Combustion Exam

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Problem 1

Consider a concentric spherical porous burner through which a pure gaseous fuel enters through the inner sphere (r_i) and oxidant enters through the outer sphere (r_0). Assume that the mass averaged velocity in the system is zero and that the fuel and air diffuse from the walls and are maintained at the walls with a mass fraction of unity. The walls are a sink to the products and they disappear at the wall. A schematic of the arrangement is shown in figure 1 below.

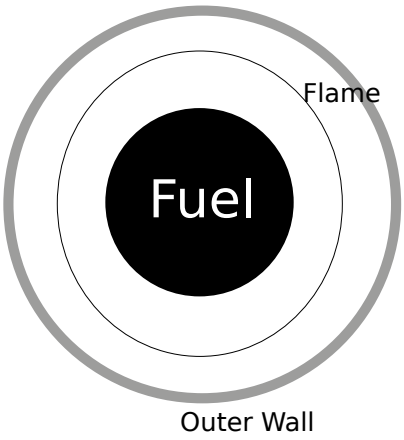

Figure 1: The spherical burner apparatus.

The governing equation for the fuel is assumed to be,

$$\frac{d}{dr}\left(r^2 \rho D \frac{dY_i}{dr}\right) = \pm r^2 w_i.$$

Where,

$$w_f = A Y_f Y_o e^{-E/RT}.$$

a) How does the fuel mass fraction (Y_i) vary functionally with radius?

We solve this spherical diffusion flame problem as a quasi-steady problem, in that we assume the fuel at the center is not shrinking as a function of time. Note that,

$$\omega = \frac{\omega_i}{\nu_i W_i} = \frac{\omega_F}{\nu_F W_F} = \frac{\omega_O}{\nu_O W_O}$$

This hints at a conserved scalar form that will decouple the chemistry from the convection-diffusion of a conserved scalar quantity. In particular,

$$\mathcal{L}(\beta) = \mathcal{L}\left(\frac{Y_O}{\nu_O W_O} - \frac{Y_F}{\nu_F W_F}\right) \Rightarrow \frac{d}{dr}\left(r^2 \rho D \frac{d\beta}{dr}\right) = 0.$$

Now, we must solve this equation. After the first integration, we have,

$$\frac{d\beta}{dr} = \frac{C_1}{r^2}.$$

This assumes constant density and transport properties. The second integration in r leaves us with,

$$\beta(r) = C_2 - \frac{C_1}{r}.$$

In other words, $Y_i \propto \frac{1}{r}$.

One comment: the boundary conditions are stated as, "the fuel and air diffuse from the walls and are maintained at the walls with mass fraction of unity". In other words, $B(r_i) \rightarrow Y_{F,i} = 1, Y_{O,i} = 0$ and $B(r_o) \rightarrow Y_{F,i} = 0, Y_{O,o} = 1$.

b) How does the temperature vary functionally with radius?

The β in part a is not unique. We can also form a conserved scalar quantity by defining $\beta = T + Y_i$. This results in precisely the same conserved scalar differential equation form,

$$\mathcal{L}(\beta) = \mathcal{L}\left(\frac{Y_O}{\nu_O W_O} - \frac{Y_F}{\nu_F W_F}\right) \Rightarrow \frac{d}{dr}\left(r^2 \rho D \frac{d\beta}{dr}\right) = 0.$$

With an identical solution, aside from different constants of integation,

$$\beta(r) = C_2' - \frac{C_1'}{r}.$$

We assume that the Damkohler number is sufficiently large that there is no (or at least, negligible) reaction leakage across the flame sheet. This is a statement that $Da \rightarrow \infty$ in,

$$\hat{\mathcal{L}}(\beta) = \text{Da } Y_f Y_O e^{-E/RT}$$

Where $\hat{\mathcal{L}}()$ is the non-dimensional form of the operator. This means we are assuming that the reactions are infinitely fast (in comparison to diffusion in the fluid) and therefore, the flame sits at an asymptotically thin region (a flame sheet) where it instantly consumes any reactants. This is the Burke-Schumann limit.

The Shvab-Zel'dovich form is then,

$$\beta_F(r) = T + Y_F = C_2' - \frac{C_1'}{r}$$
$$\beta_O(r) = T + Y_O = C_2' - \frac{C_1'}{r}$$

As a consequence of no reaction leakage,

$$Y_F = 0 \text{ when } r_f < r < r_o$$
$$Y_O = 0 \text{ when } r_i < r < r_f$$

Evaluating in these regimes then implies that the functional form of temperature must therefore also be,

$$T = k - \frac{k}{r}$$

Where k is yet another constant of integration. In other words, $T \propto \frac{1}{r}$.

d) At what value of the radius does the flame sit?

$$\begin{aligned} r_f &= \left(\frac{Y_{F,i}}{Y_{F,i}+Y_{O,o}}\right)(r_o - r_i) + r_i \\ &= \left(\frac{1}{1+Y_{O,o}}\right)(r_o - r_i) + r_i \\ &= \left(\frac{1}{1+b}\right)(r_o - r_i) + r_i \end{aligned}$$

Where b is the amount of oxidizer in the reaction. In the event that $b = 1$, then the reaction is stochiometric, and the flame will sit at a distance halfway between the fuel and oxidizer sources.

Problem 2

Formulation

Our conserved scalar equation takes the form,

$$\rho\frac{\partial\beta}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho D\frac{\partial\beta}{\partial r}\right)$$

where again, the source terms have been elimintated. This conserved scalar has the form $\beta = Y_i + T$. In order to numerically solve this, we need to discretize these equations in time and space. To do this, we will use finite difference schemes of the form,

$$\begin{aligned}\frac{\partial^2\beta}{\partial r^2} &= \frac{\beta_{j+1}-2\beta_j+\beta_{j-1}}{\Delta r^2}\\ \frac{\partial\beta}{\partial r} &= \frac{\beta_{j+1}-2beta_{j-1}}{2\Delta r}.\end{aligned}$$

Inserting these into our equation, and discretizing time using a forward euler step turns this into a system of equations we solve at every spatial location:

$$\frac{\partial\beta^{n+1}-\beta^n}{\Delta t} = D\frac{r_{i+1/2}^2\left(\frac{\beta_{j+1}-2\beta_j+\beta_{j-1}}{\Delta r}\right) - r_{i-1/2}^2\left(\frac{\beta_{j+1}-2\beta_{j-1}}{2\Delta r}\right)}{r_i^2\Delta r}.$$

Here, $r_{i+1/2}$ is the location between the cells. Note that because this is an explicit method, it is stable only when the timestep does not exceed the CFL condition,

$$\Delta t = \frac{\Delta r^2}{2D}$$

Solving the conserved scalar equation provides a time dependent profile of that quantity. For a given profile of the conserved scalar, we can then obtain solutions for T, Y_f,Y_o, Y_p, etc.

Initial and Boundary Conditions

The conserved scalar is one at the fuel inlet($\beta = 1$), and starts zero everywhere else. I set the far radius location to be 20 cm. This seemed far enough away to be 'quasi-infinite'.