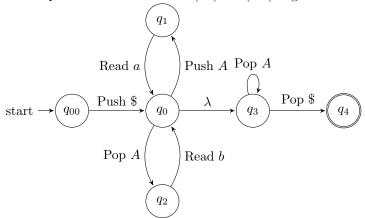
## Theory of Computation Assignment no. 8

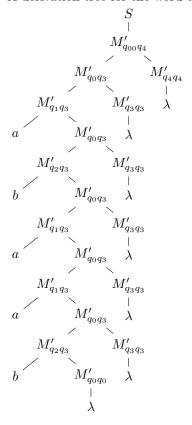
## Goktug Saatcioglu

(1) a. The simple PDA M' such that L(M) = L(M') is given below.



Where  $q_{00}$  is the intial state before the shielding symbol is pushed onto the stack,  $q_0$  is  $q_0$  from M,  $q_1$  is the intermediary step in the upper loop of M,  $q_2$  is the inermediary step in the lower loop of M,  $q_3$  is the stack empyting step and  $q_4$  is the accepting state once the stack is empty.

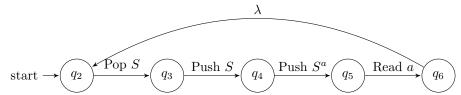
b. A derivation tree for the word w = abaab is given below.



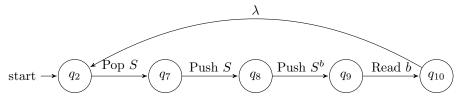
(2) The transformation of the CFG into a PDA is given below. We begin by defining the PDA with the main state and then define the transition in and out of main.

$$\operatorname{start} \longrightarrow \overbrace{q_0} \xrightarrow{\operatorname{Push} \$} \overbrace{q_1} \xrightarrow{\operatorname{Push} S} \overbrace{q_2} \xrightarrow{\operatorname{Pop} \$} \overbrace{q_3}$$

Here  $q_0$  is the initial state,  $q_1$  is the state with the shielding state,  $q_2$  is main where we begin by pushing S and  $q_3$  is the accepting state. Next, for each rule we show a set of transition out from main and back into main. We begin with the rule  $S \to aS^aS$ .



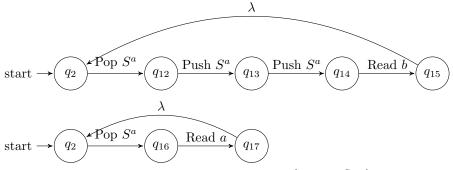
Here  $q_2$  is main and the rest of the states is the derivation rule processed in reverse. Similarly, we write the transitions for the rule  $S \to bS^bS$ .



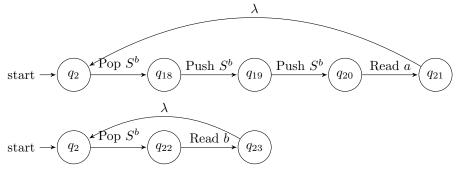
Again  $q_2$  is main and we reverse process the derivation rules. Next, we write the derivation for the rule  $S \to \lambda$  with  $q_2$  as main.

start 
$$\rightarrow q_2$$
  $q_{11}$ 

We then get two more derivations for the rule  $S^a \to a \mid a^S a S^a$  with  $q_2$  as main.



Finally, we get two more derivations for the rule  $S^b \to b \mid a^S b S^b$  with  $q_2$  as main.

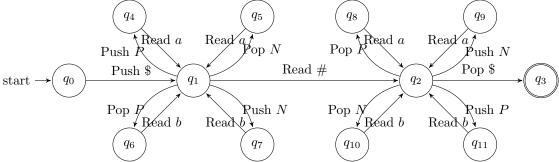


Thus, we have transformed the CFG into a PDA that accepts the same language.

- (3) (a)  $L(P) = \{u \# v \mid u, v \in \{a, b\}^* \text{ and } \#_a(u) \#_b(u) = \#_a(v) \#_b(v)\}$ 
  - (b) A data-configuration sequence of an accepting computation path in P for the word w = abb # abb is given below.

Here  $q_i$  is the *i*-th state from the left.

(c) The simple PDA P' such that L(P) = L(P') is given below.



Here  $q_0$ ,  $q_1$ ,  $q_2$  and  $q_3$  are the states given by the PDA P and the remaining states are the intermediary steps in the loops of P where for example the left-top loop of  $q_1$  which goes to  $q_4$  is the left-top loop of  $q_1$  in PDA P.

(d) A derivation tree for the word w = abb # abb is given below.

