Diffraction of Matter Waves

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***Abstract*—In 1924, de Broglie suggests that electrons can have wave-like properties. Later on, experiments proved that electrons do indeed have interference pattern. In this experiment, we have observed the interference of scattered electron beams and measured the arc length of the circle seen on the fluorescent screen. We used those calculations to measure lattice constants and we have found that $d_{10} = 0.42\ pm\ 0.06$ nm compared to the expected value 0.213 pm 0.001 nm and $d_{11} = 0.21\ pm\ 0.03$ compared to the expected value of 0.123 pm 0.001 nm**

I. Theory

Light was considered to be a wave due mostly to the interference pattern observed in the experiments. However, the opposing view has emerged after Max Planck's hypothetical oscillator and the Einstein's photoelectric effect because those theories argues quantized energy levels for light, which will imply a particle nature. It was assured after the experiments conducted by Arthur Compton during 1922-1924 that the light do indeed have a particle-like nature as well. Similarly, though in the reverse order, matter also was thought to be a particle only. De Broglie in his PhD proposed that electrons behaves like waves. Following that, Erwin Schrödinger has introduced his famous wave equation of motion for the electrons.[1]

De Broglie's hypothesis that electrons has wave properties has been proved by Davisson-Germer Experiment. In this experiment, electrons gained energy as a result of the heated filament moves in an vacuum and hit the surface of a nickel metal. Their electron detector, which is called Faraday Box, was able to detect the electrons reaching the surface from different angles. They found out that the electron beams' intensity, hiting the metal have peaks at some specific angles. This effect is owing to the wave property of the electrons. Davisson and Thomson were awarded a Nobel Prize in 1937 thanks to their findings at this experiment.[2]

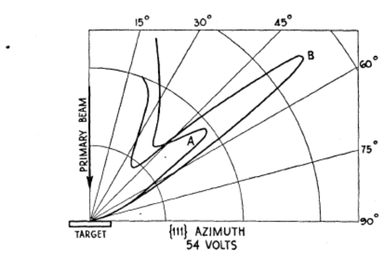

Fig. 13. The "54 volt" beam before and after heating the crystal by electron bombardment. *A*. Original condition as in Fig. 10. *B*. After heating the crystal.

Fig. 1. davgermer

De Broglie's formulation of matter having wave nature is:

$$\lambda_B = \frac{h}{p} \tag{1}$$

where h is the Planck constant, p is the momentum and λ_B is the de Broglie wavelength.

An electron accelerated in an electric field will have the following classical equation:

$$q_e V = \frac{p^2}{2m_e} \tag{2}$$

Putting (1) and (2) together:

$$\lambda_B = \frac{h}{\sqrt{2q_e V m_e}} \tag{3}$$

Heated electron beams will hit polycrystalline graphite foil placed in the path of the electrons and then scattered. The spacing between the lattice of graphite will cause a path difference between electrons so that there exists an interference pattern. Electrons passing through the target will hit the fluorescent screen and produce circles which can be seen even in the daylight. This relation happens according to The Bragg's Law:[3]

$$2dsin(\theta) = n\lambda \tag{4}$$

here d is the spacing between the lattice

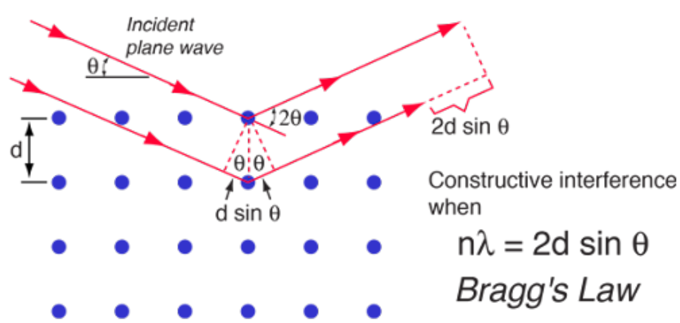

Fig. 2. bragg

now combining eqs 3 and 4 we will end up with:

$$\frac{1}{\sqrt{V}} = d\frac{\sqrt{8q_e m_e}}{h} sin(\theta) \tag{5}$$

II. Method

In the experiment, we first of all heated the electron beams and then apply a high voltages to these excited electrons so that they accelerate and move towards the target we aimed at, from which they diffracted. Due to the path difference caused by the spacing between graphite crystal, electrons interfere with each other and cause an image of two circles on the fluorescent screen.

1) voltage is set
2) electrons are heated
3) high voltage is applied to make electrons move in the desired direction
4) electrons are scattered after hitting the graphite crystal
5) interference occurs
6) circles are observed on the fluorescent screen
7) the radius of the circles' arcs are measured
8) the same process is repeated with different voltage value

III. The Experimental Setup

Here is the list of apparatus we have used in the experiment:

- Electron diffraction tube
- HV Power Supply
- Ammeter
- Plastic Tape Measure

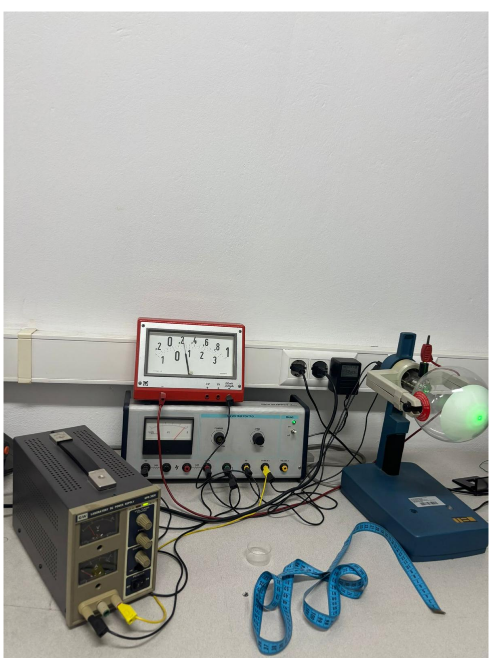

Fig. 3. apparatus

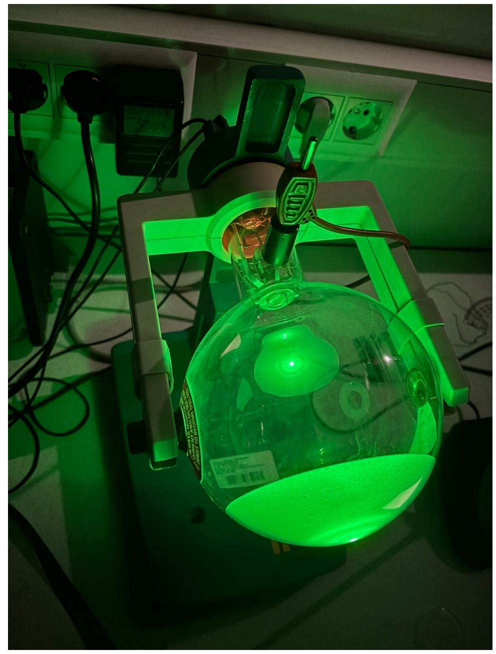

Fig. 4. Observed Diffraction Pattern on the Fluorescent Screen

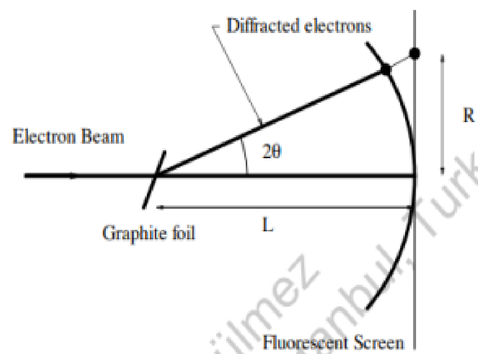

Fig. 5. the geometry of the difraction tube; here R = 4.3 pm 0.1 cm and L = 14.0 pm 0.3 cm

IV. The Data

Measured arc length are listed in the below tables for each different voltages.

TABLE I
Measurmenets of the first ring

Voltage (kV)	X_in(cm)	X_out(cm)	Y_in(cm)	Y_in(cm)
2.8 $\pm$ 0.1	1.5 $\pm$ 0.1	1.7 $\pm$ 0.1	1.4 $\pm$ 0.1	1.6 $\pm$0.1
3.0 $\pm$ 0.1	1.4 $\pm$ 0.1	1.6 $\pm$ 0.1	1.3 $\pm$ 0.1	1.5 $\pm$ 0.1
3.2 $\pm$ 0.1	1.4 $\pm$ 0.1	1.6 $\pm$ 0.1	1.4 $\pm$ 0.1	1.6 $\pm$ 0.1
3.4 $\pm$ 0.1	1.3 $\pm$ 0.1	1.6 $\pm$ 0.1	1.3 $\pm$ 0.1	1.5 $\pm$ 0.1
3.6 $\pm$ 0.1	1.3 $\pm$ 0.1	1.5 $\pm$ 0.1	1.2 $\pm$ 0.1	1.4 $\pm$ 0.1
3.8 $\pm$ 0.1	1.3 $\pm$ 0.1	1.4 $\pm$ 0.1	1.2 $\pm$ 0.1	1.4 $\pm$ 0.1
4.1 $\pm$ 0.1	1.2 $\pm$ 0.1	1.4 $\pm$ 0.1	1.3 $\pm$ 0.1	1.4 $\pm$ 0.1
4.3 $\pm$ 0.1	1.2 $\pm$ 0.1	1.4 $\pm$ 0.1	1.1 $\pm$ 0.1	1.3 $\pm$ 0.1
4.5 $\pm$ 0.1	1.1 $\pm$ 0.1	1.3 $\pm$ 0.1	1.1 $\pm$ 0.1	1.3 $\pm$ 0.1
4.7 $\pm$ 0.1	1.1 $\pm$ 0.1	1.4 $\pm$ 0.1	1.1 $\pm$ 0.1	1.3 $\pm$ 0.1

TABLE II
MEASURMENETS OF THE SECOND RING

Voltage (kV)	X_in(cm)	X_out(cm)	Y_in(cm)	Y_in(cm)
2.8 ± 0.1	2.8 ± 0.1	3.0 ± 0.1	2.6 ± 0.1	2.9 ± 0.1
3.0 ± 0.1	2.6 ± 0.1	2.8 ± 0.1	2.5 ± 0.1	2.7 ± 0.1
3.2 ± 0.1	2.5 ± 0.1	2.7 ± 0.1	2.5 ± 0.1	2.7 ± 0.1
3.4 ± 0.1	2.4 ± 0.1	2.7 ± 0.1	2.4 ± 0.1	2.6 ± 0.1
3.6 ± 0.1	2.3 ± 0.1	2.6 ± 0.1	2.3 ± 0.1	2.6 ± 0.1
3.8 ± 0.1	2.3 ± 0.1	2.5 ± 0.1	2.3 ± 0.1	2.6 ± 0.1
4.1 ± 0.1	2.2 ± 0.1	2.4 ± 0.1	2.2 ± 0.1	2.4 ± 0.1
4.3 ± 0.1	2.1 ± 0.1	2.3 ± 0.1	2.1 ± 0.1	2.3 ± 0.1
4.5 ± 0.1	2.0 ± 0.1	2.2 ± 0.1	2.0 ± 0.1	2.2 ± 0.1
4.7 ± 0.1	2.0 ± 0.1	2.3 ± 0.1	2.0 ± 0.1	2.2 ± 0.1

V. THE ANALYSIS

Equation (3) suggests that we can calculate the de Broglie wavelength of the electron for each voltage value. Here is the table for corresponding wavelengths of electrons.

TABLE III
DE BROGLIE WAVELENGTHS FOR CORRESPONDING VOLTAGES

Voltage (V)	Wavelength (λ_B(m))
2800 ± 100	$8.32 \times 10^{-12} \pm 2.44 \times 10^{-13}$
3000 ± 100	$7.65 \times 10^{-12} \pm 2.22 \times 10^{-13}$
3200 ± 100	$7.14 \times 10^{-12} \pm 2.08 \times 10^{-13}$
3400 ± 100	$6.67 \times 10^{-12} \pm 1.96 \times 10^{-13}$
3600 ± 100	$6.25 \times 10^{-12} \pm 1.86 \times 10^{-13}$
3800 ± 100	$5.87 \times 10^{-12} \pm 1.76 \times 10^{-13}$
4000 ± 100	$5.52 \times 10^{-12} \pm 1.66 \times 10^{-13}$
4100 ± 100	$5.40 \times 10^{-12} \pm 1.63 \times 10^{-13}$
4300 ± 100	$5.12 \times 10^{-12} \pm 1.56 \times 10^{-13}$
4500 ± 100	$4.87 \times 10^{-12} \pm 1.50 \times 10^{-13}$
4700 ± 100	$4.65 \times 10^{-12} \pm 1.44 \times 10^{-13}$

As table indicates, wavelengths are smaller than nanometer and it decreases with increasing voltage values.

We can also calculate the θ values by using the geometry of the setup.

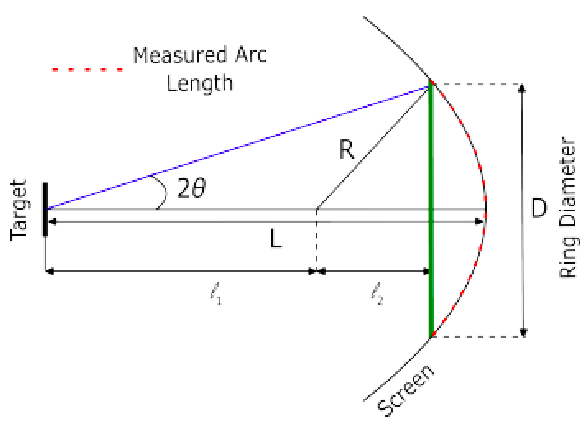

Fig. 6. the geometry of the diffraction tube

Instead of using the simplifications resulting from the assumption that θ is small, we will find it by exact calculations. In figure 6 the only thing we do not know is θ so we can get it by:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{R \cdot \sin\left(\frac{arc}{R}\right)}{L - R + R \cdot \cos\left(\frac{arc}{R}\right)}\right) \tag{6}$$

In this calculation we will take half of the average of our arc measurements by:

$$arc = \frac{X_in + X_out + Y_in + Y_out}{8} \tag{7}$$

In order to find the spacing between lattices, which is d, we can apply a linear fit to equation 5. Therefore we need to have error propagation for $\frac{1}{\sqrt{V}}$ and $sin(\theta)$. The general formula for error propagation is[3]

$$\sigma_f = \sqrt{(\frac{\partial f}{\partial x})^2(\sigma_x)^2 + (\frac{\partial f}{\partial y})^2(\sigma_y)^2 + (\frac{\partial f}{\partial z})^2(\sigma_z)^2 + ...} \tag{8}$$

the exact calculations we carried on for the error propagation of this experiment, see appendix.

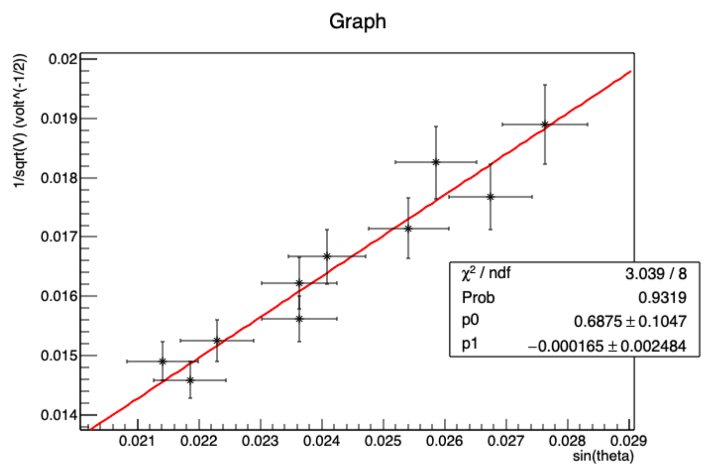

Fig. 7.

$\frac{1}{\sqrt{V}}$ vs. $\sin(\theta)$ – First Ring

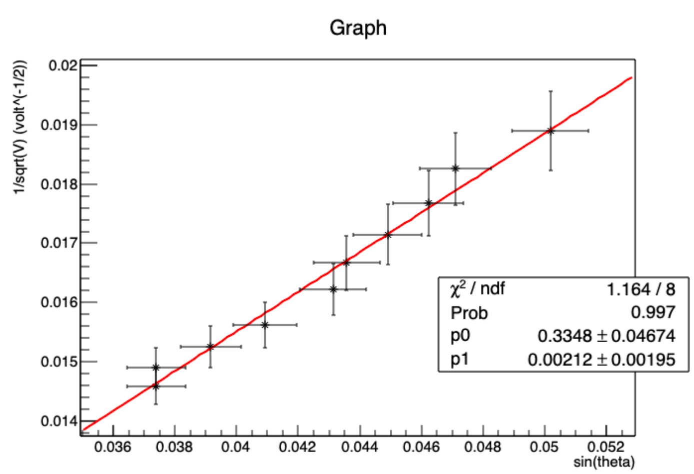

Fig. 8.

$\frac{1}{\sqrt{V}}$ vs. $\sin(\theta)$ – Second Ring

It seems like χ^2 values are reasonable for both fits. By using the equation (5) we are now able to calculate the spacing between lattices.

VI. The Result

We found out the lattice constants as $d_{10} = 0.42\ pm\ 0.06$ nm which is 3.45σ away from the expected value of 0.213 pm 0.001 nm and $d_{11} = 0.21\ pm\ 0.03$ nm which is 2.90 σ away from the expected value of 0.123 pm 0.001 nm

VII. The Conclusion

We got our results two times larger than the expected lattice constants for both rings. Although we had some difficulties measuring the arc length for,especially , lower voltage values like around 3kV, there might have been a systematic error resulting from such things as a miscalculation because getting both values to be almost two times the expected value may require more than measuring problems. We have observed 2 rings but not higher order diffraction maxima because of the low intensity of electrons. The shape is circular because electrons do not favor any direction after the scattering from crystal. Furthermore, in our calculations we have used Bragg's law which have 2 factor in front of $dsin(\theta)$ because there exists a path difference between the electrons reflecting off both planes. Besides, owing to the fact that incident wave and the reflected wave would make the same angle θ, it is 2θ that we are seeing in the figure 5. Finally, if the velocities were high enough, we would need relativistic corrections in addition to the classical calculations, which we used in the report. Those corrections would be needed because the energy and de Broglie wavelength calculations would have errors that cannot be neglected at high velocities.

VIII. Acknowledgement

Optional part that you may use to thank anyone who has contributed.

References

[1] *PL*. URL: https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#History (visited on 10/24/2024).

[2] *Davisson-Germer*. URL: http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/DavGer2.html (visited on 10/24/2024).

[3] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

[4] *MW*. URL: https://en.wikipedia.org/wiki/Matter_wave (visited on 10/24/2024).

IX. Appendix

Here is the piece of code I have used for analyzing the data for the linear fit

```
{

 const int ndata = 10;
 const float L=14.0 * 1e-2;
 const float R= 4.3 * 1e-2;
 const float sigmaR = 0.1*1e-2;
 const float sigmaL = 0.3*1e-2;
 const float sigmaArc = 0.01*1e-2;
 const float sigmaV = 100;
double h = 6.62607015 * 1e-34; // exact
double q = 1.602176634 * 1e-19; // exact
double m = 9.1093837015 * 1e-31;



 vector<double> voltages = {
  2800,3000,3200,3400,3600,3800,4100,4300,4500,4700
   };

 std::vector<std::vector<double>> mFirst = {
   {0.015, 0.017, 0.014, 0.016},
   {0.014, 0.016, 0.013, 0.015},
   {0.014, 0.016, 0.014, 0.016},
   {0.013, 0.016, 0.013, 0.015},
   {0.013, 0.015, 0.012, 0.014},
   {0.013, 0.014, 0.012, 0.014},
   {0.012, 0.014, 0.013, 0.014},
   {0.012, 0.014, 0.011, 0.013},
   {0.011, 0.013, 0.011, 0.013},
   {0.011, 0.014, 0.011, 0.013}
};

std::vector<std::vector<double>> mSecond = {
   {0.028, 0.030, 0.026, 0.029},
   {0.026, 0.028, 0.025, 0.027},
   {0.025, 0.027, 0.025, 0.027},
   {0.024, 0.027, 0.024, 0.026},
   {0.023, 0.026, 0.023, 0.026},
   {0.023, 0.025, 0.023, 0.026},
   {0.022, 0.024, 0.022, 0.024},
   {0.021, 0.023, 0.021, 0.023},
   {0.020, 0.022, 0.020, 0.022},
   {0.020, 0.023, 0.019, 0.022}
};



float arcsFirst[10],arcsSecond[10],
    sigmaThetaFirst[10],sigmaThetaSecond[10],
thetasFirst[10],thetasSecond[10],xAxisFirst[10],yAxis[10
sigmaxFirst[10],sigmaxSecond[10],sigmaY[10];

for (int i = 0; i < 10; ++i) {
   arcsFirst[i]=((mFirst[i][0]+mFirst[i][1]+mFirst[i][2]
   arcsSecond[i]=((mSecond[i][0]+mSecond[i][1]+mSecond[i

   thetasFirst[i]=(atan((sin(arcsFirst[i]/R)*R)/(L-R+R*c
   thetasSecond[i]=(atan((sin(arcsSecond[i]/R)*R)/(L-R+R

}

for (int k = 0; k < 10; ++k) {


   double partialR1 = (-(arcsFirst[k] *
       cos(arcsFirst[k] / R)) / (R * (L - R +
       R * cos(arcsFirst[k] / R)))
         + sin(arcsFirst[k] / R) / (L - R + R
             * cos(arcsFirst[k] / R))
         - (R * sin(arcsFirst[k] / R) * (-1 +
             cos(arcsFirst[k] / R) +
             (arcsFirst[k] * sin(arcsFirst[k]
             / R)) / R))
         / pow((L - R + R * cos(arcsFirst[k] /
             R)), 2))
        / (2 * (1 + (pow(R, 2) *
```

```
        pow(sin(arcsFirst[k] / R), 2)) /
        pow((L - R + R * cos(arcsFirst[k]
        / R)), 2)));



double partialL1 = -(R * sin(arcsFirst[k] /
    R)) /
      (2 * pow(L - R + R * cos(arcsFirst[k]
        / R), 2) *
      (1 + (pow(R, 2) *
        pow(sin(arcsFirst[k] / R), 2)) /
        pow(L - R + R * cos(arcsFirst[k] /
        R), 2)));



double partialArc1 = ((L - R) *
    cos(arcsFirst[k] / R) + R) /
      (2 * ((L - R) * (L - R) +
      2 * (L - R) * R * cos(arcsFirst[k] /
        R) +
      R * R * cos(arcsFirst[k] / R) *
        cos(arcsFirst[k] / R) +
      R * R * sin(arcsFirst[k] / R) *
        sin(arcsFirst[k] / R)));




double partialR2 = (-(arcsSecond[k] *
    cos(arcsSecond[k] / R)) / (R * (L - R +
    R * cos(arcsSecond[k] / R)))
       + sin(arcsFirst[k] / R) / (L - R + R
         * cos(arcsSecond[k] / R))
       - (R * sin(arcsSecond[k] / R) * (-1 +
         cos(arcsSecond[k] / R) +
         (arcsSecond[k] *
         sin(arcsSecond[k] / R)) / R))
       / pow((L - R + R * cos(arcsSecond[k]
         / R)), 2))
      / (2 * (1 + (pow(R, 2) *
        pow(sin(arcsSecond[k] / R), 2)) /
        pow((L - R + R * cos(arcsSecond[k]
        / R)), 2)));

double partialL2 = -(R * sin(arcsSecond[k]
    / R)) /
      (2 * pow(L - R + R *
        cos(arcsSecond[k] / R), 2) *
      (1 + (pow(R, 2) *
        pow(sin(arcsSecond[k] / R), 2)) /
        pow(L - R + R * cos(arcsSecond[k]
        / R), 2)));

double partialArc2 = ((L - R) *
    cos(arcsSecond[k] / R) + R) /
      (2 * ((L - R) * (L - R) +
      2 * (L - R) * R * cos(arcsSecond[k] /
        R) +
      R * R * cos(arcsSecond[k] / R) *
        cos(arcsSecond[k] / R) +
      R * R * sin(arcsSecond[k] / R) *
        sin(arcsSecond[k] / R)));


double sigmaTheta1 =
    sqrt((partialR1*sigmaR*partialR1*sigmaR)+(partialL1*sigmaL*partialL1*sigmaL)+
    (partialArc1*sigmaArc*partialArc1*sigmaArc));
    sigmaThetaFirst[k]=(sigmaTheta1);

    double sigmaTheta2 =
        sqrt((partialR2*sigmaR*partialR2*sigmaR)+(partial
    (partialArc2*sigmaArc*partialArc2*sigmaArc));
    sigmaThetaSecond[k]=(sigmaTheta2);

}



for (int n = 0; n < 10; ++n) {
  sigmaxFirst[n]=cos(thetasFirst[n])*sigmaThetaFirst[n];
  sigmaxSecond[n] =
      (cos(thetasSecond[n])*sigmaThetaSecond[n]);
  sigmaY[n] =
      ((1/pow(voltages[n],1.5))*sigmaV);

  xAxisFirst[n] = sin(thetasFirst[n]);
  xAxisSecond[n]= sin(thetasSecond[n]);

  yAxis[n] = 1/(pow(voltages[n],0.5));

}

TGraphErrors *mygraphFirst = new
    TGraphErrors(ndata,xAxisFirst,yAxis,sigmaxFirst,sigm
TGraphErrors *mygraphSecond = new
    TGraphErrors(ndata,xAxisSecond,yAxis,sigmaxSecond,si

mygraphFirst->Draw("A*");
TCanvas *c1 = new TCanvas();
TF1 *ffirst = new
    TF1("ffirst","[0]*x+[1]",0.021,0.029);
mygraphFirst->Fit(ffirst);
double slopeFirst = ffirst->GetParameter(0);
double slope_error_first =
    ffirst->GetParError(0);
double d_first = slopeFirst * h / sqrt(8 * q *
    m);
double d_error_first = slope_error_first * h /
    sqrt(8 *
q * m);
 mygraphFirst->GetXaxis()->SetTitle("sin(theta)");
 mygraphFirst->GetYaxis()->SetTitle("1/sqrt(V)
     (volt^(-1/2))");




mygraphSecond->Draw("A*");
TCanvas *c2 = new TCanvas();
TF1 *fsecond = new
    TF1("fsecond","[0]*x+[1]",0.036,0.052);
mygraphSecond->Fit(fsecond);
double slopeSecond = fsecond->GetParameter(0);
double slope_error_second =
    fsecond->GetParError(0);
double d_second = slopeSecond * h / sqrt(8 * q
    * m);
double d_error_second = slope_error_second * h
    / sqrt(8 *
q * m);
 mygraphSecond->GetXaxis()->SetTitle("sin(theta)");
 mygraphSecond->GetYaxis()->SetTitle("1/sqrt(V)
     (volt^(-1/2))");
```

```
gStyle->SetOptFit(1111);

cout<<"the lattice constant d for the first
    ring is d_first =
    "<<d_first<<"+-"<<d_error_first<<
" and for the second ring is d_second =
    "<<d_second<<"+-"<<d_error_second<< "\n";
```

```
}
```

The error calculations:

$$\frac{\partial\theta}{\partial R} = \frac{-\left(\frac{arc\cos\left(\frac{arc}{R}\right)}{R\left(L-R+R\cos\left(\frac{arc}{R}\right)\right)}\right) + \frac{\sin\left(\frac{a}{R}\right)}{L-R+R\cos\left(\frac{arc}{R}\right)} - \frac{R\sin\left(\frac{arc}{R}\right)\left(-1+\cos\left(\frac{a}{R}\right)+\frac{arc\sin\left(\frac{arc}{R}\right)}{R}\right)}{\left(L-R+R\cos\left(\frac{arc}{R}\right)\right)^2}}{2\left(1+\frac{R^2\sin^2\left(\frac{arc}{R}\right)}{\left(L-R+R\cos\left(\frac{arc}{R}\right)\right)^2}\right)}$$

$$\frac{\partial\theta}{\partial L} = \frac{-(R\sin\left(\frac{arc}{R}\right))}{2\left(L-R+R\cos\left(\frac{arc}{R}\right)\right)^2\left(1+\frac{R^2\sin^2\left(\frac{arc}{R}\right)}{\left(L-R+R\cos\left(\frac{arc}{R}\right)\right)^2}\right)}$$

$$\frac{\partial\theta}{\partial arc} = \frac{(L-R)\cos\left(\frac{arc}{R}\right)+R}{2\left((L-R)^2+2(L-R)R\cos\left(\frac{arc}{R}\right)+R^2\cos^2\left(\frac{arc}{R}\right)+R^2\sin^2\left(\frac{arc}{R}\right)\right)}$$

$$\sigma_\theta = \sqrt{(\frac{\partial\theta}{\partial R})^2(\sigma_R)^2 + (\frac{\partial\theta}{\partial L})^2(\sigma_L)^2 + (\frac{\partial\theta}{\partial arc})^2(\sigma_a rc)^2}$$

$$\sigma_{sin(\theta)} = cos(\theta).\sigma_\theta$$

$$\sigma_{V^{-1/2}} = V^{-3/2}.\sigma_V$$