

What is MST?

- Undirected graph.
- Spanning tree is a subgraph and we don't want the subgraph to have cycles in it.
- Edges have weights and when we add the weights of the edges in the subgraphs, then that's the minimum possible value of all the subgraphs that don't have cycles.
- Graph that has no cycle is called tree & the one with minimum edge weights is min.

Spanning Tree.

- Various algorithms are there like Prim's Algorithm, Kruskal's algorithm etc. But we will use Integer Programming to solve it. [If there are n vertices then we will have $(n-1)$ edges in the MST]

Formulation

c_{ij} - length/weight of edge (i,j)

$$\text{Obj: } \min \sum_{i,j} c_{ij} y_{ij}$$

$y_{ij} = \begin{cases} 0, 1 \end{cases}$ if it is included in tree
 $y_{ij} = 1$
else 0.

Constraint ①

$$\sum_{i,j} y_{ij} = n-1$$

→ Since only $n-1$ edges have to exist in the MST if more we will have cycles.]

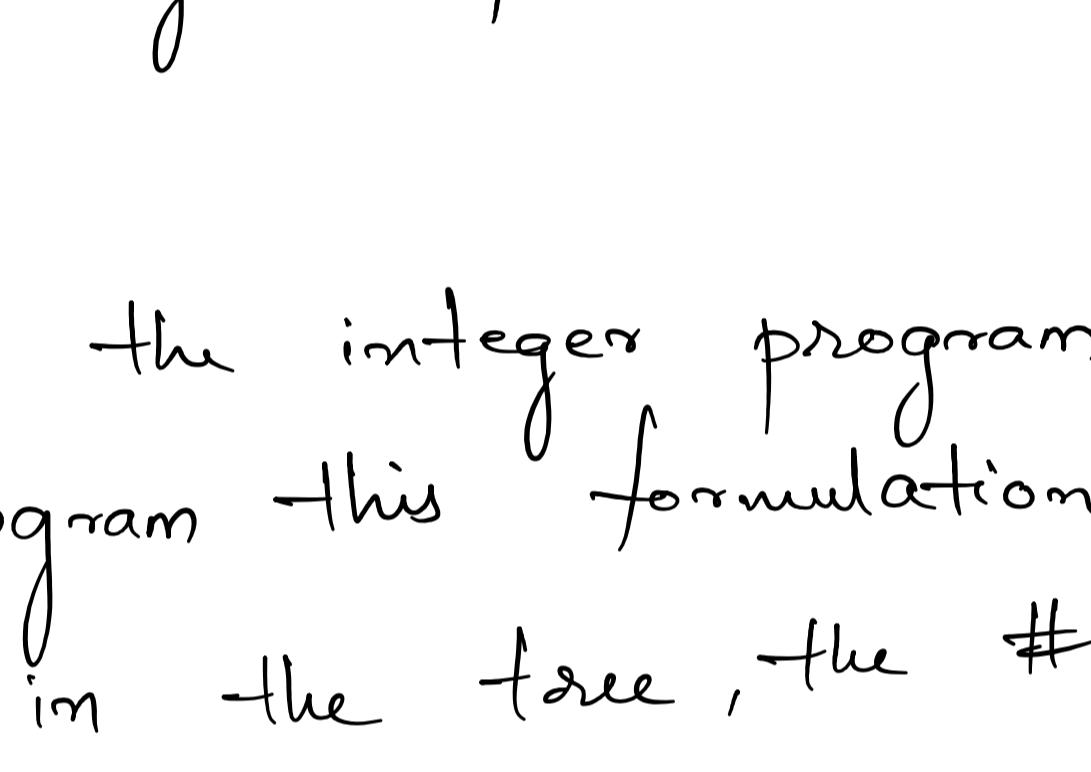
↳ Just this is not enough.

- Let's say N is node set.

S is subset of N

$$S \subseteq N$$

- In the subset we need to eliminate cycles. To do this in each subset we need to have # of edges in the subset less than or equal to $(|S|-1)$ where $|S|$ is the # of vertices in the subset.



- As a result of this we will have as many constraints as the subsets of Node set.

Constraint ②

$$\sum_{i,j \in S} y_{ij} \leq |S|-1$$

→ Every subset has # of edges \leq # of nodes in that subset.

Has 2 nodes but no edges, still satisfies $\leq |S|-1$

How do we solve this?

- We will use a method called row generation.

- In this method we will generate constraints few at a time.

- It is called row generation because we can think of each constraint as a row of the matrix and each column as a variable of the matrix.

- We solve this and check if the solution is a spanning tree. If not we add few more constraints and resolve. We proceed until we get a spanning tree. If we have arrived here we have found the solution.

- The Spanning tree that we arrive at in this way is optimal.

WHY?

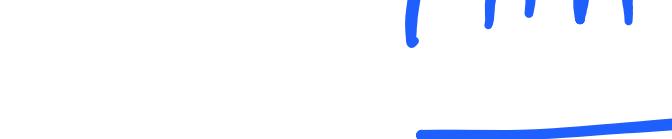
- Firstly, the solution we obtain is a feasible solution.
- Secondly, we have arrived at it by relaxing a few constraints.
- This may not be the only MST possible, there may be more MST's which could have been obtained by another set of constraints, nevertheless this is also an optimal solution.

- I will be implementing the Row Generation Algorithm in PYOMO. Check out the file mst.py for the implementation.



https://en.wikipedia.org/wiki/Minimum_spanning_tree

Sourced from Wikipedia



Two MST solutions

I shall implement the same problem from wikipedia, so that you guys can check the solution.

THANK YOU