MTM4501-Operations Research

Gökhan Göksu, PhD

Week 11

Course Content

- Definition of OR and Its History
- Decision Theory and Models
- Network Analysis
- Inventory Management Models
  - Economic Order Quantity Models
  - Production Models
- Queue Models

Inventory Models

- Inventory modeling deals with determining the level of a commodity that a business should maintain to ensure smooth operations.
- The basis of the decision is a model that balances the capital cost resulting from holding too much stock against the penalty cost due to stockouts.
- The nature of demand is a fundamental factor influencing the solution: deterministic or probabilistic.
- In real life, demand is often probabilistic, but in some cases, a simpler deterministic approach may be acceptable.

Inventory Models

- In the study of inventory models, two fundamental questions are addressed:
  - **How much** should be ordered each time?
  - When should the reorder take place?
- If there is a production process, then the following fundamental questions arise:
  - **How much** should be produced each time?
  - When should the production take place?
- The goal is to **minimize the total variable cost** over a specified period.

Inventory Costs

- **Ordering cost (Setup cost):** The cost of processing an order independent of the order quantity. For example, ordering cost would include paperwork and billing costs associated with placing an order. If a product is produced internally rather than ordered from an external source, the labor cost (including idle time) required for a production run is included in the ordering cost.
- **Holding cost:** It is generally the cost of holding an item in inventory. If the period is one year, the holding cost will be expressed in TL per unit per year. The holding cost typically includes storage costs, insurance costs, security costs, taxes, warehouse overheads, and costs associated with the possibility of deterioration, theft, or obsolescence. However, the most significant component of holding cost is usually the opportunity cost of tying up capital in inventory. For example, let's assume that one unit of a product costs 100 TL and the company can earn 15

Inventory Costs

- **Backordering cost (Shortage cost):** Costs associated with not having an item in stock when it is demanded. When a product is demanded, and it is not fulfilled at the time of demand, it is said that stockout or shortage has occurred. We say that backordering is allowed when the demand for an item is greater than the stock on hand at the time of the demand (even if it is a year later), and customers will accept delivery at a later date. The situation where backordering is allowed is generally referred to as the accumulated demand situation (e.g., car shortage at dealerships). If no customer will accept a late delivery, there is a lost sale situation (e.g., flower order). Of course, reality lies somewhere between these two extremes, but by determining the most appropriate inventory policies for both the accumulated demand and lost sale situations, a prediction should be made as to what the most appropriate inventory policy should be.

Inventory Costs (Continued)

- **Backordering cost (Stockout cost):** Many costs are associated with running out of stock. If backorders are permitted, placing backorders usually results in an additional cost. Stockouts often lead to lost sales and goodwill as customers go elsewhere to satisfy current and future demands. Stockouts can also cause a company to fall behind in other aspects of its business and may force a factory to bear the additional cost of higher overtime production. Measuring the cost of running out of stock is often more difficult than measuring ordering, purchasing, or holding costs.
- **Purchasing cost:** It includes variable labor cost associated with the production of an item, variable overhead cost, and the cost of raw materials. If goods are ordered from an external source, the unit purchase cost should include shipping charges.
- **Other costs**

Model 1: Economic Order Quantity (EOQ)

Assumptions

- *The demand rate is known and constant.*
- *The unit purchase cost is constant (No quantity discounts).*
- *The lead time (delivery time) is constant.*
- *No shortages are allowed.*

Model Parameters

- *R : Demand rate (Consumption rate per unit time)*
- c_1 *: Holding cost (TL per unit per unit time)*
- c_2 *: Ordering cost (Order cost) (TL per order)*

Model Variables

- *Q : Order quantity*
- *t : Order cycle time*

Model 1: Economic Order Quantity (EOQ)

Figure: Quantity-time graph

Total Inventory/Stock Cost = Holding Cost + Ordering Cost

Model 1: Economic Order Quantity (EOQ)

Example (Winston, pg. 857)

Göksu Airlines uses 500 rear lamps per year. Whenever an order is placed for rear lamps, a ordering cost of 5 TL is incurred. The cost of each lamp is 40 TL, and the holding cost is 8 kurus per lamp per year. Assuming that the demand occurs at a constant rate and no shortages are allowed:

a) *What is the optimal order quantity?*

b) *How many orders should be placed per year?*

c) *What should be the order cycle?*

d) *What is the total inventory cost?*

e) *What is the annual total inventory cost?*

Based on the given values

$$\begin{aligned} c2 &= 5 \text{ TL} \\ c1 &= 0,08 \text{ TL/lamp/year} \\ R &= 500 \text{ lamps/year} \end{aligned}$$

this airline should order 250 rear lamps each time the inventory reaches zero.

Model 2: Production Model (PM)

In the production process, demands are met simultaneously. Excess demand accumulates in stocks. When stocks reach a certain level called the maximum stock level, the production process is stopped. After that, demands are met from the stocks. When stocks are exhausted, the production process starts again.

Figure: Quantity-time graph

Model 2: Production Model (PM)

Model Parameters

- *K : Production rate (K > R)*
- *R : Demand rate*
- *c_1 : Holding cost (TL/unit)*
- *c_2 : Production cost - Setup cost - Cost per unit produced (TL/production)*

Model Variables

- *Q : Production quantity*
- *t : Production cycle time*
- *t_1 : Production time in one cycle*
- *t_2 : Time to meet demand from stocks*
- *Im: Maximum stock level*

Total Inventory/Stock Cost = Holding Cost + Cost Per Unit Produced

Model 2: Production Model (PM)

Example

A company has a monthly production capacity of 125 units. The customer needs 1000 units in a year. The daily unit holding cost is 10 TL, and the cost per unit produced is 67500 TL. Determine the following: (1 year = 12 months = 360 days)

a) *What is the optimum production quantity?*

b) *What is the optimum production cycle time?*

c) *What is the optimum production time in one cycle?*

d) *What is the maximum stock level?*

e) *What is the total stock cost?*

f) *How many production cycles are there in a year?*

g) *What is the annual total stock cost?*

Given:

$$K = 125 \text{ units/month} \qquad R = 1000 \text{ units/year}$$

$$c2 = 67500 \text{ TL/cycle} \qquad c1 = 10 \text{ TL} \cdot \text{unit/day}$$

Model 3: Economic Order Quantity with Shortages Allowed

Figure: Quantity-time graph

Model Parameters

- R : *Demand rate*
- c_1 : *Holding cost (TL/unit/unit time)*
- c_2 : *Ordering cost (TL/order)*
- c_3 : *Shortage cost (TL/unit/unit time)*

Model 3: Economic Order Quantity with Shortages Allowed

Model Variables

- *Q : Order quantity*
- *t : Order cycle time*
- t_1 *: Time to meet demand from stocks*
- t_2 *: Time demand queues*
- *Im : Maximum stock level (within the order cycle)*
- *S : Maximum shortage level (within the order cycle)*

Total Inventory Cost = Holding Cost + Shortage Cost + Ordering Cost

Model 3: Economic Order Quantity with Shortages Allowed

Example

Göksu Optik sells 10,000 eyeglass frames annually. This optical store orders frames from a regional supplier, charging 15 TL per frame. There is a 50 TL ordering cost for each order. Göksu Optik believes that the demand for frames can accumulate, and the cost of a frame missing for a year is 15 TL (due to potential future business loss). The annual holding cost for inventory, assuming an inventory holding value of 30 kurus, is;

a) *What is the optimal order quantity?*

b) *What is the maximum shortage that can occur?*

c) *What is the maximum stock level that will occur?*

d) *What is the length of the optimal order cycle?*

Model 4: Production Model with Shortages Allowed

Figure: Quantity-time graph

Model Parameters

- K : *Production rate* ($K > R$)
- R : *Demand rate*
- c_1 : *Holding cost (TL/unit/unit time)*
- c_2 : *Production cost (TL/production cycle)*
- c_3 : *Shortage cost (TL/unit/unit time)*

Model 4: Production Model with Shortages Allowed

Model Variables

- *Q : Production quantity*
- *t : Production cycle time*
- *Im : Maximum stock level*
- *S : Maximum shortage level*
- t_1 *: Time to accumulate stocks*
- t_2 *: Time to deplete stocks*
- t_3 *: Time to deplete the queue*
- t_4 *: Time to accumulate the queue*

Total Inventory Cost = Holding Cost + Shortage Cost + Production Cycle Cost

Model 4: Production Model with Shortages Allowed

Example

The annual demand for a product is 18,000 units. The company's production rate is 3,000 units per month. The cost of setting up a production line is 204,000 TL. The monthly holding cost for one unit is 17 TL, and the annual shortage cost is 2,000 TL. (1 year = 12 months = 360 days)

a) *What is the optimal production quantity?*

b) *What is the length of the optimal production cycle?*

c) *What is the optimal shortage level?*

d) *What is the maximum inventory level?*

e) *What is the length of a production cycle?*

f) *What is the annual total inventory cost?*