

Exercises for Lecture 11

1. Principal components analysis

Assume you are given a dataset of N samples (with each sample having D features), $X_{N \times D}$. You want to project this data down to 1D such that the variance of the projected data is maximized. Denoting the projection parameters (i.e., loading vector) with w , this problem can be written as

$$\begin{aligned} \max_w \text{Var}(Xw) \\ \text{s.t. } w^T w = 1 \end{aligned}$$

Show that the unit vector w that maximizes the variance is the eigenvector of the covariance matrix $X^T X$ with the largest eigenvalue.

You can assume that each column of X has zero mean.

2. Principal components analysis lab

Do the labs in Section 10.4 in ISLR.