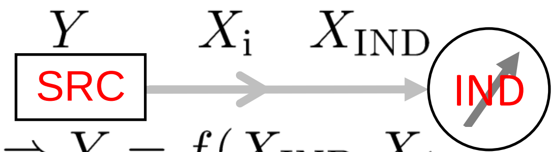
k *Expansion factor*



I.c.: Structure of the standard procedure of the GUM

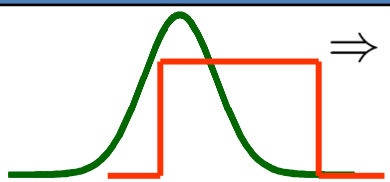
1 Collecting knowledge about the measuring process and the involved input quantities /
Gather knowledge about the measurement process and the input variables involved

2 Create a model for evaluation of the measurement /
Setting up the model for the measurement evaluation



$\Rightarrow X_{\mathrm{IND}} = g(Y, X_1, \ldots, X_{\mathrm{N}})$ $\Rightarrow Y = f(X_{\mathrm{IND}}, X_1, \ldots, X_{\mathrm{N}})$

3 Evaluating the input quantities /
Estimating the input variables
Methods: Type A and B /
Methods: Type A and B



$\Rightarrow \mathrm{PDF} \Rightarrow x_{\mathrm{i}}, u(x_{\mathrm{i}}), \nu_{\mathrm{i}}$

$x_{\mathrm{i}} = E[X_{\mathrm{i}}]$ $u(x_{\mathrm{i}}) = \sqrt{\mathrm{Var}[X_{\mathrm{i}}]}$

4 Calculation of the measured value and combining the uncertainties /
Calculating measured values and combining uncertainties $\Rightarrow y, u_{\mathrm{c}}$

5 Calculation of the expanded uncertainty
Calculating the expanded measurement uncertainty $\Rightarrow U, k$ $\Rightarrow U_{\mathrm{p}}, k_{\mathrm{p}}, \nu_{\mathrm{eff}}$

6 Indication and evaluation of the measurement result
Specifying and evaluating the measurement result $\Rightarrow y \pm U \; (k = \ldots)$

I.c.: Step 1 of the standard GUM procedure

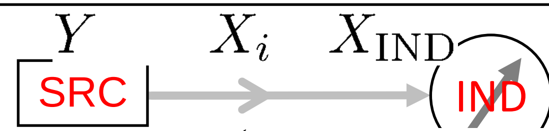
1 *Gather knowledge about the measurement process and the input variables involved*

***Collecting information about the measurement process*:**

- *Description of the measurement task*
- *Measuring principle used*
- *Measurement methods used*
- *Supplementary descriptions of the measurement procedure*

I.c.: Step 2 of the standard GUM procedure

2 *Setting up the model for the measurement evaluation*



$\Rightarrow X_{\mathrm{IND}} = g(Y, X_1, \ldots, X_N)$ $\Rightarrow Y = f(X_{\mathrm{IND}}, X_1, \ldots, X_N)$

possibly:
Setting up the mathematical
Model of the measurement and subsequent inversion

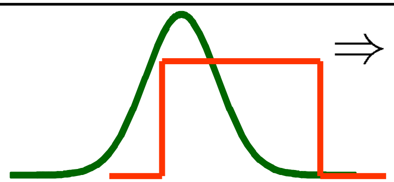
$Y \rightarrow$ *Measurement* $X_{\mathrm{IND}} = g(Y, X_1, \ldots, X_N)$ $\rightarrow X_{\mathrm{IND}}$, input X_i

or the same:
Setting up the mathematical
Model for measurement evaluation

$X_{\mathrm{IND}} \rightarrow$ *Evaluation* $Y = f(X_{\mathrm{IND}}, X_1, \ldots, X_N)$ $\rightarrow Y$, input X_i

I.c.: Step 3 of the standard GUM procedure

3 *Estimating the input variables*
Methods: Type A and B



$$\Rightarrow \mathrm{PDF} \Rightarrow x_i, u(x_i), \nu_\mathrm{i}$$

$$x_i = E[X_i] \quad u(x_i) = \sqrt{\mathrm{Var}[X_i]}$$

***Sources for the quantitative evaluation of input variables*:**

- *Results of direct measurements, comparative measurements*
- *Empirical values, results of previous evaluations*
- *Values from calibration certificates and other certificates*
- *Manufacturer's data, literature values*

I.c.: Step 3 of the standard GUM procedure

***Method type A*:**
Statistical analysis
(series of measurements)

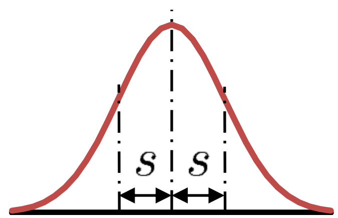
$$\bar{q} = \frac{1}{n}\sum_{j=1}^{n} q_j$$

$$s(q_j) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(q_j - \bar{q})^2}$$

$$x_i = \bar{q}$$

$$u(x_i) = s(q_j)/\sqrt{n}$$

$$\nu_i = n - 1$$



approximation for large n)
for statistical analyses: n ≥ 4

***Method type B*:**
other information

$$\nu_i = \infty$$

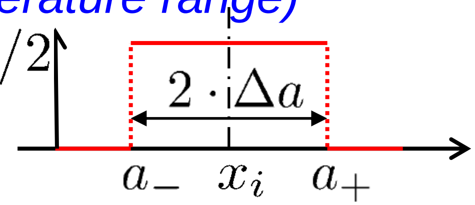
- *a single measured value*

$x_i, u(x_i)$ *or* $x_i, u(x_i), \nu_i$

- *Specification of a lower and upper limit (e.g. temperature range)*

$$x_i = (a_+ + a_-)/2$$

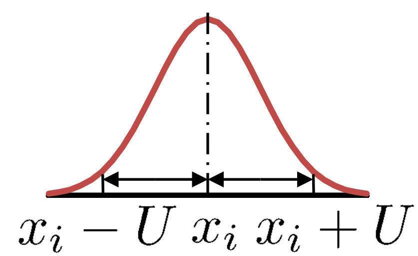
$$u(x_i) = \Delta a/\sqrt{3}$$



- *Specification in a calibration certificate*

$$x_i$$

$$u(x_i) = U/k$$



Source: http://www.ptb.de/cms/fileadmin/internet/publikationen/kessel.pdf (as at: 11.04.2013)

I.c.: Step 4 of the standard GUM procedure

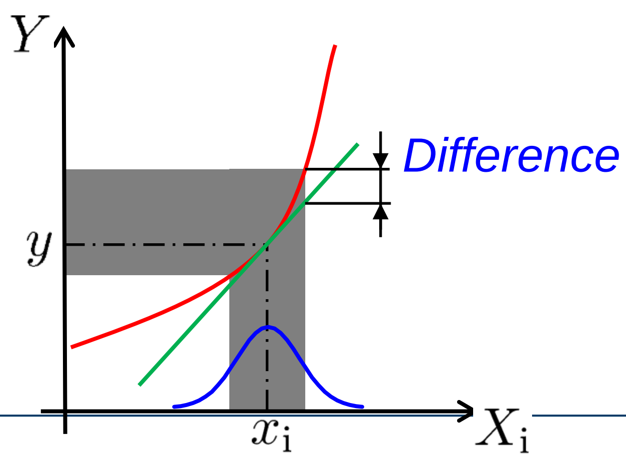
4 Calculation of the measured value and combining the uncertainties / $\Rightarrow y, u_{\mathrm{c}}$
Calculating measured values and combining uncertainties

A) *Measured value calculation*:

$$y = f(x_{\mathrm{IND}}, x_1, \ldots, x_N)$$

B) *Combining the uncertainties*:

Basic idea of the error propagation law:
Taylor series development → Linearization for operating point x $_i$



$$c_i = \left.\frac{\partial f}{\partial X_i}\right|_{x_i} = \frac{\partial f}{\partial x_i}$$

Sensitivity coefficient

□ *only possible for sufficiently linear models*

I.c.: Step 4 of the standard GUM procedure

***for uncorrelated input variables*:**

$$u_{\mathrm{c}}^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) = \sum_{i=1}^{N} c_i^2 u^2(x_i)$$

$u(x_i)$ *Standard uncertainty*

***for correlated input variables*:**

$$u_{\mathrm{c}}^2(y) = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

$u(x_i, x_j)$ ➡ *Covariance*

$$u_{\mathrm{c}}^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

Correlation coefficient

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i) u(x_j)}$$

$$u_{\mathrm{c}}^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j r(x_i, x_j) u(x_i) u(x_j)$$

I.c.: Step 5 of the standard GUM procedure

5 *Calculating the expanded measurement uncertainty* $\Rightarrow U, k$ $\Rightarrow U_{\mathrm{p}}, k_{\mathrm{p}}, \nu_{\mathrm{eff}}$

***for determination method type A*:**

Degree of freedom ν_i

= *number of observations - number of parameters to be determined*

only for a small number of observations

$\rightarrow$ *Consideration of the reliability of the standard measurement uncertainty*

***effective degree of freedom*:**

$$\nu_{\mathrm{eff}} = \frac{u(y)^4}{\sum_{i=1}^{N} \frac{u(x_i)^4}{\nu_i}}$$

Welch - Satterthwaite

$\rightarrow$ *Assessment of the reliability of the combined standard uncertainty of measurement*

I.c.: Step 5 of the standard GUM procedure

Selection of the expansion factor k or k_p :

$$U_\mathrm{p}(y) = k_\mathrm{p} \cdot u_\mathrm{c}(y) \quad \text{for} \quad k_\mathrm{p} = t_\mathrm{p}(\nu_\mathrm{eff})$$

for *p* = 68.3 %; *p* = 95.5 %; *p* = 99.73 %

- *depending on the effective degree of freedom*
- *from the t-distribution*

ν_{eff}	**3**	**4**	**5**	**10**	**20**	**50**	∞
k_p (p = 95 %)	3,18	2,78	2,57	2,23	2,09	2,01	1,96

for method type B and a large number of observations

typical values k = 2 for p = 95 % and k = 3 for p = 99 %

$$U(y) = k \cdot u_\mathrm{c}(y) \quad \text{for} \quad k = t_\mathrm{p}(\infty)$$

1.c.: Step 6 of the standard GUM procedure

6 *Specifying and evaluating the measurement result* $\Rightarrow y \pm U\ (k=2)$

general form: $y \pm U\ (k=2)$

L = (22.17 ±0.11) mm (***k*** = 2) ***or*** ***L*** = 22.17 mm ±0.11 mm (***k*** = 2)

"The standard measurement uncertainty has been determined in accordance with EA-4/02 or DKD-3 (DIN V ENV 13005)."

- *Specify measurement uncertainty with a maximum of 2 significant digits*
- *Round mathematically! (Rounding only by a maximum of 5 %)*
- *Specify the value of the result according to the significant digits of the measurement uncertainty (DIN 1333)*

I.c.: Step 4 of the standard GUM procedure

Measurement uncertainty balance:

*"Specification of a **measurement uncertainty**, the components this measurement uncertainty and its calculation and combination"*

[VIM 2012, 2.33]

$$u_i(y) = c_i \cdot u(x_i)$$

Size X_i	***Value*** x_i	***Uncertainty*** $u(x)_i$	***Distribution***	***Degree of freedom*** ν_i	***Sensitivity coefficient*** c_i	***Uncertainty contribution*** $u_i(y)$
X_1	$x_1 = \ldots$	$u(x)_1$	Form 1	ν_1	$c_1 = \partial f/\partial x_1$	$u_1(y)$
X_2	$x_2 = \ldots$	$u(x)_2$	Form 2	ν_2	$c_2 = \partial f/\partial x_2$	$u_2(y)$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
X_N	$x_N = \ldots$	$u(x)_N$	Form *N*	ν_N	$c_N = \partial f/\partial x_N$	$u_N(y)$
Y	$y = \ldots$					$u_C(y)$

I.c.: Determination of measurement uncertainty by computer simulation

Determination of the measurement uncertainty using the Monte Carlo method
(DIN EN 13005 Supplement 1:2010)

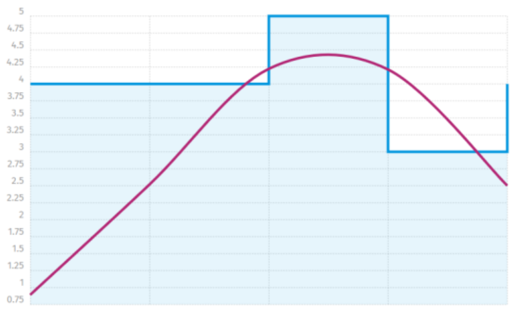
- *Application for*:
  - *non-linear model of the measurement*
  - *Deviation of the probability density function from a Gaussian distribution*
- *always returns a result for the probability density function*
- *Random experiments carried out very frequently (law of large numbers)*

I.c.: Central limit theorem

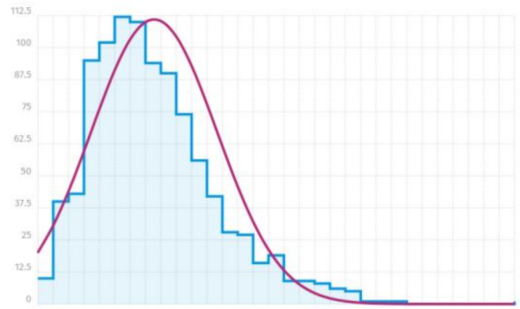
"The mean values of the samples of a population are normally distributed."

Example: Sampling distributions of the exponential distribution

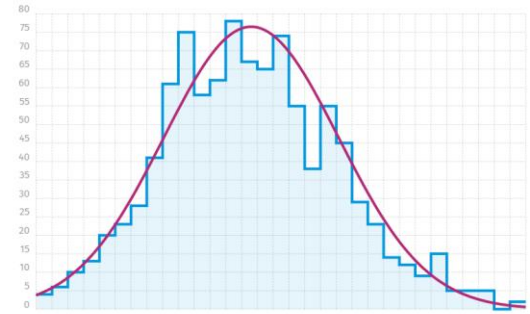
Number of samples: *n*
Sample size: *m*



$n = 20, m = 5$

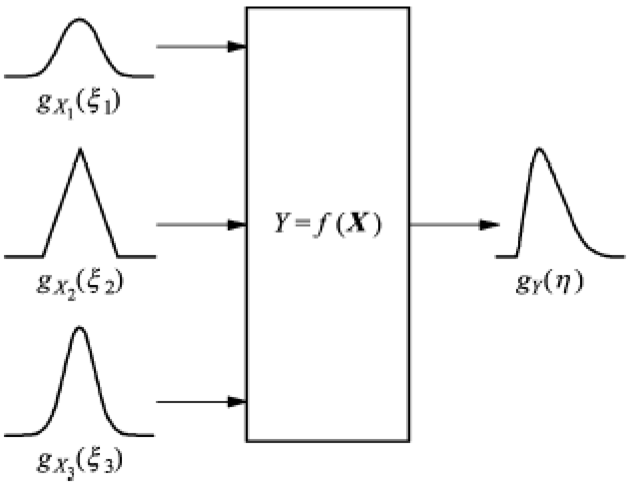


$n = 1000, m = 5$

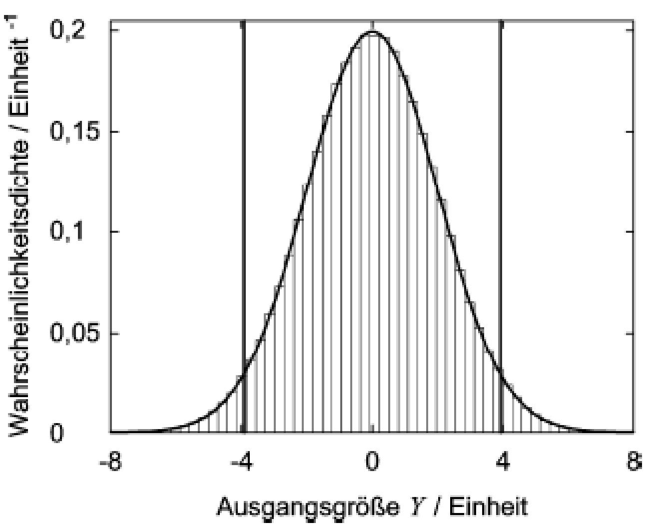


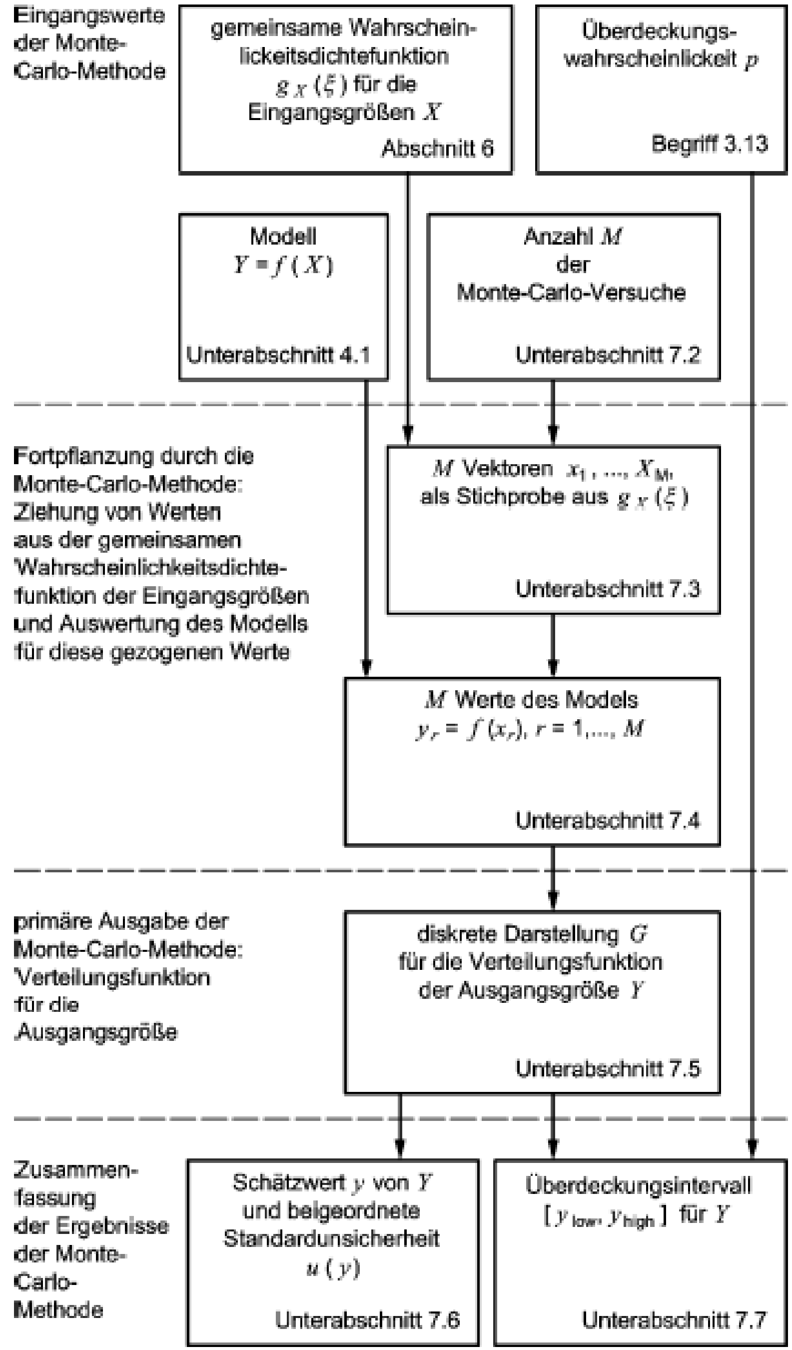
$n = 1000, m = 30$

I.c.: Monte Carlo method



[DIN V ENV 13005 Supplement 1:20





Overview of lecture content

I. Measurement technology

a. Fourier analysis

b. Evaluation of measurement results

c. Determination of measurement uncertainty according to GUM

II. MATLAB

a. Basics of programming with MATLAB

b. Visualize data

c. Symbolic Math Toolbox and useful functions

II.a.: Styleguide I

```
n = 10; l = 3;
A = (n / 4) * l^2 * cot(pi / (2*n));
```

What is calculated here?

```
numberEdges = 10; edgeLength = 3;
areaPolygon = (numberEdges/4)*edgeLength^2*cot(pi/(2*numberEdges));
```

Better!

Suggestions for the realization of meaningful naming:

- Assign meaningful names, paying attention to the scope!
- Write variables in lower case, classes and functions (scripts) in upper case
- Use compound words with alternating capitalization
- Good names increase the understanding of code enormously
- Good names are part of a good documentation of the program
- Only use English-language names

Robert C. Martin: "Clean Code: A Handbook of Agile Software Craftsmanship"
MATLAB Programming Fundamentals:
https://www.mathworks.com/help/pdf_doc/matlab/matlab_prog.pdf

II.a.: Styleguide II

```
vectorLength = sqrt(x^2 + y^2); % formula documented at ...
```

Use the comment function to provide added value for comprehensibility, e.g. on derivations for formulas or other contexts used.

Do not document any superfluous information!

Write comments in English!

```
qualityParameterZaxis = offsetParameterYaxis + weightingZaxis +
...
baseQualityParamterZaxis;
```

Long lines can be wrapped by '...' to structure code clearly. Spaces can be placed anywhere in the code between individual elements.

Style guides are always optional, but it is strongly recommended to adopt a suitable style and not to change it for a project.

II.b.: *figure* class

MATLAB *figure* class https://de.mathworks.com/help/matlab/ref/figure.html

`figure` Creates new *figure* with default settings

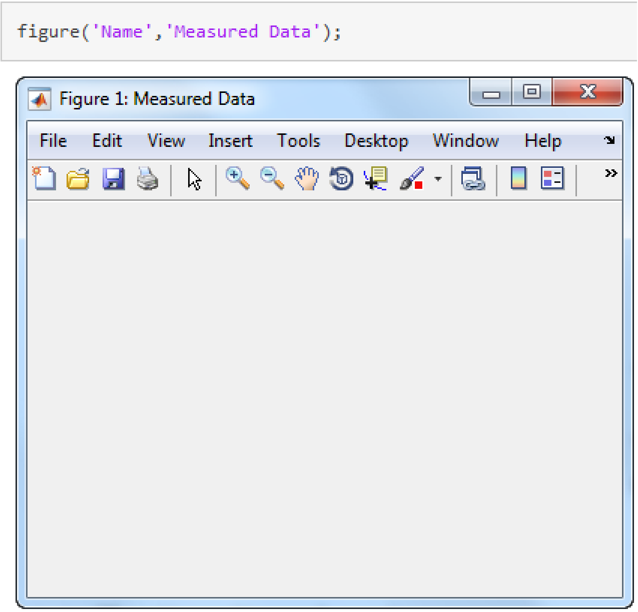
`figure(Name,Value)` Modifies the settings of the *figure*

`f = figure(___)` Returns the *figure object*

`figure(f)` Sets *figure* f as *current figure*

Current figure:
Target for graphic instructions such as *axes*

axes are automatically created in the *current figure* when a data visualization function is executed



https://de.mathworks.com/help/matlab/ref/figure.html#bvjs6cb-5

II.b.: plot() function

Overview of functions provided for visualizing data:

https://de.mathworks.com/help/matlab/creating_plots/types-of-matlab-plots.html

`plot(X,Y)` Creates 2-D line plot with x and y values (vectors)

`plot(X,Y,LineSpec)` Defines the line representation

`plot(X1,Y1,...,Xn,Yn) Represents` several x/y pairings in a plot

https://de.mathworks.com/help/matlab/ref/plot.html

`hold on` prevents existing *axes from* being deleted when a new one is created

`title` Defines a title for the selected axes object

`xlabel | ylabel | zlabel` Adds axis labeling

`xlim | ylim | zlim` Selects display range for axes manually

`legend` Defines legend entries for *axes* objects

`cla` deletes any existing data in the selected *axes* object

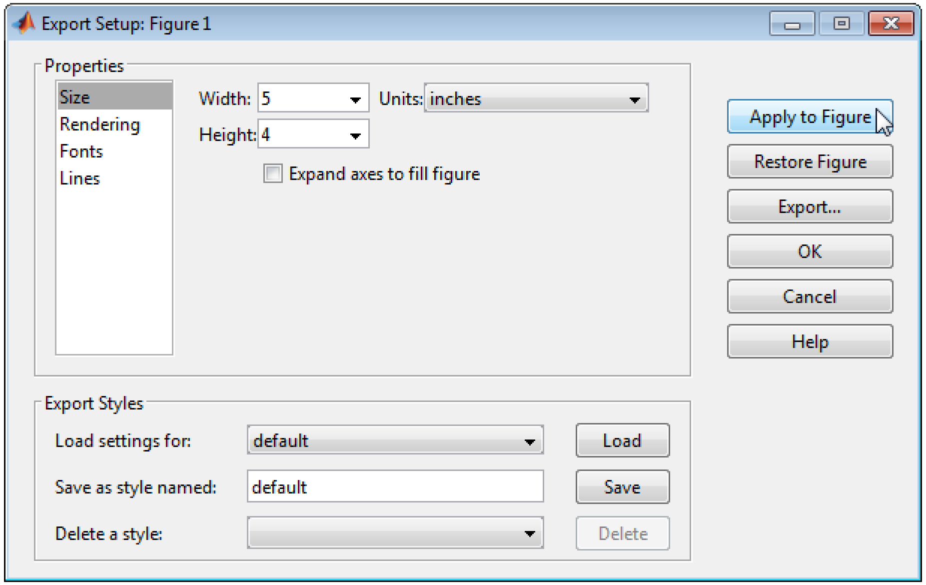
I.b.: Export of plots

After plots have been created and neatly formatted, the results must be suitably formatted and exported, e.g. for final theses

Settings for the size of the plot (e.g. adjustment to the page width of the format template of a Word document possible)

Resolution of the resulting image is adjustable to create high quality graphics

Recommendation: Save the figure as a .fig file (MATLAB format) and (uncompressed) .tiff for the respective application



https://de.mathworks.com/help/matlab/creating_plots/customize-figure-before-saving.html

II.c.: Introduction to the Symbolic Math Toolbox

Symbolic computing enables the direct implementation and processing of mathematical relationships, but generally at the expense of processing speed.

`syms x y f` Creates symbolic variables or functions

`f = x^2 + y^3;` Defines symbolic function

`g = int(f,x);` Integrates function f according to variable x symbolically, (function **`diff()`** for differentiating)

`solution = solve(g == 0,x);`Finds the zeros of g according to variable x

`f = subs(f,[x y],[1 4]);` Replaces the variables x and y in the function f with the numerical values 1 and 4 and stores the result in f again.

`result = double(f);` Converts the symbolic result of f ($1^2 + 4^3$) into a floating point number

II.c.: Useful functions for the practical part

`transformedSignal = fft(signalData);` Fourier Transformation

`transformedSignal = ifft(signalData);` inv. Fourier Transformation

`dx = gradient(x);` numerical gradient of the vector x

`y = round(x,n);` Rounds x to the next integer value or decimal place, n>0 rounds place to the right of the decimal point

`c = unique(a);` Unique values in the vector/array

`y = sqrt(x);` root of x

`y = randn(sz1,...,szn);` Matrix of normally distributed random numbers sz1 x ... x szn

`y = std(x);` standard deviation of x

`y = range(x);` Range of x

`y = struct();` create new struct

`y.a = 10;` `adds` a new field to y with the name a and value 10

`y.('a') = 10;` adds a new field with the name a and value 10 to y

`y.(a) = 10;` adds a new field to y with the value of a and the value 10

Thank you for your attention!

17.06.2024 Group A, 12:00 AM – 6:00 PM, CIP-Pool-MB

24.06.2024 Group B, 12:00 AM – 6:00 PM, CIP-Pool-MB

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Technische Fakultät

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