Applicationso of Lasers

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***Abstract*—In this experiment, we have used double-slit set-up for measuring the wavelength of the laser. We have measured it to be 632.773±13.57nm for 0.25mm separation and 603.931±9.44nm for 0.50mm slit separation, which are 1.27 σ's and 4.88 σ's away from the true value of 650nm, respectively. Then, we have calculated the width of the single slit as 0.1788±0.00252mm, being 7.46 σ's away from the true value of 0.16mm. Also, the measurements gave us my hair's thickness as 0.103±0.001mm. Finally, the experiment resulted in the measurements of the index of refraction as 1.685±0.1267 by Michelson Interferometer set-up and 1.48661 ± 0.0814069 by using Pfund's method, which are, 1.34 and 0.35 σ's away, respectively, from the known value of 1.515.**

I. Theory

Lasers are basically sources of light. Their property that differentiates them from other light sources is being coherent. The name come from the abbreviation light amplification by stimulated emission of radiation. However, the first light output created by the method of amplification by stimulated emission has been operating at microwave frequencies, hence the name maser. Charles H. Townes and graduate students are the first ones to develop a device that amplifies microwave frequencies, based on the theoretical work asserted by Joseph Weber in 1951. 7 years later the invention of the masers, the first laser was produced in 1960 by Theodre Maiman at Hughes Research Laboratories inspired from the theoretical works of earlier physicists. Today, while all the light beams having frequencies higher than microwaves are called lasers, the ones that operates at microwave or lower frequencies are called masers. [1]

The difference between any photon emission by thermal radiation and the photon creation by lasers is that the release of a photon is resulted from the nearby photon. The emissions brought about by the nearby protons are possible only if those protons have similar energies. Therefore, a chain reaction of photon creation having similar wavelengths occurs if the material having the capability of staying excited states for a long time, which are called active laser medium. A typical laser is composed of 5 different parts. Thanks to the reflector, the light oscillates inside the gain medium in which the desired frequencies are amplified. The energy is supplied typically by electrical currents.[1]

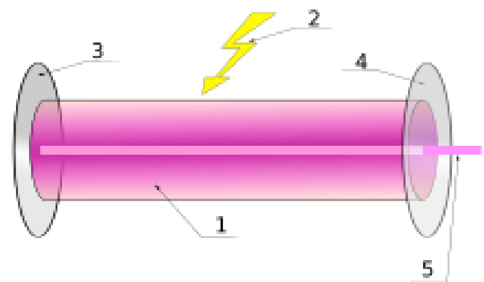

Fig. 1. 1-Gain medium,2-Laser pumping energy,3-High reflector,4-Output coupler,5-Laser beam[1]

Lasers can be used for many different purposes such as measuring the width of the thickness of a very small-sized objects such as hair, getting the index of refraction of a specific materials by different calculating methods. The wavelength of the laser can be calculated by using double-slit set-up.

A. Double-Slit Interference

As it can be seen from fig-2, there exists a path difference between the light coming out of two different slits, designated as $\delta l = dsin\theta$. This path difference causes a phase difference, which results in constructive or destructive interferences.

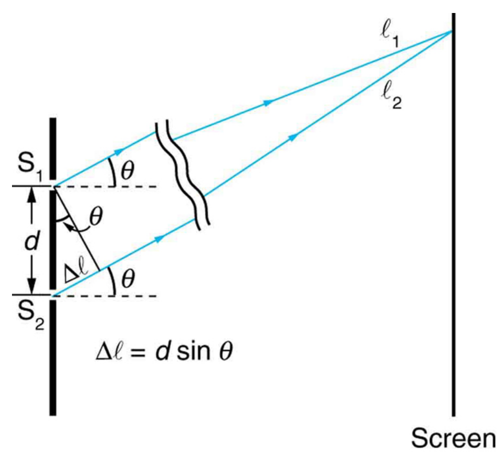

Fig. 2. Double Slit[2]

For constructive interference:

$$\Delta l = dsin\theta_m = m\lambda[2] \tag{1}$$

where m's are integers.

B. Single-Slit Interference

According to Huygens-Fresnel Principle, there also occurs interference pattern in a single slit case due to the fact that each point on the wavefront acts as a source. It is originated from the light beams moving with an angle relative to the initial direction of the light beam. Thus, constructive and destructive interference are observed. For destructive interference:

$$dsin\theta_n = n\lambda[3] \tag{2}$$

where d is the width of the slit, theta is the angle of the light beam relative to the inital direction, and n's are the order of the minimum, consisting of integers except for 0.

C. Measuring the index of refraction of a transparent solid

The phase of a light differs according to the index of refraction of the object that the light passes through. Michelson's interferometer is a useful tool to measure the index of refraction of the material by exposing this feature. [4] The optical path length inside a material that has thickness d and the relative index of refraction n, compared to the air is given as follows for normal incidence of light rays:

$$L_\perp = nd + \frac{d}{cos\theta_i} - 1[4] \tag{3}$$

here the second and third term are the additional path taken in the air. Suppose we rotate the material by an angle θ (see fig-3):

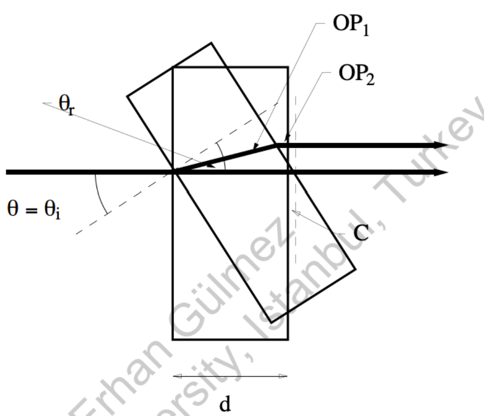

Fig. 3. Index of Refraction measurements with Michelson interferometer[4]

OP1 and OP2 are calculated as follows

$$\text{OP1} = \frac{n \cdot d}{\cos\theta_r}[4] \tag{4}$$

$$\text{OP2} = \frac{d \cdot \tan\theta_i \cdot \sin(\theta_i - \theta_r)}{\cos\theta_r}[4] \tag{5}$$

The net difference of the optical path lengths given as:

$$\Delta L = L_\theta - L_\perp \tag{6}$$

where L_θ is equal to OP1 + OP2. For destructive interference we get the following equation:

$$m = \frac{2\Delta L}{\lambda}[4] \tag{7}$$

Therefore, by combining equations 6 and 7, we get the final expression for index of refraction n as follows:

$$n = \frac{\left(d - \frac{m\lambda}{2}\right)(1 - \cos\theta)}{d(1-\cos\theta) - \frac{m\lambda}{2}} \tag{8}$$

D. Pfund's Method

The angle of refraction would be greater than the angle of incidence for the light that passing from a medium that has a lower density than the initial medium. The Snell's law is stated as:

$$n_i \sin\theta_i = n_r \sin\theta_r \tag{9}$$

If one sends a light on a solid that has a barrier at the bottom of it, the reflected light beams from the bottom would result in inner reflection after the critical angle(see fig-4). For the critical angle we have:

$$\sin\theta_c = \frac{n_r}{n_i}\sin\theta_r \tag{10}$$

we take θ_r to be 90 and n_r to be 1, then:

$$\sin\theta_c = \sqrt{\frac{\left(\frac{r}{2}\right)^2}{\left(\frac{r}{2}\right)^2 + d^2}} \tag{11}$$

Putting Together and Simplifying:

$$n = \sqrt{1 + \left(\frac{2d}{r}\right)^2}[5] \tag{12}$$

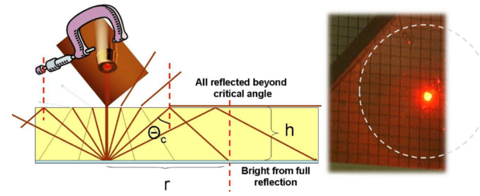

Fig. 4. Visual Representation of Pfund's Method

II. Method

We have conducted the experiment in 3 parts, determining the wavelength of the incident beam by using double slit, measuring the width of a slit in the single slit diffraction and getting the index of refraction by using michelsron interferometer and Pfund's Method.

A. Measuring the Wavelength

1) Double slit is placed with 0.25 or 0.50 mm slit separations
2) The distance from slit to the screen is measured with ruler
3) The light is created by laser
4) The first four maxima, for both sides, and the central maxima are marked
5) The differences are measured with ruler

B. Measuring the slit difference

1) A single slit, both the single slit disk with a width of 0.16mm and hair is placed to be used as a single slit
2) The distance from slit to the screen is measured with ruler
3) The light is created by laser
4) The first four minima, for both sides, and the central maxima are marked
5) The differences are measured with ruler

C. Measuring the Index of Refraction with Michelson Interferometer

1) The interferometer is adjusted so that it is perpendicular to the light
2) the solid object is placed
3) The reference angle is measured(0.3 degrees)
4) the solid is rotated so that the fringes changes, 10 times for each measurement
5) the angle that is needed for 10 different fringe changes are recorded

D. Pfund's Method

1) Water is dripped between the solid and a white plate
2) The light hits the solid
3) The radius of the circle is measured.

III. The Experimental Setup

Here is the list of apparatus we have used in different parts and the photographs of the apparatus.

A. Single Slit and Double Slit Diffraction

- Double Slits with slit widths of 0.04mm and separations 0.25mm and 0.50mm respectively
- Single Slit with the width of 0.16mm
- hair
- the holders of the slits
- Ruler
- Screen
- a diode laser with a wavelength of 650 nm

Fig. 5. Apparatus for Single and Double Slit Diffraction

B. Michelson Interferometer

- a He-Ne laser with a wavelength of 632.8 nm
- a glass plate with a known index (1.515 at 632.8nm).
- two mirrors with magnetic holders, optical benches for height adjustment, and a vertically aligned laser pointer
- Michelson Interferometer

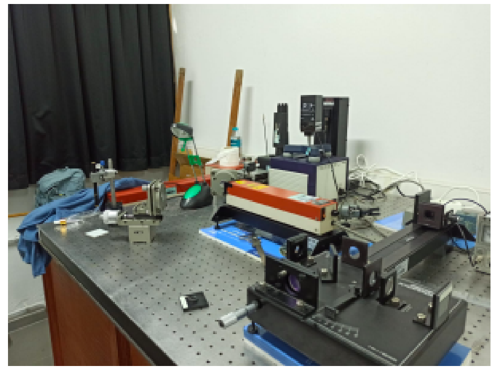

Fig. 6. Michelson Interferometer

C. Pfund's Method

- a He-Ne laser with a wavelength of 632.8 nm
- water
- a glass plate with a known index (1.515 at 632.8nm).
- Ruler

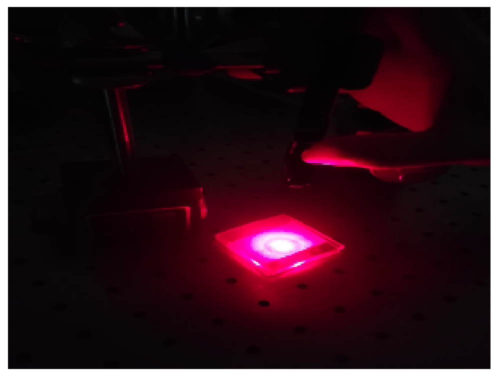

Fig. 7. Pfund's Method

IV. The Data

Here's the dataset below including all the measurements for different parts of the experiment.

TABLE I
Difference Between Maxima

n	Double slit -0.25mm,left(cm)	Double slit -0.25mm,right(cm)
1	0.4 ± 0.1	0.4 ± 0.1
2	0.8 ± 0.1	0.8 ± 0.1
3	1.1 ± 0.1	1.2 ± 0.1
4	1.5 ± 0.1	1.5 ± 0.1

TABLE II
Difference Between Maxima

n	Double slit -0.50mm,left (cm)	Double slit -0.50mm,right (cm)
1	0.2 ± 0.1	0.2 ± 0.1
2	0.4 ± 0.1	0.4 ± 0.1
3	0.5 ± 0.1	0.6 ± 0.1
4	0.7 ± 0.1	0.8 ± 0.1

TABLE III
Difference Between Minima

n	Single slit,left (cm)	Single slit -right (cm)
1	0.7 ± 0.1	0.6 ± 0.1
2	1.4 ± 0.1	1.2 ± 0.1
3	1.9 ± 0.1	1.8 ± 0.1
4	2.5 ± 0.1	2.3 ± 0.1

TABLE IV
DIFFERENCE BETWEEN MINIMA-MEASURING THICKNESS

n	Single slit,upper (cm)	Single slit -lower (cm)
1	1.3 ± 0.1	1.0 ± 0.1
2	2.8 ± 0.1	2.8 ± 0.1
3	4.2 ± 0.1	4.2 ± 0.1
4	5.5 ± 0.1	5.6 ± 0.1

TABLE V
COUNT AND CORRESPONDING ANGLE

count	Angle(degree)
0	0.3
10	3.6 ± 0.1
20	5.0 ± 0.1
30	6.1 ± 0.1
40	7.1 ± 0.1
50	7.8 ± 0.1
60	8.5 ± 0.1
70	9.2 ± 0.1
80	9.7 ± 0.1
90	10.2 ± 0.1
100	10.7 ± 0.1

V. THE ANALYSIS AND RESULTS

We have used ROOT's built-in function to calculate the values and their corresponding errors in the analysis. The general formula is used for all the error propagation calculations:

$$\sigma_f = \sqrt{(\frac{\partial f}{\partial x})^2(\sigma_x)^2 + (\frac{\partial f}{\partial y})^2(\sigma_y)^2 + (\frac{\partial f}{\partial z})^2(\sigma_z)^2 + ...} \quad (13)$$

For the calculation of the weighted averages, we have used the following formulas: Suppose for $\bar{x}$ of a set of values x_i with uncertainties σ_i,

$$\bar{x} = \frac{\sum\left(\frac{x_i}{\sigma_i^2}\right)}{\sum\left(\frac{1}{\sigma_i^2}\right)} \quad (14)$$

The uncertainty of the weighted average, $\sigma_{\bar{x}}$, is given by:

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{\sum\left(\frac{1}{\sigma_i^2}\right)}} \quad (15)$$

A. Double Slit

The corresponding λ values for double slits are given in the tables below:

TABLE VI
WAVELENGTHS FOR 0.25MM SEPARATION-DOUBLE SLIT

n	(nm)
1	661.811 ±31.2206
2	661.804 ±27.738
3	634.219 ±25.9646
4	620.419 ±25.1624

The weighted average of the wavelengths for 0.25mm separation is calculated as 632.773±13.57nm according to the equations 10 and 11, being 1.27 σ's away from the true value of 650nm.

TABLE VII
WAVELENGTHS FOR 0.50MM SEPARATION-DOUBLE SLIT

n	(nm)
1	661.813 ±35.6423
2	661.811 ±21.1927
3	606.658 ±16.4023
4	620.442 ±14.9191

The weighted average of the wavelengths for 0.50mm separation is calculated as 603.931±9.44nm according to the equations 10 and 11, being 4.88 σ's away from the true value of 650nm. Rewriting the equation 1 by writing the sinθ explicitly:

$$\lambda = \frac{d \cdot y_n}{n \cdot \sqrt{L^2 + y_n^2}} \quad (16)$$

The error propagation is done accordingly as follows:

$$\frac{\partial \lambda}{\partial d} = \frac{y_n}{n \cdot \sqrt{L^2 + y_n^2}} \quad (17)$$

$$\frac{\partial \lambda}{\partial y_n} = \frac{d}{n \cdot \sqrt{L^2 + y_n^2}} - \frac{d \cdot y_n}{n \cdot (L^2 + y_n^2)^{3/2}} \cdot y_n \quad (18)$$

$$\frac{\partial \lambda}{\partial L} = -\frac{d \cdot y_n \cdot L}{n \cdot (L^2 + y_n^2)^{3/2}} \quad (19)$$

putting together:

$$\sigma_\lambda = \sqrt{\left(\frac{\partial \lambda}{\partial d} \cdot \sigma_d\right)^2 + \left(\frac{\partial \lambda}{\partial y_n} \cdot \sigma_{y_n}\right)^2 + \left(\frac{\partial \lambda}{\partial L} \cdot \sigma_L\right)^2} \quad (20)$$

For the exact formula, see apendix.

B. Single Slit

Here are the table of corresponding measurements of single slit widths for each order of minimums:

TABLE VIII
CORRESPONDING SLIT SEPARATIONS FOR THE MEASUREMENTS

n	thickness(mm)
1	0.150 ±0.003
2	0.299 ±0.005
3	0.473 ±0.008
4	0.649 ±0.010

The weighted average of the the width of the single slit is calculated as 0.1788±0.00252mm according to the equations 10 and 11, being 7.46 σ's away from the true value of 0.16mm.

TABLE IX
MEASUREMENTS FOR HAIR THICKNESS

n	thickness(mm)
1	0.085±0.002
2	0.140±0.002
3	0.210 ±0.003
4	0.283 ±0.004

The weighted average of the thickness of my hair is calculated as 0.103±0.001mm according to the equations 10 and 11. Rewriting the equation 2 by writing sinθ explicitly

$$d = n \cdot \lambda \cdot \frac{\sqrt{L^2 + y_n^2}}{y_n} \quad (21)$$

The error propagation goes as follows:

$$\frac{\partial d}{\partial \lambda} = n \cdot \frac{\sqrt{L^2 + y_n^2}}{y_n} \tag{22}$$

$$\frac{\partial d}{\partial y_n} = -n \cdot \lambda \cdot \frac{\sqrt{L^2 + y_n^2}}{y_n^2} + n \cdot \lambda \cdot \frac{y_n}{\sqrt{L^2 + y_n^2} \cdot y_n} \tag{23}$$

$$\frac{\partial d}{\partial L} = n \cdot \lambda \cdot \frac{L}{\sqrt{L^2 + y_n^2} \cdot y_n} \tag{24}$$

Putting together:

$$\sigma_d = \sqrt{\left(\frac{\partial d}{\partial \lambda} \cdot \sigma_\lambda\right)^2 + \left(\frac{\partial d}{\partial y_n} \cdot \sigma_{y_n}\right)^2 + \left(\frac{\partial d}{\partial L} \cdot \sigma_L\right)^2} \tag{25}$$

For the exact formula, see apendix.

C. Michelson Interferometer

The calculated index of refraction values given in the table below:

TABLE X
THE INDEX OF REFRACTION OF THE SOLID WITH MICHELSON INTERFEROMETER

m	Index of Refraction
10	1.53034 ±2.56708
20	1.51838 ±1.24577
30	1.50597 ±0.804667
40	1.48272 ±0.567347
50	1.50218 ±0.478797
60	1.50447 ±0.401792
70	1.49648 ±0.337444
80	1.51445 ±0.309928
90	1.52509 ±0.28345
100	1.53 ±0.258581

The weighted average of the $n_{michelson}$ values are calculated as 1.685±0.1267 according to the equations 10 and 11, being 1.34 σ's away from the true value of 1.515

$$n = \frac{d_{\text{mich}} - \frac{m \cdot \lambda}{2}}{d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}} \tag{26}$$

$$\frac{\partial n}{\partial d_{\text{mich}}} = \frac{(1 - \cos\theta)}{d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}} - \frac{\left(d_{\text{mich}} - \frac{m \cdot \lambda}{2}\right) \cdot (1 - \cos\theta)}{\left(d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}\right)^2} \tag{27}$$

$$\frac{\partial n}{\partial \lambda} = \frac{-\frac{m}{2} \cdot (1 - \cos\theta)}{d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}} + \frac{\left(d_{\text{mich}} - \frac{m \cdot \lambda}{2}\right) \cdot \frac{m}{2} \cdot (1 - \cos\theta)}{\left(d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}\right)^2} \tag{28}$$

$$\frac{\partial n}{\partial \theta} = \frac{\left(d_{\text{mich}} - \frac{m \cdot \lambda}{2}\right) \cdot \sin\theta}{d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}} - \frac{\left(d_{\text{mich}} - \frac{m \cdot \lambda}{2}\right) \cdot d_{\text{mich}} \cdot \sin\theta}{\left(d_{\text{mich}} \cdot (1 - \cos\theta) - \frac{m \cdot \lambda}{2}\right)^2} \tag{29}$$

Putting together:

$$\sigma_n = \sqrt{\left(\frac{\partial n}{\partial d_{\text{mich}}} \cdot \sigma_{d_{\text{mich}}}\right)^2 + \left(\frac{\partial n}{\partial \lambda} \cdot \sigma_\lambda\right)^2 + \left(\frac{\partial n}{\partial \theta} \cdot \sigma_\theta\right)^2} \tag{30}$$

For the exact formula, see apendix.

D. Pfund's Method

The index of refraction by using Pfund's Method is calculated as: n_{pfund} = 1.48661 ± 0.0814069 which is 0.35 σ's away from the true value of 1.515. By rewriting the equation 12:

$$n_{\text{Pfund}} = \sqrt{1 + \left(\frac{2 \cdot d_{\text{pfund}}}{r}\right)^2} \tag{31}$$

the error propagation goes as follows:

$$\frac{\partial n_{\text{Pfund}}}{\partial d_{\text{pfund}}} = \frac{\frac{4 \cdot d_{\text{pfund}}}{r^2}}{2 \cdot \sqrt{1 + \left(\frac{2 \cdot d_{\text{pfund}}}{r}\right)^2}} \tag{32}$$

$$\frac{\partial n_{\text{Pfund}}}{\partial r} = \frac{-\frac{4 \cdot d_{\text{pfund}}^2}{r^3}}{2 \cdot \sqrt{1 + \left(\frac{2 \cdot d_{\text{pfund}}}{r}\right)^2}} \tag{33}$$

Putting together:

$$\sigma_{n_{\text{Pfund}}} = \sqrt{\left(\frac{\partial n_{\text{Pfund}}}{\partial d_{\text{pfund}}} \cdot \sigma_{d_{\text{pfund}}}\right)^2 + \left(\frac{\partial n_{\text{Pfund}}}{\partial r} \cdot \sigma_r\right)^2} \tag{34}$$

For the exact formula, see apendix.

VI. THE CONCLUSION

Our experiment, in general seems to be a success. The wavelengths of the measured values are close to the known value of the wavelength of the laser and their error seems to be acceptable. Although we have a very good outcome for the measurements of the width of the single slit as well, its more away from its known value in terms of the sigmas. There might happened some problems during the pointing the minimas. Furthermore, even though we could not find rigid scientific papers on it, it is approximated that human hair is to be between 0.016 to 0.05mm. If we regard it to be around the expected value, our measurements has approximately 10 times higher than those values. On the other hand, due to the lack of scientific evidence, we have passed the calculation of how many sigmas away the experimental value is for the thickness of hair. Finally, our measured index of refraction values seems to match with the expected ones so that we can regard that part success again. Overall, we have satisfied with the results of our experiment.

REFERENCES

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[4] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

[5] *pfund*. URL: https://www.lehigh.edu/imi/scied/docs_edu/RefractiveIndexPfundsMethod.pdf (visited on 12/20/2024).

VII. Appendix

Here is the C++ code I have used in the analysis, which is meant to be used with the help of ROOT's built-in functions.

```
{
   //SINGLE&DOUBLE SLIT VALUES

   float diffLaser = 650*1e-9;
   float diffLaser_error = 0.1*1e-7;
   float L_double = 1.511;
   float L_single = 1.497;
   float L_hair = 1.510;
   float L_double_error = 0.001;
   float L_single_error = 0.001;
   float L_hair_error = 0.001;
   float d_25 = 0.25e-3;
   float d_25_error = 0.01e-3;
   float d_50 = 0.50e-3;
   float d_50_error = 0.01e-3;
   float a = 0.16e-3;
   float a_error = 0.01e-3;

   // MICHELSON VALUES

   float michLaser= 632.8*1e-9;
   float michLaser_error = 0.1*1e-7;
   float degree_error = 0.1;
   float d_mich = 0.55*1e-2;
   float d_mich_error = 0.001*1e-2;

   // PFUND VALUES

   float r = 1.0*1e-2;
   float r_error = 0.1*1e-2;



   //DATA
   float d25Left[4]={0.4,0.8,1.1,1.5};
   float d25Right[4]={0.4,0.8,1.2,1.5};

   float d50Left[4]={0.2,0.4,0.5,0.7};
   float d50Right[4]={0.2,0.4,0.6,0.8};


   float sLeft[4]={0.7,1.4,1.9,2.5};
   float sRight[4]={0.6,1.2,1.8,2.3};

   float sUpper[4]={1.3,2.8,4.2,5.5};
   float sLower[4]={1.0,2.8,4.2,5.6};

   float
      mValues[10]={10,20,30,40,50,60,70,80,90,100};
   float
      thetas[10]={3.3,4.7,5.8,6.8,7.5,8.2,8.9,9.4,9.9,10.4};

   float pi = TMath::Pi();
  for (int i = 0; i < 10; ++i) {
     thetas[i] = thetas[i]*pi/180;
   }

   //CALCULATIONS FOR SINGLE&DOUBLE SLITS

   float
      d25[4],d50[4],single[4],thickness[4],nvalues[10],

   for (int i = 0; i < 4; ++i) {
     d25[i] =1e-2*(d25Left[i]+d25Right[i])/2;
     d50[i] = 1e-2*(d50Left[i]+d50Right[i])/2;
     single[i] = 1e-2*(sLeft[i]+sRight[i])/2;
     thickness[i] =
         1e-2*(sUpper[i]+sLower[i])/2;
   }

   float
      d25Lambdas[4],d50Lambdas[4],sWidth[4],hairWidth[4
   float
      d25Lambdas_errors[4],d50Lambdas_errors[4],sWidth_

   // DOUBLE SLIT D25
   for (int i = 0; i < 4; ++i) {
   int n = i + 1;
   float Delta_y = d25[i];
   float sqrt_term = sqrt(pow(L_double, 2) +
      pow(Delta_y, 2));

   // Calculate d25Lambdas
   d25Lambdas[i] = 1e9 * d_25 * Delta_y / (n *
      sqrt_term);

   // Partial derivatives
   float partial_d = Delta_y / (n * sqrt_term);
   float partial_Delta_y = (d_25 / (n *
      sqrt_term)) - (d_25 * Delta_y / (n *
      pow(sqrt_term, 3))) * Delta_y;
   float partial_L = -(d_25 * Delta_y *
      L_double) / (n * pow(sqrt_term, 3));

   // Error contributions
   float error_d = partial_d * d_25_error;
   float error_Delta_y = partial_Delta_y *
      (1e-2 * 0.01); // Assuming 0.01 cm
      error in Delta_y
   float error_L = partial_L * L_double_error;

   // Total error
   d25Lambdas_errors[i] = 1e9 *
      sqrt(pow(error_d, 2) +
      pow(error_Delta_y, 2) + pow(error_L,
      2));

   // Print results
   cout << "Lambda for d25[" << i << "] = " <<
      d25Lambdas[i] << "  " <<
      d25Lambdas_errors[i] << " nm" << endl;
}

   //DOUBLE SLIT D50
   for (int i = 0; i < 4; ++i) {
   int n = i + 1;
   float Delta_y = d50[i];
   float sqrt_term = sqrt(pow(L_double, 2) +
      pow(Delta_y, 2));

   // Calculate d50Lambdas
   d50Lambdas[i] = 1e9 * d_50 * Delta_y / (n *
      sqrt_term);

   // Partial derivatives
   float partial_d = Delta_y / (n * sqrt_term);
```

```
    float partial_Delta_y = (d_50 / (n *
        sqrt_term)) - (d_50 * Delta_y / (n *
        pow(sqrt_term, 3))) * Delta_y;
    float partial_L = -(d_50 * Delta_y *
        L_double) / (n * pow(sqrt_term, 3));

    // Error contributions
    float error_d = partial_d * d_50_error;
    float error_Delta_y = partial_Delta_y *
        (1e-2 * 0.01); // Assuming 0.01 cm
        error in Delta_y
    float error_L = partial_L * L_double_error;

    // Total error
    d50Lambdas_errors[i] = 1e9 *
        sqrt(pow(error_d, 2) +
        pow(error_Delta_y, 2) + pow(error_L,
        2));

    // Print results
    cout << "Lambda for d50[" << i << "] = " <<
        d50Lambdas[i] << "   " <<
        d50Lambdas_errors[i] << " nm" << endl;
}



    // SINGLE SLIT
   for (int i = 0; i < 4; ++i) {
    int n = i + 1;
    float Delta_y = single[i];
    float sqrt_term = sqrt(pow(L_single, 2) +
        pow(Delta_y, 2));

    // Calculate single slit thickness (sWidth)
    sWidth[i] = 1e3 * n * diffLaser * (n *
        sqrt_term) / Delta_y;

    // Partial derivatives
    float partial_Delta_y = -1e3 * n *
        diffLaser * (n * sqrt_term /
        pow(Delta_y, 2)) +
                       1e3 * n * diffLaser * (n
                           * Delta_y /
                           (sqrt_term *
                           Delta_y));
    float partial_L = 1e3 * n * diffLaser * (n
        * L_single / (sqrt_term * Delta_y));
    float partial_diffLaser = 1e3 * n * (n *
        sqrt_term) / Delta_y;

    // Error contributions
    float error_Delta_y = partial_Delta_y *
        (1e-2 * 0.01); // Assuming 0.01 cm
        error in Delta_y
    float error_L = partial_L * L_single_error;
    float error_diffLaser = partial_diffLaser *
        diffLaser_error;

    // Total error
    sWidth_errors[i] = sqrt(pow(error_Delta_y,
        2) + pow(error_L, 2) +
        pow(error_diffLaser, 2));

    // Print results
    cout << "Single slit thickness for order "
        << n << " = " << sWidth[i] << "   " <<
        sWidth_errors[i] << " mm" << endl;
}
```

```
    // SINGLE SLIT THICKNESS
    for (int i = 0; i < 4; ++i) {
    int n = i + 1;
    float Delta_y = thickness[i];
    float sqrt_term = sqrt(pow(L_hair, 2) +
        pow(Delta_y, 2));

    // Calculate hair thickness (hairWidth)
    hairWidth[i] = 1e3 * n * diffLaser * (n *
        sqrt_term) / Delta_y;

    // Partial derivatives
    float partial_Delta_y = -1e3 * n *
        diffLaser * (n * sqrt_term /
        pow(Delta_y, 2)) +
                       1e3 * n * diffLaser * (n
                           * Delta_y /
                           (sqrt_term *
                           Delta_y));
    float partial_L = 1e3 * n * diffLaser * (n
        * L_hair / (sqrt_term * Delta_y));
    float partial_diffLaser = 1e3 * n * (n *
        sqrt_term) / Delta_y;

    // Error contributions
    float error_Delta_y = partial_Delta_y *
        (1e-2 * 0.01); // Assuming 0.01 cm
        error in Delta_y
    float error_L = partial_L * L_hair_error;
    float error_diffLaser = partial_diffLaser *
        diffLaser_error;

    // Total error
    hairWidth_errors[i] =
        sqrt(pow(error_Delta_y, 2) +
        pow(error_L, 2) + pow(error_diffLaser,
        2));

    // Print results
    cout << "Hair thickness for order " << n <<
        " = " << hairWidth[i] << "   " <<
        hairWidth_errors[i] << " mm" << endl;
}



    // CALCULATIONS FOR MICHELSON
    for (int i = 0; i < 10; ++i) {
    float m = mValues[i];
    float cos_theta = cos(thetas[i]);
    float one_minus_cos = 1 - cos_theta;

    // Calculate index of refraction
    nvalues[i] = (d_mich - (m * michLaser / 2))
        * one_minus_cos /
              (d_mich * one_minus_cos - (m *
                  michLaser / 2));

    // Partial derivatives
    float partial_d_mich = one_minus_cos /
        (d_mich * one_minus_cos - (m *
        michLaser / 2)) -
                     ((d_mich - (m * michLaser
                         / 2)) * one_minus_cos)
                         /
                     pow(d_mich *
                         one_minus_cos - (m *
                         michLaser / 2), 2);
```

```
    float partial_michLaser = -m / 2 *
        one_minus_cos /
                          (d_mich * one_minus_cos
                              - (m * michLaser /
                              2)) +
                          ((d_mich - (m *
                              michLaser / 2)) * m
                              / 2 *
                              one_minus_cos) /
                         pow(d_mich *
                              one_minus_cos - (m
                              * michLaser / 2),
                              2);

    float partial_theta = (d_mich - (m *
        michLaser / 2)) * sin(thetas[i]) /
                       (d_mich * one_minus_cos -
                           (m * michLaser / 2)) -
                       ((d_mich - (m * michLaser
                           / 2)) * one_minus_cos
                           * d_mich *
                           sin(thetas[i])) /
                       pow(d_mich * one_minus_cos
                           - (m * michLaser / 2),
                           2);

    // Error contributions
    float error_d_mich = partial_d_mich *
        d_mich_error;
    float error_michLaser = partial_michLaser *
        michLaser_error;
    float error_theta = partial_theta *
        degree_error * TMath::Pi() / 180; //
        Convert degree error to radians

    // Total error
    nvalues_errors[i] = sqrt(pow(error_d_mich,
        2) + pow(error_michLaser, 2) +
        pow(error_theta, 2));

    // Print results
    cout << "Index of refraction for m = " << m
        << ": " << nvalues[i]
        << "  " << nvalues_errors[i] << endl;
}



    // CALCULATIONS FOR PFUND

    // Calculate Pfund index of refraction
    float nPfund = sqrt(1 + pow(2 * d_mich / r,
        2));

// Partial derivatives
float partial_d_mich = (4 * d_mich / pow(r,
    2)) / sqrt(1 + pow(2 * d_mich / r, 2));
float partial_r = (-4 * pow(d_mich, 2) /
    pow(r, 3)) / sqrt(1 + pow(2 * d_mich / r,
    2));

// Error contributions
float error_d_mich = partial_d_mich *
    d_mich_error;
float error_r = partial_r * r_error;

// Total error
float nPfund_error = sqrt(pow(error_d_mich, 2)
    + pow(error_r, 2));

// Print result
cout << "Index of refraction for Pfund = " <<
    nPfund << "  " << nPfund_error << endl;


}
```

---

the exact error propagation formula for double-slit:

$$\sigma_\lambda = \sqrt{\left(\frac{y_n}{n\cdot\sqrt{L^2+y_n^2}}\cdot\sigma_d\right)^2 + \Bigg[\Bigg(\frac{d}{n\cdot\sqrt{L^2+y_n^2}} - \frac{d\cdot y_n}{n\cdot(L^2+y_n^2)^{3/2}} \cdots}$$

the exact error propagation formula for single-slit:

$$\sigma_d = \sqrt{\left(n\cdot\frac{\sqrt{L^2+y_n^2}}{y_n}\cdot\sigma_\lambda\right)^2 + \Bigg[\Bigg(-n\cdot\lambda\cdot\frac{\sqrt{L^2+y_n^2}}{y_n^2} + n\cdot\lambda\cdot\frac{\cdots}{\sqrt{L^2 \cdots}} \cdots}$$

the exact error propagation formula for Michelson Interferometer:

$$\sigma_n = \sqrt{\left[\frac{(1-\cos\theta)}{d_{\text{mich}}\cdot(1-\cos\theta)-\frac{m\cdot\lambda}{2}} - \frac{\left(d_{\text{mich}}-\frac{m\cdot\lambda}{2}\right)\cdot(1-\cos\theta)}{\left(d_{\text{mich}}\cdot(1-\cos\theta)-\frac{m\cdot\lambda}{2}\right)^2}\right]^2\cdot\sigma_{d_{\cdots}}^2 \cdots}$$

the exact error propagation formula for Pfund's method:

$$\sigma_{n_{\text{Pfund}}} = \sqrt{\left(\frac{\frac{4\cdot d_{\text{pfund}}}{r^2}}{2\cdot\sqrt{1+\left(\frac{2\cdot d_{\text{pfund}}}{r}\right)^2}}\cdot\sigma_{d_{\text{pfund}}}\right)^2 + \left(\frac{\frac{-4\cdot d_{\text{pfund}}^2}{r^3}}{2\cdot\sqrt{1+\left(\frac{2\cdot d_{\text{pfund}}}{r}\right)^2}}\cdot\sigma_{\cdots} \cdots\right.}$$