The Hall Effect in Semiconductors

Zekeriya Gökhan Evelek
Boğaziçi University • Physics Department

***Abstract*—In this experiment we have investigated the properties of the hall effect. We have first found the resistance of the hall probe. Then we have measured hall voltage and current at constant magnetic field, for both zero and nonzero field. Then we also measured the magnetic field and hall voltage at constant current, both at the same magnitude but different signs. Those measurements are used to determine Hall Coefficients and mobility of the electrons(we have used n-type semiconductor). The weighted average of Hall Coefficient was found out to be $R_H(weighted) = -0.00089 \pm 0.00012 \frac{m^3}{As}$ and of mobility of the electrons was found out to be $\mu_e(weighted) = -0.031 \pm 0.008 \frac{m^2}{Vs}$.**

I. Theory

Although the fact that the wire carrying current in a magnetic field experiences a force was known before, it was after Maxwell's systematization of electromagnetic theory that Edwin Hall discovered the hall effect. He has questioned if the magnetic field interacts with conductors and concluded that the magnetic force results in the separation of the current in the one side so that there appears a measaureable voltage. The interesting fact is that Edwin Hall discovered this effect years before the discovery of the electrons.[1]

Moving charged particles experiences a force and it is calculated by the famous Lorentz Force Law as follows:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

where, q is the charge, E is the electric field, v is the velocity and B is the magnetic field.

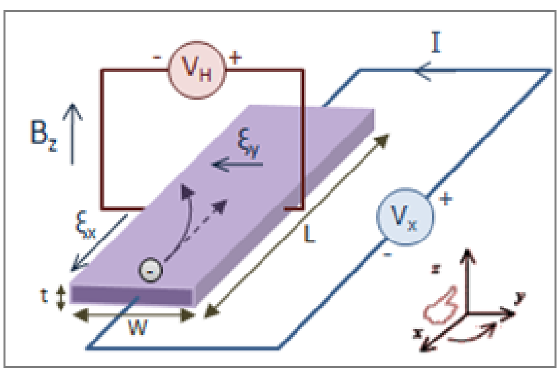

Fig. 1. Hall effect measurement setup for electrons [1]

As seen in the figure above, since in steady state F=0, then:

$$E_y + v_x B_z = 0 \tag{2}$$

and

$$E_y = \frac{V_H}{w} \tag{3}$$

By combining the equations 2 and 3 we get the hall voltage as:

$$V_H = v_x B_z w \tag{4}$$

We get the following formula for the current moving in the x direction:

$$I_x = ntw(-v_x)(-e) \tag{5}$$

Where n is the charge carrier density, t is the thickness, and w is the width, e is the elementary charge. At the end of the day, by combining the equations 4 and 5 we get the hall voltage as

$$V_H = \frac{I_x B_z}{nte} \tag{6}$$

The hall coefficient is defined as:

$$R_H = \frac{E_y}{j_x B_z} = \frac{1}{ne}[1] \tag{7}$$

where j is the current density.

However, the situation is more complicated in the case of a semiconductor, a material that exhibits properties in between a conductor and an isolator. Semiconductors in their natural state shows poor conductivity due to lack of free electrons. It is by the effect of doping that one can make a semiconductor to gain conductivity(see figure 2)[2]. If it is done by an electron acceptor it is p-type and by an electron donor then it is ntype.[3] In other words, in semiconductors, the charge carriers consists not only of a single type such as electrons, holes or ions, but there is a mixture of carriers. Semiconductors can be used in many areas by manipulating this feature. For example, they can be used in passing current in one direction more easily. Also, an electric field can be produced by combining n-type and p-type semiconductors.

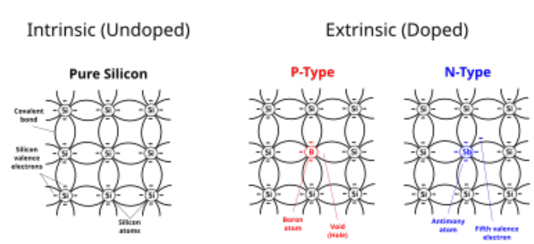

Fig. 2. Doping of a pure silicon [1]

Since the charge carriers do not solely consist of a single type anymore, the hole coefficient has a more complicated form:

$$R_H = \frac{n_h \mu_h^2 - n_e \mu_e^2}{e\left(n_h \mu_h + n_e \mu_e\right)^2}[4] \tag{8}$$

μ_h: the mobility of the hole μ_e: the mobility of the electron and by combining equations 6 and 7 we get:

$$R_H = \frac{V_H t}{IB} \tag{9}$$

If we have an n-type semiconductor, then we get the following expression for the mobility:

$$\mu_e = \frac{L}{AR_e n_e} = \frac{R_H L}{AR} \tag{10}$$

where A is the cross-section, L is length and R is the resistance.

II. Method

The following procedure was followed for the experiment:

1) The dimensions of the conductor is measured: Length: 16 ± 4mm (Current Direction), Width: 10 ± 1mm (Hall Voltage Direction), Thickness: 1.0 ± 0.2mm,
2) The resistance of the Hall probe is measured
3) Hall voltage is measured at zero magnetic field to get a reference measurement
4) Hall voltage is measured at constant but nonzero magnetic field
5) Hall voltage is measured at constant current, with the exactly same magnitude but both for positive and negative

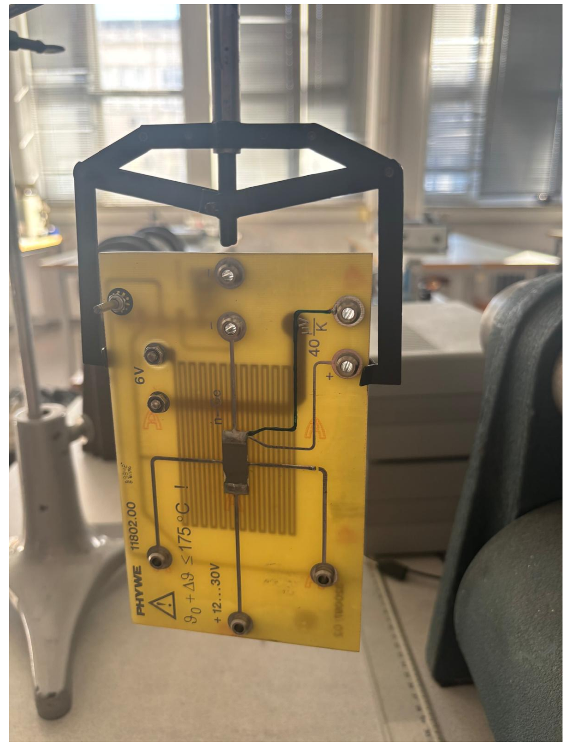

Fig. 3. Hall Probe, semiconductor n-doped Germanium

III. The Experimental Setup

Here is the setup we have used in the experiment

- Helmholtz Coil
- Teslameter
- n-doped Germanium Hall Probe
- Power Supply
- Digital Multimeters
- Connecting leads
- Ruler


Fig. 4. Helmholtz coils and teslameter probe

IV. The Data

For uncertainties of hall voltage, we have used the manual of the multimeter. Teslameter's uncertainty is $\pm\frac{5}{100}B$.[4] The rest has been taken to be the least significant figure.

TABLE I
Resistance Measurements

Current (mA)	Voltage (V)
20 $\pm$ 1.	0.907 $\pm$ 0.0484
26 $\pm$ 1.	1.154 $\pm$ 0.0607
37 $\pm$ 1.	1.650 $\pm$ 0.0855
45 $\pm$ 1.	2.031 $\pm$ 0.1046
53 $\pm$ 1.	2.369 $\pm$ 0.1215
63 $\pm$ 1.	2.827 $\pm$ 0.1444
60 $\pm$ 1.	3.126 $\pm$ 0.1593
74 $\pm$ 1.	3.366 $\pm$ 0.1713
83 $\pm$ 1.	3.765 $\pm$ 0.1913
94 $\pm$ 1.	4.280 $\pm$ 0.2440

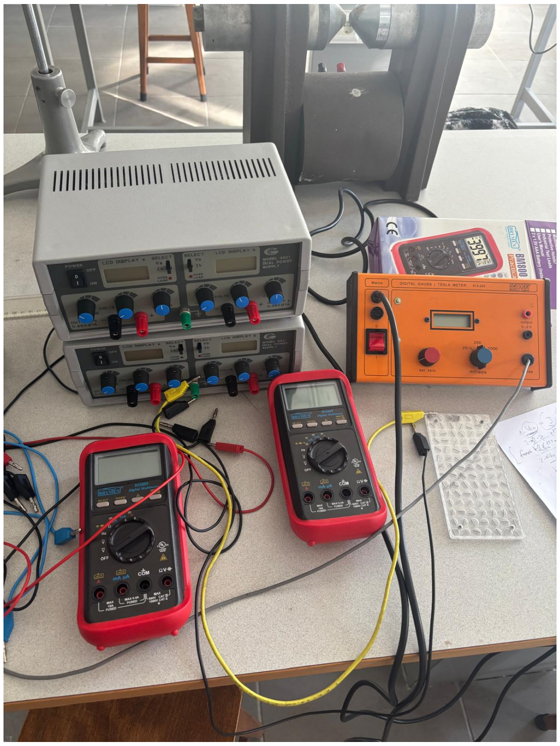

Fig. 5. Voltage and Current sources(upper left), Digital Multimeters for measuring current and voltage(lower left) and The Teslameter for measuring the magnetic field(right)

TABLE II
HALL VOLTAGE MEASUREMENTS AT B=0

Current (mA)	Hall Voltage (mV)
12. ± 1.	-0.810 ± 0.024
19. ± 1.	-1.330 ± 0.040
26. ± 1.	-1.820 ± 0.055
32. ± 1.	-2.180 ± 0.065
47. ± 1.	-3.270 ± 0.098
56. ± 1.	-3.920 ± 0.118
63. ± 1.	-4.360 ± 0.131
69. ± 1.	-4.860 ± 0.146
75. ± 1.	-5.310 ± 0.159
83. ± 1.	-5.850 ± 0.175

TABLE III
HALL VOLTAGE MEASUREMENTS AT B=139.3 MT

Current (mA)	Hall Voltage (mV)
10. ± 1.	-1.830 ± 0.055
17. ± 1.	-2.960 ± 0.089
26. ± 1.	-4.530 ± 0.136
33. ± 1.	-5.890 ± 0.177
39. ± 1.	-6.900 ± 0.207
46. ± 1.	-8.020 ± 0.241
54. ± 1.	-9.560 ± 0.287
61. ± 1.	-10.730 ± 0.322
70. ± 1.	-12.330 ± 0.370
76. ± 1.	-13.820 ± 0.415

TABLE IV
HALL VOLTAGE MEASUREMENTS AND MAGNETIC FIELD AT CONSTANT CURRENT I=53.MA

Magnetic Field (mT)	Hall Voltage (mV)
14.4 ± 0.7	-43.100 ± 1.293
26.4 ± 1.3	-47.900 ± 1.437
34.9 ± 1.7	-51.400 ± 1.542
47.3 ± 2.4	-56.300 ± 1.689
58.7 ± 2.9	-61.000 ± 1.830
70.7 ± 3.5	-65.800 ± 1.974
79.2 ± 4.0	-69.200 ± 2.076
93.2 ± 4.7	-74.900 ± 2.247
104.1 ± 5.2	-79.300 ± 2.379
126.8 ± 6.3	-88.400 ± 2.652

TABLE V
HALL VOLTAGE MEASUREMENTS AND MAGNETIC FIELD AT CONSTANT CURRENT I=-53.MA

Magnetic Field (mT)	Hall Voltage (mV)
18.7 ± 0.9	44.800 ± 1.344
26.9 ± 1.3	48.200 ± 1.446
33.2 ± 1.7	50.700 ± 1.521
41.3 ± 2.1	54.000 ± 1.620
49.9 ± 2.5	57.400 ± 1.722
61.2 ± 3.1	62.100 ± 1.863
71.2 ± 3.6	66.100 ± 1.983
84.0 ± 4.2	71.200 ± 2.136
97.5 ± 4.9	76.700 ± 2.301
112.4 ± 5.6	82.700 ± 2.481

V. THE ANALYSIS AND RESULTS

In the analysis, we have used the following error propagation formula if there is no correlation between the variables:[4] For the exact application of the formula in the analysis see the code in the appendix.

We have used the ROOT's built in function for getting linear fits and their corresponding results and errors.

$$\sigma_f = \sqrt{(\frac{\partial f}{\partial x})^2(\sigma_x)^2 + (\frac{\partial f}{\partial y})^2(\sigma_y)^2 + (\frac{\partial f}{\partial z})^2(\sigma_z)^2 + ...} \quad (11)$$

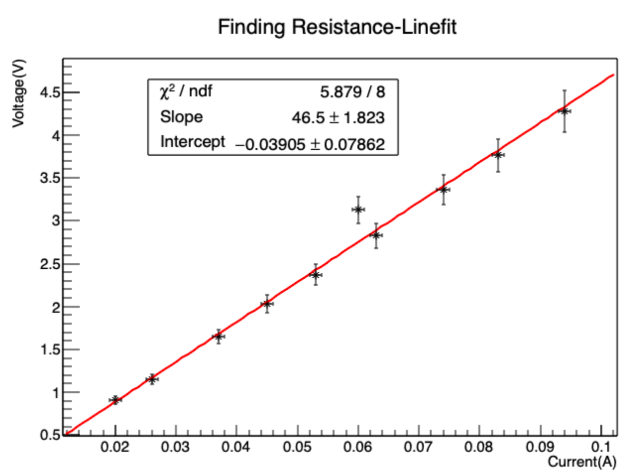

Fig. 6. Measurements for finding the resistance

The resistance was found out to be $46.5 \pm 1.8\Omega$.

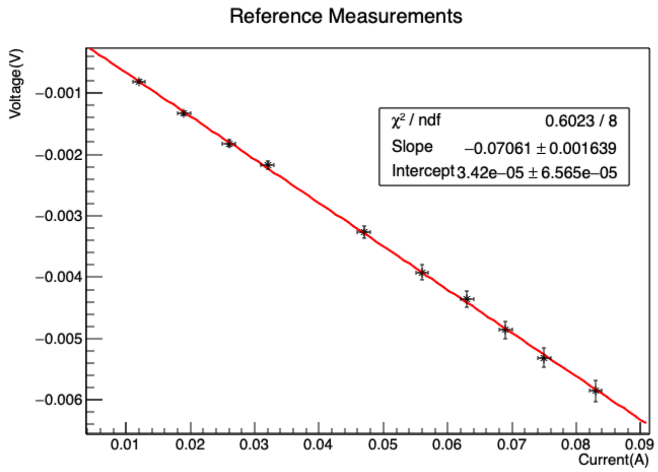

Fig. 7. Reference Measurements at B=0

From figure(7) the offset equation is found out to be as:

$$V = -0.07061I + 3.42 * 10^{-5} \tag{12}$$

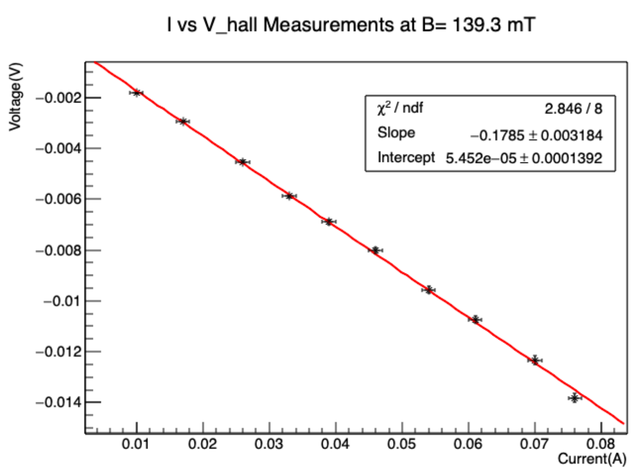

Fig. 8. The Varying Current Measurementsat B=139.3 mT

It is seen that the slope is found out to be:$-0.179 \pm 0.003 \frac{V}{A}$. By using the equations 9 and 10, we get the $R_H = -0.00128 \pm 0.00027 \frac{m^3}{As}$ and $\mu_e = -0.044 \pm 0.017 \frac{m^2}{Vs}$ respectively.

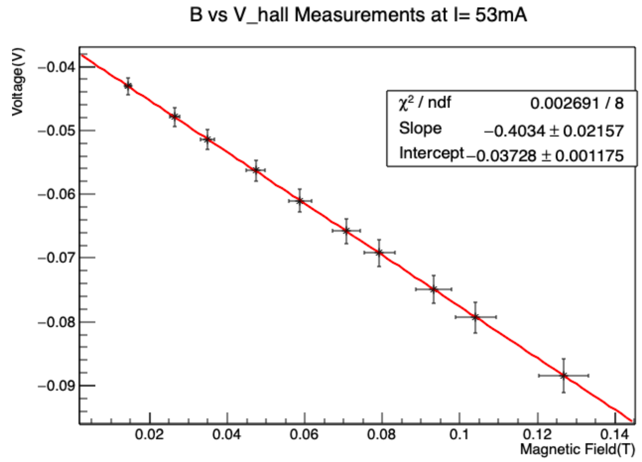

Fig. 9. Magnetic Field Measurements at I = 53mA

For the positive constant current case the slope is found out to be:$-0.403 \pm 0.022 \frac{V}{T}$. By using the equations 9 and 10, we get the $R_H = -0.00761 \pm 0.00158 \frac{m^3}{As}$ and $\mu_e = -0.262 \pm 0.101 \frac{m^2}{Vs}$ respectively.

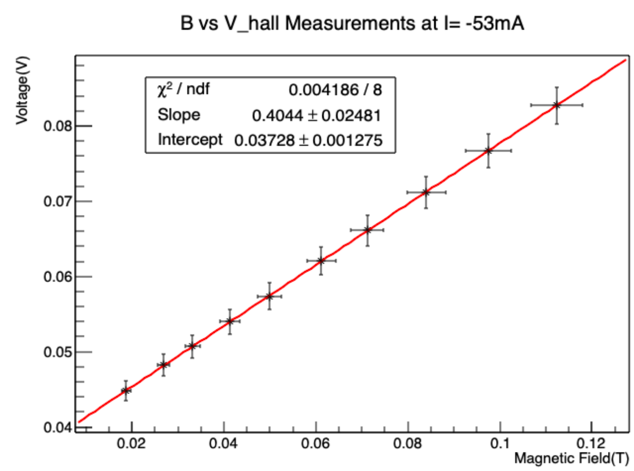

Fig. 10. Magnetic Field Measurements at I = -53mA

Now for the negative constant current case the slope is found out to be:$0.404 \pm 0.025 \frac{V}{T}$. By using the equations 9 and 10, we get the $R_H = -0.00763 \pm 0.00160 \frac{m^3}{As}$ and $\mu_e = -0.263 \pm 0.101 \frac{m^2}{Vs}$ respectively.

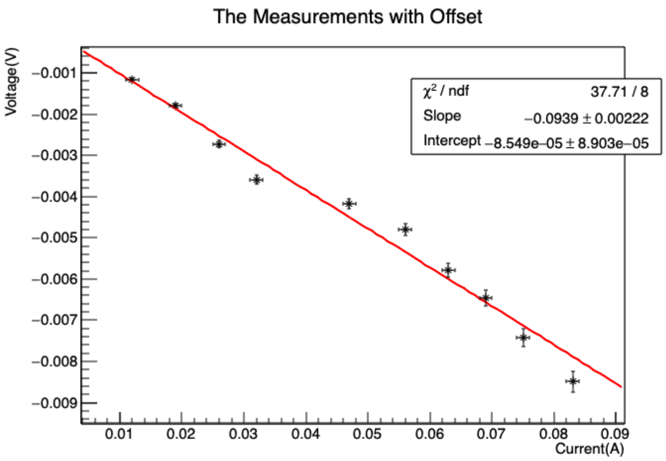

Fig. 11. The Varying Current Measurementsat B=139.3 mT with offset

Finally, by substracting the voltage values we get from the offset equation(12) from the nonzero constant magnetic field hall voltage measurements(figure 8), we found the slope as:$-0.094 \pm 0.002\frac{V}{A}$. By using the equations 9 and 10, we get the $R_H = -0.00067 \pm 0.00014\frac{m^3}{As}$ and $\mu_e = -0.023 \pm 0.009\frac{m^2}{Vs}$ respectively.

Weighted Average Formula is:

$$\bar{x}_w = \frac{\sum_{i=1}^{N} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \tag{13}$$

and the uncertainty in the Weighted Average:

$$\sigma_w = \sqrt{\frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}} \tag{14}$$

To see how those formulas are applied see the code in the appendix. By using equations 13 and 14 we get the weighted averages of Hall Coefficient and Mobility of Electrons as:

$$R_H(weighted) = -0.00089 \pm 0.00012\frac{m^3}{As} \tag{15}$$

$$\mu_e(weighted) = -0.031 \pm 0.008\frac{m^2}{Vs} \tag{16}$$

VI. The Conclusion

Hall voltage measurements at constant current seems to be a success due to their closeness to each other. Therefore, we have expected to get a similar result when we have corrected the measurements at constant nonzero magnetic field by subtracting the offset equation from the measured voltage values. This is required because the measurements in the nonzero constant magnetic field includes not only the hall effect voltage but also the V_{offset} values. In order to accomplish that, we have measured the hall voltage due only to the current itself so that we get an offset equation. On the other hand, our calculations indicate that we get the opposite effect that we have expected. In other words, our offset correction gave us Hall Coefficient and Mobility of Electron values not closer to the ones we got in the constant current cases but the difference have increased. One final note might be that after applying the offset correction, our linefit intersects with the y-axis at point closer to the origin, as we want.

References

[1] *wiki,hall*. URL: https://en.wikipedia.org/wiki/Hall_effect (visited on 11/21/2024).

[2] *wiki,semi*. URL: https://en.wikipedia.org/wiki/Hall_effect (visited on 11/21/2024).

[3] *wiki,type*. URL: https://en.wikipedia.org/wiki/Extrinsic_semiconductor#N-type_semiconductors (visited on 11/21/2024).

[4] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

VII. Appendix

Here the code I have used in the analysis of the experiment. Codes also consist of the raw data. I chose to work with the raw data in the float forms inside the codes because of the small size of the data, namely 10 data pairs for each measurements and 5 different measurements in total.

```
{
double calculateVoltUncertainty(double V)
{
V = abs(V);
double error;
if (V <= 0.4)
{
error = (3 * V / 100);
}
else if (V <= 4)
{
error = (5 * V / 100) + 0.003;
}
else if (V <= 40)
{
error = (5 * V / 100) + 0.03;
}
else if (V <= 400)
{
error = (5 * V / 100) + 0.3;
}
else
{
error = (V / 100) + 4;
}
return error;
};

double t = 1e-3;
double t_error = 0.2e-3;
double L = 16e-3;
double L_error = 4e-3;
double w = 10e-3;
double w_error = 0.2e-3;
double A = w*t;
double A_error = sqrt(pow(w_error*t,2)+pow(w
*t_error,2));

// PART 1

const int ndata =10;
```

```
float I1[ndata] =
    {0.020,0.026,0.037,0.045,0.053,0.063,0.060,0.074,0.083,0.094};
float V1[ndata] =
    {0.907,1.154,1.650,2.031,2.369,2.827,3.126,3.366,3.765,4.280};
float V1er[ndata],I1er[ndata];

for (int i = 0; i < ndata; ++i) {
   I1er[i] = 0.001;
   V1er[i] = calculateVoltUncertainty(V1[i]);
}

TGraphErrors *p1 = new
    TGraphErrors(ndata,I1,V1,I1er,V1er);
p1->Draw("A*");
p1->GetXaxis()->SetTitle("Current(A)");
p1->GetYaxis()->SetTitle("Voltage(V)");
p1->SetTitle("Finding Resistance-Linefit ");


TF1 *linefit = new TF1("linefit","[0]*x+[1]");
linefit->SetParNames("Slope", "Intercept");
p1->Fit(linefit);
gStyle->SetOptFit(1);

TCanvas *c = new TCanvas();

double R = linefit->GetParameter(0);
double Rerror = linefit->GetParError(0);

// for mobility calculations
double sigma = L/(A*R);
double sigma_error = sqrt(pow(L_error/(A*R)
,2)+pow(A_error*sigma/A,2)+pow(Rerror*
sigma/R,2));
//

// PART 2

// B = 0
float I2[ndata] =
    {0.012,0.019,0.026,0.032,0.047,0.056,0.063,0.069,0.075,0.083};
float V2[ndata] =
    {-0.00081,-0.00133,-0.00182,-0.00218,-0.00327,-0.00392,-0.00436,-0.00486,-0.00531,-0.00585};
float I2er[ndata],V2er[ndata];

for (int i = 0; i < ndata; ++i) {
   I2er[i] = 0.001;
   V2er[i] = calculateVoltUncertainty(V2[i]);
}

TGraphErrors * nonmagnetic = new
    TGraphErrors(ndata,I2,V2,I2er,V2er);
nonmagnetic->Draw("A*");
nonmagnetic->GetXaxis()->SetTitle("Current(A)");
nonmagnetic->GetYaxis()->SetTitle("Voltage(V)");
nonmagnetic->SetTitle("Reference
    Measurements");

linefit->SetParameters(-1.34,0);

nonmagnetic->Fit(linefit);
double offset = linefit->GetParameter(0);
double offset_constant =
    linefit->GetParameter(1);

cout<<"offset is "<<
    offset<<"+"<<offset_constant<<"\n";
```

```
TCanvas *c2 = new TCanvas();

// MAGNETIC

double B = 139.3*1e-3;
double B_error = B *5/100;

// finding V/I and its error

double p2_second = t/B;
double p2_seconderr =
    (t/B)*sqrt(pow((t_error/t),2) +
    pow((B_error/B),2));


float I2mag[ndata] =
    {0.010,0.017,0.026,0.033,0.039,0.046,0.054,0.061,0.0
float V2mag[ndata] =
    {-0.00183,-0.00296,-0.00453,-0.00589,-0.00690,-0.008
float I2mager[ndata],V2mager[ndata];

for (int i = 0; i < ndata; ++i) {
   I2mager[i] = 0.001;
   V2mager[i] =
       calculateVoltUncertainty(V2[i]);
}

TGraphErrors * magnetic = new
    TGraphErrors(ndata,I2mag,V2mag,I2mager,V2mager);
magnetic->Draw("A*");
magnetic->GetXaxis()->SetTitle("Current(A)");
magnetic->GetYaxis()->SetTitle("Voltage(V)");
magnetic->SetTitle("I vs V_hall Measurements
    at B= 139.3 mT");

linefit->SetParameters(-1.34,0);

magnetic->Fit(linefit);

TCanvas *c3 = new TCanvas();
//getting slope
double p2_slope = linefit->GetParameter(0);
double p2_slope_error =
    linefit->GetParError(0);

// finding RH and its error

double RH1 = p2_slope*p2_second;
double RH1_error =
    sqrt(pow(p2_slope,2)*pow(p2_seconderr,2)
+ pow(p2_second,2)*pow(p2_slope_error,2));

cout <<"RH1 is "<<RH1<<"+-"<<RH1_error<<"\n";

// mobility

double mu1 = RH1*sigma;
double mu1_error =
    sqrt(pow(RH1,2)*pow(sigma_error,2)
+ pow(sigma,2)*pow(RH1_error,2));

cout <<"mu1 is "<<mu1<<"+-"<<mu1_error<<"\n";



// PART 3
```

```
// I = 53mA

double I3p = 53*1e-3;
double I3p_error = 1e-3;

// finding t/I and its error

double p3P_second = t/I3p;
double p3P_seconderr =
    (t/I3p)*sqrt(pow((t_error/t),2) +
    pow((I3p_error/I3p),2));


float Bp[ndata] =
    {0.0144,0.0264,0.0349,0.0473,0.0587,0.0707,0.0792,0.0932,0.1041,0.1208};
float V3p[ndata] =
    {-0.0431,-0.0479,-0.0514,-0.0563,-0.0610,-0.0658,-0.0692,-0.0749,-0.0793,-0.0884};
float Bper[ndata],V3per[ndata];

for (int i = 0; i < ndata; ++i) {
   Bper[i] = Bp[i]*5/100;
   V3per[i] = calculateVoltUncertainty(V3p[i]);
}

TGraphErrors * positive = new
    TGraphErrors(ndata,Bp,V3p,Bper,V3per);
positive->Draw("A*");
positive->GetXaxis()->SetTitle("Magnetic
    Field(T)");
positive->GetYaxis()->SetTitle("Voltage(V)");
positive->SetTitle("B vs V_hall Measurements
    at I= 53mA");

linefit->SetParameters(-1.34,0);

positive->Fit(linefit);

TCanvas *c4 = new TCanvas();

//getting slope
double p3P_slope = linefit->GetParameter(0);
double p3P_slope_error =
    linefit->GetParError(0);

// finding RH and its error

double RH2 = p3P_slope*p3P_second;
double RH2_error =
    sqrt(pow(p3P_slope,2)*pow(p3P_seconderr,2)
+ pow(p3P_second,2)*pow(p3P_slope_error,2));

cout <<"RH2 is "<<RH2<<"+-"<<RH2_error<<"\n";

// mobility

double mu2 = RH2*sigma;
double mu2_error =
    sqrt(pow(RH2,2)*pow(sigma_error,2)
+ pow(sigma,2)*pow(RH2_error,2));

cout <<"mu2 is "<<mu2<<"+-"<<mu2_error<<"\n";



// I = -53mA
double I3n = -53*1e-3;
double I3n_error = 1e-3;
```

```
// finding t/I and its error

double p3N_second = t/I3n;
double p3N_seconderr =
    (t/I3n)*sqrt(pow((t_error/t),2) +
    pow((I3n_error/I3n),2));


float Bn[ndata] =
    {0.0187,0.0269,0.0332,0.0413,0.0499,0.0612,0.0712,0.
float V3n[ndata] =
    {0.0448,0.0482,0.0507,0.0540,0.0574,0.0621,0.0661,0.
float Bner[ndata],V3ner[ndata];

for (int i = 0; i < ndata; ++i) {
   Bner[i] = Bn[i]*5/100;
   V3ner[i] = calculateVoltUncertainty(V3n[i]);
}

TGraphErrors * negative = new
    TGraphErrors(ndata,Bn,V3n,Bner,V3ner);
negative->Draw("A*");
negative->GetXaxis()->SetTitle("Magnetic
    Field(T)");
negative->GetYaxis()->SetTitle("Voltage(V)");
negative->SetTitle("B vs V_hall Measurements
    at I= -53mA");

linefit->SetParameters(1.34,0);

negative->Fit(linefit);

TCanvas *c5 = new TCanvas();

//getting slope
double p3N_slope = linefit->GetParameter(0);
double p3N_slope_error =
    linefit->GetParError(0);

// finding RH and its error

double RH3 = p3N_slope*p3N_second;
double RH3_error =
    sqrt(pow(p3N_slope,2)*pow(p3N_seconderr,2)
+ pow(p3N_second,2)*pow(p3N_slope_error,2));

cout <<"RH3 is "<<RH3<<"+-"<<RH3_error<<"\n";

// mobility

double mu3 = RH3*sigma;
double mu3_error =
    sqrt(pow(RH3,2)*pow(sigma_error,2)
+ pow(sigma,2)*pow(RH3_error,2));

cout <<"mu3 is "<<mu3<<"+-"<<mu3_error<<"\n";


// WITH OFFSET


float Voff[ndata],Voffer[ndata];

for (int i = 0; i < ndata; ++i) {
   Voff[i] = V2mag[i]-
       (offset*I2mag[i]+offset_constant);
   Voffer[i] =
       calculateVoltUncertainty(V2mag[i]-
       (offset*I2mag[i]+offset_constant));
```

```
}


TGraphErrors * withoff = new
    TGraphErrors(ndata,I2,Voff,I2er,Voffer);
withoff->Draw("A*");
withoff->GetXaxis()->SetTitle("Current(A)");
withoff->GetYaxis()->SetTitle("Voltage(V)");
withoff->SetTitle("The Measurements with
    Offset");

linefit->SetParameters(-1.34,0);

withoff->Fit(linefit);

TCanvas *c6 = new TCanvas();

//getting slope
double offset_slope = linefit->GetParameter(0);
double offset_slope_error =
    linefit->GetParError(0);

// finding RH and its error

double RH4 = offset_slope*p2_second;
double RH4_error =
    sqrt(pow(offset_slope,2)*pow(p2_seconderr,2)
+ pow(p2_second,2)*pow(offset_slope_error,2));

cout <<"RH4 is "<<RH4<<"+-"<<RH4_error<<"\n";

// mobility

double mu4 = RH4*sigma;
double mu4_error =
    sqrt(pow(RH4,2)*pow(sigma_error,2)
+ pow(sigma,2)*pow(RH4_error,2));

cout <<"mu4 is "<<mu4<<"+-"<<mu4_error<<"\n";

double RH_weighted = (RH1 / pow(RH1_error, 2)
    + RH2 / pow(RH2_error, 2) + RH3 /
    pow(RH3_error, 2) + RH4 / pow(RH4_error,
    2)) /
                (1 / pow(RH1_error, 2) + 1 /
                    pow(RH2_error, 2) + 1 /
                    pow(RH3_error, 2) + 1 /
                    pow(RH4_error, 2));

double RH_weighted_error =
    sqrt(1/(1/pow(RH1_error,2)+1/pow(RH2_error,2)+1/pow(RH3_error,2)+1/pow(RH4_error,2)));

double mu_weighted = (mu1 / pow(mu1_error, 2)
    + mu2 / pow(mu2_error, 2) + mu3 /
    pow(mu3_error, 2) + mu4 / pow(mu4_error,
    2)) /
                (1 / pow(mu1_error, 2) + 1 /
                    pow(mu2_error, 2) + 1 /
                    pow(mu3_error, 2) + 1 /
                    pow(mu4_error, 2));

double mu_weighted_error =
    sqrt(1/(1/pow(mu1_error,2)+1/pow(mu2_error,2)+1/pow(mu3_error,2)+1/pow(mu4_error,2)));



cout <<"RH weighted is
    "<<RH_weighted<<"+-"<<RH_weighted_error<<"\n";
cout <<"mu weighted is
    "<<mu_weighted<<"+-"<<mu_weighted_error<<"\n";


}
```