CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)
FOR n=0 (assume true):
$a_0 = -2^1 = -2$
$a_1 = -2^2 = 3(-2) + 2 = 4$ ✓
FOR n=k (assume true):
$a_k = -2^{k+1}$
$a_{k+1} = -2^{k+2} = 3(-2^{k+1}) + 2^{k+1}$ (simplify)
$-2^{k+2} = -2(2^{k+2}$
$-2^{k+2} = -2^{k+2}$ ✓
(b) Find the solution with $a_0 = 1$.
(Solution)
$a_n = a_n^{(h)} + a_n^{(p)}$
$(a_n^{(h)})$ $r - 3 = 0 \rightarrow a_n^{(h)} = \alpha 3^n$
(a_n^p) FORM OF THE SOLUTION (GUESS): $A.2^n$
$A.2^n - 3.A.2^n - 1 = 2^n$
$(-1/2)A.2^n = 2^n$
(-1/2)A = 1 , A=-2 $\rightarrow a_n^p = -2^{n+1}$
$a_n = \alpha 3^n - 2^{n+1}$
$a_0 = 1 = \alpha - 2 \rightarrow \alpha = 3$
$a_n = 3.3^n - 2^{n+1}$

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)
$f(n) = f(n)^{(h)} + f(n)^{(p)}$
$(f(n)^{(h)})r^2 - 4r + 4 = 0 \rightarrow r = 2$ (double root)
$f(n)^{(h)} = \alpha 2^n + \beta n 2^n$
$(f(n)^{(p)})$ FORM OF THE SOLUTION (GUESS): $A.2^n n$
$A2^n n - 4.A2^{(n-1)}(n-1) + 4.A2^{(n-2)}(n-2) = 2^n$
$An - 2An - 2A + An - 2A = 1$
-4A=1 → A=-1/4
f(n)=$\alpha 2^n + \beta n 2^n + (-1/4)2^n n$
f(0)=2=α
f(1)=5=4 + β.2 -1 → $\beta = 1$
$f(n) = 2.2^n + n2^n + (-1/4).2^n n$

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)
$r^2 - 2r + 2 = 0$
$\Delta = b^2 - 4ac = 4 - 8 = -4$
$$r_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = (2 \pm \sqrt{-4})/2 = 1 \pm i$$

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)
$a_n = \alpha(1+i)^n + \beta(1-i)^n$
$a_0 = 1 = \alpha + \beta \rightarrow \alpha = 1 - \beta$
$a_1 = 2 = \alpha(1+i) + \beta(1-i) \rightarrow (1-\beta)(1+i) + \beta(1-i) = 2$
$1 + i - \beta - i\beta + \beta - i\beta = 2$ (simplify)
$i - 2i\beta = 1 \rightarrow \beta = \dfrac{1+i}{2}, \alpha = \dfrac{1-i}{2}$
$a_n = \dfrac{1-i}{2}(i+1)^n + \dfrac{1+i}{2}(i-1)^n$