

MTM3691-Theory of Linear Programming

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Week 7



Course Content

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 - ▶ Standard Form
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Standard Form

LP problems need to be converted to standard form. In a standard form,

- ▶ all variables must satisfy the non-negativity condition,
- ▶ all constraints must be in equality form and have a non-negative, constant right-hand side.

When converting to standard form, it may be necessary to define a new variable or rearrange the constraints:

- ▶ Inequalities are multiplied by -1 to get non-negative right-hand side.
- ▶ Inequalities are converted into equations by adding or subtracting non-negative variables (slack or surplus).
- ▶ Unrestricted variables are rewritten with two new non-negative variables.

All LP problems can be written in standard form!!!

Standard Form: Free Variables with Upper Bounds

Free variables with upper bounds of the form $x_i \leq u_i$ are converted to standard form as follows:

$$x_i \leq u_i \implies 0 \leq u_i - x_i \implies x'_i \geq 0 \text{ where } x'_i = u_i - x_i$$

Example

$$\begin{aligned} \max Z &= x_1 + x_2 \\ \text{subject to } &-x_1 + 2x_2 \leq 8 \\ &x_1 + 2x_2 \leq 10 \\ &x_1 \leq 8, \ x_2 \geq 0 \end{aligned}$$

Standard Form: Free Variables with Upper Bounds

Example

$$\begin{aligned}\max Z &= -x_1 + 2x_2 \\ \text{subject to } &-3x_1 + 4x_2 \leq 24 \\ &x_1 + 2x_2 \leq 16 \\ &0 \leq x_1 \leq 9, x_2 \geq 0\end{aligned}$$

Standard Form: Free Variables with Lower Bounds

Free variables with lower bounds of the form $\ell_i \leq x_i$ are converted to standard form as follows:

$$\ell_i \leq x_i \quad \implies \quad 0 \leq x_i - \ell_i \quad \implies \quad x'_i \geq 0 \text{ where } x'_i = x_i - \ell_i$$

Example

$$\begin{aligned} \max Z &= -x_1 + 3x_2 \\ \text{subject to } &-2x_1 + 3x_2 \leq 24 \\ &x_1 + 2x_2 \leq 8 \\ &x_1 \geq -8, \ x_2 \geq 0 \end{aligned}$$

Standard Form: Variables with Interval Bounds

Variables with interval bounds of the form $\ell_i \leq x_i \leq u_i$ are converted to standard form as follows:

$$\begin{aligned}\ell_i \leq x_i \leq u_i &\implies 0 \leq x_i - \ell_i \leq u_i - \ell_i &\implies 0 \leq x'_i \leq u_i - \ell_i \\ & &\implies x'_i \geq 0, x''_i \geq 0, \\ & &\text{where } x''_i = u_i - \ell_i - x'_i\end{aligned}$$

Example

$$\begin{aligned}\max Z &= -x_1 + 3x_2 \\ \text{subject to } &-x_1 + 2x_2 \leq 8 \\ &x_1 + 2x_2 \leq 12 \\ &4 \leq x_1 \leq 10, x_2 \geq 0\end{aligned}$$

Standard Form: Free Variables

- ▶ Sometimes a variable can be given without any bounds. These types of variables are called free variables.
- ▶ To obtain standard form from a free variable, the free variable is expressed as the difference of two non-negative variables. So, if x_i is a free variable, then we get

$$x_i = u_i - v_i, \quad 0 \leq u_i, \quad 0 \leq v_i.$$

Example

Put the following problem in standard form and write starting basic feasible solution.

$$\begin{aligned} \max z &= 10x_1 - 5x_2 - 2x_3 \\ \text{subject to } &-2x_1 + 3x_2 + 3x_3 \leq 10 \\ &x_1 + x_2 - 5x_3 = 8 \\ &x_1 \geq -3 \\ &-1 \leq x_2 \leq 1 \\ &x_3 \text{ free} \end{aligned}$$

Standard Form: Free Variables

Example

$$\begin{aligned}\max z &= 10x_1 - 5x_2 - 2x_3 \\ \text{subject to } &-2x_1 + 3x_2 + 3x_3 \leq 10 \\ &x_1 + x_2 - 5x_3 = 8 \\ &x_1 \geq -3 \\ &-1 \leq x_2 \leq 1 \\ &x_3 \text{ free}\end{aligned}$$