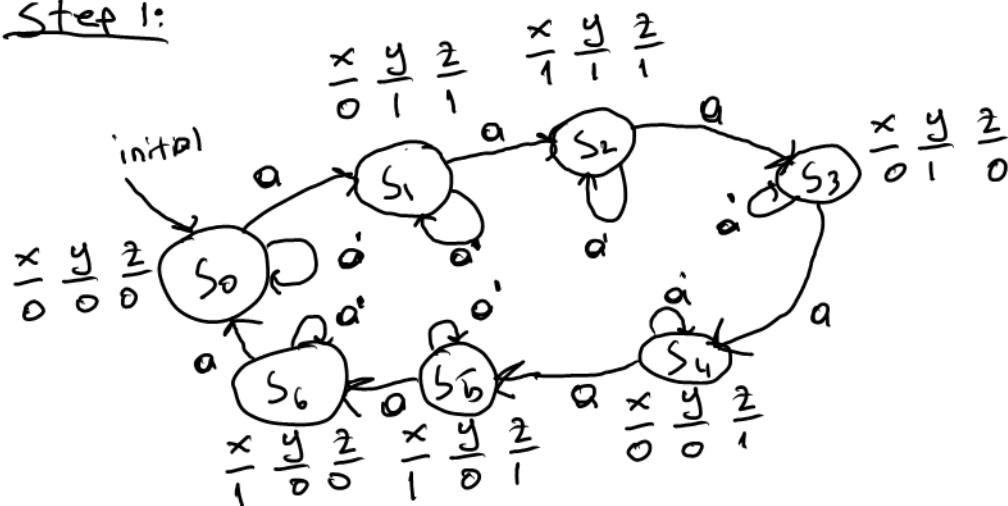


Step 1:



Step 2:

7 states: S_0 to S_6

Input: a

Output: x, y, z

3 bit register (input: $n_2 n_1 n_0$, output: $S_2 S_1 S_0$)

Output of combinational circ. input of register
 vice versa

Step 3: Encode states

$$S_0(000) = \begin{matrix} S_2 & S_1 & S_0 \\ 0 & 0 & 0 \end{matrix}$$

$$S_1(011) = \begin{matrix} 0 & 0 & 1 \end{matrix}$$

$$S_2(111) = \begin{matrix} 0 & 1 & 0 \end{matrix}$$

$$S_3(010) = \begin{matrix} 0 & 1 & 1 \end{matrix}$$

$$S_4(001) = \begin{matrix} 1 & 0 & 0 \end{matrix}$$

$$S_5(101) = \begin{matrix} 1 & 0 & 1 \end{matrix}$$

$$S_6(100) = \begin{matrix} 1 & 1 & 0 \end{matrix}$$

Step 4: Generate state table

| | INPUT | | | | OUTPUT | | | | | | |
|--------------------|-------|-------|-------|-----|--------|-----|-----|-------|-------|-------|--|
| | S_2 | S_1 | S_0 | a | x | y | z | n_2 | n_1 | n_0 | |
| State ₀ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | |
| State ₁ | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | |
| | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | |
| State ₂ | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | |
| | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | |
| State ₃ | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | |
| | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | |
| State ₄ | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | |
| | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | |
| State ₅ | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | |
| | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | |
| State ₆ | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | |
| | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | |
| Empty | 1 | 1 | 1 | 0 | x | x | x | x | x | x | |
| | 1 | 1 | 1 | 1 | x | x | x | x | x | x | |

Step 5:

Karnaugh Maps:

for x:

| $S_2 S_1 \backslash S_0 a$ | 00 | 01 | 11 | 10 |
|----------------------------|----|----|----|----|
| 00 | | | | |
| 01 | 1 | 1 | | |
| 11 | 1 | 1 | x | x |
| 10 | | | 1 | 1 |

$\rightarrow S_1 S_0'$ (purple box)
 $\rightarrow S_2 S_0$ (green box)

$$x = S_1 S_0' + S_2 S_0$$

for y:

| $S_2 S_1 \backslash S_0 a$ | 00 | 01 | 11 | 10 |
|----------------------------|----|----|----|----|
| 00 | | | 1 | 1 |
| 01 | 1 | 1 | 1 | 1 |
| 11 | | | x | x |
| 10 | | | | |

$\rightarrow S_2' S_0$ (green box)
 $\rightarrow S_2' S_1$ (red box)

$$y = S_2' S_1 + S_2' S_0 = S_2' (S_1 + S_0)$$

for z:

| $S_2 S_1 \backslash S_0 a$ | 00 | 01 | 11 | 10 |
|----------------------------|----|----|----|----|
| 00 | | | 1 | 1 |
| 01 | 1 | 1 | | |
| 11 | | | x | x |
| 10 | 1 | 1 | 1 | 1 |

$\rightarrow S_1' S_0$ (green box)
 $\rightarrow S_2' S_1 S_0'$ (red box)
 $\rightarrow S_2 S_1$ (purple box)

$$\begin{aligned}
 z &= S_2' S_1 S_0' + S_2 S_1' + S_1' S_0 \\
 &= S_1' (S_2 + S_0) + S_2' S_1 S_0'
 \end{aligned}$$

for n_2 :

| $S_2 S_1 \backslash S_0 a$ | 00 | 01 | 11 | 10 |
|----------------------------|----|----|----|----|
| 00 | | | | |
| 01 | | | 1 | |
| 11 | 1 | 1 | x | x |
| 10 | 1 | 1 | 1 | 1 |

$\rightarrow S_1 S_0 a$ (green box)
 $\rightarrow S_2 a'$ (red box)
 $\rightarrow S_2 S_1$ (purple box)

$$\begin{aligned}
 n_2 &= S_1 S_0 a + S_2 a' + S_2 S_1' \\
 &= S_2 (S_1' + a') + S_1 S_0 a
 \end{aligned}$$

for n_1 :

| $S_2 S_1 \backslash S_0 a$ | 00 | 01 | 11 | 10 |
|----------------------------|----|----|----|----|
| 00 | | | 1 | |
| 01 | 1 | 1 | | 1 |
| 11 | 1 | | X | X |
| 10 | | | 1 | |

$\rightarrow S_1' S_0 a$ (purple)
 $\rightarrow S_1 S_0 a'$ (orange)
 $\rightarrow S_2' S_1 S_0'$ (blue)
 $\rightarrow S_1 S_0' a'$ (green)

0011, 0100, 0101, 0110, 1011, 1100

$$\begin{aligned}
 n_1 &= S_2' S_1 S_0' + S_1 S_0' a' + S_1 S_0 a' + S_1' S_0 a \\
 &= S_1 a' (S_0 + S_0') + S_2' S_1 S_0' + S_1' S_0 a \\
 &= S_1 a' + S_2' S_1 S_0' + S_1' S_0 a \\
 &= S_1 (S_2' S_0' + a') + S_1' S_0 a
 \end{aligned}$$

for n_0 :

| $S_2 S_1 \backslash S_0 a$ | 00 | 01 | 11 | 10 |
|----------------------------|----|----|----|----|
| 00 | | 1 | | 1 |
| 01 | | 1 | | 1 |
| 11 | | | X | X |
| 10 | | | 1 | 1 |

$\rightarrow S_0 a'$ (purple)
 $\rightarrow S_2' S_0 a$ (green)
 $\rightarrow S_2 S_1' S_0' a$ (red)

$$\begin{aligned}
 n_0 &= S_2 S_1' S_0' a + S_2' S_0' a + S_0 a' \\
 &= S_0' a (S_2 S_1' + S_2') + S_0 a' \\
 &\quad \underbrace{S_2 S_1' + S_2'}_{S_2 \text{ NAND } S_1} \\
 &= S_0' a (S_2 \text{ NAND } S_1) + S_0 a'
 \end{aligned}$$

