A Web Application for Solving Multi-objective Integer Programming Problems

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Table of contents

The Research Project

The research project

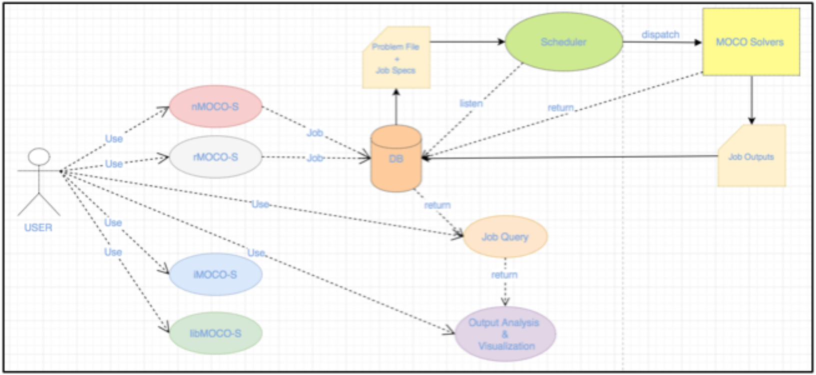
- **Title:** Nondominated Points of Multi-objective Integer Programming Problems: Analysis, Approaches and Applications
- **Funding:** Scientific and Technological Research Council of Turkey (TUBITAK)
- **Members:** Murat Koksalan, Banu Lokman, Gokhan Ceyhan, Sami Ozarik, Ilgin Dogan
- **Goals:**
  - Develop *generic* algorithms to find all or representative nondominated points of *any* MOIP with *any* number of objective function
  - Provide *cloud services* to research communities by embedding these algorithms in a web application
  - Provide tools for visualization of nondominated points
- The project is **on-going**.

www.onlinemoco.com/MOIP/

- **In development phase**
- Applications
  - **nMOCO-S:** Finds all nondominated points
  - **rMOCO-S:** Finds a representative set of nondominated points
  - **iMOCO-S:** Integrates user preferences
  - **libMOCO-S:** A library for MOIP instances
- Technology
  - Client side technologies: **html5, javascript, jQuery**
  - Server side technologies: **javaservlet**
  - Web server: **Apache Tomcat**
  - Database: **Apache Derby** and **JDBC**
  - Solvers: coded in **C++**, use **IBM ILOG CPLEX** API

Use case

How a user intereacts with the system:



Generating all nondominated points

nMOCO-S

MOIP:

$$\begin{aligned} \text{“Max”} \quad & \mathbf{z(x)} = \{z_1(\mathbf{x}), z_2(\mathbf{x}), ..., z_m(\mathbf{x})\} \\ \text{s.t.} \quad & \mathbf{x} \in X \end{aligned}$$

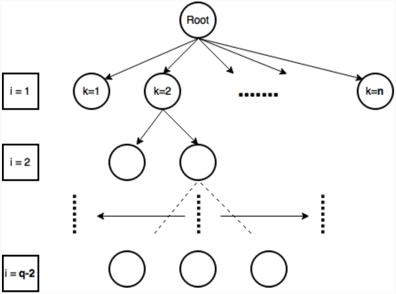
where $z_i(\mathbf{x})$ is a continuous function of $\mathbf{x}$, $X \subseteq \mathbb{R}^n$ is compact and x is a vector of **integer-valued** decision variables. We denote the image of X in the objective space as Z.

- Generates all nondominated points for MOIPs
- Can also be applied as an approximate method for MOMIPs
- Based on Lokman and Koksalan (2013)
- **The idea:** The search region is partitioned and reduced progressively by removing the regions that are dominated by previously found nondominated points.

nMOCO-S

Enhancement:

- An effective data structure to handle any number of objective functions
- A smarter way of detecting the status of a search region



- **N-ary tree**
- **Tree height** = number of objectives (q) - 2
- **Number of child nodes** $\leq$ number of generated nondominated points (N)

nMOCO-S

Tree elements

- **Node:** defines a search space
- **Branching point:** a nondominated point already generated
- **Branching criterion:** a criterion index of the branching point
- **Bound vector:** the *branching criterion* values of *branching points* of the nodes on the path from the root node to the current node

So, at node **k**:

- k_i is the *branching point* index at level i, $i = 1, 2, \ldots, q-2$
- $z_i^{k_i}$ is the *branching criterion* value at level i
- $b^k = (z_1^{k_1}, z_2^{k_2}, \ldots, z_i^{k_i}) \quad \forall i = 1, 2, \ldots (q-3)$ and
  $b^k = (z_1^{k_1}, z_2^{k_2}, \ldots, z_i^{k_i}, z_{i+1}^{k_{i+1}})$ for $i = q-2$ where
  $z_{i+1}^{k+1} = \max_{j \in N} \left\{ z_{i+1}^j : z_l^j \geq b_l^k, l = 1, 2, \ldots, i \right\}$

nMOCO-S

Special structure

- $z_q^1 \geq z_q^2 \geq, \ldots, z_q^N$
- Let C^k be the child nodes of node k. Then,

$$S^k = \bigcup_{c \in C^k} S^c$$

- Let L^k and R^k be the left and right siblings of node k. For nodes except the leaf nodes:

$$S^l \subseteq S^k \subseteq S^r, l \in L^k, r \in R^k$$

- At leaf nodes,

$$b_{q-1}^l \geq b_{q-1}^k \geq b_{q-1}^r, l \in L^k, r \in R^k$$

What kind of problems can nMOCO-S solve?

- For MOIPs, any problem for which there are effective algorithms to solve single objective case
- We use IBM ILOG CPLEX as the single objective solver
  - **MILP**
  - **MIQP**
  - **MIQCP**
- For MOMIPs, our algorithm can generate a set of nondominated points which $\epsilon-$ dominates the nondominated set

Generating representative nondominated points

rMOCO-S

- **The idea:** Find a subset of nondominated points that *represents* the nondominated frontier well enough.
- Based on Ceyhan et al. (2014)
- **How to represent:** *Masin ve Bukchin (2008): Coverage gap*

$$\alpha = \max_{z \in ND} \left\{ \min_{y \in R} \left\{ \max_{1 \leq i \leq p} z_i - y_i \right\} \right\}$$

Corollary

If the coverage gap of subset R is α^*, then R α^*-dominates all nondominated points. $\forall z \in ND, \exists y \in R : z_i \leq y_i + \alpha^*, \forall i = 1, ..., p$.

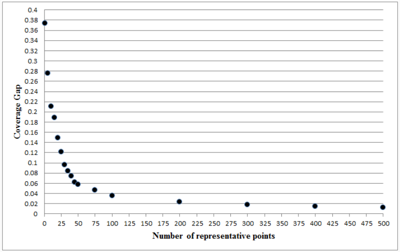
rMOCO-S

Approach 1:

- Find worst covered nondominated point at each iteration which gives the coverage gap of the available subset
- Stop when the desired coverage gap is achieved

The procedure:

- Define coverage gap as an additional objective function to be maximized
- Use nMOCO-S with $q + 1$ objective functions



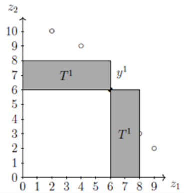
rMOCO-S

Approach 2:

- Identify the desired coverage gap, Δ
- Find a set of nondominated points such that $\alpha \leq \Delta$

The procedure:

- Utilize Δ actively throughout the algorithm
- Construct territories around the previously generated points that are inadmissible for the new point

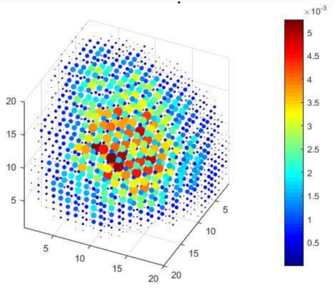


- Tighten the bounds in the leaf nodes of nMOCO-S tree with Δ

rMOCO-S

New features:

- Minimize density weighted coverage gap



- Try to achieve a coverage gap with the smallest possible set
  - Criteria for sorting the search spaces
  - Scalarization of objective functions for the search problem

Visualization of nondominated points

gMOCO-S

The needs:

- To capture the global characteristics of the nondominated frontier

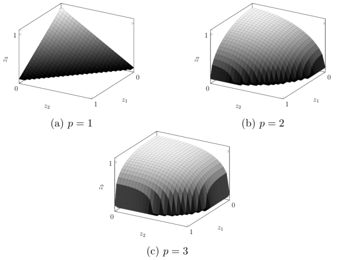


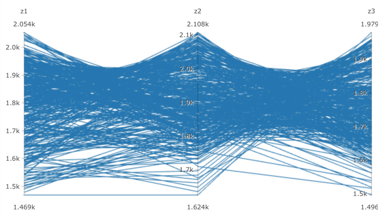
Figure 7: Three dimensional L_p surfaces with different p values

- To investigate the local trade-offs

gMOCO-S

The challenges:

- There can be too many points to visualize



- More than three criteria

gMOCO-S

- We developed *shiny* web application in *R* language
- We use a graphics library offered by *plotly*
- Source code is available in our *github* account
- Users should create an account in *shiny.io* and deploy our source code

Demo

Demo: How to solve a MOKP instance with nMOCO-S?

Summary

Summary

- A need for publicly available multi-objective solvers
- There are recent studies on this direction (e.g. *vOpt-Solver*)
- We try to contribute to this need by providing our MOIP solvers on the web as a *cloud-service*
- A collection of MOIP instances that could be used for benchmarking
- A source code for tools of visualization

Questions?

`http://onlinemoco.com/MOIP/`

`https://github.com/gokhanceyhan`

`https://onlinemoco.shinyapps.io/gMOCO/`