

# General ODE systems

```
In[ ]:= dIsdt = R[Iw, Is, Iww] - d Is -  $\Pi$ s[Ds, Dw, Dww] Is -  $\eta$  Is ;
dIwdt = (1 - p)  $\eta$  Is - (d +  $\alpha$ w) Iw -  $\Pi$ w[Ds, Dw, Dww,  $\beta$ w] Iw;
dIwwdt = p  $\eta$  Is - (d +  $\alpha$ ww) Iww -  $\Pi$ ww[Ds, Dw, Dww,  $\beta$ ww] Iww;
dDsdt = B[Ds, Dw, Dww, Is, Iw, Iww] -  $\mu$  Ds - ( $\lambda$ ww +  $\lambda$ w) Ds;
dDwdt = ( $\lambda$ w + (1 - q)  $\lambda$ ww) Ds - ( $\mu$  +  $\sigma$ w) Dw - ((1 - q)  $\lambda$ ww +  $\lambda$ w) Dw;
dDwwdt = q  $\lambda$ ww Ds + ((1 - q)  $\lambda$ ww +  $\lambda$ w) Dw - ( $\mu$  +  $\sigma$ ww) Dww;
dWdt = fw Dw + fww Dww -  $\delta$  W -  $\eta$  Is;

forceInf = { $\eta$  →  $\gamma$  W,  $\lambda$ w → h ( $\rho$  +  $\beta$ w) Iw,  $\lambda$ ww → h ( $\rho$  +  $\beta$ ww) Iww};
odesRes = {dIsdt, dIwdt, dIwwdt, dDsdt, dDwdt, dDwwdt, dWdt};
varRes = {Is, Iw, Iww, Ds, Dw, Dww, W};
```

```
In[ ]:= dDsdt + dDwdt + dDwwdt // FullSimplify
```

```
Out[ ]:= -Ds  $\mu$  - Dw ( $\mu$  +  $\sigma$ w) - Dww ( $\mu$  +  $\sigma$ ww) + B[Ds, Dw, Dww, Is, Iw, Iww]
```

# R0 using next generation matrix

```
In[ ]:= varPar = {Iw, Iww, Dw, Dww, W};
dDwdt = ( $\lambda$ w + (1 - q)  $\lambda$ ww) Ds - ( $\mu$  +  $\sigma$ w) Dw - ((1 - q)  $\lambda$ ww +  $\lambda$ w) Dw;
Mmatrix = {{-d -  $\alpha$ w -  $\Pi$ w[Ds, Dw, Dww,  $\beta$ w], 0, 0, 0, (1 - p)  $\gamma$  Is},
{0, -d -  $\alpha$ ww -  $\Pi$ ww[Ds, Dw, Dww,  $\beta$ ww], 0, 0, p  $\gamma$  Is},
{h ( $\rho$  +  $\beta$ w) Ds, (1 - q) h ( $\rho$  +  $\beta$ ww) Ds, - $\mu$  -  $\sigma$ w - ((1 - q)  $\lambda$ ww +  $\lambda$ w), 0, 0},
{0, q h ( $\rho$  +  $\beta$ ww) Ds, ((1 - q)  $\lambda$ ww +  $\lambda$ w), - $\mu$  -  $\sigma$ ww, 0},
{0, 0, fw, fww, - $\delta$  -  $\gamma$  Is}};
{{dIwdt, dIwwdt, dDwdt, dDwwdt, dWdt} = Mmatrix.varPar} /. forceInf //
FullSimplify
Fmatrix = {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, fw, fww, 0}};
Vmatrix = {{d +  $\alpha$ w +  $\Pi$ w[Ds, Dw, Dww,  $\beta$ w], 0, 0, 0, -(1 - p)  $\gamma$  Is},
{0, d +  $\alpha$ ww +  $\Pi$ ww[Ds, Dw, Dww,  $\beta$ ww], 0, 0, -p  $\gamma$  Is}, {-
h ( $\beta$ w +  $\rho$ ) Ds, -(1 - q) h ( $\beta$ ww +  $\rho$ ) Ds,  $\mu$  +  $\sigma$ w + ((1 - q)  $\lambda$ ww +  $\lambda$ w), 0, 0}, {0,
-q h ( $\rho$  +  $\beta$ ww) Ds, -((1 - q)  $\lambda$ ww +  $\lambda$ w),  $\mu$  +  $\sigma$ ww, 0}, {0, 0, 0, 0,  $\delta$  +  $\gamma$  Is}};
Fmatrix - Vmatrix == Mmatrix // FullSimplify
```

```
Out[ ]:= {True}
```

```
Out[ ]:= True
```

```

In[ ]:= Mmatrix // MatrixForm
Out[ ]//MatrixForm=

$$\begin{pmatrix} -d - \alpha w - \Pi w [Ds, Dw, Dww, \beta w] & 0 & 0 \\ 0 & -d - \alpha ww - \Pi ww [Ds, Dw, Dww, \beta ww] & 0 \\ Ds h (\beta w + \rho) & Ds h (1 - q) (\beta ww + \rho) & -\lambda w - (1 - q) \lambda ww - \mu - \sigma w \\ 0 & Ds h q (\beta ww + \rho) & \lambda w + (1 - q) \lambda ww \\ 0 & 0 & f w \end{pmatrix}$$


In[ ]:= Fmatrix // MatrixForm
Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f w & f w w & 0 \end{pmatrix}$$


In[ ]:= Vmatrix // MatrixForm
Out[ ]//MatrixForm=

$$\begin{pmatrix} d + \alpha w + \Pi w [Ds, Dw, Dww, \beta w] & 0 & 0 & 0 \\ 0 & d + \alpha ww + \Pi ww [Ds, Dw, Dww, \beta ww] & 0 & 0 \\ -Ds h (\beta w + \rho) & Ds h (-1 + q) (\beta ww + \rho) & \lambda w + (1 - q) \lambda ww + \mu + \sigma w & 0 \\ 0 & -Ds h q (\beta ww + \rho) & -\lambda w - (1 - q) \lambda ww & \mu + \sigma w \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


In[ ]:= R0 = \gamma Is \frac{p q h (\beta ww + \rho)}{\alpha ww + d + \Pi ww [Ds, Dw, Dww, \beta ww]} \frac{Ds}{\mu + \sigma ww} \frac{f ww}{\delta + \gamma Is} +
\gamma Is \left( \frac{(1 - p) (\beta w + \rho) h}{\alpha w + d + \Pi w [Ds, Dw, Dww, \beta w]} + \frac{p (1 - q) (\beta ww + \rho) h}{\alpha ww + d + \Pi ww [Ds, Dw, Dww, \beta ww]} \right)
\frac{Ds}{\mu + \sigma w} \frac{f w}{\delta + \gamma Is};
R0 == (Eigenvalues[Fmatrix.Inverse[Vmatrix]] [[5]] /. \lambda w \to 0 /. \lambda ww \to 0) //
FullSimplify
Out[ ]:= True

```

# Stability of the disease-free equilibrium with linear birth function

```

In[ ]:= func0 = {R[Iw, Is, Iww] \to r (Is + Iw + Iww), \Pi s [Ds, Dw, Dww] \to \rho (Ds + Dw + Dww),
\Pi w [Ds, Dw, Dww, \beta w] \to (\rho + \beta w) (Ds + Dw + Dww),
\Pi ww [Ds, Dw, Dww, \beta ww] \to (\beta ww + \rho) (Ds + Dw + Dww),
B [Ds, Dw, Dww, Is, Iw, Iww] \to \rho c (Ds + Dw + Dww) (Is + Iw + Iww)};
sysfunc0 = odesRes /. forceInf /. func0;

```

```
In[ ]:= solSusceptibleFunc0 = Solve[
  Thread[sysfunc0 == 0] /. {Iww -> 0, Iw -> 0, Dww -> 0, Dw -> 0, W -> 0}, varRes][[2]]
```

**Solve:** Equations may not give solutions for all "solve" variables.

$$\text{Out[ ]} = \left\{ I_s \rightarrow \frac{\mu}{c \rho}, D_s \rightarrow -\frac{d-r}{\rho} \right\}$$

```
In[ ]:= jacob0 = D[sysfunc0, {varRes}];
```

```
In[ ]:= jacob0 /. {Iww -> 0, Iw -> 0, Dww -> 0, Dw -> 0, W -> 0, Is -> \frac{\mu}{c \rho}, Ds -> -\frac{d-r}{\rho}} //
```

**MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & r & r & -\frac{\mu}{c} & -\frac{\mu}{c} & -\frac{\mu}{c} & -\frac{\gamma \mu}{c \rho} \\ 0 & -d - \alpha w + \frac{(d-r)(\beta w + \rho)}{\rho} & 0 & 0 & 0 & 0 & \frac{(1-p)\gamma \mu}{c \rho} \\ 0 & 0 & -d - \alpha ww + \frac{(d-r)(\beta ww + \rho)}{\rho} & 0 & 0 & 0 & \frac{p \gamma \mu}{c \rho} \\ -c(d-r) & -c(d-r) + \frac{h(d-r)(\beta w + \rho)}{\rho} & -c(d-r) + \frac{h(d-r)(\beta ww + \rho)}{\rho} & 0 & \mu & \mu & 0 \\ 0 & -\frac{h(d-r)(\beta w + \rho)}{\rho} & -\frac{h(1-q)(d-r)(\beta ww + \rho)}{\rho} & 0 & -\mu - \sigma w & 0 & 0 \\ 0 & 0 & -\frac{h q (d-r)(\beta ww + \rho)}{\rho} & 0 & 0 & -\mu - \sigma ww & 0 \\ 0 & 0 & 0 & 0 & fw & fww & -\delta - \frac{\gamma \mu}{c \rho} \end{pmatrix}$$

```
In[ ]:= jacob0 /. {Iww -> 0, Iw -> 0, Dww -> 0, Dw -> 0, W -> 0, Is -> \frac{\mu}{c \rho}, Ds -> -\frac{d-r}{\rho}} //
```

**Eigenvalues**

Out[ ]:=

$$\left\{ -\sqrt{d-r} \sqrt{\mu}, \sqrt{d-r} \sqrt{\mu}, \frac{\text{Root}\left[-c^4 d^2 fw h \beta w \beta ww \gamma \mu^2 \rho^{22} + \dots 326 \dots + \#1^5 \&, 1\right]}{c \rho^5}, \right. \\ \frac{\text{Root}\left[-c^4 d^2 fw h \beta w \beta ww \gamma \mu^2 \rho^{22} + \dots 326 \dots + \#1^5 \&, 2\right]}{c \rho^5}, \\ \frac{\text{Root}\left[-c^4 d^2 fw h \beta w \beta ww \gamma \mu^2 \rho^{22} + \dots 326 \dots + \#1^5 \&, 3\right]}{c \rho^5}, \frac{1}{c \rho^5} \text{Root}\left[ \right. \\ -c^4 d^2 fw h \beta w \beta ww \gamma \mu^2 \rho^{22} + c^4 d^2 fw h p q \beta w \beta ww \gamma \mu^2 \rho^{22} - c^4 d^2 fww h p q \beta w \beta ww \gamma \mu^2 \rho^{22} + \\ \dots 322 \dots + (-c d \beta w \rho^4 + c r \beta w \rho^4 - c d \beta ww \rho^4 + c r \beta ww \rho^4 + \gamma \mu \rho^4 + 2 c r \rho^5 + c \alpha w \rho^5 + \\ c \alpha ww \rho^5 + c \delta \rho^5 + 2 c \mu \rho^5 + c \rho^5 \sigma w + c \rho^5 \sigma ww) \#1^4 + \#1^5 \&, 4 \left. \right], \frac{1}{c \rho^5} \text{Root}\left[ \right. \\ -c^4 d^2 fw h \beta w \beta ww \gamma \mu^2 \rho^{22} + c^4 d^2 fw h p q \beta w \beta ww \gamma \mu^2 \rho^{22} - c^4 d^2 fww h p q \beta w \beta ww \gamma \mu^2 \rho^{22} + \\ \dots 322 \dots + (-c d \beta w \rho^4 + c r \beta w \rho^4 - c d \beta ww \rho^4 + c r \beta ww \rho^4 + \gamma \mu \rho^4 + 2 c r \rho^5 + \\ c \alpha w \rho^5 + c \alpha ww \rho^5 + c \delta \rho^5 + 2 c \mu \rho^5 + c \rho^5 \sigma w + c \rho^5 \sigma ww) \#1^4 + \#1^5 \&, 5 \left. \right] \left. \right\}$$

large output

show less

show more

show all

set size limit...

```
In[ ]:= (R0 /. func0 /. solSusceptibleFunc0 /. {Dw -> 0, Dww -> 0});
```

**R0func0cond =**

```
Reduce[% > 1 && fw > 0 && fww > 0 && r > 0 && d > 0 && r > d && \gamma > 0 &&
\mu > 0 && 0 < p < 1 && 0 < q < 1 && c > 0 && \delta > 0 && \beta ww > 0 && \beta w > 0 &&
\alpha w > 0 && \alpha ww > 0 && \rho > 0 && \sigma w > 0 && \sigma ww > 0 && h > 0 && h \le 1, Reals];
```

Simplify the expressions for  $\gamma$

```

$$\gamma_{\text{cond}} = \text{R0func0cond}[[14]][[1]][[5]][[2]];$$


$$\text{In}[ ] := \left( \frac{(c \delta \rho ((r-d) \beta w + (\alpha w + r) \rho) ((r-d) \beta ww + (\alpha ww + r) \rho) (\mu + \sigma w) (\mu + \sigma ww))}{\mu} \right.$$


$$1 / \left( f w h r ((1-p) \alpha ww \rho (\beta w + \rho) + p (1-q) \alpha w \rho (\beta ww + \rho) + \right.$$


$$(1-p q) r (\beta w + \rho) (\beta ww + \rho)) (\mu + \sigma ww) -$$


$$(\alpha w \rho + r (\beta w + \rho)) (\mu + \sigma w) ((-f w w h p q + \mu + \sigma ww) (\beta ww + \rho) r + \alpha ww \rho (\mu + \sigma ww)) +$$


$$d^2 (f w h ((1-p q) \beta w \beta ww + p (1-q) \beta w \rho + (1-p) \beta ww \rho) (\mu + \sigma ww) -$$


$$\beta w (\mu + \sigma w) (-f w w h p q (\beta ww + \rho) + \beta ww (\mu + \sigma ww))) +$$


$$d (f w h (-2 \times (1-p q) r \beta w \beta ww + (-1+p (-1+2 q)) r \beta w \rho - (1-p) \alpha ww \rho (\beta w + \rho) -$$


$$p (1-q) \alpha w \rho (\beta ww + \rho) + r \rho ((-2+p+p q) \beta ww + (-1+p q) \rho)) (\mu + \sigma ww) +$$


$$(\mu + \sigma w) (-f w w h p q (\beta ww + \rho) (2 r \beta w + (r + \alpha w) \rho) + (2 r \beta w \beta ww +$$


$$(r + \alpha ww) \beta w \rho + (r + \alpha w) \beta ww \rho) (\mu + \sigma ww))) \Big) == \gamma_{\text{cond}} // \text{FullSimplify}$$

```

$\text{Out}[ ] = \text{True}$

```

$$\text{In}[ ] := \text{R0func0cond}[[14]][[1]][[2]];$$


$$\text{fwcond} = \text{R0func0cond}[[14]][[1]][[2]][[2]];$$


$$\text{In}[ ] := \text{fwcond} // \text{FullSimplify}$$


$$\text{Out}[ ] := -((d \beta w - \alpha w \rho - r (\beta w + \rho)) (\mu + \sigma w) (-f w w h p q r (\beta ww + \rho) +$$


$$(r \beta ww + (r + \alpha ww) \rho) (\mu + \sigma ww) + d (f w w h p q (\beta ww + \rho) - \beta ww (\mu + \sigma ww)))) /$$


$$(h (d - r) ((-1+p) \alpha ww \rho (\beta w + \rho) + p (-1+q) \alpha w \rho (\beta ww + \rho) + (-1+p q) r (\beta w + \rho)$$


$$(\beta ww + \rho) - d ((-1+p q) \beta w \beta ww + p (-1+q) \beta w \rho + (-1+p) \beta ww \rho)) (\mu + \sigma ww)))$$


$$\text{In}[ ] := \text{fwcond} == \frac{(d \beta w - \alpha w \rho - r (\beta w + \rho)) (\mu + \sigma w)}{h (d - r) (\mu + \sigma ww)}$$


$$((( -f w w h p q (\beta ww + \rho) + \beta ww (\mu + \sigma ww)) (r - d) + (\alpha ww + r) \rho (\mu + \sigma ww)) /$$


$$((1-p) \alpha ww \rho (\beta w + \rho) + p (1-q) \alpha w \rho (\beta ww + \rho) + (1-p q) (r - d) (\beta w + \rho) (\beta ww + \rho) +$$


$$d \rho ((1-p) \beta w + \rho + p ((1-q) \beta ww - q \rho))) // \text{FullSimplify}$$

```

$\text{Out}[ ] = \text{True}$

```

$$\text{In}[ ] := \text{R0func0cond}[[14]][[1]][[1]][[5]] == \text{R0func0cond}[[14]][[2]][[1]][[2]]$$

```

$\text{Out}[ ] = \text{True}$

```

$$\text{In}[ ] := \text{fwwcond} = \text{R0func0cond}[[14]][[1]][[1]][[5]] // \text{FullSimplify}$$

```

```

$$\text{Out}[ ] := \frac{(d \beta ww - \alpha ww \rho - r (\beta ww + \rho)) (\mu + \sigma ww)}{h p q (d - r) (\beta ww + \rho)}$$

```

$\text{In}[ ] :=$

# Nonlinear birth function for intermediate host

```

In[ ]:= func1 = {R[Iw, Is, Iww] → r (1 - k (Is + Iw + Iww)) (Is + Iw + Iww),
  Πs[Ds, Dw, Dww] → ρ (Ds + Dw + Dww), Πw[Ds, Dw, Dww, βw] →
  (ρ + βw) (Ds + Dw + Dww), Πww[Ds, Dw, Dww, βww] → (ρ + βww) (Ds + Dw + Dww),
  B[Ds, Dw, Dww, Is, Iw, Iww] → ρ c (Ds + Dw + Dww) (Is + Iw + Iww)};

sysfunc1 = odesRes /. forceInf /. func1;
jacob1 = D[sysfunc1, {varRes}];

solSusceptibleFunc1 = Solve[
  Thread[sysfunc1 == 0] /. {Iww → 0, Iw → 0, Dww → 0, Dw → 0, W → 0}, varRes][[3]]

```

 **Solve:** Equations may not give solutions for all "solve" variables.

$$\text{Out[ ]} = \left\{ \text{Is} \rightarrow \frac{\mu}{c \rho}, \text{Ds} \rightarrow -\frac{k r \mu + c d \rho - c r \rho}{c \rho^2} \right\}$$

In[ ]:=

```

In[ ]:= Reduce[(Ds /. solSusceptibleFunc1) > 0 &&
  k > 0 && r > 0 && c > 0 && μ > 0 && d > 0 && ρ > 0]

```

$$\text{Out[ ]} = r > 0 \&\& 0 < d < r \&\& \rho > 0 \&\& \mu > 0 \&\& k > 0 \&\& c > \frac{k r \mu}{-d \rho + r \rho}$$

```

In[ ]:= jacob1 /. {Iww → 0, Iw → 0, Dww → 0, Dw → 0, W → 0, Is → 0, Ds → 0} // Eigenvalues

```

$$\text{Out[ ]} = \{-d + r, -d - \alpha w, -d - \alpha ww, -\delta, -\mu, -\mu - \sigma w, -\mu - \sigma ww\}$$

## Stability of the equilibrium

$$\text{In[ ]} = \text{eigval} = \text{Eigenvalues}\left[\text{jacob1} /. \left\{ \text{Iww} \rightarrow 0, \text{Iw} \rightarrow 0, \text{Dww} \rightarrow 0, \text{Dw} \rightarrow 0, \text{W} \rightarrow 0, \text{Is} \rightarrow \frac{\mu}{c \rho}, \right. \right. \\ \left. \left. \text{Ds} \rightarrow \frac{-k r \mu + c (-d + r) \rho}{c \rho^2}, \sigma w \rightarrow \sigma, \sigma ww \rightarrow \sigma, \alpha w \rightarrow \alpha, \alpha ww \rightarrow \alpha, \text{fww} \rightarrow \epsilon \text{fw} \right\} \right];$$

Coefficients

$$\text{Out[ ]} = \text{A3} = c^2 \rho^7 \\ (\gamma \mu \rho - k r \mu (\beta w + \beta ww + 2 \rho) + c \rho (-d (\beta w + \beta ww) + r (\beta w + \beta ww + 2 \rho) + \rho (2 \alpha + \delta + \mu + \sigma)))$$

$$\text{Out[ ]} = \text{A2} = \\ c^4 \rho^{14} \left( k^2 r^2 \mu^2 (\beta w + \rho) (\beta ww + \rho) - k r \mu \rho (\gamma \mu (\beta w + \beta ww + 2 \rho) + c (2 r (\beta w + \rho) (\beta ww + \rho) - \right. \\ \left. d (\beta ww \rho + \beta w (2 \beta ww + \rho)) + \rho (\beta w + \beta ww + 2 \rho) (\alpha + \delta + \mu + \sigma)) \right) + \\ c \rho^2 (\gamma \mu (-d (\beta w + \beta ww) + r (\beta w + \beta ww + 2 \rho) + \rho (2 \alpha + \mu + \sigma)) + \\ c (d^2 \beta w \beta ww + r^2 (\beta w + \rho) (\beta ww + \rho) + \\ r \rho (\beta w + \beta ww + 2 \rho) (\alpha + \delta + \mu + \sigma) + \rho^2 (\alpha^2 + \delta (\mu + \sigma) + 2 \alpha (\delta + \mu + \sigma)) - \\ \left. d (r \beta ww \rho + r \beta w (2 \beta ww + \rho) + (\beta w + \beta ww) \rho (\alpha + \delta + \mu + \sigma)) \right) \right)$$

$$\text{In[ ]} = \text{Dsfoo} = c \rho (r - d) - k r \mu;$$

$$\text{B4} = 1$$

$$\text{Out[ ]} = 1$$

```
In[ ]:= B3 ==  $\rho^7 \left( \text{Dsfoo} (\beta w + \beta ww + 2 \rho) + \gamma \mu \rho + c \rho^2 (2 d + 2 \alpha + \delta + \mu + \sigma) \right) // \text{FullSimplify}$ 
```

```
Out[ ]:= True
```

```
In[ ]:=  $\frac{B2}{\rho^{14}} == \left( c \rho^2 (\alpha + \delta + \mu + \sigma + d) + \gamma \mu \rho \right) (\beta w + \beta ww + 2 \rho) \text{Dsfoo} +$   

 $\text{Dsfoo}^2 (\beta w + \rho) (\beta ww + \rho) + c \rho^3 \left( d \gamma \mu + (d c \rho + \gamma \mu) (2 \alpha + \mu + \sigma + d) + \right.$   

 $\left. d c \rho (2 \delta + \mu + \sigma) + \rho c (\alpha^2 + \delta (\mu + \sigma) + 2 \alpha (\delta + \mu + \sigma)) \right) // \text{FullSimplify}$ 
```

```
Out[ ]:= True
```

```
In[ ]:= foo1 =  $\alpha \rho^2 (\alpha \gamma \mu + 2 (\gamma \mu + c \delta \rho) (\mu + \sigma) + c \alpha \rho (\delta + \mu + \sigma))$  ;  

foo2 =  

 $c \rho^2 (\alpha \gamma \mu + (\gamma \mu + c \delta \rho) (\mu + \sigma) + c \alpha \rho (\delta + \mu + \sigma)) (\text{Dsfoo} (\beta w + \beta ww + 2 \rho) + d 2 \rho c \rho) +$   

 $(\beta w + \rho) (\beta ww + \rho) \text{Dsfoo}^2 (\gamma \mu + c \rho (\delta + \mu + \sigma)) +$   

 $((\beta w + \beta ww + 2 \rho) \text{Dsfoo} (\gamma \mu + c \rho (\delta + \mu + \sigma)) + d c \rho^2 (\gamma \mu + c \rho (\delta + \mu + \sigma))) d c \rho^2$  ;  

foo3 =  $((1 - p) \beta w + \rho + p (\beta ww + q (\epsilon - 1) (\beta ww + \rho)))$  ;
```

```
 $\frac{B1}{\rho^{22}} == -fw \gamma \mu h c \rho^2 \text{Dsfoo} \text{foo3} + c^2 \rho^2 \text{foo1} + \text{foo2} // \text{Simplify}$ 
```

```
Out[ ]:= True
```

```
In[ ]:=  $(1 - p) \beta w + \rho (1 - p q) + p \beta ww (1 - q) == \text{foo3} /. \epsilon \rightarrow 0 // \text{Simplify}$ 
```

```
Out[ ]:= True
```

```
In[ ]:= foo4 =  $\text{Dsfoo} (\text{Dsfoo} (1 + p q (\epsilon - 1)) (\beta w + \rho) (\beta ww + \rho) + c \rho^2 (d + \alpha) \text{foo3})$  ;  

foo5 =  $(\gamma \mu + c \delta \rho) (\text{Dsfoo}^2 (\beta w + \rho) (\beta ww + \rho) +$   

 $c \rho^2 (d + \alpha) \text{Dsfoo} (\beta ww + \rho) + \text{Dsfoo} (\beta w + \rho) c \rho^2 (d + \alpha) + c^2 \rho^4 (d + \alpha)^2) (\mu + \sigma)$  ;
```

```
 $\frac{B0}{c^9 \rho^{31}} == -fw h \gamma \mu \text{foo4} + \text{foo5} // \text{Simplify}$ 
```

```
Out[ ]:= True
```

## Condition for basic reproduction

```
In[ ]:= R0 /. func1 /. solSusceptibleFunc1 /. {Iww → 0, Iw → 0, Dww → 0,  

Dw → 0, W → 0,  $\sigma w \rightarrow \sigma$ ,  $\sigma ww \rightarrow \sigma$ ,  $\alpha w \rightarrow \alpha$ ,  $\alpha ww \rightarrow \alpha$ ,  $fww \rightarrow \epsilon fw$ };  

R0func1cond = Reduce[ $\% > 1 \ \&\& \ fw > 0 \ \&\& \ \epsilon > 0 \ \&\& \ \beta w > 0 \ \&\& \ \beta ww > 0 \ \&\& \ 0 < q < 1 \ \&\&$   

 $0 < p < 1 \ \&\& \ r > d \ \&\& \ r > 0 \ \&\& \ d > 0 \ \&\& \ c > 0 \ \&\& \ \rho > 0 \ \&\& \ \mu > 0 \ \&\& \ \gamma > 0 \ \&\&$   

 $\sigma > 0 \ \&\& \ \delta > 0 \ \&\& \ \alpha > 0 \ \&\& \ k > 0 \ \&\& \ c > \frac{k r \mu}{-d \rho + r \rho} \ \&\& \ h > 0 \ \&\& \ h \leq 1$ ];
```

In[\*]:= R0func1cond[[17]][[2]] // FullSimplify

Out[\*]= 
$$\frac{(\gamma \mu + c \delta \rho) (k r \mu (\beta w + \rho) - c \rho (-d \beta w + r \beta w + (r + \alpha) \rho))}{(k r \mu (\beta w w + \rho) - c \rho (-d \beta w w + r \beta w w + (r + \alpha) \rho)) (\mu + \sigma)} /$$

$$(h \gamma \mu (k r \mu + c (d - r) \rho) (k r (1 + p q (-1 + \epsilon)) \mu (\beta w + \rho) (\beta w w + \rho) -$$

$$c \rho (d (-1 + p) \beta w w \rho + r (1 + p q (-1 + \epsilon)) (\beta w + \rho) (\beta w w + \rho) -$$

$$d \beta w (\beta w w + p q \beta w w (-1 + \epsilon) + p (1 + q (-1 + \epsilon)) \rho) +$$

$$\alpha \rho (\beta w - p \beta w + \rho + p (\beta w w + q \beta w w (-1 + \epsilon) + q (-1 + \epsilon) \rho)))$$