Numerical results for trophical transmission model with multiple infections

In[486]:=
```
SetDirectory [NotebookDirectory []]
<< Tools`
ParallelNeeds ["Tools`" ](* Import package Tools that define useful functions *)
```
Out[486]=
```
/Users/phuongnguyen/Work/multipleinfections/code
```

ODEs

Dynamics of intermediate hosts

In[489]:=
```
dIsdt = R[Iw, Is, Iww ] − d Is − Πs[Ds, Dw, Dww ] Is − η Is; (*Susceptible host *)
dIwdt = (1 − p) η Is − (d + αw) Iw − Πw[Ds, Dw, Dww, βw] Iw;
(* Host infected with one parasite *)
dIwwdt = p η Is − (d + αww) Iww − Πww[Ds, Dw, Dww, βww] Iww;
(* Host infected with two parasites *)
```

Dynamics of definitive hosts

In[7]:=
```
dDsdt = B[Ds, Dw, Dww, Is, Iw, Iww] - μ Ds - (λww + λw) Ds; (*Susceptible host*)
dDwdt = (λw + (1 - q) λww) Ds - (μ + σw) Dw - ((1 - q) λww + λw) Dw;
(*Host infected with one parasite*)
dDwwdt = q λww Ds + ((1 - q) λww + λw) Dw - (μ + σww) Dww;
(* Host infected with two parasites*)
```

Dynamics of free-living parasites

In[10]:=
```
dWdt = fw Dw + fww Dww - δ W - η Is;
```

Setting force of infection, and list of parameters for solving the odes

In[11]:=
```
forceInf = {η -> γ W, λw -> h (ρ + βw) Iw, λww -> h (ρ + βww) Iww};
odesRes = {dIsdt, dIwdt, dIwwdt, dDsdt, dDwdt, dDwwdt, dWdt};
varRes = {Is, Iw, Iww, Ds, Dw, Dww, W};
vartRes = {Is[t], Iw[t], Iww[t], Ds[t], Dw[t], Dww[t], W[t]};
```

Testing that the total dynamics of definitive host does not involve transmission

In[15]:=
```
dDsdt + dDwdt + dDwwdt // FullSimplify
```
Out[15]=
```
- Ds μ - Dw (μ + σw) - Dww (μ + σww) + B[Ds, Dw, Dww, Is, Iw, Iww]
```

Reproduction ratio R0

This is the result from the file trophicaltransmission_analytical_git.nb

In[16]:=

$$R0 = \gamma \, ls \frac{p\,q\,h\,(\beta ww + \rho)}{\alpha ww + d + \Pi ww[Ds, Dw, Dww, \beta ww]} \frac{Ds}{\mu + \sigma ww} \frac{fww}{\delta + \gamma \, ls} + \gamma \, ls \left(\frac{(1-p)(\beta w + \rho) h}{\alpha w + d + \Pi w[Ds, Dw, Dww, \beta w]} + \frac{p\,(1-q)(\beta ww + \rho) h}{\alpha ww + d + \Pi ww[Ds, Dw, Dww, \beta ww]} \right) \frac{Ds}{\mu + \sigma w} \frac{fw}{\delta + \gamma \, ls};$$

Graph format and parallel computation

In[17]:=

```
includeFrame = True;
imageSize = 600;
frameStyle = Directive [Black, Thickness [0.003 ]];
labelStyle = {Black, FontSize −> 24};
colorlist = ColorData [97, "ColorList" ]
colorstable = ColorData [97, "ColorList" ][[{1, 2, −2}]]
ihostCol = ColorData [97, "ColorList" ][[1]];
 dhostCol = ColorData [97, "ColorList" ][[4]];
fCol1 = ColorData [97, "ColorList" ][[3]];
fCol2 = ColorData [97, "ColorList" ][[−1]];
pointsize = 0.02;
figResolution = 300;
colbifur = {Black, Black, Black, Black };
colbifurfilling =
  {Lighter [Gray, 0.1 ], Lighter [Gray, 0.3 ], Lighter [Gray, 0.5 ], Lighter [Gray, 0.7 ]}
panellabel = 250;
thick = 0.003;
coldat = "Pastel";
intorder = 5;
```

Out[21]= {■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■}

Out[22]= {■, ■, ■}

Out[30]= {■, ■, ■, ■}

Linear birth function for intermediate host

Setting birth function

```
In[35]:= func0 = {R[Iw, Is, Iww] → r (Is + Iw + Iww), Πs[Ds, Dw, Dww] → ρ (Ds + Dw + Dww),
           Πw[Ds, Dw, Dww, βw] → βw (Ds + Dw + Dww),
           Πww[Ds, Dw, Dww, βww] -> βww (Ds + Dw + Dww),
           B[Ds, Dw, Dww, Is, Iw, Iww] → ρ c (Ds + Dw + Dww) (Is + Iw + Iww)};
         sysfunc0 = odesRes /. forceInf /. func0;
         sysNDSolvefunc0 = MakeSystem[varRes, t, sysfunc0];
```

Solving the odes with linear birth function

```
In[38]:= solsds = Solve[Thread[sysfunc0 == 0] /.
           {Iw -> 0, Iww -> 0, Dw -> 0, Dww -> 0, W -> 0}, varRes][[2]]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[38]= $\left\{Is \to \frac{\mu}{c\,\rho},\ Ds \to -\frac{d-r}{\rho}\right\}$

Jacobian matrix

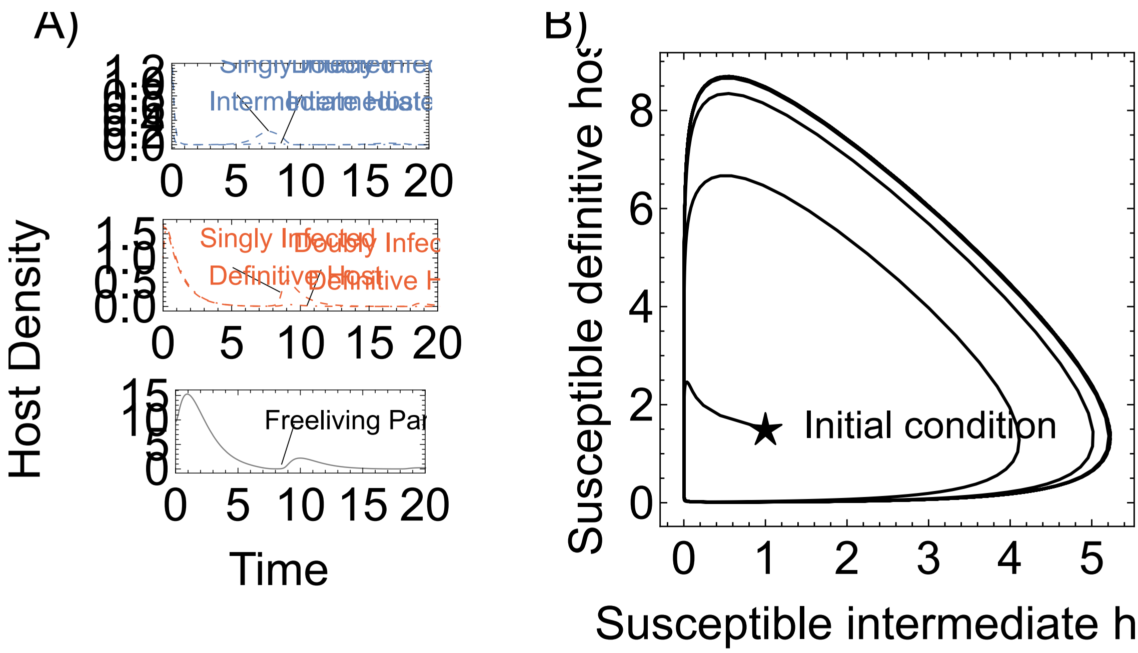
```
In[39]:= Jmatfunc0 = D[sysfunc0, {varRes}];
```

Ecological trajectories (supplementary figure)

```
(*Set parameters *)
prEco0 = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0, αww -> 0, βw → 1.5,
    βww -> 1.5, p -> 0.1, c → 1.4, μ → 0.9, σw → 0, σww -> 0,
    q -> 0.01, fw → 6.5, fww -> 7.5, δ → 0.9, h -> 0.8};
maxt = 100;

(*Set initial conditions for population dynamics *)
init0 = {Is[0] == 1, Iw[0] == 1, Iww[0] == 0.3,
    Ds[0] == 1.5, Dw[0] == 1.7, Dww[0] == 1, W[0] == 10};

(*Solving the equilibrium *)
sols0 = NSolve[Thread[(odesRes /. func0 /. forceInf /. prEco0) == 0], varRes];
(*Solving the dynamics of the odes *)
ndsol0 = NDSolve[Join[sysNDSolvefunc0 /. prEco0, init0], varRes, {t, 0, maxt}];

(*Set the range for the graph *)
range = {{-1, 20}, {-1, 20}};

(* Annotattion for the different lines *)
```

```
epilogParasite01 =
  {{Black, Line[{{8.5, 0.03}, {10.1, 0.81}}]}, {Black, Line[{{7.5, 0.23}, {5.1, 0.81}}]},
   {ihostCol, Text[Style["Doubly Infected \n Intermediate Host", FontSize → 14],
      {10., 1.}, {-0.8, 0}]}, {ihostCol, Text[Style[
        "Singly Infected \n Intermediate Host", FontSize → 14], {4., 1.}, {-0.8, 0}]}};
epilogParasite02 = {{Black, Line[{{10.5, 0.03}, {11.5, 0.75}}]},
   {Black, Line[{{8.5, 0.3}, {5.1, 0.8}}]},
   {dhostCol, Text[Style["Doubly Infected \n Definitive Host", FontSize → 14],
      {10.9, 0.9}, {-0.8, 0}]}, {dhostCol, Text[
      Style["Singly Infected \n Definitive Host", FontSize → 14], {4., 1.}, {-0.8, 0}]}};
epilogParasite03 = {{Black, Line[{{9.5, 9.03}, {8.5, 0.98}}]},
   {Black, Text[Style["Freeliving Parasites", FontSize → 14], {9., 10.}, {-0.8, 0}]}};

(*Plot the dynamics with time for infected intermediate hosts *)
p11 = Plot[Evaluate[{Iw[t], Iww[t]} /. ndsol0],
   {t, 0, maxt}, PlotRange → {{0, 20}, All}, AspectRatio -> 1/3,
   ImageSize → imageSize/3, Frame -> True, FrameStyle → frameStyle,
   LabelStyle → labelStyle, PlotStyle → {{ihostCol, Dashed, Thickness[0.005]},
      {ihostCol, DotDashed, Thickness[0.005]}}, Epilog → epilogParasite01];
(* Plot the dynamics with time for infected definitive hosts *)
p12 = Plot[Evaluate[{Dw[t], Dww[t]} /. ndsol0],
   {t, 0, maxt}, PlotRange → {{0, 20}, All}, AspectRatio -> 1/3,
   ImageSize → imageSize/3, Frame -> True, FrameStyle → frameStyle,
   LabelStyle → labelStyle, PlotStyle → {{dhostCol, Dashed, Thickness[0.005]},
      {dhostCol, DotDashed, Thickness[0.005]}}, Epilog → epilogParasite02];
(*Plot the dynamics with time for free-living parasites *)
p13 = Plot[Evaluate[{W[t]} /. ndsol0], {t, 0, maxt},
   PlotRange → {{0, 20}, All}, AspectRatio -> 1/3, ImageSize → imageSize/3,
   Frame -> True, FrameStyle → frameStyle, LabelStyle → labelStyle,
   PlotStyle → {Gray, Thickness[0.005]}, Epilog → epilogParasite03];
(* Combine the graphs of dynamics with respect to time *)
p1 = GraphicsColumn[{p11, p12, p13}, Spacings → 0, ImageSize -> {900, 1000},
   Epilog -> {Text[Style["Time", Black, 24], {Center, -215}],
      Rotate[Text[Style["Host Density", Black, 24], {0.5, Center}], 90 Degree]}];

(*Annotation for initial condition and equilibrium *)
epilogHost0 = {{Black, Text[Style[★, FontSize → 24], {1., 1.5}, {0, 0}]},
   {Black, Text[Style["Initial condition", FontSize → 20], {1., 1.5}, {-1.3, -0.2}]}};

(*Phase plane of susceptible intermediate and definitive hosts *)
p2 = ParametricPlot[{Evaluate[{Is[t], Ds[t]} /. ndsol0]}, {t, 0, maxt}, AspectRatio → 1,
   PlotRange → All, Frame → includeFrame, PlotStyle → Black, FrameStyle → frameStyle,
   FrameLabel → {Style["Susceptible intermediate host", Black, 24],
      Style["Susceptible definitive host", Black, 24]},
   ImageSize → imageSize * 1.5, LabelStyle → labelStyle, Epilog → epilogHost0];
(*Combine graphs of infected and susceptible hosts *)
GraphicsRow[{p1, p2}, Spacings → 0, Epilog →
  {Text[Style["A)", Black, 24], {100, -10}], Text[Style["B)", Black, 24], {900, -10}]}]
(*Export["diseasefree_linear.pdf", %, ImageResolution -> figResolution]*)
```

Out[55]=



Nonlinear birth function for intermediate host

Setting birth function

In[56]:=
```
func1 = {R[Iw, Is, Iww ] → r (1 − k (Is + Iw + Iww)) (Is + Iw + Iww), Πs[Ds, Dw, Dww ] →
      ρ (Ds + Dw + Dww), Πw[Ds, Dw, Dww, βw] → (ρ + βw) (Ds + Dw + Dww),
    Πww[Ds, Dw, Dww, βww] −> (ρ + βww) (Ds + Dw + Dww),
    B[Ds, Dw, Dww, Is, Iw, Iww ] → ρ c (Ds + Dw + Dww) (Is + Iw + Iww)};

sysfunc1 = odesRes /. forceInf /. func1;
sysNDSolvefunc1 = MakeSystem [varRes, t, sysfunc1 ];
Thread [sysfunc1 == 0] /. {Iw −> 0, Iww −> 0, Dw −> 0, Dww −> 0, W −> 0};
```

Equilibrium of susceptible intermediate and definitive host at disease free scenario

In[60]:=
```
diseasefree = Solve[%, {Is, Ds}][[3]];
```

Reproduction ratio

In[61]:=
```
R0func1 = R0 /. func1 /. Dw -> 0 /. Dww -> 0 /. Iw -> 0 /. Iww -> 0 // Simplify;
```

Jacobian matrix

In[62]:=
```
Jmatfunc1 = D[sysfunc1, {varRes }];
```

Ecological trajectories (Figure 3)

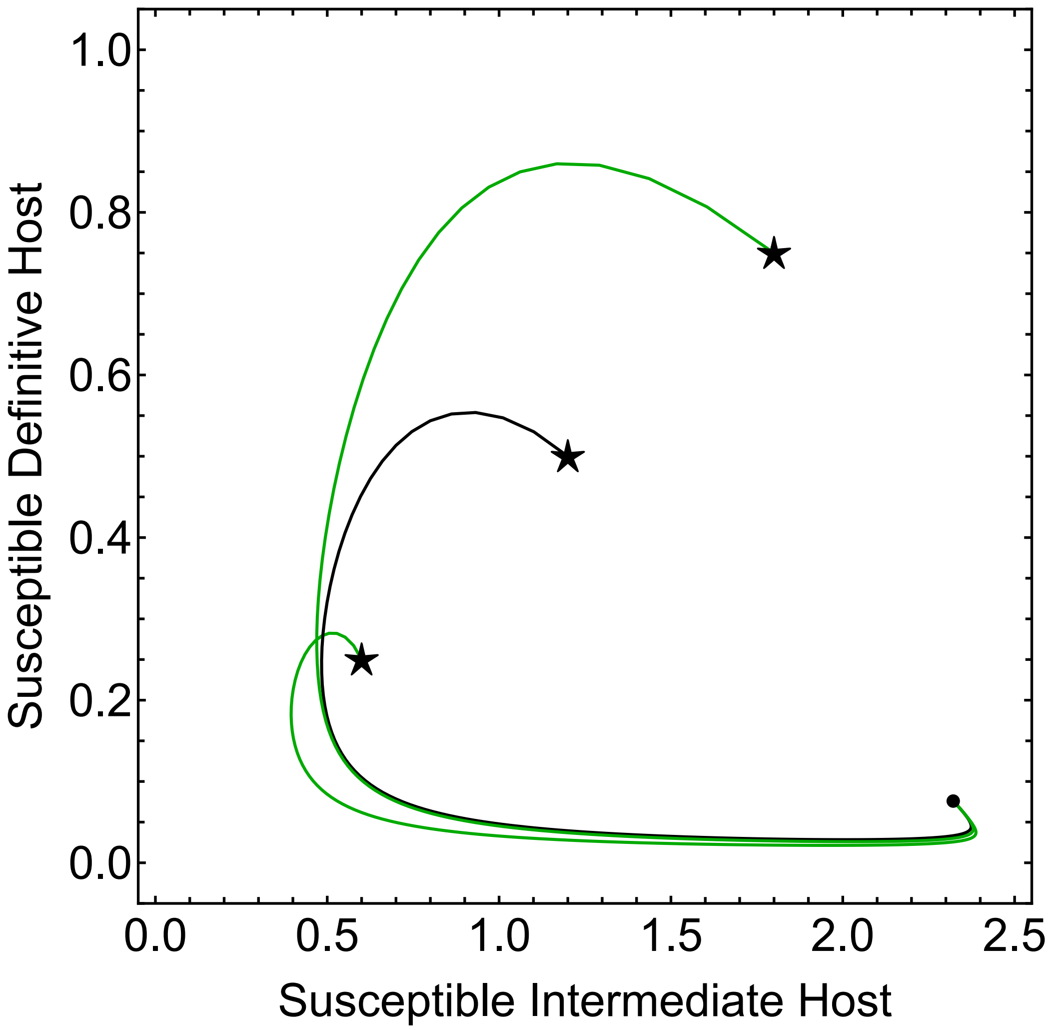
Disease free equilibrium

In[65]:=
```
(*Set parameters *)
prEcoNL = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0, αww -> 0,
    βw → 1.5, βww -> 1.5, p -> 0.05, c → 1.4, μ → 3.9, σw → 0, σww -> 0,
    q -> 0.05, fw → 7.5, fww -> 7.5, δ → 0.9, k -> 0.26, h -> 0.8};
maxt = 100;

(*Solving equilibrium *)
solsNL = NSolve[Thread[(sysfunc1 /. prEcoNL) == 0], varRes];
(*Set range for the graph *)
range1 = {{-0.05, 2.55}, {-0.05, 1.05}};

(*Set different initial conditions for population dynamics *)
ics = {
    {Is[0] == 0.6, Iw[0] == 0.2, Iww[0] == 0.3,
      Ds[0] == 0.25, Dw[0] == 0.7, Dww[0] == 0.1, W[0] == 0.8},
    {Is[0] == 1.2, Iw[0] == 0.2, Iww[0] == 0.3, Ds[0] == 0.5,
      Dw[0] == 0.7, Dww[0] == 0.1, W[0] == 0.8},
    {Is[0] == 1.8, Iw[0] == 0.2, Iww[0] == 0.3, Ds[0] == 0.75,
      Dw[0] == 0.7, Dww[0] == 0.1, W[0] == 0.8}};

(*Solving the odes at different initial conditions *)
ndsolNL = Table[NDSolve[Join[sysNDSolvefunc1 /. prEcoNL, ics[[i]]],
      varRes, {t, 0, maxt}], {i, 1, Length[ics], 1}];

(*Population dynamics at equilibrium 1 *)
ieq1 = Evaluate[Is[maxt] /. ndsolNL][[1]];
deq1 = Evaluate[Ds[maxt] /. ndsolNL][[1]];

(*Annotation for initial equilibrium and starting condition 1 *)
epilogHost1 = {{Black, PointSize[pointsize], Point[{{ieq1, deq1}}]},
    {Black, Text[Style[★, FontSize → 24], {1.2, 0.5}, {0, 0}]},
    {Black, Text[Style["Initial condition", FontSize → 14], {1.2, 0.5}, {-1.2, -0.2}]},
    {Black, Text[Style["Equilibrium", FontSize → 14], {ieq1, deq1}, {1.2, -0.2}]}};

(*Phase plane for susceptible intermediate and definitive hosts *)
ppHost1 = ParametricPlot[Evaluate[{Is[t], Ds[t]} /. ndsolNL], {t, 0, maxt}, AspectRatio → 1,
    Frame → includeFrame, PlotStyle → Black, FrameStyle -> frameStyle,
    PlotRange → range1, FrameLabel → {"Density of \n Susceptible Intermediate Host",
      "Density of \n Susceptible Definitive Host" },
    LabelStyle → labelStyle, ImageSize → imageSize/1.1, Epilog → epilogHost1];

(*Annotation for different lines *)
epilogParas11 = {{ihostCol, Text[Style["Singly Infected \n Intermediate Hosts",
        FontSize → 14], {2, 0.6}, {-1.05, 0}]},
    {ihostCol, Text[Style["Doubly Infected \n Intermediate Hosts", FontSize → 14],
      {7, 0.2}, {-1.05, 0}]}};
```

```
epilogParas12 = {{dhostCol, Text [Style ["Doubly Infected \n Definitive Hosts",
        FontSize → 14], {7, 0.2 }, {−1.05, 0 }]},
    {dhostCol, Text [Style ["Singly Infected \n Definitive Hosts", FontSize → 14],
      {0.2, 0.6 }, {−1.05, 0 }]}};
epilogParas13 = {{Black, Text [Style ["Freeliving Parasites", FontSize → 14],
      {0.75, 0.8 }, {−1.05, 0 }]}};

(*Plot dynamics with respect to time for intermediate host,
definitive host and free −living parasite *)
spcl = ndsolNL [[2]];
ppParasite11 = Plot [Evaluate [{Iw [t], Iww [t]} /. spcl ],
    {t, 0, maxt }, AspectRatio → 1/3, PlotRange → {{0, 10 }, {−0.1, 1.2 }},
    PlotStyle → {{ihostCol, Dashed }, {ihostCol, DotDashed }}, Frame → includeFrame,
    ImageSize → imageSize, FrameStyle → frameStyle, Epilog → epilogParas11 ];
ppParasite12 = Plot [Evaluate [{ Dw [t], Dww [t]} /. spcl ], {t, 0, maxt },
    AspectRatio → 1/3, PlotRange → {{0, 10 }, {−0.1, 1.2 }},
    PlotStyle → {{dhostCol, Dashed }, {dhostCol, DotDashed }}, Frame → includeFrame,
    ImageSize → imageSize, FrameStyle → frameStyle, Epilog → epilogParas12 ];
ppParasite13 = Plot [Evaluate [{W [t]} /. spcl ], {t, 0, maxt }, AspectRatio → 1/3,
   PlotRange → {{0, 10 }, {−0.1, 1.2 }}, PlotStyle → Gray, Frame → includeFrame,
   ImageSize → imageSize, FrameStyle → frameStyle, Epilog → epilogParas13 ];
ppParasite1 = GraphicsColumn [{ppParasite11, ppParasite12, ppParasite13 },
   Spacings → 0, ImageSize → {600, 600 }];
```

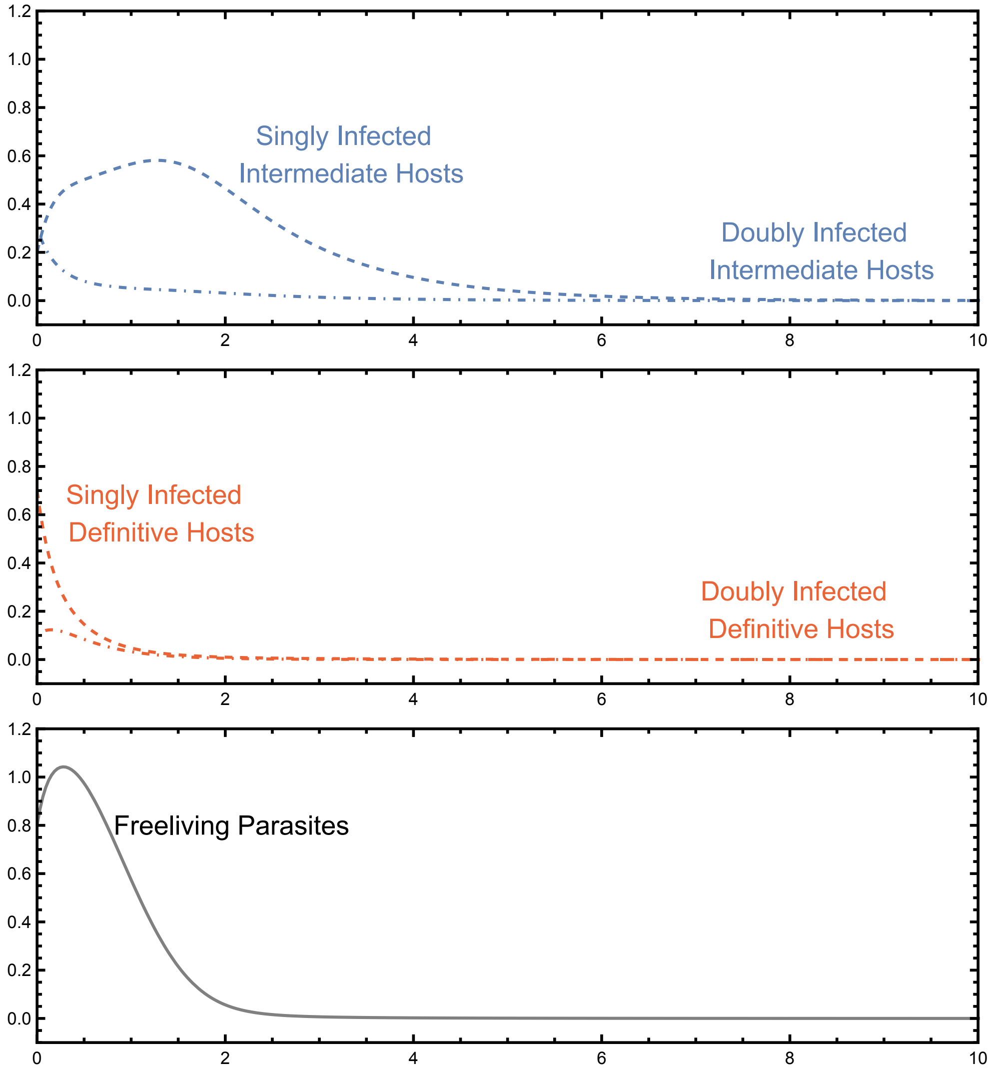
In[86]:=
```
epilogHost1 =
  {{PointSize[Large], Point[Partition[Riffle[ieq1, deq1], 2]]}, Table[
    {Black, Text[Style[★, FontSize → 24], {ics[[i, 1, 2]], ics[[i, 4, 2]]}, {0, 0}]},
    {i, 1, Length[ics], 1}] };
ppHost1 = ParametricPlot[{Evaluate[{Is[t], Ds[t]} /. ndsolNL[[1]]],
   Evaluate[{Is[t], Ds[t]} /. ndsolNL[[2]]],
   Evaluate[{Is[t], Ds[t]} /. ndsolNL[[3]]]}, {t, 0, maxt}, AspectRatio → 1,
  Frame → includeFrame, PlotStyle → {Darker[Green], Black, Darker[Green]},
  FrameStyle -> frameStyle, PlotRange → range1, FrameLabel →
   {"Susceptible Intermediate Host", "Susceptible Definitive Host"},
  LabelStyle → labelStyle, ImageSize → imageSize / 1.1, Epilog → epilogHost1]
```

Out[87]=


1.0
0.8
0.6
0.4
0.2
0.0
Susceptible Definitive Host
0.0
0.5
1.0
1.5
2.0
2.5
Susceptible Intermediate Host

In[88]:= `ppParasite1`

Out[88]=



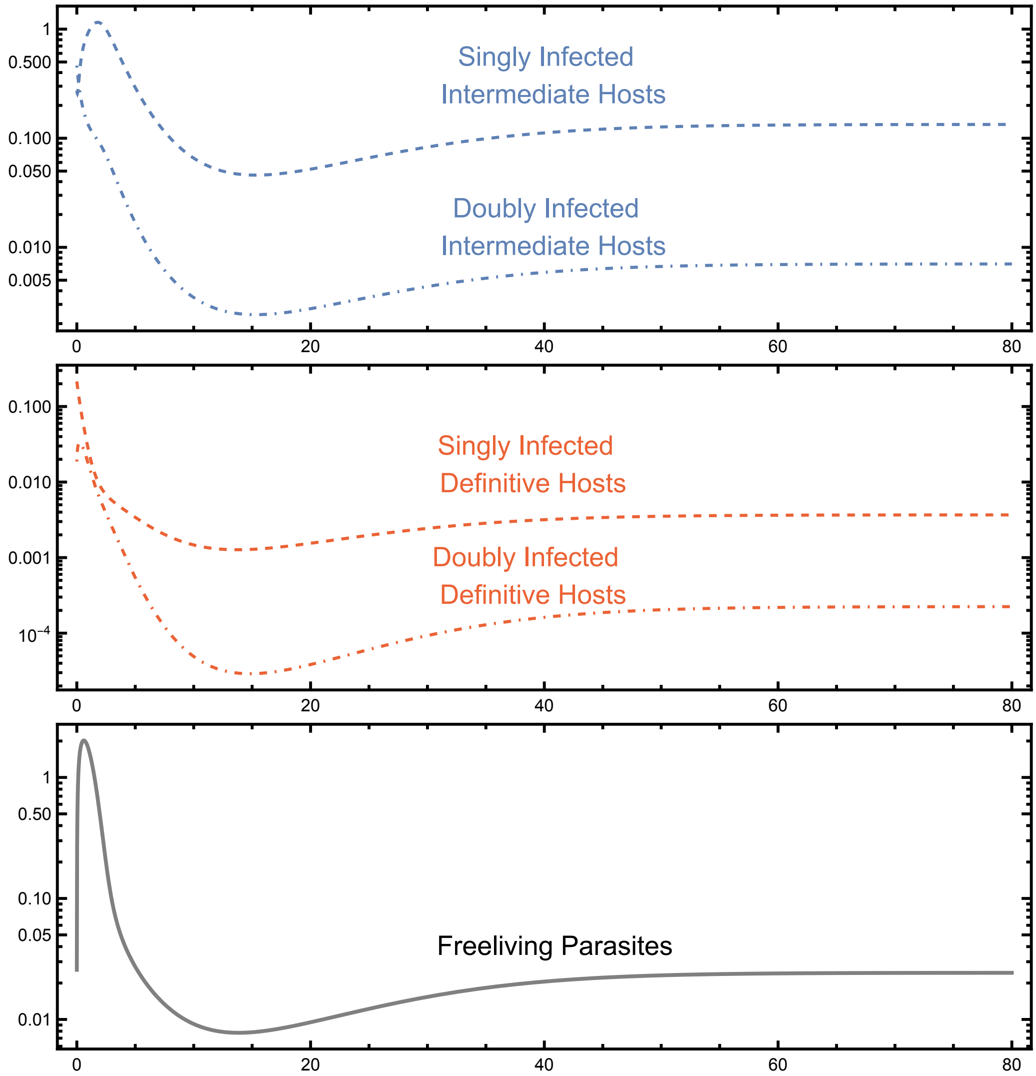
Disease stable equilibrium

In[89]:=
```
(*Set parameters *)
prEcoDSNL = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, α → 0, βw → 1.5, βww -> 1.5,
     p -> 0.05, c → 1.4, μ → 3.9, σ → 0, q -> 0.05,
     fw → 45, fww -> 45, δ → 0.9, k -> 0.26, h -> 0.6};
condsimplifyNL = {αw -> α, αww -> α, σw -> σ, σww -> σ};
maxt = 80;

(*Set range for the plot and initial values *)
range2 = Full;
inits =
   {{Is[0] == 0.9, Iw[0] == 0.27287415391003156`, Iww [0] == 0.46321506893225517`,
      Ds[0] == 0.9, Dw[0] == 0.214956687585793`, Dww [0] == 0.018729116483236274`,
      W[0] == 0.025702772691754472` }, {Is[0] == 0.4588921303535818`,
```

```
      Iw[0] == 0.27287415391003156`, Iww[0] == 0.46321506893225517`,
      Ds[0] == 0.4460456255182149`, Dw[0] == 0.214956687585793`,
      Dww[0] == 0.018729116483236274`, W[0] == 0.025702772691754472`},
    {Is[0] == 1.45, Iw[0] == 0.27287415391003156`, Iww[0] == 0.46321506893225517`,
      Ds[0] == 1.2, Dw[0] == 0.214956687585793`,
      Dww[0] == 0.018729116483236274`, W[0] == 0.025702772691754472`}};
(*Solving the ode for different initial values *)
ndsolDS =
   Table[NDSolve[Join[sysNDSolvefunc1 /. condsimplifyNL /. prEcoDSNL, inits[[i]]],
      varRes, {t, 0, maxt}], {i, 1, Length[inits], 1}];

(*Get the dynamics of initial condition 2 *)
ieq2 = Evaluate[Is[maxt] /. ndsolDS][[1]];
deq2 = Evaluate[Ds[maxt] /. ndsolDS][[1]];

(*Annotation for different lines *)
epilogParas21 = {{ihostCol, Text[Style["Singly Infected \n Intermediate Hosts",
          FontSize → 14], {30, -1.}, {-1.05, 0}]},
    {ihostCol, Text[Style["Doubly Infected \n Intermediate Hosts", FontSize → 14],
        {30, -4.2}, {-1.05, 0}]}};
epilogParas22 = {{dhostCol, Text[Style["Singly Infected \n Definitive Hosts",
          FontSize → 14], {30, -4.1}, {-1.05, 0}]},
    {dhostCol, Text[Style["Doubly Infected \n Definitive Hosts", FontSize → 14],
        {30, -7.5}, {-1.05, 0}]}};
epilogParas23 = {{Black, Text[Style["Freeliving Parasites", FontSize → 14],
        {30, -3.2}, {-1.08, 0}]}};
trajectStyle = {{Gray, Thick}};
spcl = ndsolDS[[2]];

(*Plot trajectories of infected intermediate hosts,
definitive hosts, and free-living parasite with respect to time *)
ppParasite21 = LogPlot[Evaluate[{Iw[t], Iww[t]} /. spcl], {t, 0, maxt},
    AspectRatio → 1/3, PlotStyle → {{ihostCol, Dashed}, {ihostCol, DotDashed}},
    PlotRange -> All, Frame -> includeFrame, FrameStyle → frameStyle,
    ImageSize → imageSize, Epilog → epilogParas21];
ppParasite22 = LogPlot[Evaluate[{Dw[t], Dww[t]} /. spcl], {t, 0, maxt},
    AspectRatio → 1/3, PlotStyle → {{dhostCol, Dashed}, {dhostCol, DotDashed}},
    PlotRange -> All, Frame -> includeFrame, FrameStyle → frameStyle,
    ImageSize → imageSize, Epilog → epilogParas22];
ppParasite23 = LogPlot[Evaluate[{W[t]} /. spcl], {t, 0, maxt}, AspectRatio → 1/3,
    PlotStyle → trajectStyle, PlotRange -> All, Frame -> includeFrame,
    FrameStyle → frameStyle, ImageSize → imageSize, Epilog → epilogParas23];


ppParasite2 = GraphicsColumn[{ppParasite21, ppParasite22, ppParasite23},
    Spacings -> 0, ImageSize → {600, 600}];
```

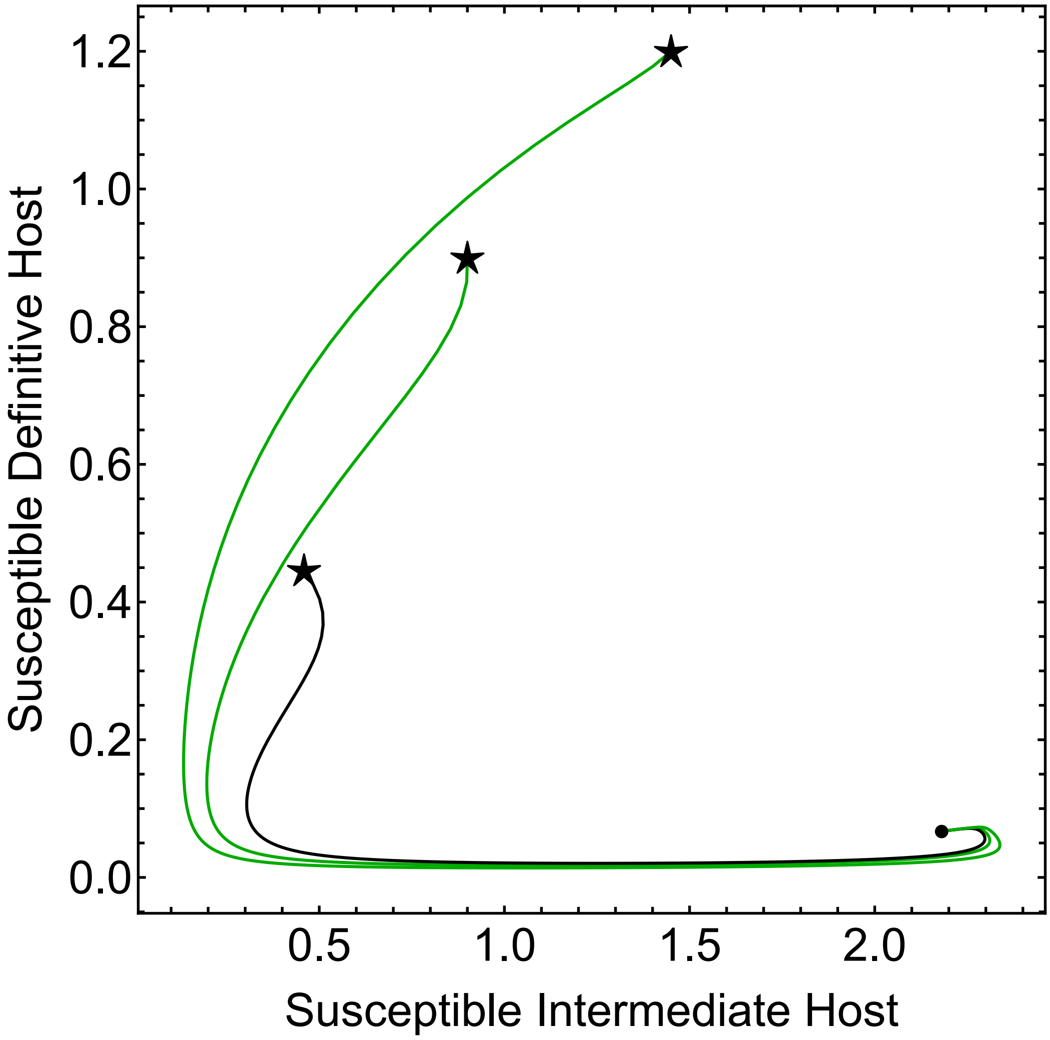
```
In[106]:= ppParasite2
```

Out[106]=


Singly Infected
Intermediate Hosts
Doubly Infected
Intermediate Hosts
Singly Infected
Definitive Hosts
Doubly Infected
Definitive Hosts
Freeliving Parasites

In[109]:=
```
(*Plot phase plane of susceptible intermediate and definitive hosts*)
epilogHost2 = {{PointSize[Large], Point[Partition[Riffle[ieq2, deq2], 2]]},
   Table[{Black, Text[Style[★, FontSize → 24],
      {inits〚i, 1, 2〛, inits〚i, 4, 2〛}, {0, 0}]}, {i, 1, Length[inits], 1}] };
ppHost2 = ParametricPlot[{Evaluate[{Is[t], Ds[t]} /. ndsolDS〚1〛],
   Evaluate[{Is[t], Ds[t]} /. ndsolDS〚2〛],
   Evaluate[{Is[t], Ds[t]} /. ndsolDS〚3〛]}, {t, 0, maxt}, AspectRatio → 1,
  Frame → includeFrame, PlotStyle → {Darker[Green], Black, Darker[Green]},
  FrameStyle -> frameStyle, PlotRange → range2, FrameLabel →
   {"Susceptible Intermediate Host", "Susceptible Definitive Host"},
  LabelStyle → labelStyle,  ImageSize → imageSize / 1.1, Epilog → epilogHost2]
```

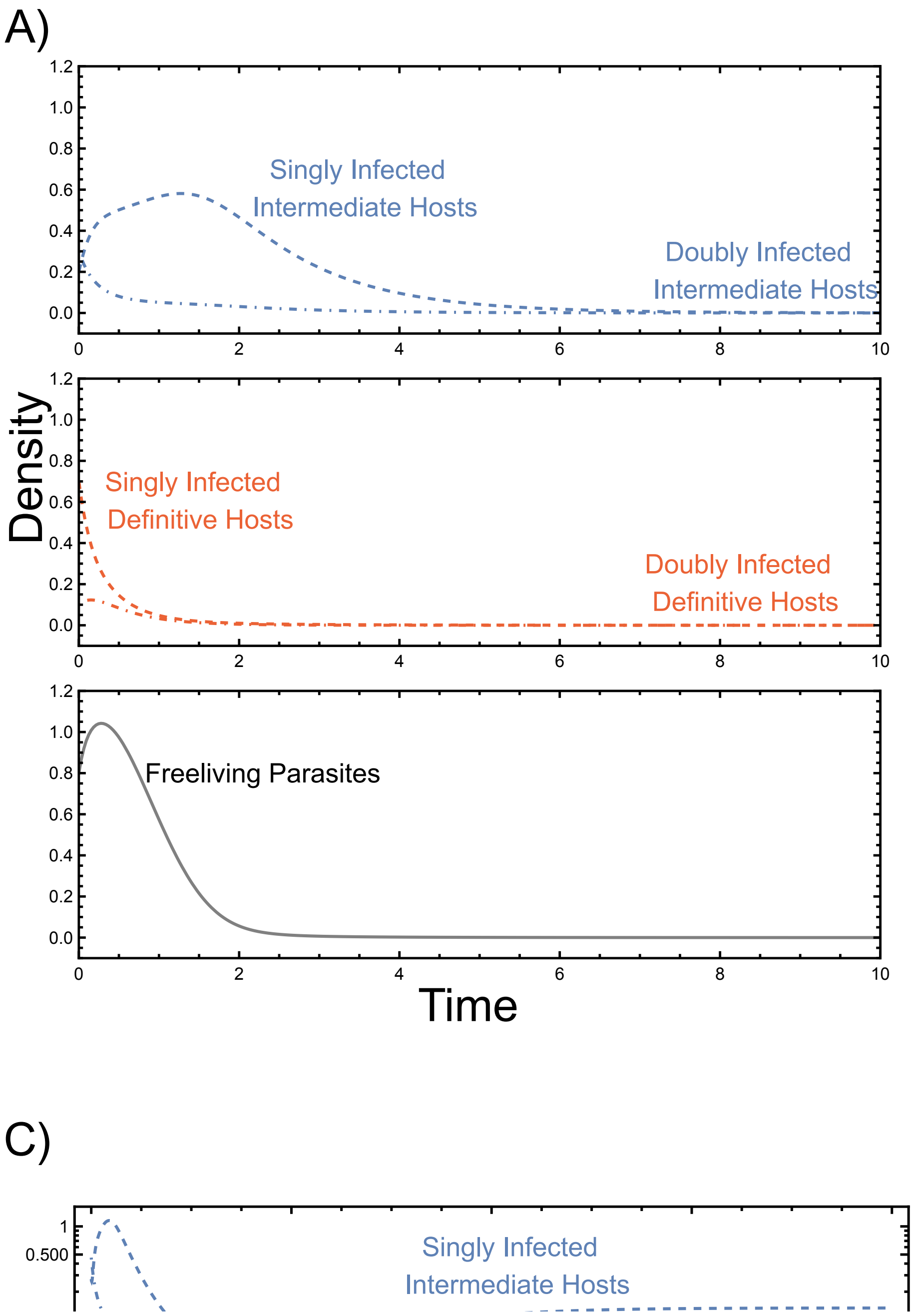
Out[110]=

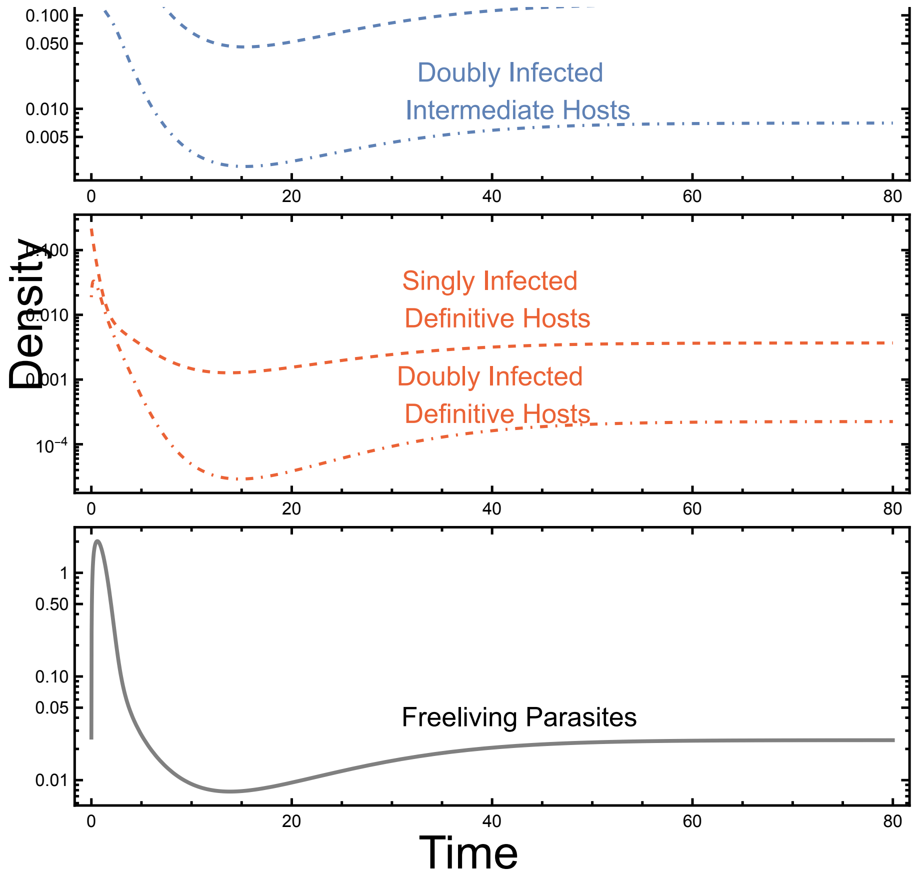


Combined disease free and disease stable ecology plot (Figure 3)

In[111]:=
```
epilogAxesCombiPanel = {Text[Style["A)", Black, 24], {100, -10}],
    Text[Style["Time", Black, 24], {330, -520}],
    Rotate[Text[Style["Density", Black, 24], {100, -240}], 90 Degree],
    Text[Style["B)", Black, 24], {660, -10}], Text[Style["C)", Black, 24], {100, -590}],
    Text[Style["Time", Black, 24], {330, -1120}],
    Rotate[Text[Style["Density", Black, 24], {100, -840}], 90 Degree],
    Text[Style["D)", Black, 24], {660, -590}]};
GraphicsGrid[{{ppParasite1, ppHost1}, {ppParasite2, ppHost2}},
  ImageSize → {1200, 1200}, Spacings → 0, Epilog → epilogAxesCombiPanel]
(*Export["ecotraject_nonlinear.pdf", %, ImageResolution -> figResolution]*)
```

Out[112]=





Bifurcation for reproduction in single infection f_w ($f_{ww} = \epsilon f_w$) (Figure 5)

In[113]:=
```
fwstart = 33;
fwend = 45;
fwstep = 0.1;
fwrange = Range[fwstart, fwend, fwstep];
```

Parameter set 1 (bistability) Figure 5 Panel B-C

CALCULATE BIFURCATION

```
In[117]:= parfw1 = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0,
        αww -> 0, βw → 1.5, βww -> 1.5, p -> 0.05, c → 1.4, μ → 3.9,
        σw → 0, σww -> 0, q -> 0.05, δ → 0.9, k -> 0.26, ϵ -> 4, h -> 0.6};
   (*Find all solutions of the system with the given parameter values *)
   solsfwAll1 =
     NSolve[Thread[(odesRes /. func1 /. forceInf /. fww -> ϵ fw /. parfw1 /. fw -> 36) == 0],
       varRes, Reals];

   (*Select disease circulation solutions *)
   fwAllPositive1 = ParallelTable[NSolvePositive[sysfunc1 /. fww -> ϵ fw,
         parfw1, fw -> i, varRes, eq], {i, fwstart, fwend, fwstep}];
   (*Select disease free solutions *)
   solsfwzero1 =
     Select[solsfwAll1, (Is /. #) > 0 && (Iw /. #) == 0 && (Iww /. #) == 0 && (Ds /. #) > 0 &&
         (Dw /. #) == 0 && (Dww /. #) == 0 && (W /. #) == 0 &][[1]];
```

CALCULATE DYNAMICS

```
In[121]:= maxt1 = 300;
   init1 = {Is[0] == 0.1588921303535818`,
        Iw[0] == 0.27287415391003156`, Iww[0] == 0.46321506893225517`,
        Ds[0] == 0.4460456255182149`, Dw[0] == 0.214956687585793`,
        Dww[0] == 0.018729116483236274`, W[0] == 0.025702772691754472`};
   solsbistable1 = NDSolve[Join[sysNDSolvefunc1 /. fww -> ϵ fw /. parfw1 /. fw -> 39 /.
          ϵ -> 4, init1], varRes, {t, 0, maxt1}];
   maxt2 = 300;
   init2 = {Is[0] == 1.1588921303535818`,
        Iw[0] == 0.27287415391003156`, Iww[0] == 0.46321506893225517`,
        Ds[0] == 0.4460456255182149`, Dw[0] == 0.214956687585793`,
        Dww[0] == 0.018729116483236274`, W[0] == 0.025702772691754472`};
   solsbistable2 = NDSolve[Join[sysNDSolvefunc1 /. fww -> ϵ fw /. parfw1 /. fw -> 39 /.
          ϵ -> 4, init2], varRes, {t, 0, maxt2}];
```

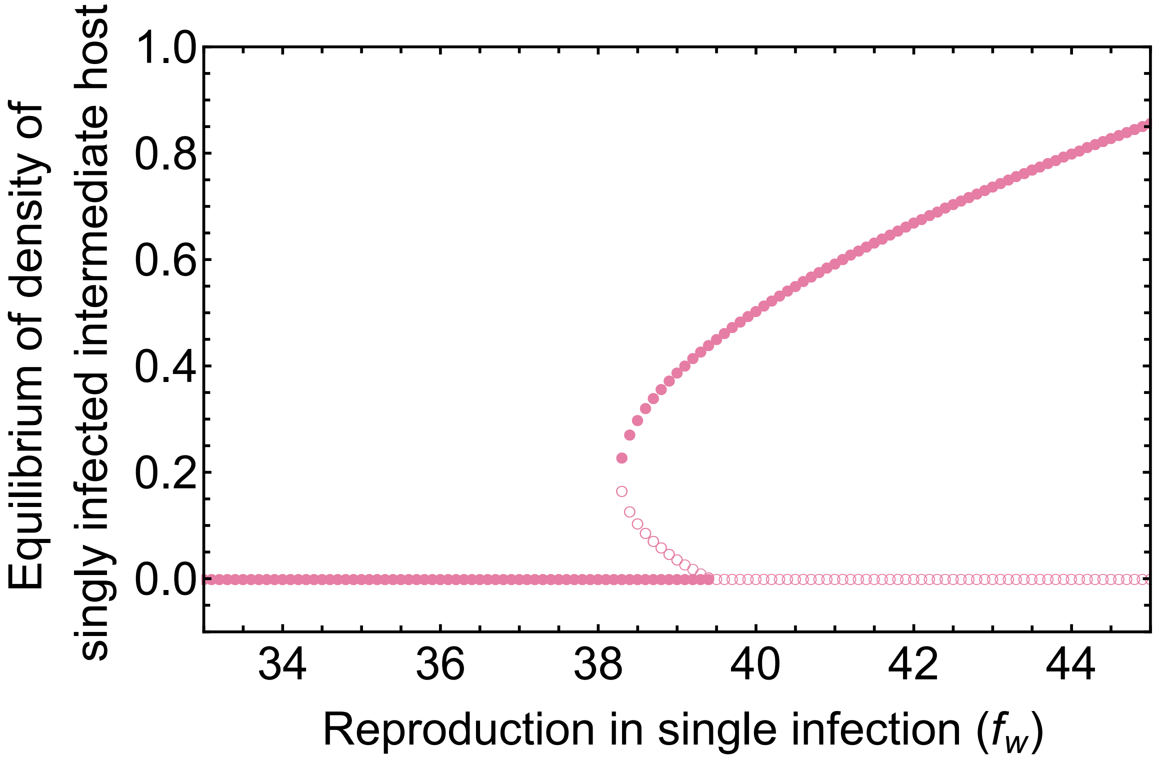
PLOT

In[127]:=
```
fwlist = fw /. Flatten [fwAllPositive1, 1 ];
eqfwlist = eq /. Flatten [fwAllPositive1, 1 ];
eqfwzero = ConstantArray [solsfwzero1, Length [fwrange ]];
col1 = ColorData ["GeologicAges", "ColorList" ][[45]];
colLine1 = ColorData ["GeologicAges", "ColorList" ][[27]]
colLine2 = ColorData ["GeologicAges", "ColorList" ][[25]]
thickness = 0.005;
rangelw = {{fwstart, fwend }, {−0.1, 1. }};

lwfwlist = lw /. eqfwlist;

sdat1 = MakeListPlotData [fwlist, lwfwlist ];
markers1 = ListStableMark [Jmatfunc1 /. fww −> ϵfw,
      parfw1, fw, fwlist, eqfwlist, { "✱", "○", "●" }];

zdat1 = MakeListPlotData [fwrange, lw /. eqfwzero ];
markerz1 = ListStableMark [Jmatfunc1 /. fww −> ϵfw,
      parfw1, fw, fwrange, eqfwzero, { "✱", "○", "●" }];

plw = ListPlot [Join [sdat1, zdat1 ], PlotMarkers −> Join [markers1, markerz1 ],
    PlotStyle → col1, PlotRange → rangelw, ImageSize → imageSize,
    Frame → includeFrame, FrameLabel → {" Reproduction in single infection (fw)",
       "Equilibrium of density of \n singly infected intermediate host" },
    LabelStyle → labelStyle, FrameStyle → frameStyle ]
Export ["reproduction _bifurcationB.pdf", %, ImageResolution −> figResolution ]

parfw1Dynamics =
  LogLogPlot [{Evaluate [lw [t]] /. solsbistable1, Evaluate [lww [t]] /. solsbistable1,
      Evaluate [lw [t]] /. solsbistable2, Evaluate [lww [t]] /. solsbistable2 },
    {t, 0, maxt1 }, PlotStyle → {{colLine1, Dashed, Thickness [thickness ∗2]},
       {colLine1, DotDashed, Thickness [thickness ∗ 2]}, {colLine2, Dashed,
         Thickness [thickness ]}, {colLine2, DotDashed, Thickness [thickness ]}},
    PlotRange −> {{0, maxt1 }, {−0.1, 1.9 }}, Frame −> includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle,
    ImageSize → imageSize, FrameTicksStyle → Directive [Black, 20 ],
    FrameLabel → {"Time", "Dynamics of singly and doubly \n infected intermediate host" }]
Export ["reproduction _bifurcationD.pdf", %, ImageResolution −> figResolution ]
```

Out[131]= 

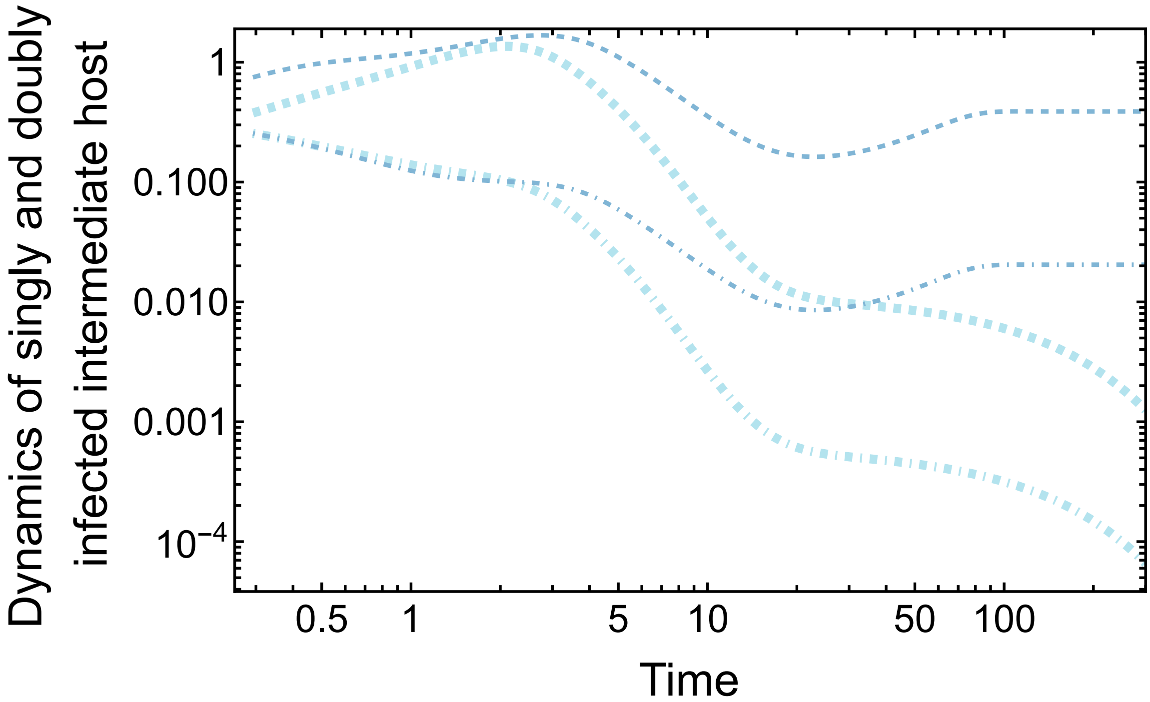
Out[132]= 

Out[140]=



Out[141]= `reproduction_bifurcationB.pdf`

Out[142]=



Out[143]= `reproduction_bifurcationD.pdf`

Parameter set 2 (single equilibrium) Figure 5 Panel B

CALCULATION

```
In[144]:= parfw2 = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0,
            αww -> 0, βw → 1.5, βww -> 1.5, p -> 0.05, c → 1.4, μ → 3.9,
            σw → 0, σww -> 0, q -> 0.05, δ → 0.9, k -> 0.26, ϵ -> 1, h -> 0.6};
          (*Find all solutions of the system with the given parameter values *)
          solsfwAll2 =
            NSolve[Thread[(odesRes /. func1 /. forceInf /. fww -> ϵ fw /. parfw2 /. fw -> 36) == 0],
              varRes, Reals];

          (*Select disease circulation solutions *)
          solsfw2 =
            Select[solsfwAll2, (Is /. #) > 0 && (Iw /. #) > 0 && (Iww /. #) > 0 && (W /. #) > 0 &][[1]];

          (*Select disease free solutions *)
          solsfwzero2 =
            Select[solsfwAll2, (Is /. #) > 0 && (Iw /. #) == 0 && (Iww /. #) == 0 && (Ds /. #) > 0 &&
                (Dw /. #) == 0 && (Dww /. #) == 0 && (W /. #) == 0 &][[1]];

          (*Select disease circulation solutions *)
          fwAllPositive2 = ParallelTable[NSolvePositive[sysfunc1 /. fww -> ϵ fw,
                parfw2, fw -> i, varRes, eq], {i, fwstart, fwend, fwstep}];

          ••• Part: Part 1 of {} does not exist.
```

PLOTTING

```
In[149]:= rangeIw = {{fwstart, fwend}, {-0.01, 0.2}};
          fwlist = fw /. Flatten[fwAllPositive2, 1];
          eqfwlist = eq /. Flatten[fwAllPositive2, 1];
          eqfwzero = ConstantArray[solsfwzero2, Length[fwrange]];
          col2 = ColorData["GeologicAges", "ColorList"][[35]];

          Iwfwlist = Iw /. eqfwlist;

          sdat3 = MakeListPlotData[fwlist, Iwfwlist];
          markers3 = ListStableMark[Jmatfunc1 /. fww -> ϵ fw,
            parfw2, fw, fwlist, eqfwlist, {"✱", "○", "●"}];
          zdat3 = MakeListPlotData[fwrange, Iw /. eqfwzero];
          markerz3 = ListStableMark[Jmatfunc1 /. fww -> ϵ fw,
            parfw2, fw, fwrange, eqfwzero, {"✱", "○", "●"}];

          ppIw = ListPlot[Join[sdat3, zdat3], PlotMarkers -> Join[markers3, markerz3],
            PlotStyle → col2, PlotRange → rangeIw, Frame → includeFrame,
            ImageSize → imageSize, FrameLabel → {"Reproduction in single infection (f_w)",
              "Equilibrium of density of \n singly infected intermediate host"},
            LabelStyle → labelStyle, FrameStyle → frameStyle (*,
            PlotLabel →Pane["B) f_ww = f_w", Alignment →Left, ImageSize →panellabel]*)];
          Export["reproduction_bifurcationC.pdf", %, ImageResolution -> figResolution]

Out[160]= reproduction_bifurcationC.pdf
```

Bifurcation ϵ and f_w ($f_{ww} = \epsilon f_w$) Figure 5 Panel A

CALCULATE

```
parϵfw = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0,
    αww -> 0, βw → 1.5, βww -> 1.5, p -> 0.05, c → 1.4, μ → 3.9,
    σw → 0, σww -> 0, q -> 0.05, δ → 0.9, k -> 0.26, h -> 0.6};
ϵstart = 0;
ϵend = 6;
ϵinterval = 0.025;
fwstart = 33;
fwend = 45;
fwinterval = 0.25;

ϵfwAllresults = ParallelTable[NSolveCodim2Positive[
      sysfunc1 /. fww -> ϵ fw, parϵfw, {fw -> i}, {ϵ -> j}, ϵfw, eq, varRes, 12],
    {i, fwstart, fwend, fwinterval}, {j, ϵstart, ϵend, ϵinterval}];
```

(kernel 5) NSolve::sfail : Subsystem could not be solved for

$$\frac{172209\,Ds}{183067} - \frac{117425\,Dw}{183067} - \frac{116690\,Dww}{183067} - \frac{138285\,Is}{183067} + \frac{169206\,Iw}{183067} - \frac{151916\,Iww}{183067} - \frac{185189\,W}{183067}$$

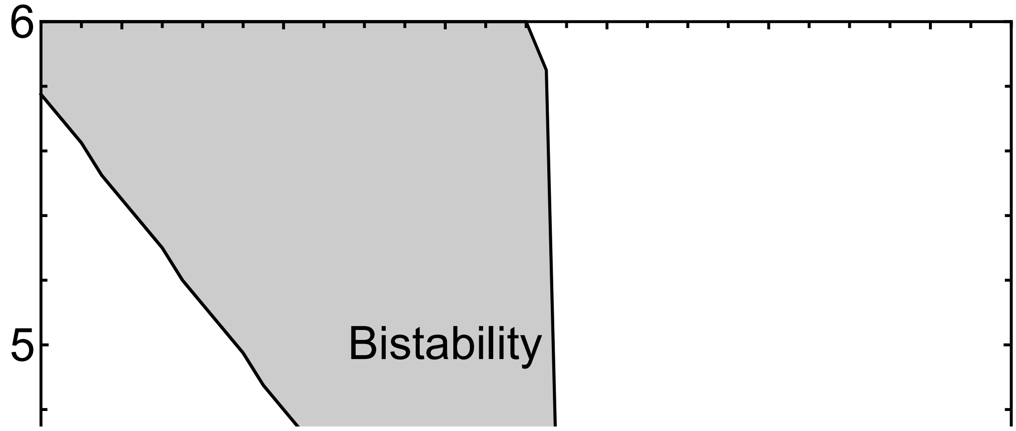
at value -1.8595047574076894. The likely cause is failure to detect zero due to low precision. The likely effect is the loss of one or more solutions. Increasing WorkingPrecision might prevent some solutions from being lost.

(kernel 3) NSolve::sfail : Subsystem could not be solved for

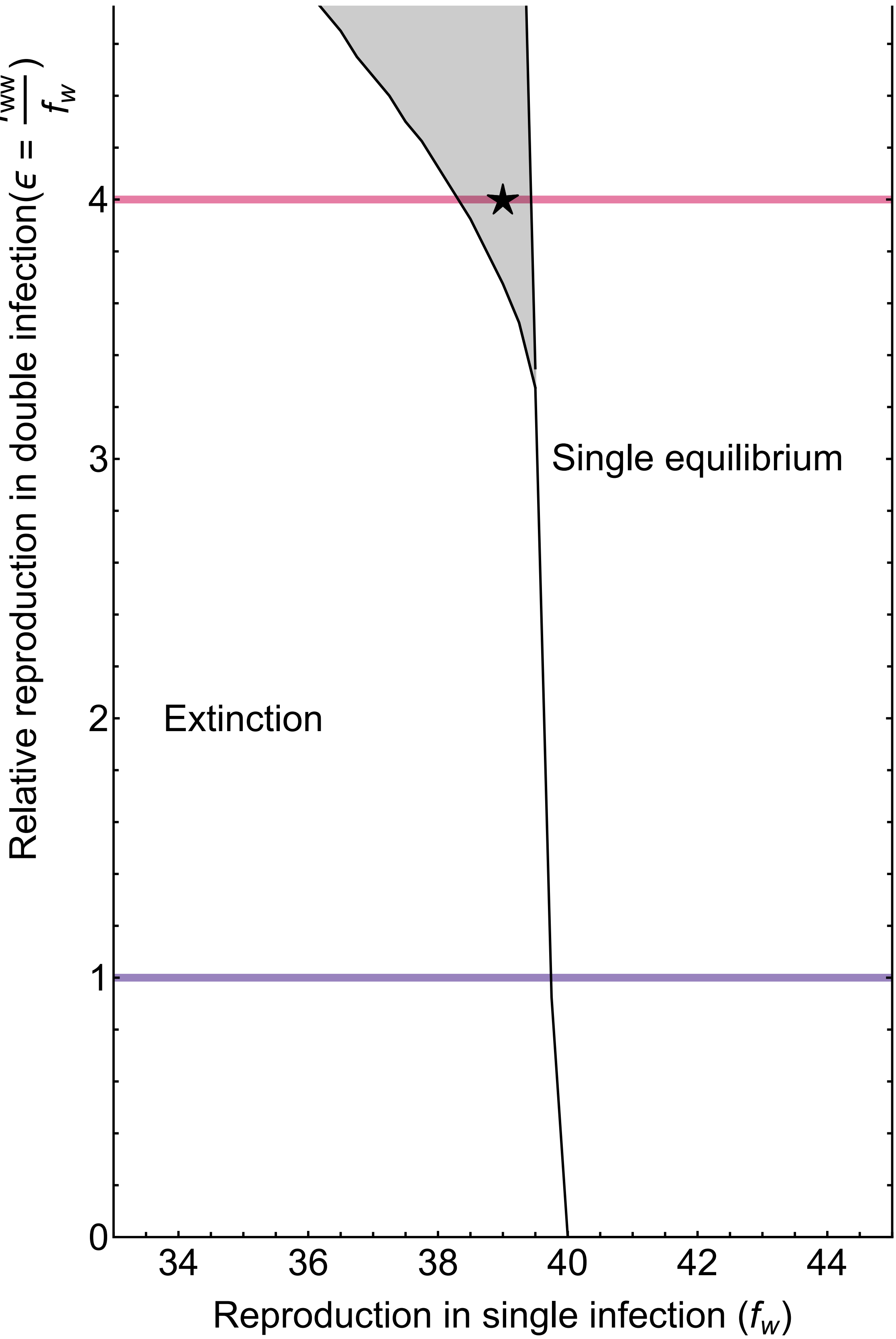
$$\frac{172209\,Ds}{183067} - \frac{117425\,Dw}{183067} - \frac{116690\,Dww}{183067} - \frac{138285\,Is}{183067} + \frac{169206\,Iw}{183067} - \frac{151916\,Iww}{183067} - \frac{185189\,W}{183067}$$

at value -1.8594515089778949. The likely cause is failure to detect zero due to low precision. The likely effect is the loss of one or more solutions. Increasing WorkingPrecision might prevent some solutions from being lost.

```
In[169]:= eqϵfw = eq /. Flatten[ϵfwAllresults, 2];
ϵfwlist = ϵfw /. Flatten[ϵfwAllresults, 2];
rangeϵfw = {{fwstart, fwend}, {ϵstart, ϵend}};

marklist = ListStableMarkTwoParameters[
    Jmatfunc1 /. fww -> ϵ fw, parϵfw, {ϵ, fw}, ϵfwlist, eqϵfw, colorstable, True];
MakeListPlotData[ϵfwlist[[All, 1]], ϵfwlist[[All, 2]]];
p0 =
  ListPlot[%, PlotStyle -> marklist, PlotRange → rangeϵfw, ImageSize → imageSize,
    Frame -> includeFrame, FrameLabel → {"ϵ", "Reproduction in single infection f_w"},
    GridLines → {{1}, {}}, GridLinesStyle -> Directive[Thick,
      Red], LabelStyle → labelStyle, FrameStyle → frameStyle, AspectRatio → 1];
GetBoundaryLineBiStable[ϵfwlist];
p1 = ListLinePlot[%, PlotRange → rangeϵfw, Frame → includeFrame,
    FrameStyle → frameStyle, FrameLabel → {"Reproduction in single infection (f_w)",
      "Relative reproduction in double infection (ϵ = f_ww/f_w)"},
    LabelStyle → labelStyle, GridLines → {{}, {{1, col2}, {4, col1}}},
    GridLinesStyle → Thickness[0.01], PlotStyle -> Black, Filling -> {1 -> {2}}];
GetBoundaryLineSingle[ϵfwlist];
p2 = ListLinePlot[%, PlotRange → rangeϵfw,
    Frame → includeFrame, FrameStyle → frameStyle, PlotStyle -> Black];
ip = ip = ListLinePlot[{{39, 4}}, PlotRange → rangeϵfw, PlotMarkers -> {"★", 28},
     Frame → includeFrame, FrameStyle → frameStyle,
     PlotStyle -> Black, ImageSize → imageSize, LabelStyle → labelStyle];
GraphicsGrid[{{p0, Show[{p1, p2}]}}];
pϵfw = Show[{p1, p2, ip}, ImageSize -> {imageSize, imageSize * 2},
  Epilog -> {Text[Style["Extinction", FontSize → 24], {35, 2}],
                              Text[Style["Single equilibrium", FontSize → 24], {42, 3}],
     Text[Style["Bistability", FontSize → 24], {38, 5.}]},
  AspectRatio -> 2/1, LabelStyle -> labelStyle,
  ImagePadding -> {{90(*left*), 5(*right*)}, {65(*bottom*), 10(*top*)}}]
Export["reproduction _bifurcationA.pdf", %, ImageResolution -> figResolution]
```



Out[181]=



Out[182]= `reproduction_bifurcationA.pdf`

Contour plot (Figure 4)

Manipulation ratio representation

In[222]:=
```
βwwstart = 0.1;
βwwend = 3;
βwwstep = 0.03;
ϵstart = 0.5;
ϵend = 12;
ϵstep = 0.2;
alpha = 0.6;
```

Changing reproduction in single infection

fw = 37, βw = 1.65 (Figure 6 Panel D)

CALCULATION

In[245]:=
```
(*Set parameters *)
parmanipβϵ = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0, αww -> 0,
    p -> 0.05, c → 1.4, μ → 3.9, σw → 0, σww -> 0, q -> 0.05,
    δ → 0.9, k -> 0.26, fw -> 37, h -> 0.6, βw -> 1.65};
range1 = {{βwwstart / (βw /. parmanipβϵ), βwwend / (βw /. parmanipβϵ)},
   {ϵstart, ϵend + 0.1}}
```

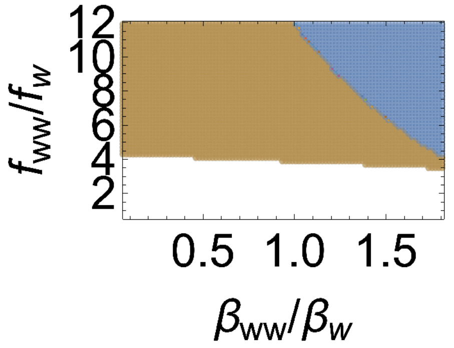
Out[246]=
```
{{0.0606061, 1.81818}, {0.5, 12.1}}
```

```
(*Bifurcation value*)
manipβϵAllResults = ParallelTable[NSolveCodim2Positive[sysfunc1 /. fww -> ϵ fw,
     parmanipβϵ, {βww -> x}, {ϵ -> y}, βϵ, eq, varRes, 15],
    {x, βwwstart, βwwend, βwwstep}, {y, ϵstart, ϵend, ϵstep}];
```

PLOT

```
equilibrium = eq /. Flatten [manipβϵAllResults, 2 ];
parslist = βϵ /. Flatten [manipβϵAllResults, 2 ];

(*Create markers for equilibrium *)
marklist = ListStableMarkTwoParameters [Jmatfunc1 /. fww -> ϵ fw,
    parmanip βϵ, {βww, ϵ}, parslist, equilibrium, colorstable, True ];
MakeListPlotData [parslist [[All, 1]]/(βw /. parmanip βϵ), parslist [[All, 2]]];
(*Plot equilibrium as points with blue as stable points and orange as unstable points *)
pβϵ = ListPlot [%, PlotStyle -> marklist,
    PlotRange → range1, ImageSize → imageSize, Frame -> includeFrame,
    FrameLabel → {"βww/βw", "fww/fw"}, LabelStyle → labelStyle ];
(*MapThread to get ratio as xaxis *)
(*Connect points at boundary *)
MapThread [#1/#2 &,
   {GetBoundaryLineBiStable [parslist ], { ConstantArray [{βw /. parmanip βϵ, 1},
       Length @GetBoundaryLineBiStable [parslist ][[1, All ]]], ConstantArray [
       {βw /. parmanip βϵ, 1}, Length @GetBoundaryLineBiStable [parslist ][[2, All ]]]}}];
(*Draw upper boundary *)lpβϵ1 = ListLinePlot [%, PlotRange → range1, Frame →
     includeFrame, FrameStyle → frameStyle, LabelStyle → labelStyle, PlotStyle ->
     {{Thick, Thickness [thick ], colbifur [[3]]}, {Thick, Thickness [thick ], colbifur [[3]]}},
    Filling -> {1 -> {{2}, Directive [Opacity [alpha ], colbifurfilling [[3]]]}},
    ImageSize → imageSize ];
GetBoundaryLineSingle [parslist ]/ConstantArray [{βw /. parmanip βϵ, 1},
    Length @GetBoundaryLineSingle [parslist ]];
(*Draw lower boundary *)lpβϵ2 = ListLinePlot [%, PlotRange → range1,
    Frame → includeFrame, FrameStyle → frameStyle, LabelStyle → labelStyle,
    PlotStyle -> {Thick, Thickness [thick ], colbifur [[3]]},
    FrameLabel → {"Relative manipulation in double infections (βww/βw)",
       "Relative reproduction \n in double infections (fww/fw)"},
    Filling → Bottom, FillingStyle → Directive [LightGray, HatchFilling []]];
lβϵ = Show [{lpβϵ2, lpβϵ1}];
GraphicsGrid [{{pβϵ, lβϵ}}]
```

Out[257]=

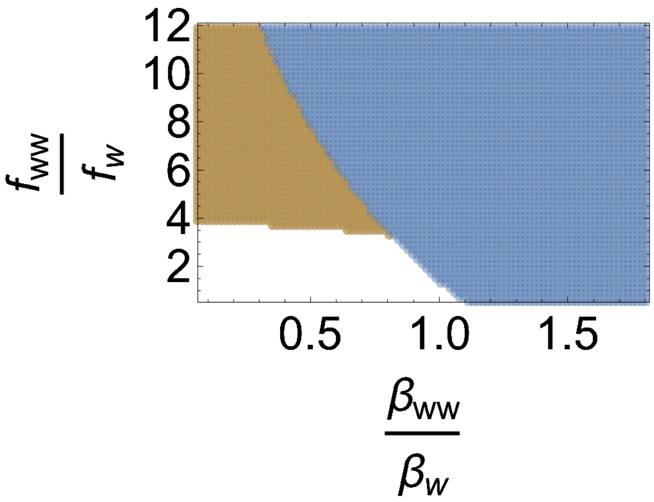


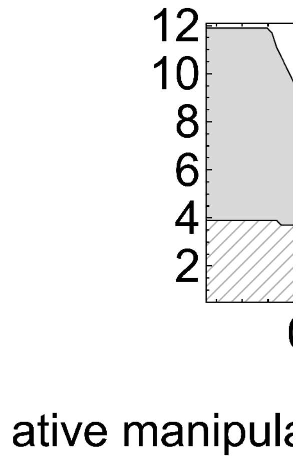
fw = 38, βw = 1.65 (Figure 6 Panel B)

```
(*Set parameters *)
parmanipβϵ1 = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0, αww -> 0,
    p -> 0.05, c → 1.4, μ → 3.9, σw → 0, σww -> 0, q -> 0.05,
    δ → 0.9, k -> 0.26, fw -> 38, h -> 0.6, βw -> 1.65};
(*Equilibrium values *)manipβϵ1AllResults = ParallelTable[NSolveCodim2Positive[
      sysfunc1 /. fww -> ϵfw, parmanipβϵ1, {βww -> x}, {ϵ -> y}, βϵ, eq, varRes, 15],
    {x, βwwstart, βwwend, βwwstep}, {y, ϵstart, ϵend, ϵstep}];
```

```
equilibrium = eq /. Flatten[manipβϵ1AllResults, 2];
parslist = βϵ /. Flatten[manipβϵ1AllResults, 2];

(*Mark equilibrium as stable and unstable *)
marklist = ListStableMarkTwoParameters[Jmatfunc1 /. fww -> ϵ fw,
    parmanipβϵ1, {βww, ϵ}, parslist, equilibrium, colorstable, True];
(*Plot unstable and stable equilibrium *)
MakeListPlotData[parslist[[All, 1]]/(βw /. parmanipβϵ1), parslist[[All, 2]]];
pβϵ1 = ListPlot[%, PlotStyle -> marklist, PlotRange → range1, ImageSize → imageSize,
    Frame -> includeFrame, FrameLabel → {"βww/βw", "fww/fw"}, LabelStyle → labelStyle];
(*Connect boundary points *)
(*MapThread to get ratio as xaxis *)
MapThread[#1/#2 &,
  {GetBoundaryLineBiStable[parslist], {ConstantArray[{βw /. parmanipβϵ1, 1},
      Length@GetBoundaryLineBiStable[parslist][[1, All]]], ConstantArray[
      {βw /. parmanipβϵ1, 1}, Length@GetBoundaryLineBiStable[parslist][[2, All]]]}}];
(*Upper boundary *)lpβϵ1 = ListLinePlot[%, PlotRange → range1, Frame → includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle, FrameLabel →
     {"Relative manipulation in double infections (βww/βw)", " \n "}, PlotStyle ->
     {{Thick, Thickness[thick], colbifur[[3]]}, {Thick, Thickness[thick], colbifur[[3]]}},
    Filling -> {1 -> {{2}, Directive[Opacity[alpha], colbifurfilling[[3]]]}},
    ImageSize → imageSize];
(*Lower boundary *)
GetBoundaryLineSingle[parslist]/
  ConstantArray[{βw /. parmanipβϵ1, 1}, Length@GetBoundaryLineSingle[parslist]];
lpβϵ2 = ListLinePlot[%, PlotRange → range1, Frame → includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle,
    PlotStyle -> {Thick, Thickness[thick], colbifur[[3]]},
    Filling → Bottom, FillingStyle → Directive[LightGray, HatchFilling[]]];
lβϵ1 = Show[{lpβϵ1, lpβϵ2}];
GraphicsGrid[{{pβϵ1, lβϵ1}}]
```

Out[268]=



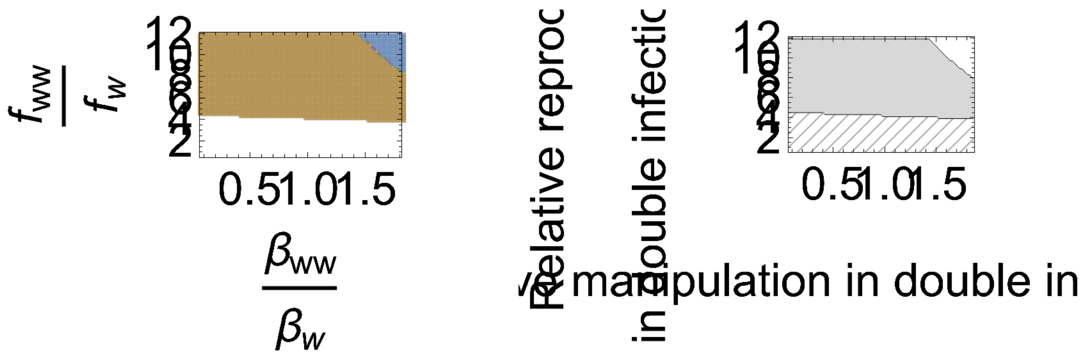


fw = 37, βw = 1.60 (Figure 6 Panel C)

```
(*Set parameters *)
parmanipβϵ2 = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0, αww -> 0,
    p -> 0.05, c → 1.4, μ → 3.9, σw → 0, σww -> 0, q -> 0.05,
    δ → 0.9, k -> 0.26, fw -> 37., h -> 0.6, βw -> 1.60};
(*Calculate equilibrium *)manipβϵ2AllResults = ParallelTable[NSolveCodim2Positive[
      sysfunc1 /. fww -> ϵfw, parmanipβϵ2, {βww -> x}, {ϵ -> y}, βϵ, eq, varRes, 15],
    {x, βwwstart, βwwend, βwwstep}, {y, ϵstart, ϵend, ϵstep}];
range2 = {{βwwstart /(βw /. parmanipβϵ2), βwwend /(βw /. parmanipβϵ2)},
    {ϵstart, ϵend + 0.1}};
```

```
equilibrium = eq /. Flatten [manipβϵ2AllResults, 2 ];
parslist = βϵ /. Flatten [manipβϵ2AllResults, 2 ];

(*Mark stable and unstable equilibrium *)
marklist = ListStableMarkTwoParameters [Jmatfunc1 /. fww -> ϵ fw,
    parmanipβϵ2, {βww, ϵ}, parslist, equilibrium, colorstable, True ];
MakeListPlotData [parslist [[All, 1]]/(βw /. parmanipβϵ2), parslist [[All, 2]]];
(*Plot stable and unstable equilibrium *)
pβϵ2 = ListPlot [%, PlotStyle -> marklist, PlotRange → range1, ImageSize → imageSize,
    Frame -> includeFrame, FrameLabel → {"βww/βw", "fww/fw"}, LabelStyle → labelStyle ];
(*Connect boundary points *)
(*MapThread to get ratio as xaxis *)
MapThread [#1/#2 &,
   {GetBoundaryLineBiStable [parslist ], { ConstantArray [{βw /. parmanipβϵ2, 1},
       Length @GetBoundaryLineBiStable [parslist ][[1, All ]]], ConstantArray [
       {βw /. parmanipβϵ2, 1}, Length @GetBoundaryLineBiStable [parslist ][[2, All ]]]}}];

(*Upper boundary *)lpβϵ1 = ListLinePlot [%, PlotRange → range2,
    Frame → includeFrame, FrameStyle → frameStyle, LabelStyle → labelStyle,
    FrameLabel → {"Relative manipulation in double infections (βww/βw)",
       " Relative reproduction \n in double infections (fww/fw) "},
    PlotStyle -> {{Thickness [thick ], colbifur [[3]]}, {Thickness [thick ], colbifur [[3]]}},
    Filling -> {1 -> {2}}, FillingStyle → Directive [Opacity [alpha ], colbifurfilling [[3]]],
    ImageSize → imageSize ];
(*Lower boundary *)
GetBoundaryLineSingle [parslist ]/
   ConstantArray [{βw /. parmanipβϵ2, 1}, Length @GetBoundaryLineSingle [parslist ]];
lpβϵ2 = ListLinePlot [%, PlotRange → range2, Frame → includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle,
    PlotStyle -> {Thickness [thick ], colbifur [[3]]}, Filling → Bottom,
    FillingStyle → Directive [LightGray, HatchFilling []]];
lβϵ2 = Show [{lpβϵ1, lpβϵ2}];
GraphicsGrid [{{pβϵ2, lβϵ2}}]
```

Out[295]=

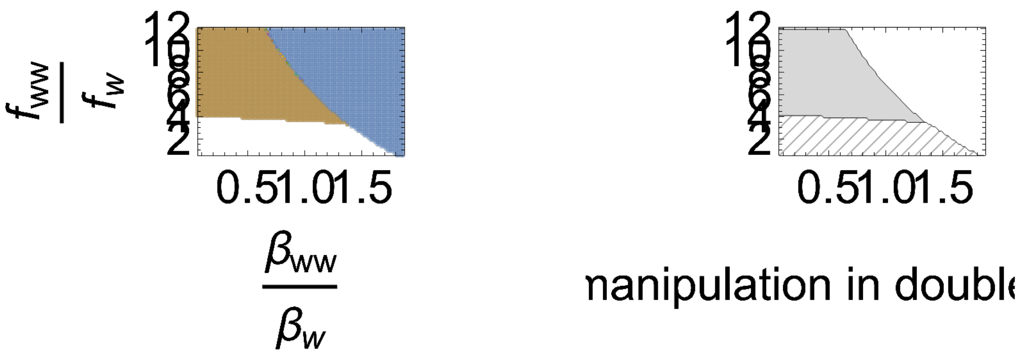


fw = 38, βw = 1.60 (Figure 6 Panel A)

```
(*Set parameters *)
parmanipβϵ3 = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0, αww -> 0,
    p -> 0.05, c → 1.4, μ → 3.9, σw → 0, σww -> 0, q -> 0.05,
    δ → 0.9, k -> 0.26, fw -> 38, h -> 0.6, βw -> 1.60};
(*Calculate equilibrium *)manipβϵ3AllResults = ParallelTable[NSolveCodim2Positive[
      sysfunc1 /. fww -> ϵfw, parmanipβϵ3, {βww -> x}, {ϵ -> y}, βϵ, eq, varRes, 15],
    {x, βwwstart, βwwend, βwwstep}, {y, ϵstart, ϵend, ϵstep}];
```

```
equilibrium = eq /. Flatten [manipβϵ3AllResults, 2 ];
parslist = βϵ /. Flatten [manipβϵ3AllResults, 2 ];

(*Mark stable and unstable equilibrium *)
marklist = ListStableMarkTwoParameters [Jmatfunc1 /. fww -> ϵ fw,
      parmanip βϵ3, {βww, ϵ}, parslist, equilibrium, colorstable, True ];
MakeListPlotData [parslist [[All, 1]]/(βw /. parmanip βϵ3), parslist [[All, 2]]];
pβϵ3 = ListPlot [%, PlotStyle -> marklist, PlotRange → range2, ImageSize → imageSize,
      Frame -> includeFrame, FrameLabel → {"βww/βw", "fww/fw"}, LabelStyle → labelStyle ];

(*Connect boundary points *)
(*MapThread to get ratio as xaxis *)
MapThread [#1/#2 &,
   {GetBoundaryLineBiStable [parslist ], { ConstantArray [{βw /. parmanip βϵ3, 1},
         Length @GetBoundaryLineBiStable [parslist ][[1, All ]]], ConstantArray [
         {βw /. parmanip βϵ3, 1}, Length @GetBoundaryLineBiStable [parslist ][[2, All ]]]}}];
(*Upper boundary *)lpβϵ1 = ListLinePlot [%, PlotRange → range2,
      Frame → includeFrame, FrameStyle → frameStyle, LabelStyle → labelStyle,
      FrameLabel → {"Relative manipulation in double infections (βww/βw)", " \n "},
      PlotStyle -> {{Thickness [thick ], colbifur [[3]]}, {Thickness [thick ], colbifur [[3]]}},
      Filling -> {1 -> {2}}, FillingStyle → Directive [Opacity [alpha ], colbifurfilling [[3]]],
      ImageSize → imageSize ];
(*Lower boundary *)
GetBoundaryLineSingle [parslist ]/
   ConstantArray [{βw /. parmanip βϵ3, 1}, Length @GetBoundaryLineSingle [parslist ]];
lpβϵ2 = ListLinePlot [%, PlotRange → range2, Frame → includeFrame,
      FrameStyle → frameStyle, LabelStyle → labelStyle,
      PlotStyle -> {Thickness [thick ], colbifur [[3]]}, Filling → Bottom,
      FillingStyle → Directive [LightGray, HatchFilling []]];
lβϵ3 = Show [{lpβϵ1, lpβϵ2}];
GraphicsGrid [{{pβϵ3, lβϵ3}}]
```

Out[306]=

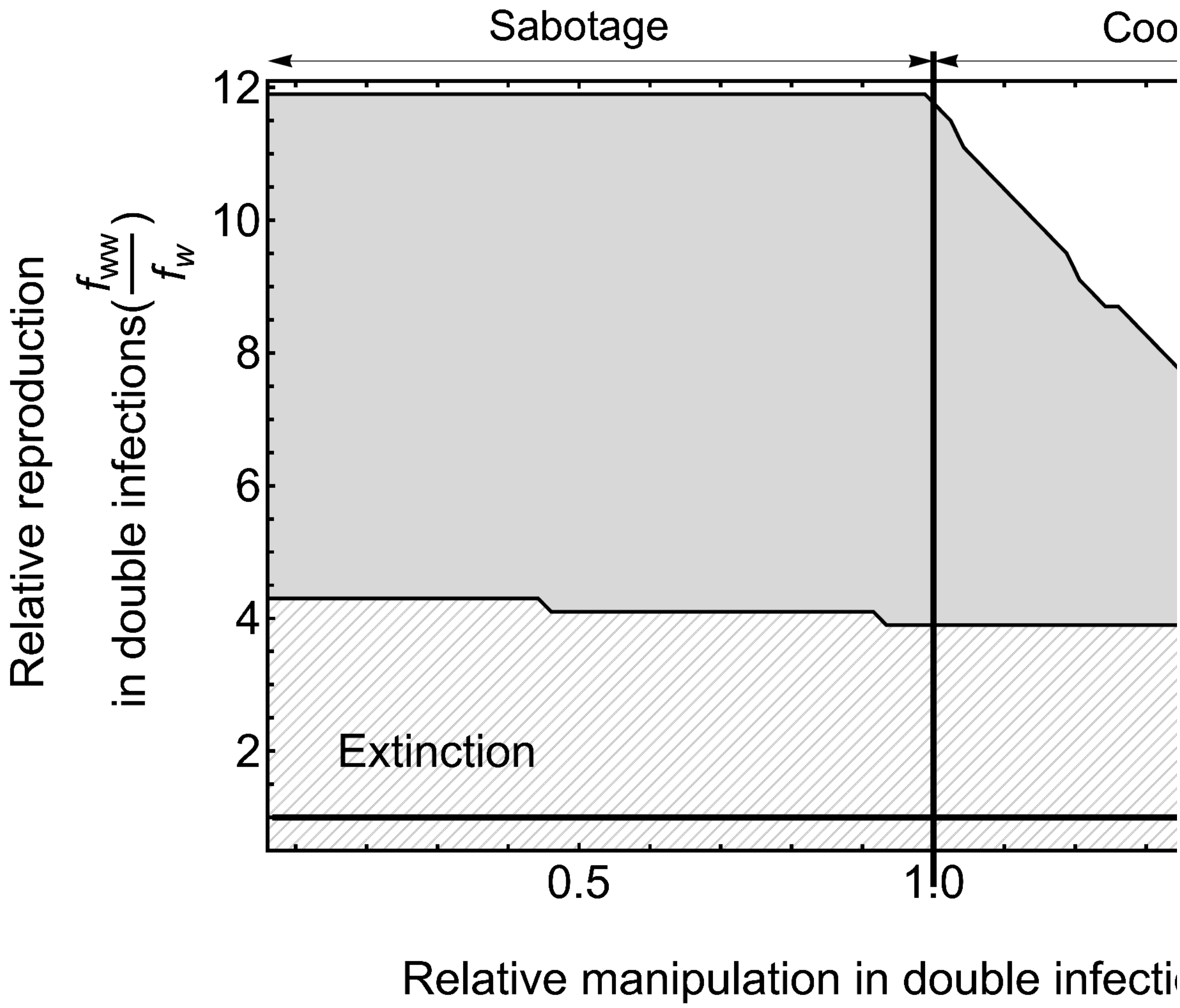


Combined graph (Figure 6)

In[361]:=
```
line = {Graphics [{Thickness [0.005 ], Black, Line [{{1, range [[2]][[1]]}, {1, 12.5 }}]}],
    Graphics [{Thickness [0.005 ], Black, Line [{{range [[1]][[1]], 1}, {2.25, 1}}]}]};
arrowsize = {−0.02, 0.02 }
arrow1 = Graphics [{Arrowheads [arrowsize ], Arrow [{{1, 12.4 }, {range1 [[1]][[1]], 12.4 }}],
        Text [Style ["Sabotage", FontSize → 22], {0.5, 12.9 }]}];
arrow2 = Graphics [{Arrowheads [arrowsize ], Arrow [{{1, 12.4 }, {range1 [[1]][[2]], 12.4 }}],
        Text [Style ["Cooperation", FontSize → 22], {1.4, 12.9 }]} ];
arrow3 = Graphics [{Arrowheads [arrowsize ], Arrow [{{1.9, 1}, {1.9, range1 [[2]][[2]]}}],
        Rotate [Text [Style ["Enhancement", FontSize → 22], {1.96, 6.7 }], −90 Degree ]} ];
arrow4 = Graphics [{Text [Style ["Suppression", FontSize → 22], {2.01, 0.55 }]} ];
plotIβϵfull1 = Show [{ Iβϵ, line, arrow1, arrow2, arrow3, arrow4 }, PlotRangePadding -> 0,
    PlotRangeClipping -> False, ImagePadding -> {{150, 200 }, {100, 45 }},
            Epilog -> { Text [Style ["Extinction", FontSize → 24], {0.3, 2. }],
        Text [Style ["Single \n equilibrium", FontSize → 24], {1.6, 9.5 }]},
    ImageSize -> {1000, 650 }]
Export ["ratio _reproduction _manipulationA.pdf", %, ImageResolution -> figResolution ]
plotIβϵfull1 = Show [{ Iβϵ1, line, arrow1, arrow2, arrow3, arrow4 }, PlotRangePadding -> 0,
    PlotRangeClipping -> False, ImagePadding -> {{110, 200 }, {100, 45 }},
            Epilog -> { Text [Style ["Extinction", FontSize → 24], {0.3, 2. }],
        Text [Style ["Single \n equilibrium", FontSize → 24], {1.6, 9.5 }]},
    ImageSize -> {1000, 650 }]
Export ["ratio _reproduction _manipulationB.pdf", %, ImageResolution -> figResolution ]

arrow1 = Graphics [{Arrowheads [arrowsize ], Arrow [{{1, 12.4 }, {range2 [[1]][[1]], 12.4 }}],
        Text [Style ["Sabotage", FontSize → 22], {0.5, 12.9 }]}];
arrow2 = Graphics [{Arrowheads [arrowsize ], Arrow [{{1, 12.4 }, {range2 [[1]][[2]], 12.4 }}],
        Text [Style ["Cooperation", FontSize → 22], {1.4, 12.9 }]} ];
arrow3 = Graphics [{Arrowheads [arrowsize ], Arrow [{{1.96, 1}, {1.96, range2 [[2]][[2]]}}],
        Rotate [Text [Style ["Enhancement", FontSize → 22], {2.02, 6.7 }], −90 Degree ]} ];
arrow4 = Graphics [{Text [Style ["Suppression", FontSize → 22], {2.07, 0.55 }]} ];
plotIβϵfull1 = Show [{ Iβϵ2, line, arrow1, arrow2, arrow3, arrow4 }, PlotRangePadding -> 0,
    PlotRangeClipping -> False, ImagePadding -> {{150, 200 }, {100, 45 }},
            Epilog -> { Text [Style ["Extinction", FontSize → 24], {0.3, 2. }],
        Text [Style ["Single \n equilibrium", FontSize → 24], {1.7, 11.2 }]},
    ImageSize -> {1000, 650 }]
Export ["ratio _reproduction _manipulationC.pdf", %, ImageResolution -> figResolution ]
plotIβϵfull1 = Show [{ Iβϵ3, line, arrow1, arrow2, arrow3, arrow4 }, PlotRangePadding -> 0,
    PlotRangeClipping -> False, ImagePadding -> {{110, 200 }, {100, 45 }},
            Epilog -> { Text [Style ["Extinction", FontSize → 24], {0.3, 2. }],
        Text [Style ["Single \n equilibrium", FontSize → 24], {1.6, 9.5 }]},
    ImageSize -> {1000, 650 }]
Export ["ratio _reproduction _manipulationD.pdf", %, ImageResolution -> figResolution ]
```

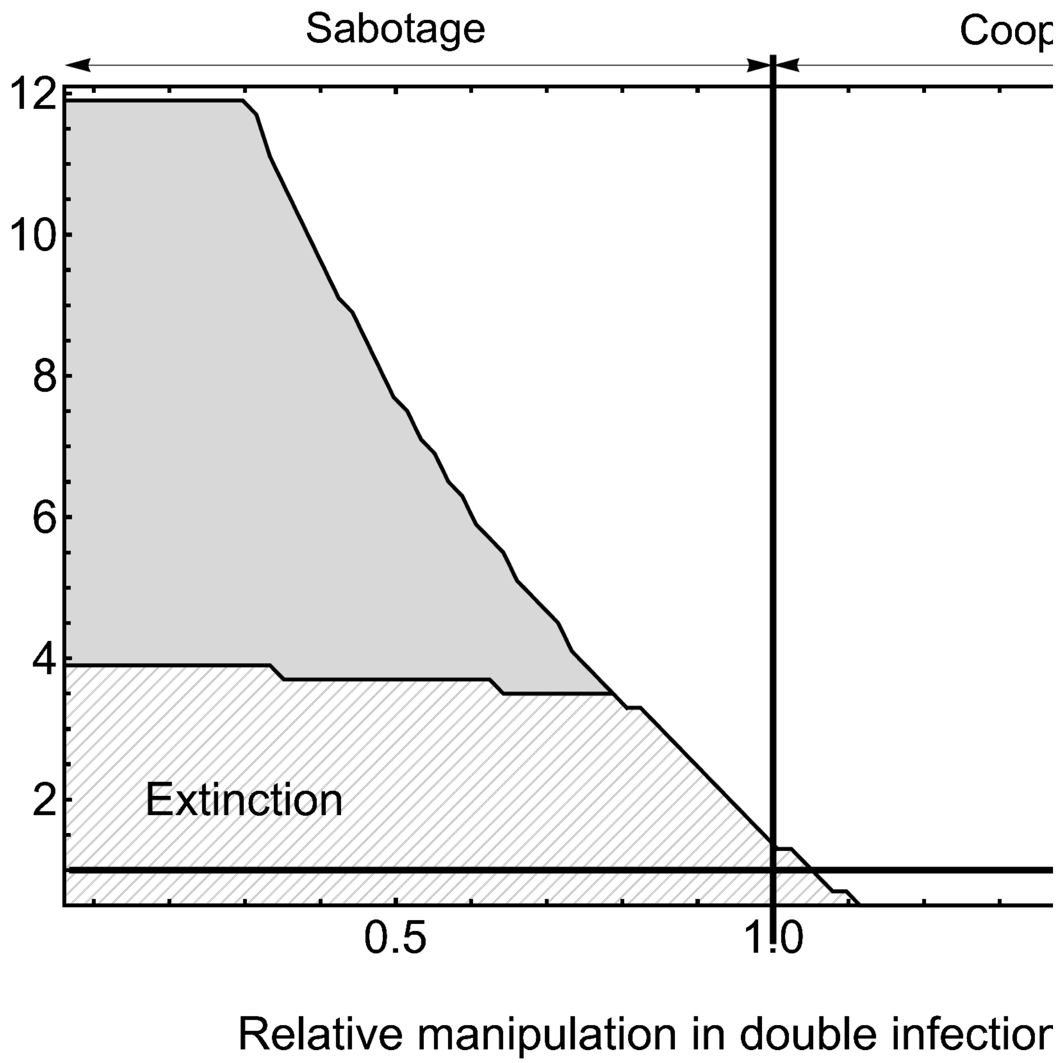
Out[362]=
```
{-0.02, 0.02}
```

Out[367]=



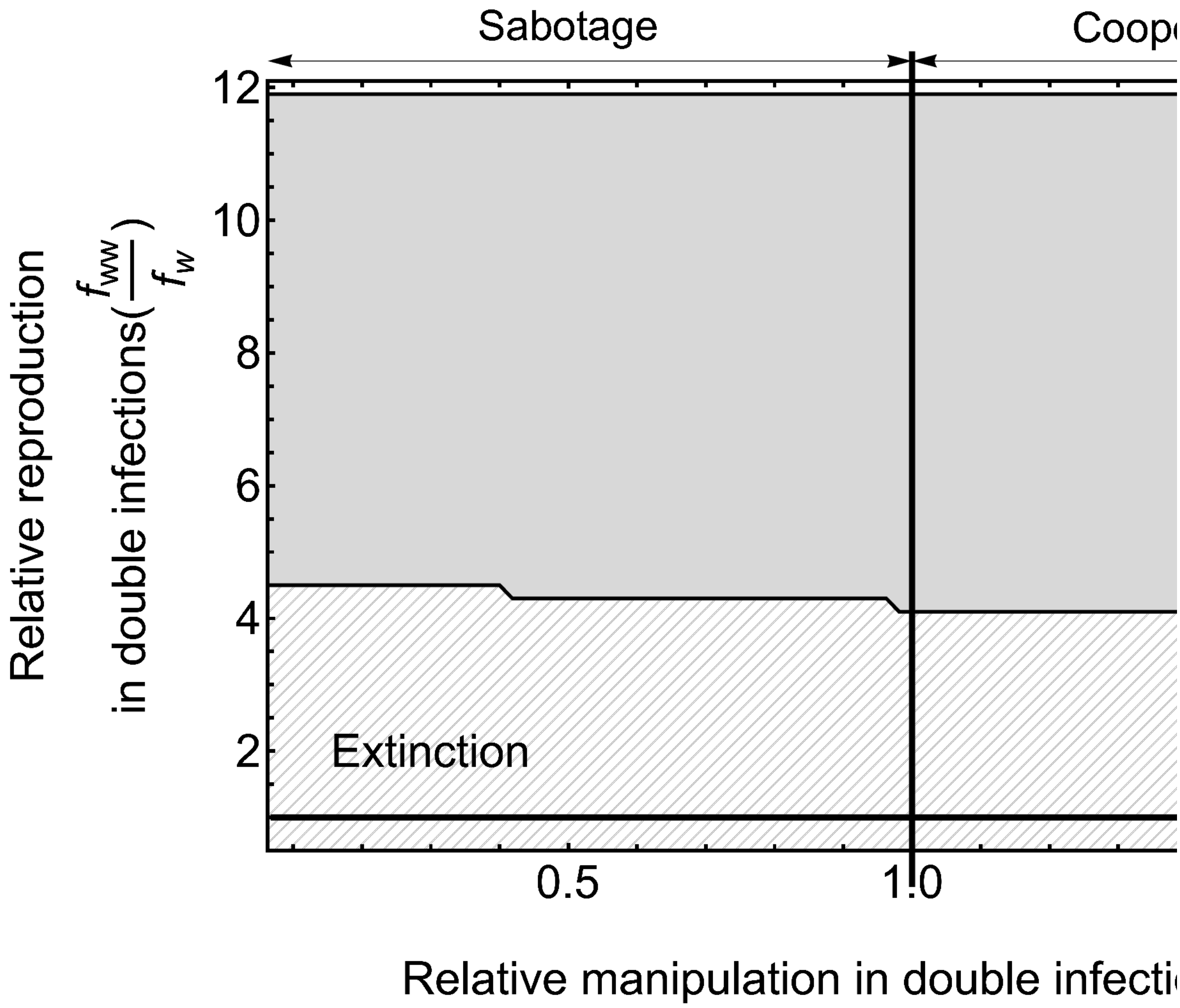
Out[368]= ratio_reproduction_manipulationA.pdf

Out[369]=



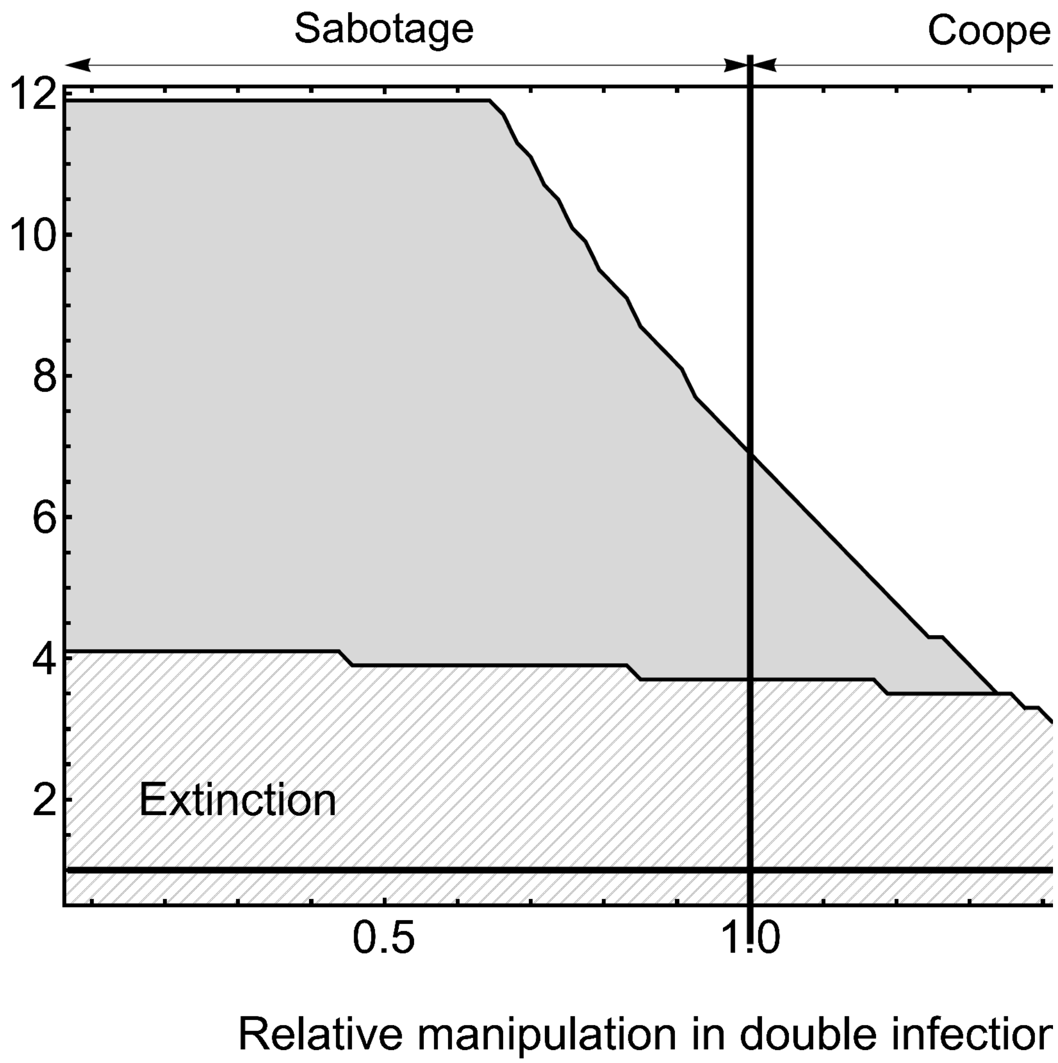
Out[370]= ratio_reproduction_manipulationB.pdf

Out[375]=



Out[376]= ratio_reproduction_manipulationC.pdf

Out[377]=



Out[378]= `ratio_reproduction_manipulationD.pdf`

Changing p (Figure 7 Panel A)

```
βwwstart = 0.1;
βwwend = 6.8;
βwwstep = 0.05;
coldat = "Rainbow";
```

Out[ ]= `0.05`

CALCULATION

```
parmanipβp = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0,
    αww -> 0, c → 1.4, μ → 3.9, σw → 0, σww -> 0, q -> 0.05,
    δ → 0.9, k -> 0.26, fw -> 38, h -> 0.6, βw -> 1.4, ϵ -> 4.5};
manipβpAllResults = ParallelTable [NSolveCodim2Positive [sysfunc1 /. fww -> ϵfw,
      parmanipβp, {βww -> x}, {p -> y}, βp, eq, varRes, 15 ],
    {x, βwwstart, βwwend, βwwstep }, {y, 0, 1, 0.007 }];
```

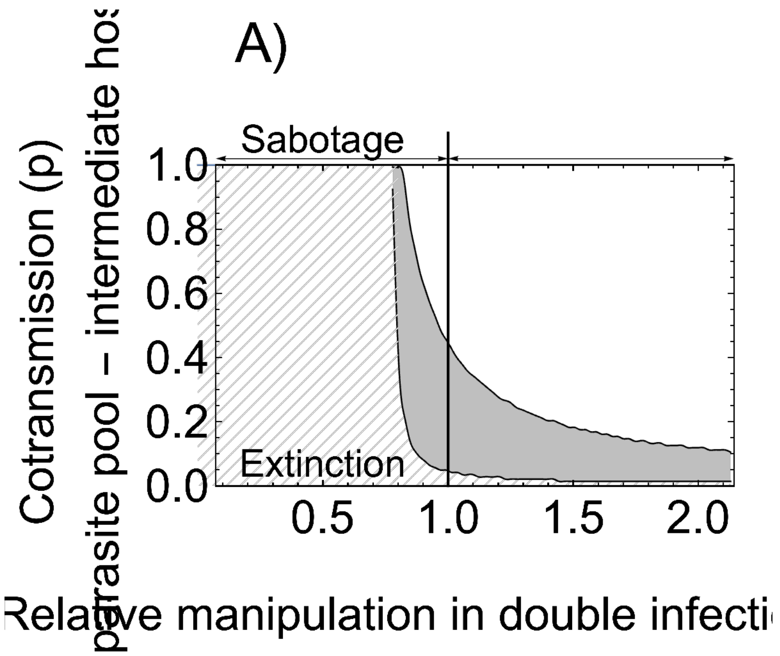
PLOT

```
In[448]:= equilibrium = eq /. Flatten [manipβpAllResults, 2 ];
parslist = βp /. Flatten [manipβpAllResults, 2 ];
range = {{βwwstart /(βw /. parmanipβp), βwwend /(βw /. parmanipβp) }, {0, 1}};
marklist = ListStableMarkTwoParameters [Jmatfunc1 /. fww -> ϵfw,
    parmanipβp, {βww, p }, parslist, equilibrium, colorstable, True ];
MakeListPlotData [parslist [[All, 1]]/(βw /. parmanipβp), parslist [[All, 2]]];
pβp = ListPlot [%, PlotStyle -> marklist, PlotRange → range, ImageSize → imageSize,
    Frame -> includeFrame, FrameLabel → {"βww/βw", "p"}, LabelStyle → labelStyle ];
(*MapThread to get ratio as xaxis *)
MapThread [#1/#2 &,
   {GetBoundaryLineBiStable [parslist ], {ConstantArray [{βw /. parmanipβp, 1},
        Length @GetBoundaryLineBiStable [parslist ][[1, All]]], ConstantArray [
        {βw /. parmanipβp, 1}, Length @GetBoundaryLineBiStable [parslist ][[2, All]]]}}];
lpβp1 = ListLinePlot [%, PlotRange → range, Frame → includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle,
    FrameLabel → {"Relative manipulation in double infection (βww/βw)",
       "Cotransmission (p) \n parasite pool - intermediate host" },
    PlotStyle -> {{ Thickness [thick ], Black }, {Thickness [thick ], Black }},
    Filling -> {1 -> {2}}, FillingStyle → colbifurfilling [[3]], ImageSize → imageSize,
    PlotLabel → Pane ["A)", Alignment → Left, ImageSize → panellabel ],
    InterpolationOrder → intorder ];
GetBoundaryLineSingle [parslist ]/ConstantArray [{βw /. parmanipβp, 1},
    Length @GetBoundaryLineSingle [parslist ]];
lpβp2 = ListLinePlot [%, PlotRange → range, Frame → includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle,
    PlotStyle -> {Thickness [thick ], Black }, InterpolationOrder → intorder,
    Filling → Bottom, FillingStyle → Directive [LightGray, HatchFilling []]];
line = {Graphics [{Thickness [0.005 ], Black, Line [{{1, 0}, {1, 1.1 }}]}]};
arrow1 = Graphics [
    {Arrowheads [arrowsize ], Arrow [{{1., 1.02 }, { βwwstart /(βw /. parmanipβp), 1.02 }}],
      Text [Style ["Sabotage", FontSize → 20], {0.5, 1.08 }]}];
arrow2 = Graphics [{Arrowheads [arrowsize ],
      Arrow [{{1., 1.02 }, {βwwend /(βw /. parmanipβp), 1.02 }}],
      Text [Style ["Cooperation", FontSize → 20], {2.9, 1.08 }]} ];
```

```
lv = ListLinePlot[{{{0., 1}, {0.8, 1}}}, PlotRange → range, Frame → includeFrame,
    FrameStyle → frameStyle, LabelStyle → labelStyle,
    PlotStyle -> {Thickness[thick], Black}, Filling → Bottom,
    FillingStyle → Directive[LightGray, HatchFilling[]]];
lβp = Show[{lpβp1, lpβp2, line, lv, arrow1, arrow2 },
    Epilog -> {Text[Style["Extinction", FontSize → 20], {0.5, 0.07}],
      Text[Style["Single \n equilibrium", FontSize → 20], {3.3, 0.6}]},
    PlotRangePadding -> 0, PlotRangeClipping -> False,
    ImagePadding -> {{110, 20}, {100, 45}}, ImageSize -> imageSize/1.5];
GraphicsGrid[{{pβp, lβp}}];
```

In[464]:= lβp

Out[464]=



Changing q (Figure 7 Panel B)

CALCULATION

In[379]:=
```
parmanipβq = {ρ → 1.2, d → 0.9, r → 2.5, γ → 2.9, αw → 0,
    αww -> 0, p -> 0.05, c → 1.4, μ → 3.9, σw → 0, σww -> 0,
    δ → 0.9, k -> 0.26, fw -> 38, h -> 0.6, βw -> 1.45, ϵ -> 4.5};
manipβqAllResults = ParallelTable[NSolveCodim2Positive[sysfunc1 /. fww -> ϵfw,
      parmanipβq, {βww -> x}, {q -> y}, βq, eq, varRes, 15],
    {x, βwwstart, βwwend, βwwstep}, {y, 0, 1, 0.007}];
```

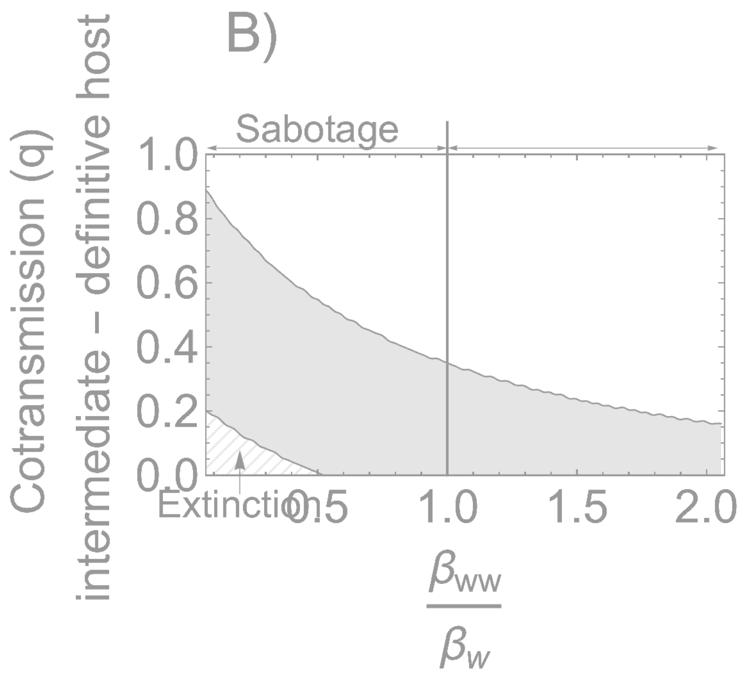
PLOT

```
equilibrium  = eq /. Flatten [manipβqAllResults, 2 ];
parslist  = βq /. Flatten [manipβqAllResults, 2 ];
range = {{βwwstart /(βw /. parmanip βq), βwwend /(βw /. parmanip βq) }, {0, 1}};
arrowsize  = {−0.02, 0.02 };

marklist  = ListStableMarkTwoParameters  [Jmatfunc1 /. fww −> ϵ fw,
     parmanip βq, {βww, q }, parslist, equilibrium, colorstable, True  ];
MakeListPlotData [parslist [[All, 1]]/(βw /. parmanip βq), parslist [[All, 2]]];
pβq = ListPlot [%, PlotStyle −> marklist, PlotRange → range, ImageSize → imageSize,
     Frame −> includeFrame, FrameLabel → {"βww/βw", "q"}, LabelStyle → labelStyle ];
Join [Thread @{parslist [[All, 1]]/(βw /. parmanip βq), parslist [[All, 2]]}, Idensity, 2 ];
MapThread [#1/#2 &,
   {GetBoundaryLineBiStable [parslist ], { ConstantArray [{βw /. parmanip βq, 1},
        Length @GetBoundaryLineBiStable [parslist ][[1, All ]]], ConstantArray [
        {βw /. parmanip βq, 1}, Length @GetBoundaryLineBiStable [parslist ][[2, All ]]]}}];
lpβq1 = ListLinePlot [%, PlotRange → range, Frame → includeFrame,
     FrameStyle → frameStyle, LabelStyle → labelStyle,
     FrameLabel → {"βww/βw", "Cotransmission (q) \n intermediate − definitive host" },
     PlotStyle −> {{Thickness [thick ], Black }, {Thickness [thick ], Black }},
     Filling −> {1 −> {2}}, FillingStyle → colbifurfilling [[3]], ImageSize → imageSize,
     PlotLabel → Pane ["B)", Alignment → Left, ImageSize → panellabel ],
     InterpolationOrder → intorder ];
GetBoundaryLineSingle [parslist ]/ConstantArray [{βw /. parmanip βq, 1},
     Length @GetBoundaryLineSingle [parslist ]];
lpβq2 = ListLinePlot [%, PlotRange → range, Frame → includeFrame,
     FrameStyle → frameStyle, LabelStyle → labelStyle,
     PlotStyle −> {Thickness [thick ], Black }, InterpolationOrder → intorder,
     Filling → Bottom, FillingStyle → Directive [LightGray, HatchFilling []]];
line = {Graphics [{Thickness [0.005 ], Black, Line [{{1, 0}, {1, 1.1 }}]}]};
arrow1 = Graphics [
     {Arrowheads [arrowsize ], Arrow [{{1., 1.02 }, { βwwstart /(βw /. parmanip βp), 1.02 }}],
       Text [Style ["Sabotage", FontSize → 20], {0.5, 1.08 }]}];
arrow2 = Graphics [{Arrowheads [arrowsize ],
       Arrow [{{1., 1.02 }, {βwwend /(βw /. parmanip βp) − 0.1, 1.02 }}],
       Text [Style ["Cooperation", FontSize → 20], {2.9, 1.08 }]} ];
arrow3 = Graphics [{Arrow [{{0.2, −0.06 }, {0.2, 0.08 }}]}];
lβq = Show [{lpβq1, lpβq2, line, arrow1, arrow2, arrow3 },
   Epilog −> {Text [Style ["Extinction", FontSize → 20], {0.2, −0.09 }],
       Text [Style ["Single \n equilibrium", FontSize → 20], {3.3, 0.6 }]},
   PlotRangePadding −> 0, PlotRangeClipping −> False,
   ImagePadding −> {{110, 20 }, {110, 45 }}, ImageSize −> imageSize /1.5 ]
(*GraphicsGrid [{{pβq, lβq}}]*)
```

Out[469]= {-0.02, 0.02}

Join: Expression Idensity at position 2 is expected to have head List for all expressions at level 2.

Out[482]=



Combined graph

```
GraphicsGrid [{{lβp, lβq}}, ImageSize → imageSize ∗2.8, Spacings -> -60]
Export ["coinfect _transmission.pdf", %, ImageResolution -> figResolution ]
```

Out[465]=