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Abstract

In Chapter 1, I present a model of the market for news with rational voters who seek information to make a better decision and a profit-maximizing media outlet who maximizes viewership and generate some interior bias which is consistent with the empirical literature. The innovation of our model is that bias emerges even though there is no exogenous asymmetry in the game. The first driver of the emergence of bias is the way we define it, that is the difference between the numbers of journalists a media outlet hires from two opposite political camps. In most of the literature, bias is defined over reporting strategies only. The second driver of the bias is the resulting convexity of the media outlet's profit function. Having a continuum of voters with uniformly distributed priors to attract, a media outlet might choose to hire more of a certain type of journalists to receive better information about a certain state depending on its budget and technology level. This is the same reason why voters prefer receiving information from a media outlet while they have access to a public signal such as social media. It is because media outlets can hire credible journalists, columnists or reporters to distribute more precise information. We also provide some comparative statics which say surprisingly that higher income and technology of media outlets trigger bias whereas higher precision of a public signal alleviate it.

In Chapter 2, I analyze how much information rent a long-lived agent can extract

against an infinite sequence of short-lived players in a moral hazard game played repeatedly. Given the existence of a committed type, we show that the long-lived agent can attain even higher payoffs than the mixed Stackelberg payoff. We assume that principals do not observe past play. Instead, they observe a review score (an aggregate review score as in Yelp, Foursquare ...) announced by a central mechanism called review platform. By censoring the past actions of the long-lived player, the platform enables her to attain the highest payoff that can ever be achieved.

In Chapter 3, I provide empirical evidence for our results found in Chapter 2. Those are, a better reputation is in line with a higher incentive to produce fake reviews as opposed to what the prior literature put forward. I design a structural estimation model using a data set scraped from Yelp. The structured parameters are found by OLS estimation.

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I would also like to thank Henrique Roscoe de Oliveira. He has been an invaluable mentor throughout my Ph.D. years. He did not only help me have a clear and broader picture of my research and understand where it stands in the Microeconomics literature, but also he was there whenever I needed his counsel.

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Chapter 1

Bias Out of The Blue

1.1 Introduction

Media is considered the fourth pillar of democracy. It is an informative bridge between governing bodies and public. For this reason, it is important for media to be independent of the other three bodies.

Empirical research on media began in the 1930s mostly affected by Hitler's and Mussolini's use of media as a heavy propaganda tool. Hence, there is no surprise that the conventional wisdom was that the bias is a result of ideological media outlets trying to manipulate public opinion. However, this view has been challenged by recent empirical studies. Shapiro & Gentzkow [23] suggests that media bias should be explained with the idea of profit-maximizing outlets comforting to consumer beliefs. Our model follows their approach on firm side meaning that the media outlet we consider is purely profit-maximizing. Some blame Fox News or other right-wing media for the outcome of the 2016 presidential elections in the US. However, this does not mean that those outlets are taking an ideological stance. A New York Magazine article outlining mostly the details of Roger Ailes case by Gabriel Sherman [41] also states that Fox News owner Rupert Murdoch instructed Fox News executives to take down Donald Trump. However, as the primary results came out, there was a shift in Fox News's attitude towards Trump. We think that this cannot be explained by simply a sudden change of political views of the Fox News owner. We believe that media firms are worth too much to risk for a political view.

There are different views also on consumer side. One is that consumers demand biased news because of its entertainment value. For example, a liberal voter might enjoy reading unfavorable news about a Republican candidate. Consistent with the literature, we call those individuals who receive entertainment value from news reports biased and those who care about media only as an information source rational. Mullainathan & Shleifer [37] claims that the only way media bias exists is with biased readers. This kind of models in which consumers are not rational, the bias of a consumer is represented by an additive term in the utility function which captures the consumer's extra payoff from her subjective beliefs being reinforced. The other view contains economically a more standard approach in which a consumer takes an action and receive a payoff depending only on her action and the true state of the world. Then, media serves as an information source which helps individuals make better decisions and has a welfare-improving role. We take the latter approach on consumer side. However, it is worthwhile to note that rational consumers who have different opinions about a state of the world process the same message differently than each other. This leads to beliefs diverging and polarization that we observe especially in the political environment of the U.S. This paper contains a static model for the time being so we do not offer any asymptotic analysis.

Given our approaches on the supply and the demand side, the question becomes how media bias emerges with profit-maximizing firms and rational individuals. The answer that Shapiro & Gentzkow [22] came up with is non-uniform belief. Consider a binary state of the world. If a decision maker's belief over the right state occurring is strictly more than half, the media outlet would slant its reports to the right. Our model removes their assumption of belief asymmetry. Instead, we have a continuum of voters whose beliefs over a binary state are uniformly distributed with full support. We show that it is still possible to see media bias.

In this chapter, we design a model of a news market where a media outlet maximizes the number of his audience who has subjective prior beliefs and consumes news reports to inform their private decisions. The media outlet has the option to hire different types of journalists. Each type of journalist has a network on different candidate. So a left-wing type of journalist brings favorable news about L and unfavorable news about R and vice versa for a right-wing type of journalist. In real politics, majority of journalists are associated with either the right or the left wing. Moreover, those journalists who are considered to be from the left-wing (the right-wing) have closer relations with Liberal (Republican) candidate campaigns. This is what we base our assumption on that certain type of journalist brings certain type of message to the outlet. Note that apart from being one of the types, the journalists are identical in terms of hiring cost and precision to preserve the symmetry that we promised.

Given subjective prior beliefs, voters might value certain types of signals more than others and hence prefer the outlets which employ those signals. However, the prior belief distribution is symmetric around the uniform one so media outlets that seek to maximize profits do not trivially choose to slant their reports. In short, the question that we seek to answer is what is the best catering strategy to uniformly distributed subjective priors.

Our result says that the optimal strategy of the media outlet depends on the convexity of the cost function associated with acquiring signals. The convexity of the cost function favors unbiasedness, whereas the opposite leads to bias. If the cost function is concave or linear, the equilibrium is extremely biased. Otherwise, we might have interior or no bias. Our contribution is to generate bias out of the blue, without any exogenous asymmetry.

This paper has also a say on increasing trend of media bias. There is no consensus that competition among media outlets leads to more extreme biases in different directions, namely polarization even though majority of the recent literature says it does. Hamilton [26] documents that the media underwent fundamental changes in the U.S. between 1870 and 1920 in the direction of non-partisan reporting accompanied with rising competition. This contradicts the idea of positive relation between bias and competition. Our model predicts that competition does not change the intensity of bias whereas it leads to polarization.

There is no doubt that social media has become a significant source of information in news market. Even though we do not have any strong stance or claim on how social media functions in this paper, we think it is worth to mention because there are some parts that the reader might relate to social media. Sometimes, it is the case that we observe reports earlier on Twitter or on other social platforms by a tweet or a port of first witnesses of a scene and then see it published on mass media. Nevertheless, people still choose to receive information from news outlets or confirm the information that they gathered from social media. This implies that media outlets offer something that readers find valuable on top of the information coming from social media. Media outlets have resources dedicated to picking up credible evidence when they display a report whereas social media contains a lot of unreliable information which might be hard to distinguish from reliable ones. This distinctive feature of media outlets that makes them attract voters is precision and credibility in our model. They offer exclusive reporting and framing. People do want to know what Maureen Dowd, David Brooks or Jonah Goldberg thinks on some certain issues and they believe the information their favorite columnists deliver more than anything else. Moreover, in real life, the state of the world is not usually binary and neither the message space is. This might lead to disinformation originating from social media because there are many individual users who post biased reports. Some think that social media is a platform which brings millions together and so the precision of information coming from social media goes to 1. There are also some who think that social media is platform not producing information but allowing people share their subjective opinions. If the reader would like to interpret the public signal in our model as social media, the approach would something in between the former and the latter.

The rest of the chapter is organized as follows. We review the media literature in Section 1.2. Section 1.3 describes the model. In Section 1.4, we solve the model and present our main result. In Section 1.5, we display comparative statics with computer simulations. Section 1.6 concludes the chapter.

1.2 Literature Review

Even though empirical studies go back to as early as 1940s (Lazarsfeld et. al., [31]), theoretical literature on media bias began and has been growing recently. A significant question that researchers have been trying to answer in media literature is if bias is originating from the supply (Anderson & Mclaren [2], Baron [3], Besley [5]) or demand side, or both. Recent studies suggest that it is coming from the demand side. Mullainathan and Shleifer [37] claim that unless consumers enjoy entertainment value of news, there is no room for bias. By enjoying entertainment value, what we mean is that consumers get some extra payoff from media firms reinforcing their priors. Shapiro and Gentzkow ([22]) and Burke [7] use a fact that has been useful in media literature to improve this result. That is, a Bayesian consumer who is uncertain about the quality of information infers that the source is of higher quality if the information caters to the consumer's prior belief. Then, in order to maximize the posterior of the consumer over the source being of higher quality, a media outlet slants its reports to whichever direction the consumer's prior is leaned to. They use the term rational consumers to refer to those whose utility does not include any entertainment value as it is a common practice in the literature. In other words, rational consumers simply seek the truth. However, Shapiro & Gentzkow [22] has a single consumer in their model.

With multiple consumers, it is complicated to characterize the interior solution that empirical studies point at (Shapiro & Gentzkow [23], Groseclose [25], Durante [14]). Dziuda [15] does that to some extent with a biased expert and an uninformed decision maker with uniformly distributed intrinsic preference. Since the timeline is a cheap talk setting, the decision maker is not influenced by the reports that are unfavorable to the expert's preference. As we mentioned earlier, our model relaxes the assumption of the sender being biased.

Earlier empirical studies (Glasser, Allen, and Blanks [24], Pritchard [39] relied on judgement to measure the slant. Groseclose & Milyo [25] made an important contribution by proposing a new measure of ideological content. They counted the times that a media outlet cites various think tanks. Their results are interesting, or maybe not: they say that there is a significant liberal bias in the U.S. media. All of the news outlets they analysed except Fox News' Special Report received a score to the left of the average member of Congress. Similarly, Shapiro & Gentzkow [23] compare phrase frequencies in a newspaper to identify whether that newspaper's language is more similar to that of a congressional Republican or Democrat. Another method to measure bias is to count the number of coverages of issues that are owned by the Democratic or the Republican Party based on a predetermined issue ownership list (Latham [30], Puglisi [40]). Chiang [9] develop an econometric model to show that voters rely on the media for information during campaigns but the extent of this reliance depends upon the degree and direction of any bias. To be more precise, endorsements for the Democratic candidate from left-leaning newspapers are less influential than are endorsements from neutral or right-leaning newspapers. This result supports our rationality assumption on the consumer side.

Our model is similar to both Sobbrio [42] and Oliveros and Vardy [38] in terms of information acquisition. Bias is defined over the choice of sources that information is received by media outlets. However, we relax the assumption of a strictly positive probability of a partisan voter and ideological editors.

There is a number of dynamic models as well. (Bernhardt et. al. [4], Acemoglu [1]). For the time being, we do not focus on asymptotic properties of belief distribution of consumers and leave the model static. Also, it is worthwhile that even though the media outlet chooses what type of journalists he will hire more and less in this model, the resulting bias is not editorial (Druckman [13], Entman [17]). For a deeper and more comprehensive survey of the media literature, see Stromberg [43]).

1.3 The Model

1.3.1 The Environment

There is a continuum of voters and a media outlet. The mass of voters is normalized to unity. There is a binary state of the world $S \in \{L, R\}$. Consider L as representing that a liberal candidate is the right choice to be elected and R that a republican candidate is. A more common way that this is introduced in the literature is that there are two candidates and voters want to choose the candidate with higher valence. Voters are unsure about which candidate has higher valence and they all have different opinions on that.

1.3.2 The Players

A type of a voter is identified by her subjective belief θ on the state being right. Beliefs are distributed uniformly on $[0, 1]$ with full support. An alternative way to identify a voter would be by an intrinsic (Dziuda [15]) or an idiosyncratic preference (Sobbrio [42]) which both capture an euclidean distance between her preference and true state of the world instead of a subjective prior. In that case, we would assume a common prior based on the true distribution of the state. As long as the distribution of intrinsic or idiosyncratic preferences is the same with the distribution of subjective priors, the results would be the same. As our results do not depend on whether a voter is identified by her prior or intrinsic or idiosyncratic preference, we prefer going with the former approach without loss of generality.

A media outlet has a uniform belief over the two states so the probability that he assigns to the state being R or L is 0.5. However, his action is independent of his belief over the state because his profit only depends on viewership which we will formalize in the next section. Therefore, the prior belief of the media outlet does not play any role in this game. We assume a uniform prior for the media outlet for the sake of consistency of his prior with those of voters.

1.3.3 The Game and The Payoffs

1.3.3.1 Information Acquisition

Every player in this model has access to a public signal $y \in \{l, r\}$ with a precision $\pi = Pr(y = l|S = L) = Pr(y = r|S = R) \in [0.5, 1]$. One can interpret this as social media or an agent's own ability to gather information from friends, colleagues, neighbours and so on. In addition to the public signal, the media outlet has an option to hire certain journalists and reporters who conduct investigations to acquire information with perfect precision ($Pr(s = l|S = L) = Pr(s = r|S = R) = 1$) on a certain state. As an example, consider New York Times hiring more leftist staff to receive better information about news that favor the left state. This part of information acquisition structure is essentially no different than what Kamenica & Gentzkow [28] has.

The media outlet does not have enough budget to hire all of the journalists out there. Let us assume that he has to pay a fixed amount of salary for every journalist that he hires and N is the maximum number of journalists that he can hire given its budget constraint. Let N_l and N_r be the numbers of left-wing and right-wing journalists that the outlet hires. We will normalize N_l and N_r by $n_l = \frac{N_l}{N}$ and $n_r = \frac{N_r}{N}$ so that n_l and n_r are in $[0, 1]$. The reason we do this normalization is that we assume that the media outlet has a general form Cobb-Douglas production function: $f(k, n_{s\in\{l,r\}}) = k^\beta n^\alpha_{s\in\{l,r\}}$ where α and β are the elasticities of labor and endowment, and the endowment is fixed. Endowment can be interpreted as any other means of the media outlet to acquire information. The reason for calling it endowment is that we implicitly assume that a media outlet's ability to receive information net of his labor work is positively related to his total endowment. Consider New York

Times having contracts with Associated Press, United Press International and some individual reporters ("nightcrawlers") to gather information. As a media outlet is endowed with more resources, his ability to gather information will be higher.

We define bias as $b = n_r - n_l$, the difference between the number of right-wing and left-wing journalists that the outlet has. A positive b implies right bias and a negative one implies left bias. The media outlet chooses a tuple (n_l, n_r). By choosing any arbitrary (n_l, n_r), the outlet guarantees to receive perfect signal with probability $f(k, n_l)$ when the true state is left and with probability $f(k, n_r)$ when the true state is right. Let $\sigma_l = f(k, n_l)$ and $\sigma_r = f(k, n_r)$.

Even though the information acquisition is as in standard Bayesian persuasion case, the media outlet is not committed to reporting his signal truthfully. However, Blackwell's (1951) theorem says that an experiment γ that provides information about the state of the world is preferred by every decision maker to an experiment γ' if and only if γ' is a garble of γ. Hence, the media outlet does not have incentive to garble his signals and so we consider the media outlet's strategy as simply choosing (σ_l, σ_r).

1.3.3.2 The Media Outlet's Decision and Payoff

Since $n_l + n_r \leq 1$, it follows that

$$\sigma_l^{\frac{1}{\alpha}} + \sigma_r^{\frac{1}{\alpha}} \leq k^{\frac{\beta}{\alpha}} \tag{1.1}$$

Let us call $q = k^{\frac{\beta}{\alpha}}$ and interpret the inequality (1.1) as the media outlet being endowed with a limited resource q that can be spent on acquiring information. It allocates this resource between acquiring perfect signals about left and right states. Let us restrict q to be weakly smaller than 1 in order to secure σ_l and σ_r from exceeding 1. Inequality (1.1) represents the cost function in terms of (σ_l, σ_r) so let $C(\sigma_l, \sigma_r) = \sigma_l^{\frac{1}{\alpha}} + \sigma_r^{\frac{1}{\alpha}}$. We can either say that the media outlet's decision is to choose (n_l, n_r) such that $n_l + n_r \leq 1$ or (σ_l, σ_r) such that $C(\sigma_l, \sigma_r) \leq q$. We prefer the latter for the sake of simplicity. Note that (n_l, n_r) is on a discrete space. However, we will work on continuous space for the ease of computations. It will not change the essence of our results as long as N is not too small.

Figure 1.1 displays the media outlet's strategy (σ_l, σ_r) from the perspective of voter with prior θ. Nature determines the state to be R with probability θ or L with probability $1 - \theta$. Suppose the state is R. Then, with probability σ_r, the game moves on to the node where the journalists of the outlet brings him right message and the outlet truthfully reports this message; or with probability $1 - \sigma_r$, it goes to the node where the outlet does not receive any message from its staff and reports the public signal which is right with probability π and left with probability $1 - \pi$. Note that the report space of the outlet is $\{l, r\}$, so it does not have the option to not report or report any other news than r or l. The other parts of the tree are interpreted similarly.

The media outlet maximizes its profit which is assumed to be an increasing affine function of the total utility of its audience. It is worthwhile to note that a more common assumption in the literature is that media outlet's profit is an affine function of the measure of its audience to capture the fact that the majority of outlets' profits come from advertising. We capture the very same situation. However, advertising revenue is based on the number of page views. We believe that not every reader views the same number of pages on a media outlet. Our implicit assumption is that the number of page views by a voter is an affine function of her payoff from receiving information from the media outlet. Therefore, the outlet maximizes the viewership instead of readership. In our understanding, viewership is represented by total utility

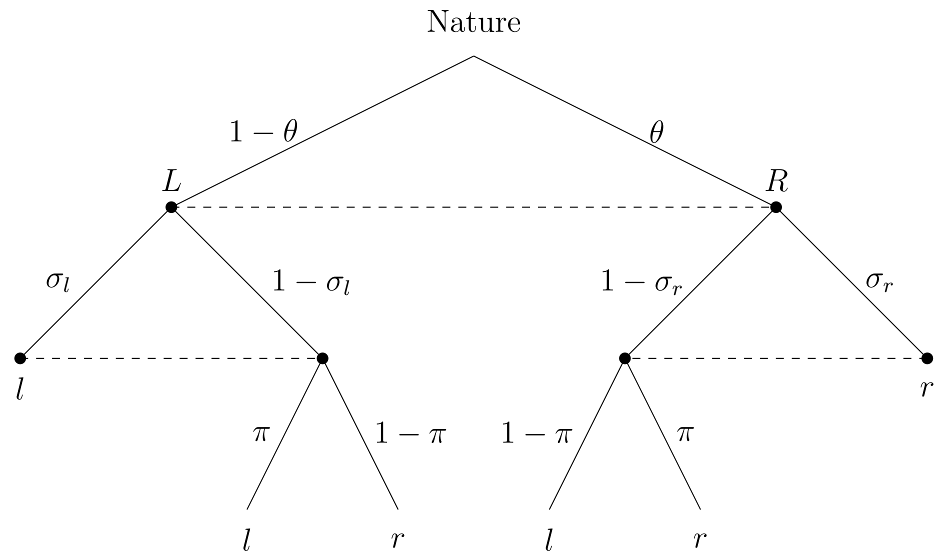

Figure 1.1. How voter with prior θ sees the media outlet's strategy (σ_l, σ_r).

of the audience whereas readership is represented by measure of the audience. We think that it is reasonable or even necessary to assume that a more dedicated reader visits more pages, read more columns which increase the advertising revenue of the outlet.

We furthermore assume that there is no subscription fee or any price to view the reports so the cost of reading the outlet is arbitrarily small as it is the case for most of the digital news outlets. Thus, voters choose to receive information from the media outlet whenever the outlet yields them a strictly positive extra payoff. The outlet's problem can be written as

$$\begin{aligned} &\max_{\sigma_l,\sigma_r} V = \int_0^1 \mathbb{1}_{(\theta|\pi,\sigma_l,\sigma_r)} \times U(\theta,\pi,\sigma_l,\sigma_r)d\theta \\ &st \;\; C(\sigma_l,\sigma_r) \leq q \end{aligned} \tag{1.2}$$

where $\mathbb{1}_{(\theta|\pi,\sigma_l,\sigma_r)}$ is 1 if the voter θ reads the media outlet with the strategy (σ_l, σ_r), 0 otherwise and $U(\theta,\pi,\sigma_l,\sigma_r)$ is the net payoff of voter with prior θ from receiving information from the outlet given the precision of public signal π and the outlet's strategy (σ_l, σ_r).

1.3.3.3 Voter's Decision and Payoff

Each voter makes a decision between two actions or votes which we denote by $d = (L, R)$. Correctly matching her vote to the state yields payoff 1, whereas mismatching results in payoff 0.

1.3.3.4 Voter With No Information

If the voter with prior θ did not have access to any information, her payoff would be

$$E[u(\theta)] = \begin{cases} \theta & \text{if } \theta \in [0.5, 1] \\ 1 - \theta & \text{if } \theta \in [0, 0.5] \end{cases} \tag{1.3}$$

The first and second lines of equation (1.3) are voter's expected payoffs when she votes for the right and the left candidate respectively. It is obvious that those whose

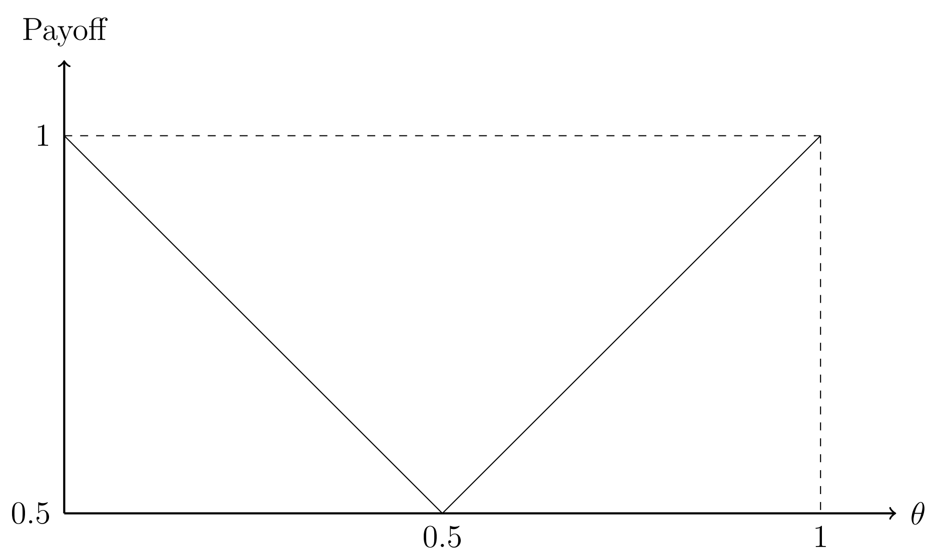

Figure 1.2. Payoff distribution of voters with respect to the priors without any information.

prior is greater than 0.5 votes for the right and those whose prior is less than 0.5 votes for the left. The median voter whose prior is 0.5 is indifferent and it does not matter for which candidate she votes and if she chooses to receive information from the outlet because any belief has zero measure. Note that there is no cost of voting since it would not play a role in our bias analysis. We will make it clear why a positive cost of voting would not play any role in our model in the subsequent section. Figure 1.2 exhibits the payoff distribution of voters without any information source. Horizontal axis accounts for the priors and the vertical represents the payoffs.

1.3.3.5 Voter With Access To The Public Signal

A voter observes one message which is the public signal in this case. Then, her action space becomes:

1. always voting for L,
2. always voting for R,
3. voting for L after public signal l and voting for R after public signal r.

We denote these actions by (L,L), (R,R) and (L,R). Notice that voting for L after public signal r and voting for R after public signal l ((R,L)) is strictly dominated unless π and θ are both 0.5.

Observing the public signal yields some extra payoff to some moderate voters who take the last action (L,R). While partisan voters are not persuaded with the public signal and vote for their favorite candidate no matter what the public signal is, some moderates vote for the candidate that the public signal favors. If a voter is moderate enough to benefit from the public signal, her expected payoff is

$$E[U(\theta,\pi)] = Pr(y=l) \times Pr(S=L|y=l) + Pr(y=r) \times Pr(S=R|y=r) \quad (1.4)$$

$$
\begin{aligned}
&= [(1-\theta)\pi + \theta(1-\pi)]\frac{(1-\theta)\pi}{(1-\theta)\pi + \theta(1-\pi)} \\
&+ [\theta\pi + (1-\theta)(1-\pi)]\frac{\theta\pi}{\theta\pi + (1-\theta)(1-\pi)} \\
&= \pi
\end{aligned}
$$

Otherwise, her payoff remains either as θ or $1-\theta$ depending on if she is a partisan rightist or leftist. Therefore, the utility for any voter with prior θ is given by

$$
E[u(\theta,\pi)] = \begin{cases} \theta & \text{if } \theta \in [\pi, 1] \\ \pi & \text{if } \theta \in [1-\pi, \pi] \\ 1-\theta & \text{if } \theta \in [0, 1-\pi] \end{cases} \tag{1.5}
$$

The extra payoff by the public signal for voters is displayed by Figure 1.3.

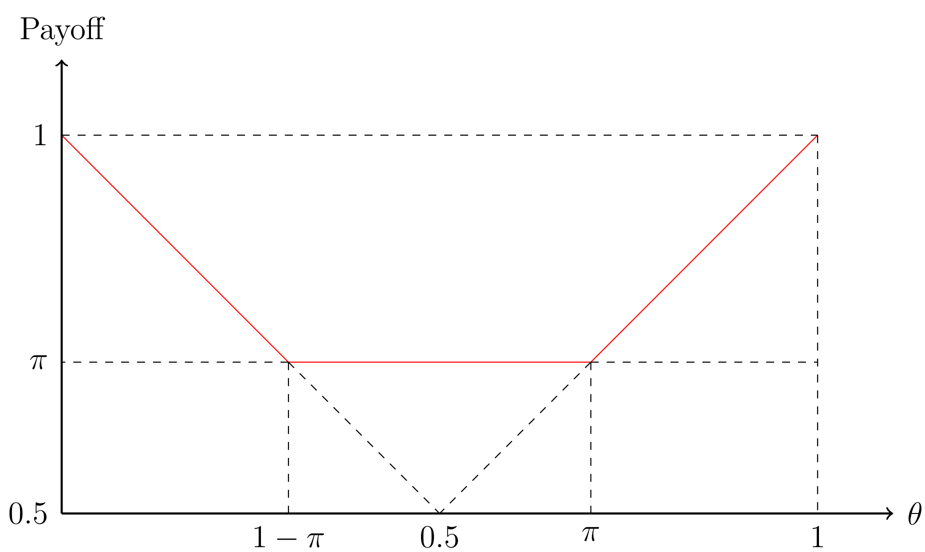

Figure 1.3. Payoff distribution of voters with respect to the priors having access to the public signal.

1.3.3.6 Voter With Access To The Public Signal and The Media

This time, a voter observes one public signal and one media report. Her strategy space then becomes

1. (L,L,R,L),
2. (R,L,R,R) and
3. (L,L,R,R)

where the first component is her vote when the media report and the public signal are both l; the second is her vote when the media report is l and the public signal is r; the third is her vote when the media report is r and the public signal is l; and the fourth is her vote when the media report and the public signal are both r. As before, it is easy to find that all other strategies other than those above are strictly dominated again. In addition, when the media report and the public signal differ, a voter is sure that the state is what the media outlet reports because it is for certain that the report came from a journalist who has perfect precision.

We can think of media as sending two types of messages: conclusive and inconclusive. We define a conclusive message as one which is different than the public signal. Therefore, conclusive messages completely identify the state. We define an inconclusive message as one which is the same as the public signal. This means that unless σ_l or σ_r is 1, inconclusive messages do not identify the state.

Given a conclusive message by the media outlet, a voter's action is a no-brainer. If the message is inconclusive, some moderate voters' decisions will be influenced by the media so even receiving an inconclusive message from the outlet yields extra payoff to them. There may also be some stubborn voters with partisan right-wing or left-wing beliefs that do not change their voting behavior whatever the inconclusive message is so the media sending inconclusive message does not affect their payoffs.

For instance, suppose the public signal delivered l and everyone observes this. This can be interpreted as a situation when there is a mild perception on an issue in the society. Let us say that the media outlet reports r. Then, the media's information becomes very valuable to all because it implies that the state is R for certain. We need to consider 4 cases separately: media reporting r, public signal delivering l; media reporting l, public signal delivering r; media reporting r, public signal delivering r; media reporting l, public signal delivering l.

The probabilities of the 4 cases happening according to a voter with prior θ are

$$Pr(r,l|\theta) = \theta\sigma_r(1-\pi) \tag{1.6}$$

$$Pr(l,r|\theta) = (1-\theta)\sigma_l(1-\pi) \tag{1.7}$$

$$Pr(r,r|\theta) = \theta\pi + (1-\theta)(1-\sigma_l)(1-\pi) \tag{1.8}$$

$$Pr(l,l|\theta) = (1-\theta)\pi + \theta(1-\sigma_r)(1-\pi) \tag{1.9}$$

and the posteriors associated with the 4 cases follow as

$$Pr(R|\theta,r,l) = 1 \tag{1.10}$$

$$Pr(R|\theta,l,r) = 0 \tag{1.11}$$

$$Pr(R|\theta,r,r) = \frac{\theta\pi}{\theta\pi + (1-\theta)(1-\sigma_l)(1-\pi)} \tag{1.12}$$

$$Pr(R|\theta,l,l) = \frac{\theta(1-\sigma_r)(1-\pi)}{(1-\theta)\pi + \theta(1-\sigma_r)(1-\pi)} \tag{1.13}$$

If a voter is moderate enough so that her vote is influenced by an inconclusive message, her expected payoff becomes

$$E[U(\theta, \pi, \sigma_l, \sigma_r)] = \theta\sigma_r(1-\pi) + (1-\theta)\sigma_l(1-\pi) + \pi \tag{1.14}$$

If a voter is partisan enough to vote for the right candidate independent of conclusive messages, her payoff is

$$E[U(\theta, \pi, \sigma_l, \sigma_r)] = \theta + (1-\theta)\sigma_l(1-\pi) \tag{1.15}$$

Finally, if a voter is partisan leftist, her payoff is as follows

$$E[U(\theta, \pi, \sigma_l, \sigma_r)] = 1 - \theta + \theta\sigma_r(1-\pi) \tag{1.16}$$

In general, we can write the utility of any voter with θ as

$$E[U(\theta, \pi, \sigma_l, \sigma_r)] = \begin{cases} \theta + (1-\theta)\sigma_l(1-\pi) & \text{if } \theta \in [\frac{\pi}{1-\sigma_r(1-\pi)}, 1] \\ \theta\sigma_r(1-\pi) + (1-\theta)\sigma_l(1-\pi) + \pi & \text{if } \theta \in [1 - \frac{\pi}{1-\sigma_l(1-\pi)}, \frac{\pi}{1-\sigma_r(1-\pi)}] \\ 1 - \theta + \theta\sigma_r(1-\pi) & \text{if } \theta \in [0, 1 - \frac{\pi}{1-\sigma_l(1-\pi)}] \end{cases} \tag{1.17}$$

Figure 1.4 shows the payoff distribution of voters with access to the public signal and the media. The red line refers to Figure 1.3 which shows the payoffs with the public signal. If there is also media, the payoffs go up to the green line. The middle piece being positively sloped means that σ_r is higher than σ_l. Note that as long as σ_l or σ_r is positive, the measure of moderates whose votes are influences by inconclusive messages $(\bar{\theta} - \underline{\theta})$ is higher than that of those voters whose votes are affected by the public signal.

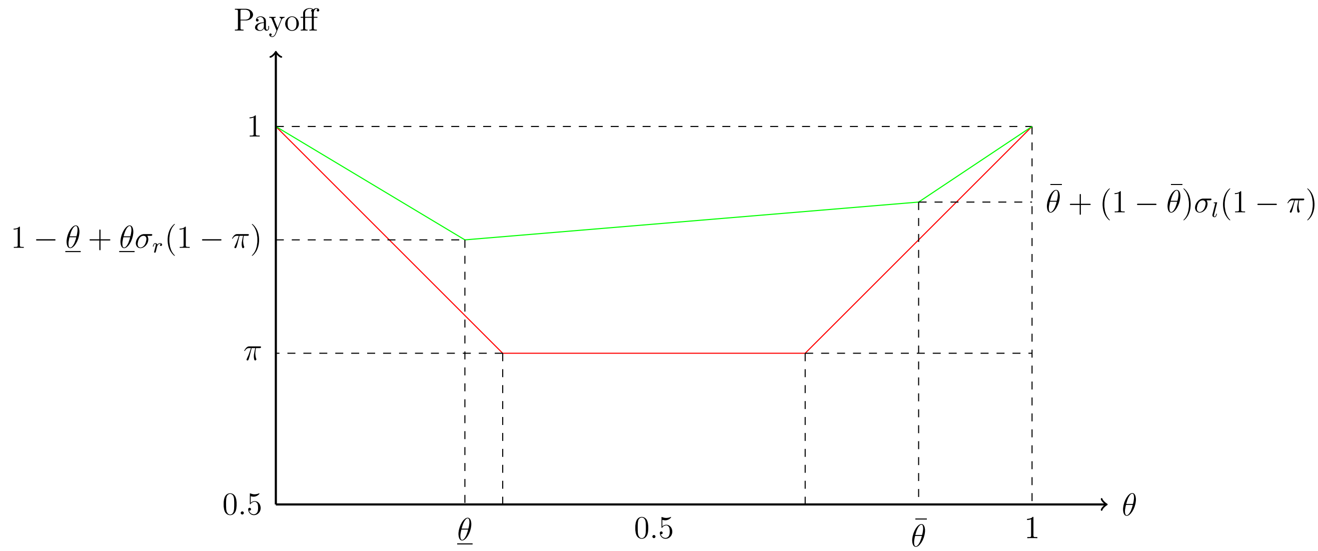

Figure 1.4. Payoff distribution of voters with access to the public signal and the media by the green line.

1.3.3.7 Strictly Positive Cost Of Voting

So far, we have assumed that voter turnout is 100% because there is no cost of voting and a voter's payoff depends only on her own action and the true state so we do not have the pivotal voter issue. We relax the first assumption here. Suppose that cost of voting is ε. With compulsory voting, all the lines in Figures 1.3 and 1.4 would be shifted down by ε as shown in the upper panel of Figure 1.5 so the profit of the outlet which is represented by the gray shaded area would remain the same for a given strategy. Hence, the problem of the outlet would not be affected. If voting is not compulsory, a voter has the option to abstain. Then, her action space becomes

(L,L,R,L), (L,L,R,A), (L,L,R,R), (A,L,R,A), (A,L,R,R) and (R,L,R,R)

where A denotes abstain. Of course, there are some strictly dominated strategies which have not been exhibited. Similar to the previous analysis, left partisan voters

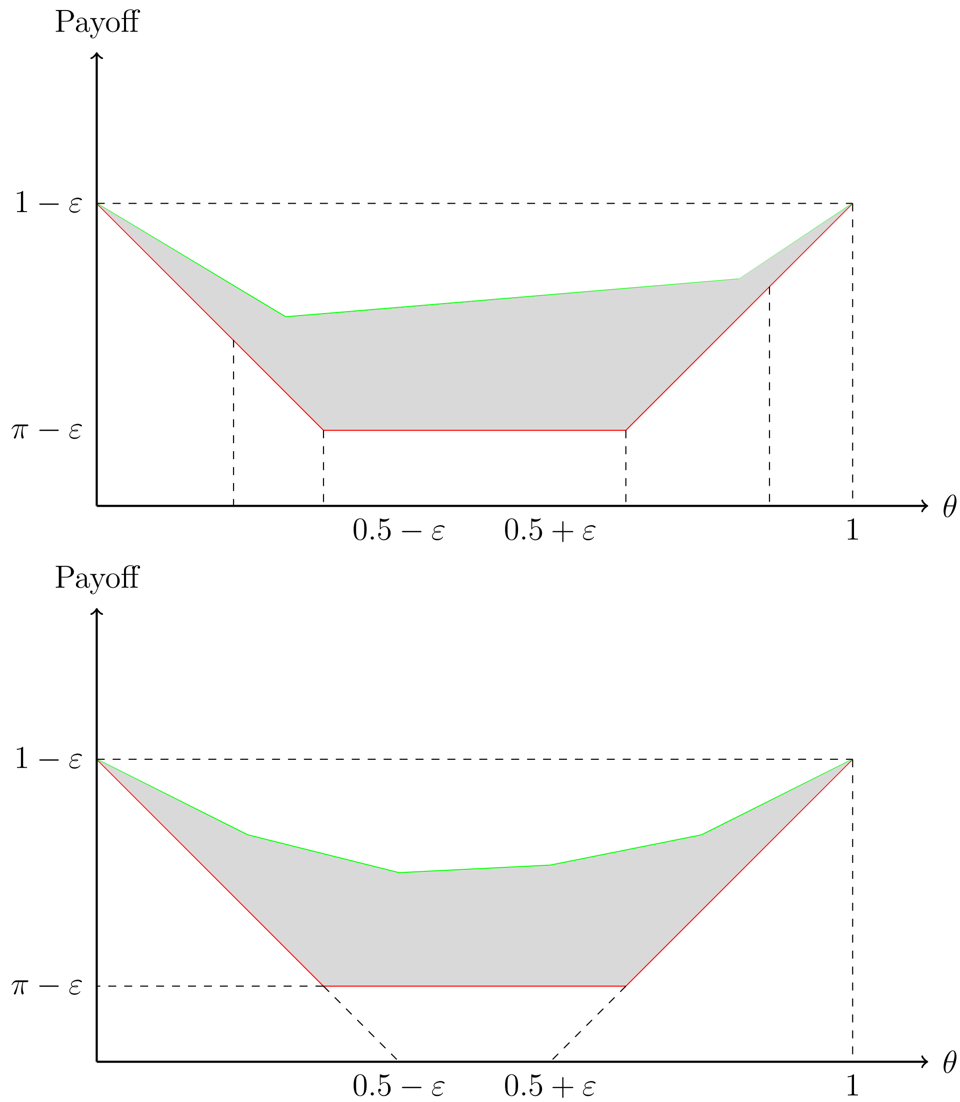

Figure 1.5. Payoff distribution of voters with cost of voting ε: Upper panel for compulsory voting, the lower for with abstaining option

play (L, L, R, L). Moderately left-wing voters play (L, L, R, A). The next switch occurs between (L, L, R, A) and either (L, L, R, R) or (A, L, R, A). The rest follows similarly. As a result, the payoff distribution is as shown in the lower panel of Figure 1.5. As we will see in the next section, the main driver of our result is the convexity of the green line which persists in the existence of cost of voting. Hence, we assume that it is zero for the ease of exposition.

1.4 Equilibrium Analysis

First, let us draw some results from equations (1.5) and (1.17). By comparing the second line of both equations, we see that the media outlet yields a strictly better payoff to any voter than the public signal as long as he spends some of his endowment.

1.4.1 Profit Of The Media Outlet

Figure 1.6 displays the profit of the media outlet for an arbitrary strategy (σ_l, σ_r). The red and the green lines represent the payoffs with access to the public signal and with access to the media along with the public signal respectively as in Figures 1.3 and 1.4. Then, the gray shaded region becomes the profit of the outlet which we will denote by V. We can write V with respect to (σ_l, σ_r) as the following:

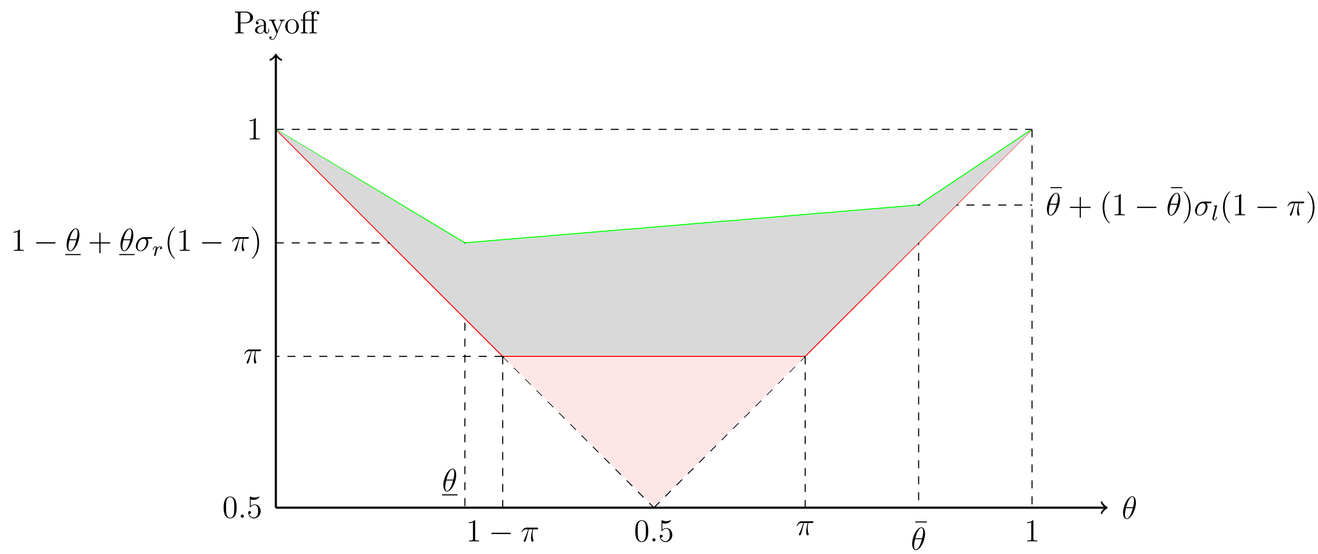

Figure 1.6. Payoff distribution of voters with respect to the priors given a strategy of the outlet and the resulting profit of the outlet represented by the gray shaded region.

$$V = \frac{1}{2}\sigma_r(1-\pi)\Big(1-\frac{\pi}{1-\sigma_l(1-\pi)}\Big)^2 \tag{1.18}$$

$$
\begin{aligned}
&+ \frac{1}{2}\Big[(1-\pi)\Big(\frac{\sigma_r+\sigma_l}{2}\Big) + \pi - 0.5 \\
&+ \sigma_r(1-\pi)\Big(1 - \frac{\pi}{1-\sigma_l(1-\pi)}\Big)\Big]\Big[\frac{\pi}{1-\sigma_l(1-\pi)} - 0.5\Big] \\
&+ \frac{1}{2}\Big[(1-\pi)\Big(\frac{\sigma_r+\sigma_l}{2}\Big) + \pi - 0.5 \\
&+ \sigma_l(1-\pi)\Big(1 - \frac{\pi}{1-\sigma_r(1-\pi)}\Big)\Big]\Big[\frac{\pi}{1-\sigma_r(1-\pi)} - 0.5\Big] \\
&+ \frac{1}{2}\sigma_l(1-\pi)\Big(1 - \frac{\pi}{1-\sigma_r(1-\pi)}\Big)^2 \\
&- \frac{(2\pi-1)(\pi-0.5)}{2}
\end{aligned}
$$

which can now be simplified as

$$
\begin{aligned}
V = \frac{2\pi-1}{4}\Big(\frac{\pi}{1-\sigma_l(1-\pi)} + \frac{\pi}{1-\sigma_r(1-\pi)} - 2\pi\Big) \\
+ \frac{\pi(1-\pi)}{4}\Big(\frac{\sigma_l}{1-\sigma_l(1-\pi)} + \frac{\sigma_r}{1-\sigma_r(1-\pi)}\Big)
\end{aligned}
\tag{1.19}
$$

1.4.2 Boundary Solutions

From equation (1.19), it follows

$$\frac{\partial V}{\partial \sigma_r} = \frac{\pi^2(1-\pi)}{2[1-\sigma_r(1-\pi)]^2} \tag{1.20}$$

and

$$\frac{\partial V}{\partial \sigma_l} = \frac{\pi^2(1-\pi)}{2[1-\sigma_l(1-\pi)]^2} \tag{1.21}$$

which are both positive. Moreover, we have:

$$\frac{\partial^2 V}{\partial \sigma_r^2} = \frac{\pi^2(1-\pi)^2}{[1-\sigma_r(1-\pi)]^3} \tag{1.22}$$

and

$$\frac{\partial^2 V}{\partial \sigma_l^2} = \frac{\pi^2(1-\pi)^2}{[1-\sigma_l(1-\pi)]^3} \tag{1.23}$$

which are both positive as well. We also have

$$\frac{\partial^2 V}{\partial \sigma_r \sigma_l} = 0 \tag{1.24}$$

which makes us conclude that $V(\sigma_l, \sigma_r)$ is convex and separable.

The partial derivatives of V being positive results in the following lemma.

Lemma 1.4.1. *Budget constraint of the media outlet binds.*

Proof. (1.20) and (1.21) being positive means that the objective function is increasing with the inputs. Thus, as long as α is a finite number, the lemma follows. □

Since we take the outlet's strategy as (σ_l, σ_r) instead of (n_l, n_r), let us modify the definition of bias as $b = \sigma_r - \sigma_l$. Then, we have the following definitions:

Definition 1. Given that the budget constraint binds, the unique unbiased strategy is $\left(\left(\frac{q}{2}\right)^\alpha, \left(\frac{q}{2}\right)^\alpha\right)$.

Definition 2. Extreme bias is either $(0, q^\alpha)$ or $(q^\alpha, 0)$.

1.4.3 Non-convex Cost

As the driver of emergence of bias, we have convexity of the objective function defined over a feasible set which is convex or concave depending on the cost function. Let us assume for now that $\alpha \geq 1$ so that the cost function is non-convex. This leads to the following lemma.

Lemma 1.4.2. *For $\alpha \geq 1$, the equilibrium strategy is extreme bias.*

Proof. Since we have a symmetric and convex objective function defined over a symmetric weakly concave set, the solution is at the corners which represent extreme bias strategies. $\square$

1.4.4 Convex Cost

If α is smaller than 1, we have to deal with a non-convex optimization that we cannot solve analytically. Nevertheless, we are able to give a proof for the following proposition.

Proposition 1. *There exists an $\underline{\alpha} < 0.5$ such that for every $\alpha > \underline{\alpha}$, there exist π and q which make the equilibrium strategy is biased.*

Proof. One might consider writing σ_l in terms of σ_r, plug it in the objective function and search for a global maximum. However, there is a neater way which is gradient approach. Consider the gradient of the value function in equation (1.19) at any (σ_l, σ_r):

$$\frac{\frac{\partial V}{\partial \sigma_r}}{\frac{\partial V}{\partial \sigma_l}} = \frac{[1 - \sigma_l(1 - \pi)]^2}{[1 - \sigma_r(1 - \pi)]^2} \tag{1.25}$$

Also, consider the gradient of the cost function

$$\frac{\frac{\partial C}{\partial \sigma_r}}{\frac{\partial C}{\partial \sigma_l}} = \frac{\sigma_r^{\frac{1-\alpha}{\alpha}}}{\sigma_l^{\frac{1-\alpha}{\alpha}}} \tag{1.26}$$

At the unbiased strategy when $\sigma_l = \sigma_r$, (1.25) and (1.26) are both 1. This means that the unbiased strategy is a critical point. However, we do not know neither if it is a local or global nor if it is a maximum or minimum. In order to know if the unbiased strategy is a global maximum or minimum, we first need to know if the function (1.19) is concave or convex around the unbiased strategy. Note that (1.25) and (1.26) are both symmetric around the unbiased strategy. Therefore, we will only restrict attention to cases where $\sigma_r > \sigma_l$.

Notice that (1.26) is decreasing with α whenever $\sigma_r > \sigma_l$. This increases the odds to have an unbiased equilibrium as α decreases, which is intuitive because as the cost function gets more convex, we approach to the unbiased solution. Thus, if there is any, the unbiased equilibrium is at lower α. Hence, it is enough to show that there exists an unbiased equilibrium when $\alpha = 0.5$. Our claim can be formulated as

$$\frac{[1-\sigma_l(1-\pi)]^2}{[1-\sigma_r(1-\pi)]^2} > \frac{\sigma_r}{\sigma_l} \tag{1.27}$$

for any $\sigma_r > \sigma_l$. It follows that

$$\begin{aligned} & 2(\sigma_r^2 - \sigma_l^2)(1-\pi) > (\sigma_r - \sigma_l) + (\sigma_r^3 - \sigma_l^2)(1-\pi)^2 \\ \Rightarrow\ & 2(\sigma_r + \sigma_l)(1-\pi) > 1 + (\sigma_r^2 + \sigma_r\sigma_l + \sigma_l^2)(1-\pi)^2 \\ \Rightarrow\ & 2(\sigma_r + \sigma_l)(1-\pi) > 1 + (q + \sigma_r\sigma_l)(1-\pi)^2 \qquad (1.28) \end{aligned}$$

Right-hand side of inequality (1.28) converges to $4\sqrt{q/2}(1-\pi)$ and the left-hand side to $1 + 3q/2(1-\pi)^2$ as σ_r approaches down to σ_l. Let $q = 1$ and $\pi = 0.5$ as the simulations in the next section suggest that tendency to be biased increases with q and decreases with π. Those values makes inequality (1.28) hold, which completes the proof. $\square$

1.5 Simulations and Comparative Statics

As non-convex optimization problems usually rely on heuristic approach, we present some simulations to give a sense of how the equilibrium looks like for particular cases. First, we would like to present how the equilibrium changes with respect to q. Upper-left panel displays how the equilibrium shifts with different values of q. The horizontal axis represents bias values. As q increases, maximum bias that can be reached goes up. In order for bias to reach 1 or -1, q must be 1 so that the strategy can be $(1, 0)$ or $(0, 1)$. Otherwise, by Lemma 1.4.1, The maximum that the bias can reach in absolute terms is less than 1. The public signal precision and the convexity of the cost function are fixed at $\pi = 0.5$ and $\alpha = 0.8$. The graphs suggest that the equilibrium tends to be more biased as q increases and that higher endowed outlets have higher advertising revenues.

Upper-right panel fixes π and q to be 0.5. As we argued before, higher α leads to bias. The bottom panel fixes q to be 0.5 and α to be 0.8. The simulation suggests that as precision of the public signal goes up, the profit of the media outlet drops and it tends to be less biased. Notice that there is a parameter set $(\pi = 0.5, \alpha = 0.8, q = 0.5)$ which is present in all the panels and it is associated with the same interior bias solution in each panel.

1.6 Conclusion

We design a model of the market for news with rational consumers and a profit-maximizing media outlet in which media bias arises endogenously depending on the marginal cost of acquiring information. The novelty of our paper is introducing a public signal and the way we define the bias. These two innovations provide us the

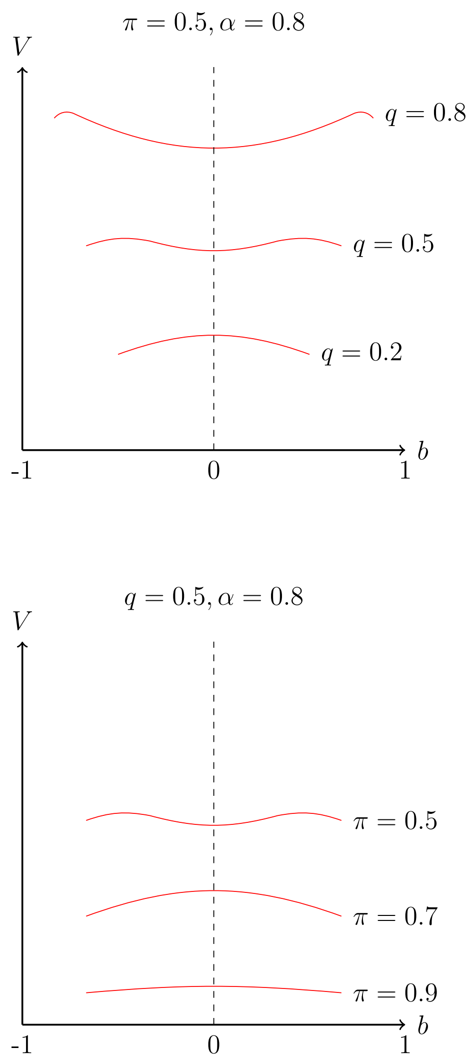

Figure 1.7. Profit of the outlet with respect to different parameter sets.

convexity of the profit function of the outlet which is the main driver of the bias. In contrast to the existing literature, our model does not have any exogenous asymmetry in the sense that voters have a common utility function, the distribution of priors is symmetric and journalists have symmetric 2 types.

One possible extension of this model might be analysing industrial organization aspects. To be more specific, solving this model for duopoly and oligopoly would be more helpful to understand the media polarization that we observe in the U.S. and to find out the impact of competition in the bias.

Chapter 2

Sustainable Reputations with Biased Review Platforms

2.1 Introduction

Some goods do not reveal their quality at the time of the purchase. Therefore, a certain degree of trust is required for trade to happen. However, trust is susceptible to exploitation. As an example, imagine a souvenir shop in a tourist destination so the probability of a customer comeback is low. With the quality of an item not observable by buyers at the time of a purchase, a seller is tempted to reduce the quality. As a result, rational consumers hesitate to shop at this store even when the quality is at its highest. The inefficiency described here has two sources. One is the seller's lack of commitment and the other is the inability of buyers to monitor the quality.

Online review platforms tackle the latter by providing a buyer with reviews from the past experiences, and an aggregate summary such as an average score. However, the reviews carry partial information. As a result, sellers can still build a reputation for being trustworthy.

In addition to the imperfectness of information on review platforms, there is also a question of trustworthiness. Mayzlin, Dover and Chevalier [36] show that when posting a review has no cost as in TripAdvisor, hotels feed made-up reviews in order to inflate their rating or malign a competitor. In light of this fact, we will analyze in this paper how much information rent a patient long-run player can gain against myopic short-run players in a repeated game with a review platform.

To the best of our knowledge, Fudenberg & Levine [19] presents the first model in which a long-run player with an unknown type repeatedly plays a simultaneous-move game against a sequence of short-lived opponents. Their result states that if there is a positive probability that the long-run player will always play the pure Stackelberg strategy, then her payoff in any Nash equilibrium exceeds a bound that converges to the pure Stackelberg payoff as her discount factor approaches one. Ekmekci [16] improves the highest attainable payoff to the long-run player by constructing a finite rating system. The rating system ensures that the long-run player attains almost her mixed Stackelberg payoff. To put it in a nutshell, the earlier literature lacks the characterization of an equilibrium that yields payoffs to the long-run player greater than or equal to the mixed Stackelberg payoff. Our paper demonstrates that it is possible that the mixed Stackelberg payoff can be attained. We construct a review platform which is similar to Ekmekci's [16] rating system and show that if there is a positive probability that the long-run player is a commitment type who plays the pure Stackelberg strategy every period, then the strategic type can achieve strictly higher payoffs than the mixed Stackelberg payoff.

The existence of a crazy role-model who takes a fixed action that other sellers are motivated to imitate is crucial in our model. The idea of a role-model approach was introduced by Kreps, Milgrom, Roberts & Milson [29].

Most of the studies where the seller's type is unknown take the complete information case as the benchmark. Fudenberg, Kreps & Maskin [18] show that the highest payoff to a long-run player cannot exceed the pure Stackelberg payoff in a repeated complete information game played against a sequence of myopic buyers.

Even though the early literature improves upon the complete information benchmark by adding crazy committed types, it lacks a characterization of equilibrium that yields payoffs close to the (mixed) Stackelberg payoff to the long-run player.

Later studies provide characterizations of such equilibria that attain almost the Stackelberg payoff to the long-run player. Pei [12] defines different types for the long-run player over the cost of producing a high-quality product. As a result, he characterizes an equilibrium in which the long-run player achieves a payoff arbitrarily close to her Stackelberg payoff. Liu [33] makes similar improvements to the long-run player's payoff by imposing a cost to short-run players for information acquisition.

Cripps, Mailath and Samuelson [11] (CMS hereafter) presents a negative result by showing that in games with imperfect monitoring, it is impossible to maintain a permanent reputation. However, there are various ways to sustain reputation effects for a long time. One of them is imposing a stochastic process governing the type of the long-run player through time. Holmstrom [27], Cole, Dow and English [10], Mailath and Samuelson [35], Phelan [8], Wiseman [44] and [45] are some examples of stochastic type. In our model, the type of the long-run player is determined once and for all before the game begins.

In contrast to CMS, reputation dynamics do not necessarily degenerate in review platforms despite the abundance of noisy but informative feedbacks. The reason for the persistent reputation dynamics is that information acquired by buyers may not be accumulated. An explanation for why the information may not be accumulated is that most of the time buyers only look at an aggregate summary rather than the complete history of reviews. Yelp provides a 5-star rating system based on the reviews that the restaurants receive. By looking only at an aggregate measure, buyers miss most of the past data and this behavior allows the seller to build reputation even in the distant future. Another possible explanation is driven by the fact that some review platforms such as eBay provide censored information. Doing so hinders buyers from stripping out the noise in the reviews.

Best and Quigley [6] considers a piece of similar machinery to ours to get around the CMS result. A review aggregator keeps myopic buyers uncertain about the past actions of a seller by garbling the history. Since they examine a persuasion game similar to Kamenica & Gentzkow [28], their technique does not apply to our model.

As we mentioned, we follow Ekmekci [16]'s lead in proposing a mechanism that determines the information revealed to the short-run player each period. We call that mechanism our review platform. Hence, one can interpret the information at a given period as an aggregate measure observed by the short-run player. A lot of past data is censored in our model. The reason for the censoring may be either due to how the review platform is designed or to the fact that buyers find it too costly to gather all the information available in the platform. An example for the former is eBay showing feedbacks given to a seller only in the most recent month.

In this paper, we focus on a specific form of information censoring. A central authority of a review platform announces a score from a finite set based on the past performance of the seller. The first score of the game is determined randomly. At the end of each period, unless the score is at its lowest, it may remain the same, increase or decrease by one depending on the past actions of the long-run player. We deviate from Ekmekci [16] in one way, which is allowing for a jump of two scores if the current score is the lowest. Even though this jump may seem inconsequential when the number of total scores is large, it significantly increases the highest payoff attainable to the long-run player. The review platform we design will condition the scores only on the actions of the long-run player except the initial period when there is no action observed.

Our review platform is a reminiscent of the machinery in Kamenica & Gentzkow [28]. Every score delivered to the buyers is associated with a posterior belief. Yet in our model, the posterior beliefs never hit the boundaries of the unit simplex. Posterior belief at a boundary refers to a complete information game where the payoff to the long-run player is dramatically lower than what we find.

It is fair to take the Stackelberg payoff as an upper bound as to how much payoff the long-run player can achieve. However, in the presence of a crazy type who is committed to a fixed action, the opportunistic long-run player can enjoy higher payoffs than the Stackelberg payoff. To fix this idea, suppose that buyers will purchase a product only if it meets a certain quality criteria and they mistakenly believe that the seller is likely to be committed to producing the high quality of a product, which is actually her dominated action. In this case, without any information revelation, the opportunistic type can constantly cheat buyers by producing the low quality. The Stackelberg action in this example would be mixing the low quality with adequate high quality in order to make buyers indifferent. Therefore, the opportunistic type enjoys strictly higher than the Stackelberg payoffs.

The new upper bound that we set on the payoff to the long-run player depends on the prior belief of buyers about her type. As is usual in the literature, we would like to consider small probabilities of the long-run player being a crazy type. We show that even when the prior belief is very small, the long-run player achieves strictly higher than the Stackelberg payoff, in contrast to the previous literature.

Besides the highest attainable payoff to the long-run player, another focus of attention of the literature is efficiency. According to Fudenberg & Levine [20], the inefficiency may be severe if the imperfection of the monitoring is large. Yet the review platform we propose offers equilibria with efficient allocations even though the monitoring is quite limited.

In our model, the long-run player is either a commitment type who plays the pure Stackelberg action or a normal type who has a moral hazard problem. The review platform observes the past play and the past scores, then announces a new score. The rule governing the transition from the past scores to the new one is called a transition rule.

We construct the transition rule as a function of a parameter and show the existence of a value of the parameter for which our results are satisfied. We cannot write down its value explicitly except asymptotically as discount factor goes to one and the number of possible scores goes to infinity.

In the next section, we describe the model in detail. In Section 2.3, we present the first step of our result. We present our result in two steps for the sake of clarity. In Section 2.4, we go over an example. Section 2.5 illustrates numerically the nature of the equilibrium and concludes. Section 2.6 concludes our result with the second step. Finally, Section 2.7 analyzes the payoffs to the short-run players.

2.2 Model

We study a product choice game played repeatedly between a long-run player (Player 1, Seller, she) and an infinite sequence of short-run players (Player 2, Buyer, he). At every period, a new short-run player enters the game and lives for one period.

2.2.1 The stage game

	B	N
H	$p-c$,v_H-p	$-c$,0
L	p,v_L-p	0,0

The pure actions available to player 1 are high and low effort: $A_1 = \{H, L\}$. Let $\Delta(A_1)$ denote the set of all probability distributions over A_1 and α_1 be a generic element of $\Delta(A_1)$.

Player 2's pure actions are buy or not: $A_2 = \{B, N\}$. Similarly, the set of all actions for player 2 is $\Delta(A_2)$ and α_2 denotes a generic element of $\Delta(A_2)$.

The players move simultaneously. The normal form of the stage game is depicted above. It is a standard product choice game where a buyer pays a fixed price p in order to buy the good and receives a value v_H if the seller exerted high effort and v_L if she exerted low effort. Buyers are interested in trade only if a certain amount of high effort is put by the seller: $v_H > p, v_L < p$. The seller has to incur a constant cost of c to exert high effort. Therefore, she has a dominant action of exerting low effort. She receives a payment of p which is higher than her cost if the product is sold. Hence, trade always makes her better off. In the unique Nash equilibrium of the stage game, player 1 exerts low effort and player 2 does not buy and the unique Nash equilibrium payoff profile is normalized to be $(0, 0)$.

Let us highlight the crucial conditions implied by the assumptions we made on the normal form game. Note that these conditions also exist in Ekmekci [16].

Condition 1: The incentive to exert low effort $c > 0$ is independent of whether the good is sold or not. This assumption is made for the sake of simplicity. We will discuss in more detail how it makes the construction easier later.

Condition 2: Trade is always profitable to the seller. This condition is one of those which allow us to have a punishment and reward phase for the seller depending on her past actions. The review platform punishes her with a higher probability of no-trade state in future.

Condition 3: There exists a non-degenerate α_1 such that for any α_1' with $\alpha_1'(H) \geq \alpha_1(H)$, the best response of player 2 to α_1' includes buying: $\alpha_1'(H)v_H + \alpha_1'(L)v_L \geq p$.

Condition 4: Player 1 prefers committing to exerting high effort to her Nash equilibrium payoff: $p - c > 0$.

2.2.2 Incomplete Information

For the sake of simplicity, we assume that there are only two types, commitment and normal as we call them. Prior to time 0, nature chooses player 1 to be either the commitment type with probability λ or the normal type with probability $1 - \lambda$. Let us denote the type space as $\Omega = \{n, c\}$.

The payoff structure of the normal type of player 1 is as shown in the stage game. The commitment type, on the other hand, exerts high effort with probability 1 at every period.

2.2.3 Repeated game with review platform

The stage game is played repeatedly between a seller and an infinite sequence of buyers with a different buyer each period. Player 1 discounts the future with $\delta < 1$. Player 2 lives for one period and only observes the outcome in his own period.

A review platform observes the past play and announces a public score $s \in S = \{1, 2, ..., K\}$ at the beginning of each period. The set of scores, S, is finite. The initial score is chosen randomly.

The review platform R is denoted as $\{S, \pi_0, P\}$ where S is a finite set of scores,

$\pi_0 \in \Delta(S)$ is the distribution that the initial score is drawn from and P is a transition rule. A transition rule maps histories that consist of past actions and scores to a distribution of probabilities over S. For our purpose, it is sufficient to focus on Markovian transition rules. We define a Markovian transition rule as a map $P : S \times A \to \Delta(S)$.

At time 0, s_0 is announced according to π_0. Then, players move simultaneously to play the stage game. The review platform observes the actions taken by the players and updates the score for time 1. This process is repeated infinitely.

2.2.4 Equilibrium definition:

At time t before the stage game is played, player 1 has a private history of h_1^t which consists of all past actions $\{(a_{1,0}, a_{2,0}), ..., (a_{1,t-1}, a_{2,t-1})\}$, and review scores $\{s_0, ...s_t\}$. Player 2 does not observe anything other than the current public score of s_t, so h_2^t only consists of s_t. Let H_1^t and H_2^t denote the set of all possible histories for player 1 and 2. Then, we can write $H_1^0 = S$, $H_1^t = (\Pi_{i=0}^{i=t} S \times \Pi_{i=1}^{i=t} A)$ for $t > 1$ and $H_2^t = S$ where $A = A_1 \times A_2$.

Let $H_1 = \cup_{t=0}^{\infty} H_1^t$ be the set of all possible histories for player 1. We use σ_1 to denote the strategy of player 1 which maps player 1's histories to the set of probability distributions over her action space, $\sigma_1 : H_1 \to \Delta(A_1)$. Since there are only two pure actions, we can identify a mixed strategy by the probability of playing one of the actions, say H: $\sigma_1 := \sigma_1(H)$. The strategy of a player 2 that lives at time t is a map $\sigma_{2,t} : S \to \Delta(A_2)$. Similarly, we will use $\sigma_{2,t}$ to denote $\sigma_{2,t}(B)$ from now on. Let σ_2 be the collection of all period-t strategies. The strategy spaces of player 1 and player 2 are Σ_1 and Σ_2 respectively.

Let $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$ denote the stage game payoffs of player 1 and 2 when the actions a_1 and a_2 are taken.

A review platform R, a strategy profile (σ_1, σ_2) and the type model (Ω, λ) altogether induce a probability distribution Z over action profiles and scores. The discounted average payoffs to the normal type of player 1 and period-t player 2 at time t are:

$$U_1(\sigma_1, \sigma_2|h_1^t) = E^Z\Big((1-\delta)\sum_{t=0}^{\infty}\delta^t u_1(a_1, a_2)|h_1^t\Big)$$
$$U_{2t}(\sigma_1, \sigma_2|h_2^t) = E^Z\Big(u_2(a_1, a_2|h_2^t)\Big)$$

Definition 1: A strategy profile (σ_1, σ_2) along with the beliefs $\lambda_t(s)_{t\in\mathbb{N}, s\in S}$ is a Perfect Bayesian Equilibrium if for all t,

(i) $U_1(\sigma_1, \sigma_2|h_1^t) \geq U_1(\sigma_1', \sigma_2|h_1^t) \;\; \forall\sigma_1' \in \Sigma_1, \;\; \forall h_1^t \in H_1$

(ii) $U_{2t}(\sigma_1, \sigma_2|h_2^t) \geq U_{2t}(\sigma_1, \sigma_2'|h_2^t) \;\; \forall\sigma_2' \in \Sigma_2, \;\; \forall h_2^t \in S$

(iii) The beliefs of player 2 are updated according to Bayes' rule.

The review platform we are going to construct does not allow for any out of equilibrium path. Therefore, $Z(s)$ is strictly positive for all $s \in S$. Player 1 knows her type but player 2 will need to update his belief using Bayes' rule when he observes a score.

2.3 First-step Result

Let B_2 denote the best response correspondence of player 2. It can be written as

$$B_2(\alpha_1) = \{\alpha_2 \in \Delta(A_2)|U_2(\alpha_1, \alpha_2) \geq U_2(\alpha_1, \alpha_2') \; \forall\alpha_2' \in \Delta(A_2)\}$$

Let $\bar{v}(\sigma_1)$ denote the Stackelberg payoff of player 1. That is,

$$\bar{v} = \max_{\alpha_1 \in \Delta(A_1)} \max_{\alpha_2 \in B_2(\alpha_1)} U_1(\alpha_1, \alpha_2)$$

and let α_s denote the Stackelberg strategy. Then, we can write

$$\bar{v} = \alpha_s(p-c) + (1-\alpha_s)p \tag{2.1}$$

The key here is that the Stackelberg payoff is a concept which has nothing to do with the different types of player 1. With a commitment type in the game, who is fine with receiving a less payoff than $\bar{v}$, the normal type can extract the rest of the remaining surplus. Then, let us define the adjusted Stackelberg payoff to the normal player 1. That is,

$$\bar{V} = \frac{\bar{v} - \lambda(p-c)}{1-\lambda} \tag{2.2}$$

which comes from the condition that average payoff to player 1 is $\bar{v}$ in its maximum: $\lambda(p-c) + (1-\lambda)\bar{V} = \bar{v}$. It is easy to check that $\bar{V}$ is strictly greater than $\bar{v}$. Finally, we assume that the prior belief λ is strictly less than α_s. Otherwise, hiding all the information from the buyers would be an easy solution. We chose to present our result in two steps. The first step assumes that the platform knows the type of player 1. We will get rid of this assumption in Theorem 2.6.1.

Theorem 2.3.1. *For any $\lambda \in (0, \alpha_s)$ and $u < \bar{V}$, there exists a $\bar{K}$ such that for all $K > \bar{K}$, there exists a $\bar{\delta}$ such that for all $\delta > \bar{\delta}$ there exists a review platform $R^* = \{S, \pi^0, P^*\}$ and a stationary Perfect Bayesian Equilibrium $\{\sigma_1^*(s), \sigma_2^*(s), \lambda^*(s)\}_{s \in \{1,\ldots,K\}}$ where the payoff to the normal type of the long-run player is at least u after every* *history.*

Proof Sketch: We present here the sketch of the proof and leave the formal proof to the appendix.

(i): The transition rule of the review platform induces a steady state distribution over the scores for each of the types. We first show that for the commitment type, each steady-state probability of the scores from 1 to $K/2$ is arbitrarily close to 0 for some large values of δ and K.

(ii): Next, by using (i), we show in the appendix that the probability of visiting score 1 for the normal player 1 is arbitrarily close to 0 for some large values of K and δ.

(iii): According to our equilibrium strategy, score 1 is the only state in which trade does not occur and (ii) states that the probability of visiting such a state even for the normal type is almost 0. The next step is to show that there is no unnecessary reputation built-up. In other words, we show that there exists a transition rule such that the posterior belief upon observing score K is equal to α_s.

(iv): Knowing the type of player 1, the platform chooses the posteriors to be equal to the steady-state probabilities. To make this idea more concrete, consider the case when the platform did not know the type of player 1. If the review platform did not know the true type of player 1, the early posteriors would be close to the prior belief λ and it would take an infinite amount of updates for the posteriors to reach the steady-states. Therefore at any point in time, the posterior at the highest score K would be less than the minimal belief for trade to take place. Therefore no buyer would buy at the highest score when player 1 does not exert any effort and our equilibrium unravels. In the equilibrium that we construct, player 2 is indifferent between buying and not buying at all the scores in which trade exists. As such, any perturbation of the transition rule or of the equilibrium strategy of player 1 will cause inefficiencies. Therefore, we assumed that the review platform knows player 1's type so it can choose the initial distribution to be equal to the steady-state distribution over type space and scores. □

Note that $\bar{V}$ is strictly greater than the Stackelberg payoff $\bar{v}$ for all $\lambda > 0$. Therefore, Theorem 1 states that even for small probabilities of a presence of the commitment type, the payoff the normal type of player 1 is higher than the Stackelberg payoff.

2.4 Example

Consider the stage game below. The payoff matrix satisfies all the conditions we

	B	N
H	1,2	−1,0
L	2,−2	0,0

stated before. The unique Nash equilibrium outcome of the stage game is $(0, 0)$. The Stackelberg strategy of player 1 is to play H with probability 0.5. Hence, her Stackelberg payoff is 1.5.

Figure 2.1 visualizes all payoff profiles of the stage game. $(1, 2)$ is the outcome when player 1 is committed to her pure Stackelberg action, and $(1.5, 0)$ is when she is committed to her mixed Stackelberg strategy in a complete information game. Overall, 1.5 is the highest feasible payoff to player 1. We do not claim to obtain any non-feasible payoff. However, we do make the distinction between the payoff to the commitment and the payoff to the normal type of player 1. The equilibrium we propose yields an average payoff profile arbitrarily close to $(1.5, 0)$. In other words, player 1 extracts all the information rent that is shared by the commitment and the normal type of player 1. Given that player 2 buys almost surely, the commitment type's payoff is arbitrarily close to 1. Hence, given $\lambda < 0.5$, the profile of the normal type of player 1 and player 2 will be arbitrarily close to a point on the line between $(1.5, 0)$ and $(2, -2)$, marked by the blue point. This gives player 1 a strictly higher payoff than 1.5.

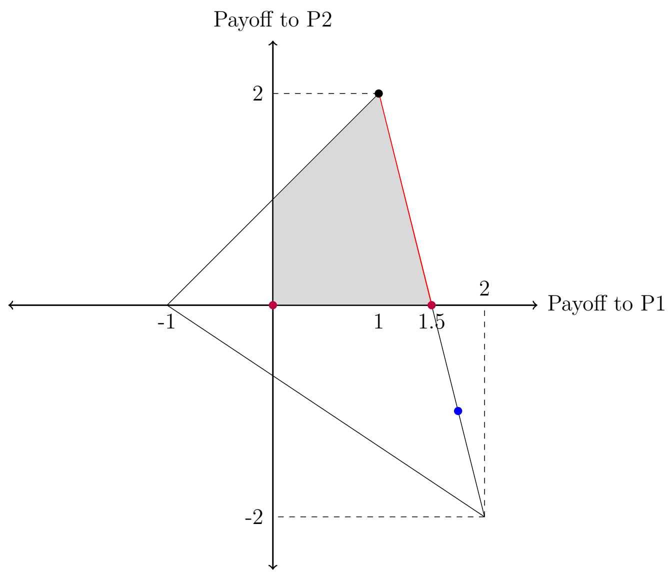

Figure 2.1. Payoff profile space of the stage game

The prior belief λ on player 1 being the commitment type is 1/3. $\lambda = 0$ represents the complete information case. In the complete information game, the highest payoff player 1 can achieve is 1. This is because if there is no commitment type, player 1 needs to exert high effort with positive probability whenever player 2 is buying. As a result, exerting high effort whenever her product is purchased is her best reply, from which her stage game payoff cannot exceed the payoff of 1, so is her average discounted payoff. This proof does not apply to the incomplete information game because with the existence of a commitment type, there will be some scores under which the best response of player 1 when her product is purchased is cheating player 2 by exerting low effort.

In the incomplete information game, Ekmekci [16] attains almost the Stackelberg payoff of 1.5 for player 1.

Let $S = \{1, 2, ..., K\}$ be the set of scores. Our theorem says that for any $u < 1.75$, there exists a $\bar{K}$ such that for all $K > \bar{K}$, there exists a $\bar{\delta}$ such that for all $\delta > \bar{\delta}$, there exists a review platform and a Perfect Bayesian Equilibrium where the payoff to the normal type of the long-run player is at least u after every history.

Step 1: Constructing the transition rule P. [1] Recall that the review platform updates the score based on whether H or L is observed. The commitment type always plays H and the normal type is free to choose between the two. We construct two transition matrices for high and low effort as displayed in Figure 2.2. Each matrix consists of transition probabilities from the most recent score to the new score given the action played by player 1. The rows represent the most recent score and the columns represent the new score. Consider the 2nd row. The punishment for playing L is losing the probability of upgrade which is 2/3 and having it added to the probability of staying at the same score. For all the scores except the lowest one, the score can be downgraded or upgraded by one, or remain the same. When the score is 1 at its lowest, it either jumps to 3 or remains at 1. This jump that we allow for, unlike Ekmekci [16], makes the computations harder but improves the highest payoff to player 1 significantly.

The first matrix is ergodic but the second is not. If L was to be played in all the states, there would be no way of reaching the high states. However, in the equilibrium we construct, there is always a positive probability of H being played except the highest score. Therefore, averaging the transition matrix for H, and that for L; we obtain an ergodic transition matrix. Hence there is a unique stationary distribution

[1] Referring to the appendix for the general form of the transition probabilities, we note that the transition rule depends on a parameter η. We show in the appendix that a value of η with the appropriate properties can always be found.

$$a_1 = H : \begin{bmatrix} 5/12 & 0 & 7/12 & 0 & \dots & & & \\ 1/6 & 1/6 & 2/3 & 0 & \dots & & & \\ & \ddots & \ddots & \ddots & & & & \\ \dots & 0 & 1/6 & 1/6 & 2/3 & 0 & \dots & \\ & & & \ddots & \ddots & \ddots & & \\ & & \dots & 0 & 1/6 & 1/6 & 2/3 & \\ & & & \dots & 0 & 1/6 & 5/6 & \end{bmatrix}$$

$$a_1 = L : \begin{bmatrix} 5/12 & 0 & 7/12 & 0 & \dots & & & \\ 1/6 & 5/6 & 0 & \dots & & & & \\ & \ddots & \ddots & \ddots & & & & \\ \dots & 0 & 1/6 & 5/6 & 0 & \dots & & \\ & & & \ddots & \ddots & \ddots & & \\ & & \dots & 0 & 1/6 & 5/6 & 0 & \\ & & & \dots & 0 & 1/6 & 5/6 & \end{bmatrix}$$

Figure 2.2. The transition probabilities based on the action of player 1

for each of them. Let $\pi_c^* \in \Delta(S)$ and $\pi_n^* \in \Delta(S)$ denote the stationary distributions of the commitment and the normal type respectively.

Step 2: Equilibrium strategies. Consider the strategies $\sigma_1^* : S \to [0,1]$ and $\sigma_2^* : S \to [0,1]$:

$$\sigma_1^*(s) = \begin{cases} 0 & \text{if } s \in \{1, K\} \\ \frac{0.5-\lambda^*(s)}{1-\lambda^*(s)} & \text{otherwise} \end{cases}$$

where $\lambda^*(s)$ is the posterior belief of player 2 on player 1 being the commitment type upon observing the score s. It is formed according to Bayes' rule:

$$\lambda^*(s) = \frac{\lambda \pi_c(s)}{\lambda \pi_c(s) + (1-\lambda)\pi_n(s)}$$

and

$$\sigma_2^*(s) = \begin{cases} 0 & \text{if } s = 1 \\ \\ 1 & \text{otherwise} \end{cases}$$

The equilibrium we are proposing has 3 phases:

(i): Score 1 serves a punishment phase. Player 2 does not buy the product. Condition 2 states that player 1 strictly prefers trade so no-trade is a punishment. The transition rule at this score is independent of player 1's action. Hence she plays L.

(ii): Middle scores from 2 to K-1 serve as reputation building phase. Player 1 puts the least amount of effort necessary for player 2 to buy the product. Hence, buying is one of the best responses for player 2.

(iii): Score K serve as the reputation milking phase. Since player 2 thinks that player 1 is likely to be the commitment type, he buys the product. The transition rule at this score is independent of actions again, therefore player 1 plays L.

Step 3: Perturbing P to make σ_1^* optimal. Given σ_2^* and $\delta < 1$, the proposed strategy σ_1^* for the normal type of player 1 is not optimal. We need to perturb P so that the difference in the continuation values of any two adjacent scores must be $1.5(1-\delta)/\delta$. In order to understand this argument, let us calculate the difference between expected continuation values to player 1 from playing H and L. At any middle score $s \in \{2, 3, ..., K-1\}$:

$$V(s|H) = (1-\delta)u_1(H, B) + \delta[2/3V(s+1) + 1/6V(s) + 1/6V(s-1)]$$

$$V(s|L) = (1-\delta)u_1(L, B) + \delta[5/6V(s) + 1/6V(s-1)]$$

Since player 1 builds reputation at middle scores by mixing H and L, we must have $V(s) = V(s|H) = V(s|L)$ which implies $V(s+1) - V(s) = 1.5(1-\delta)/\delta$.

Let $P(i, j, a_1)$ denote the probability of moving from score i to score j when action a_1 is played. We perturb P in the following way:

$$P^*(s, s-1, a_1) = 1/6 + (K-s)(1-\delta)/\delta$$

$$P^*(s, s, a_1) = \begin{cases} 1/6 - (K-s)(1-\delta)/\delta & \text{if } s > 1 \text{ and } a_1 = 1 \\ 5/6 - (K-s)(1-\delta)/\delta & \text{if } s > 1 \text{ and } a_1 = 0 \\ 5/12 + (K-s)(1-\delta)/(2\delta) & \text{if } s = 1 \end{cases}$$

$$P^*(1, 3, a_1) = 7/12 - (K-s)(1-\delta)/(2\delta)$$

For any fixed K, the perturbed transition probabilities are well defined for δ sufficiently close to 1. The perturbed transition rule ensures that given player 2's strategy σ_2^*, σ_1^* is optimal.

Step 4: Fixing K and δ. We have shown that given the proposed σ_2^*, σ_1^* is optimal. Now we will show that for some (K, δ), σ_2^* is optimal given the beliefs held by player 2. For any middle score and the posterior upon observing that score, σ_1^* is high enough for player 2 to buy the product. In order for player 2 to buy at the score of K as well, we need the reputation of player 1 at the highest score, $\lambda^*(K)$, to be at least 0.5. The perturbed transition rule together with σ_1^* induces a posterior belief upon observing each score. Suppose that K is 50 and δ is 0.9996. With given parameters, $\lambda^*(K)$ becomes 0.501. Hence, player 2 buys the product even though the normal type of player 1 exerts low effort. Note that, as K increases, $\lambda^*(K)$ may drop below 0.5. However, as we show in the appendix, there is always a transition rule which makes $\lambda^*(K)$ at least 0.5. We could have constructed a transition rule to obtain a much higher reputation than 0.5 at the highest score as Ekmekci [16] did. However, while constructing the transition rule and the equilibrium, we economize on the reputations that player 1 would have because higher reputation implies a higher amount of effort having been spent. Therefore, we would like the belief at the score of K as close as possible to 0.5. We show in the appendix that there is a review platform that makes $\lambda^*(K)$ equal to 0.5

Step 5: Early periods. If the review platform did not observe the type of player 1, the early scores announced by the platform would not be as informative. In order to understand this argument better, consider the very first period at time 0. Since there would be nothing to observe yet, the initial score would have no information and thus the prior belief would not be updated. Yet the posterior beliefs would converge to the steady state beliefs with time. We cannot overcome this issue by any perturbation because player 2 is indifferent in all scores bigger than 1. Hence, we assume that the review platform can choose an initial distribution depending on player 1's type. That is,

$$\pi^0(w, s) = \begin{cases} \pi_c^*(s) & \text{if } \omega = c \\ \pi_n^*(s) & \text{if } \omega = n \end{cases}$$

2.4.1 No-jump Case

In this section, we are going to analyze the example in more detail to show the improvement in the payoff to the long-run player relative to Ekmekci [16]. To start with, as we mentioned earlier, we first change his setup by allowing for jumps between the review scores. However, there is another essential step forward to our results. That is economizing the reputations. We present two equilibria. The first one is

$$
a_1 = H : \begin{bmatrix} 5/8 & 3/8 & 0 & \dots & & & \\ 1/8 & 5/8 & 1/4 & 0 & \dots & & \\ & \ddots & \ddots & \ddots & & & \\ \dots & 0 & 1/8 & 5/8 & 1/4 & 0 & \dots \\ & & & \ddots & \ddots & \ddots & \\ & & \dots & 0 & 1/8 & 5/8 & 1/4 \\ & & & \dots & 0 & 1/8 & 7/8 \end{bmatrix}
$$

and

$$
a_1 = L : \begin{bmatrix} 5/8 & 3/8 & 0 & \dots & & & \\ 1/8 & 7/8 & 0 & \dots & & & \\ & \ddots & \ddots & & & & \\ \dots & 0 & 1/8 & 7/8 & 0 & \dots & \\ & & & \ddots & \ddots & \ddots & \\ & & \dots & 0 & 1/8 & 7/8 & 0 \\ & & & \dots & 0 & 1/8 & 7/8 \end{bmatrix}
$$

.

Figure 2.3. The transition probabilities without jump

the most economizing equilibrium given a review platform with no jump as in as the rating system of Ekmekci [16]. In the second one, player 1 plays H and L with equal probabilities whenever he is indifferent as she does in Ekmekci [16] but under a review platform with a jump as in ours. To make this part more clear, we are going to find v_1 and v_2 in Table 2.1 where the payoff row represents the payoff in the limit to player 1. According to the example, our review platform can achieve payoffs to

	Example 1	Example 2	Ekmekci	Us
Economized Reputations	Yes	No	No	Yes
Jump	No	Yes	No	Yes
Payoff	v_1	v_2	1.5	1.75

Table 2.1. Payoffs to the long-run player under different settings and equilibria

the long-run player arbitrarily close to 1.75 whereas Ekmekci [16] obtains almost 1.5. Suppose the transition probabilities without the perturbation are as follows in Figure 2.3: Furthermore, we propose the same strategies whenever player 1 is indifferent for

$s < K$:

$$\sigma_1^*(s) = \begin{cases} 0 & \text{if } s \in \{1, K\} \\ \frac{0.5-\lambda^*(s)}{1-\lambda^*(s)} & \text{otherwise} \end{cases}$$

Then, as long as K is greater than 3, the posterior belief at K is going to be higher than 0.5, hence player 1 is able to cheat player 2 by exerting no effort. In this equilibrium, there is no reputation wasted. In other words, player 1 is never exerting more effort than necessary. However, if we wrote down the recursive formula and computed the value functions, each payoff would be less than 1.5. Finding the same value functions as in Ekmekci [16] as a result of the recursive equations makes perfect sense analytically because the only thing we changed is the randomization probabilities which should not affect the outcome. The intuition though for the values not increasing is that as player 1 exerts less effort, with the platform Ekmekci [16] designed, the probability of visiting the lowest score in which no trade occurs significantly increases because his rating system does not have the upward shift. The upward shift is; between any pair of adjacent scores, the steady-state probability of the higher score is greater than the lower one. Recall that in our platform, that probability is almost 0. Hence, our platform enables the long-run player to economize his reputation without the worry over no-trade possibility.

Next, we are going to look at another equilibrium when the platform has the same attributes as ours but player 1 is not smart enough to economize on his reputations. Suppose the transition probabilities are as shown at the beginning of this section in Figure 2.2, and player 1 plays H and L with probability 0.5 whenever he is indifferent. Recall that in order to achieve the properties of our equilibrium, the number of total scores K must be a large number. Moreover, the posterior belief at the highest score is 0.5 so that player 2 is indifferent. With player 1 exerting more effort at the middle scores, the probability of her visiting the highest score will be higher than before, decreasing the posterior belief below 0.5 In order to bring it back up to 0.5, we would need to update the transition probabilities by the parameter η (see Appendix for more information on η). In our example, when the review platform is designed as we proposed and player 1 economizes perfectly all her reputations, η goes to 1.5 as the total number of scores K increases for δ close enough to 1. Therefore, for a value of η slightly bigger than 1.5, there are values of K and δ such that the payoff to the long-run player is almost 1.75. When player 1 is playing H and L with probability 0.5 whenever she is indifferent, we would need to update η to be slightly bigger than a value of around 1.66. Remember that our set of recursive equations in the Appendix gives us the following formula for the value functions:

$$V(s) = 2 - \frac{\eta - 1}{2} - \frac{(K - s)(1 - \delta)\eta}{\delta}$$

Hence, the values converge to 1.67 for every score. The updated version of Table 2.1 is shown below. As we see, the jump between the review scores is essential in passing

	Example 1	Example 2	Ekmekci	Us
Economized Reputations	Yes	No	No	Yes
Jump	No	Yes	No	Yes
Payoff	1.5	1.67	1.5	1.75

Table 2.2. Payoffs to the long-run player under different settings and equilibria

the threshold of mixed Stackelberg payoff. However, it is not enough to attain the maximum information rent available. In order for that to happen, we need player 1 to be smart enough to distribute her reputations over the scores wisely. Our equilibrium represents such a situation and the resulting payoff is the highest that she could ever attain in any setting. It is worth to note that the platform in Example 2 and the one we proposed are not identical. Nonetheless, the functional forms are. The review platform has been defined on only one input argument which is η. The updated η in Example 2 gives us the highest attainable payoff to the long-run player given the restriction that she exerts high effort with probability half whenever she is indifferent.

2.5 A Numerical Illustration

As in the example, let us fix K at 50 and δ at 0.9996. With η equal to 1.505, the resulting posterior beliefs look like in Figure 2.4. Note that Lemma 3 provides an analytical solution for each belief. That is, $\lambda^*(s) = 0.5^{K-s+1}$. As a result, the beliefs

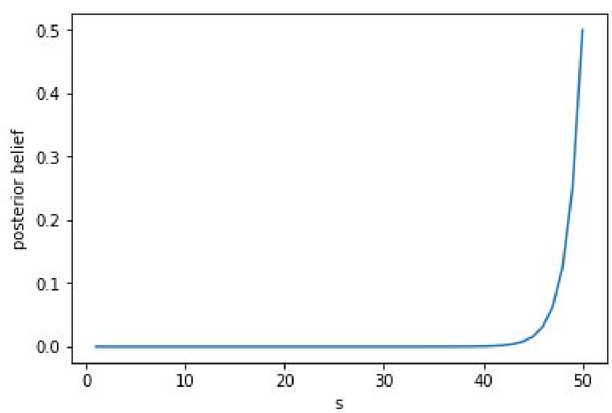

Figure 2.4. Steady state posterior belief distribution over the scores

float right above zero for all except the very top scores. This is intuitive because, at the steady state, the probability of being at one of the top scores is higher than one of the lower scores. At all the low scores where the posterior belief is small, the normal type of player 1 plays H with a probability close to 0.5.

Figure 2.4 implies that reputation building is slower at low scores, which is a realistic finding because buyers hesitate to trust when the review scores are low.

The recent literature has attempted to explore the limits of attainable payoffs to the long-run player and we believe that this paper sets a new bar by reaching payoffs that were considered to be non-feasible.

2.6 Main Result

Our results do not rely on the platform knowing the type of player 1. In this section, we demonstrate that attaining as high payoffs to the long-run player as we have is still possible with the review platform who is unaware of the type of the long-run player. However, our new equilibrium will depart from the previous one in a way. That is, we would need time-dependent strategies for the long-run player for some initial finite number of periods.

When the designer of the review platform does not know the type of the long-run player, the current transition rule does not work. In order to understand why it does not, let us consider the very first period. In the first period, there is no past action played and the platform's only option is to condition the score on the type of player 1. If the designer does not have access to that information, the first score will be uninformative. Hence, the belief will not be updated. As the game progresses and the platform observes more actions, the beliefs will get closer to their steady-state levels. However, they will never be equal to those. This fact makes it hard to economize the reputations. The reason is that what we mean by economizing the reputations is exerting such an amount of effort to make player 2 indifferent given the belief he holds. Since a posterior belief upon a given score is never equal to the posterior beliefs when the same score was observed before, we cannot maintain a stationary equilibrium which yields as high payoff as we have found. Therefore, we need the long-run player to play non-stationary strategies. Nevertheless, this is not enough either. Even though we can take care of the middle scores by imposing non-stationary strategies, there is no way to fix the issue at score K. That is, we cannot enable the normal player 1 to cheat because, in the cheating state, she strictly prefers action L so cannot randomize between the two actions. With the uninformative initial score, the posterior belief at score K will always be less than what is needed in order for player 2 to buy the product unconditionally. Therefore, our proposed equilibrium will not work in this case. In addition to non-stationary strategies, we also need to tweak the platform to make the posterior belief at score K slightly higher than what it was before. As such, after a finite amount of periods, the posterior belief will exceed the minimum belief player 2 ought to have to buy unconditionally. Let us call that finite period N. Up until N, player 2 does not buy if the normal player 1 plays according to σ_1^* because as we said there will be some scores under which the beliefs will never be high enough to sustain σ_1^* and score K is one of them. Without a reward period, we can not incentivize the normal player 1 to put any effort at the middle scores. For this reason, we need to update the strategies and the review platform. We let stage-game Nash equilibrium be played until period N and adjust the platform such that the unconditional probability of announcing every score stays unchanged even though the normal player 1 is playing a completely different strategy. The way we do it is as the following. Remember that the normal player 1 had a transition rule P_n^* equal to a combination of $P^*(H)$ and $P^*(L)$, that is $\sigma_1^* P^*(H) + (1-\sigma_1^*)P^*(L)$ with some abuse of notation. Let us denote the new adjusted transition rule function by P^{**}. P_c^{**} will be the same as $P^*(H)$ so we set $P^{**}(H) = P^*(H)$. On the other hand, P_n^{**} is $P^{**}(L)$ until period N now, therefore we set $P^{**}(L) = P_n^* = \sigma_1^* P^*(H) + (1-\sigma_1^*)P^*(L)$. To put it in simple terms, the platform announces the scores until period N as if the normal player 1 is playing according to σ_1^* even though she is not. From N on, she will be playing time-dependent strategies due to the argument we made previously. As a result, we can assert the following theorem.

Theorem 2.6.1. *For any* $\lambda \in (0, \alpha_s)$ *and* $u < \bar{V}$*, there exists a* $\bar{K}$ *such that for all* $K > \bar{K}$*, there exists a* $\bar{\delta}$ *such that for all* $\delta > \bar{\delta}$ *there exists a review platform* $R^{**} = \{S, \pi_0^{**}, P^{**}\}$ *and a Perfect Bayesian Equilibrium* $\{\sigma_{1,t}^{**}(s), \sigma_{2,t}^{**}(s), \lambda_t^{**}(s)\}_{t=0,1,2,\ldots;s\in\{1,\ldots,K\}}$ *where the payoff to the normal type of the long-run player is at least* u *after every history.*

Sketch of the proof: We provide here a sketch of the proof. The formal proof can be seen in the appendix.

i) Constructing the platform: In order to make player 2 strictly prefer buying at the score of K, we tweak the transition rule by slightly increasing the probability of staying at K when H is played. Since the normal type does not play H at the score K, this tweak does not affect the transition rule regarding the normal type. No matter how small the tweak is, there will be a time period before infinity that the belief upon observing K will exceed the α_s and stay above it forever.

ii) Equilibrium strategies: Let us call N the period when the belief upon observing the score of K would exceed α_s the first time. Note that K is not necessarily observed in the period N. Since N is finite, we do not worry about what happens before N by considering the values of δ approaching 1. As a result, we propose no-trade strategy profile until N. From N onward, we propose again the minimal effort level for each middle score, that makes player 2 indifferent. For scores 1 and K, the normal player 1 still exerts no effort.

iii) The intuition behind the optimality of the strategies: As we discussed, the platform not knowing the type of player 1 brought an issue to our equilibrium where player 2 was indifferent at all the scores expect the lowest one when there is no trade. We solved the issue for the score of K by tweaking the platform to make the option of buying strictly preferable. However, there may always be some other scores that need to be taken care of. By having introduced the time-dependent strategies, we do that. However, the transition rule regarding the normal type of player 1 is distorted with the time-dependent strategies. Therefore, we need to make sure that the belief upon observing K remains above α_s. We do that in the appendix.

iv) The periods until N: Up until N, we proposed no-trade strategies to be played. Then, with the given transition rule, the posterior belief at the score of K would be nowhere near α_s. Actually, it would be much higher than that because having exerted no effort thus far, the probability of visiting the highest score for the normal type would be quite low compared to what it would be if she played the minimal effort to make player 2 indifferent at every middle score. In order to solve this problem, we update the transition rule as follows. Before the time N, the platform announces a score as if the normal player 1 is following a strategy that prescribes a positive amount of effort for every middle score. This strategy is nothing but what the time-dependent strategies of the periods after N are converging to. In other words; even though the normal type is not putting any effort, the platform is covering her by keeping the unconditional probability of observation of the higher scores up.

2.7 The size of the jump

In the example we gave, the jump needed to sustain the target payoffs was from score 1 to score 3. We consider that as having size 2. Basically, having size 1 means that there is no jump. Even though this is what differentiates this paper from Ekmekci [16], we do not want to design an unrealistic review platform with an unreasonable jump. Therefore, in this section, we analyze how the minimum size of the jump changes with the model settings.

Combining equations 2.1 and 2.2, and plugging in the condition $\alpha_s = \frac{p-v_L}{v_H-v_L}$; we find

$$\bar{V} = \frac{p}{1-\lambda} - \frac{(p-v_L)c}{(v_H-v_L)(1-\lambda)} - \frac{\lambda(p-c)}{1-\lambda} \tag{2.3}$$

We need to sustain the value in 2.3 for every score s. As we expressed before, the problem is when the score 1 because that is the only state where there is no trade. Let us find the value of being at score 1 in general terms:

$$V(1) = \delta[\gamma V(1+j) + (1-\gamma)V(1)] \tag{2.4}$$

Our indifference condition for the long-run player to be randomizing at all the middle scores requires

$$V(s+j) - V(s) = j\frac{\eta(1-\delta)c}{\delta} \tag{2.5}$$

where j is the size of the jump. Substituting 2.5 into 2.4 yields

$$V(1) = j\gamma c\eta \tag{2.6}$$

Note that we ignore the perturbation terms for the sake of this analysis as it is assumed to be negligibly small.

By plugging in the asymptotic value of η, we find

$$V(1) = j\gamma c\Big(1 + \frac{c(\alpha_s-\lambda)}{\alpha_s(1-\lambda)}\Big) \tag{2.7}$$

The expression above for V(1) has to be arbitrarily close to $\bar{V}$ of equation 2.3.

We already know one example where the size of the jump has to be at least 2. At this point, we give an example where size 1 is enough.

Suppose that $v_H = 7$, $v_L = 3$, $p = 5$, $c = 4$, and $\lambda = 0.2$. The stage game becomes As a result, α_s is still 0.5 and $\bar{V}$ is 3.5. Equation 2.7 yields that $V(1)$ is $16j\gamma$. The

	B	N
H	1,2	−4,0
L	5,−2	0,0

only restriction we have on γ is that it be strictly smaller than 1. Hence, the size of the jump, j, does not need to be greater than 1.

By taking the derivative of $V(s)$ with respect to c, v_H and v_L one by one, we conclude that the minimum size of the jump required is positively correlated with v_H and λ; negatively correlated with v_L and c. The sign of the derivative with respect to the price is ambiguous as it depends on the other parameters.

2.8 Regulation

As most of the previous literature, we focus on how a long-run player benefits from reputation effects. However, the purpose of a review platform is supposed to make short-run buyers better off. The reason the literature has focused on the payoffs to the seller is that the non-trivial results lie in the search of those payoffs. Nevertheless, some earlier literature analyzes if it serves the buyers to reveal full information or conceal some of it in order to avoid the cold start problem pointed out by Lillethun [32]. The cold start problem is when buyers are certain that they are facing a strategic seller and therefore afraid to buy. This implies that it may be better for short-run players to be uninformed of some information known by the platform. In our model, depending on our assumption, the platform may observe first the type of the seller and then certainly observes her action each period. We find that a cold start problem exists if the platform knows and reveals the type of seller to the buyers. We need to elaborate on the cold start problem in our context because its definition is slightly different here. We define the cold start problem as the existence of the equilibrium of no-trade. As such, if the platform knows and reveals the type of the buyer, the game turns into a complete information game where no-trade equilibrium is possible. However, if the type of the seller is not revealed, we can use the Fudenberg / Levine [19] that the probability of no-trade even in the worst equilibrium for the seller is arbitrarily small. We propose a simple review platform which is fully informative on the most recent action of the seller in order to obtain a grim-trigger equilibrium. The platform has two possibles scores, the lower of which is announced if the seller plays L and the higher of which is announced when the seller plays H; and the punishment is no-trade. Under such a review platform, the only stationary equilibrium is the seller exerting high effort and the buyers buying every period.

Theorem 2.8.1. *There exists a review platform that survives the cold start problem. It is defined for values of δ close to 1 as R:*

i) R does not know or reveal w,

ii) $K = 2$,

iii) $P(1, 2, H) = P(2, 2, H) = 1$ and $P(1, 1, L) = P(2, 1, L) = 1$.

Proof: See Appendix.

Chapter 3

Empirical Evidence

3.1 The Equilibrium's Observable Characteristics

Before we move on to the next section which is going to demonstrate the empirical evidence of our equilibrium, we are going to present some equilibrium properties.

First, we are going to look at the effort levels and the posterior beliefs based on score together.

For this analysis, all our calculations are in regard of the long-run equilibrium characteristics. We pin down η by the condition that $\lambda^*(K)$ is approximately equal to α_s. Moreover, we know that the effort level at score K-1, $\sigma_1^*(K-1)$, is written in terms of $\lambda^*(K-1)$. Going further, we can write $\lambda^*(K-1)$ in terms of the transition probabilities, π_c^* and π_n^*. Finally, referring to the equations 9 and 10, we are back to an expression with respect to $\sigma_1^*(K-1)$. As a result, we find $\sigma_1^*(K-1) = \frac{\alpha_s}{1+\alpha_s}$. Plugging this expression into first equation we had in hand for $\sigma_1^*(K-1)$ (equation 14), we find that $\lambda^*(K-1)$ is equal to α_s^2. By iterating downwards, we find $\lambda^*(s) = \alpha_s \lambda^*(s+1)$ which implies $\lambda^*(s) = \alpha_s^{K+1-s}$ for scores from K all the way down to the score that is jumped from score 1. Additionally, we find

$$\sigma_s^*(s) = \frac{\alpha_s - \alpha_s^{K+1-s}}{1 - \alpha_s^{K+1-s}}.$$

Taking the derivative of $\sigma_s^*(s)$ with respect to score, we find that the effort level is decreasing with score:

$$\begin{aligned}\frac{d\sigma_s^*(s)}{ds} &= -\frac{(1-\alpha_s)log_{\alpha_s}^{K+1-s}\alpha_s^{K+1-s}}{(1-\alpha_s^{K+1-s})^2} \\ &< 0.\end{aligned}$$

We are going to dig deeper at the moment but we can also conclude that the second derivative is also negative and thus, $\sigma^*(s)$ is concave function. Our only take-away is that it is decreasing.

In real life, a review platform obviously does not assign a score to a business regardless of customer reviews. Based on the previous literature, what happens is that the business owner fabricate fake reviews in order to inflate their ratings. We assume that customers leave a noisy feedback on a business. This review is added up to the average rating of a business. The next customer who is to decide whether to go the restaurant or not looks at the average star or most recent scores in order to have an idea about the restaurant. After the visit, this customer may also leave a review. Upon a review, the business owner evaluates the overall rating of the restaurant and fabricate some good reviews to inflate the rating if need be. We claim that the observable variables affected by this honest reviewing and fabrication processes will reflect the patterns suggested by our equilibrium.

In order to make a solid analysis, we need to know if a review is fake or not. We could have developed an algorithm to predict if a review is fake or not but using our own unsupervised learning technique to strengthen our hypothesis would be very questionable. While we thought that it was impossible to find information from an unbiased authority on whether online reviews on a review platform are fake or not, we got lucky. Yelp has its own algorithm to detect fake reviews. This brings a huge opportunity to our work in the direction of empirical analysis. We can observe if a review is fake or not. Of course, there may be a question of how trustworthy the Yelp algorithm is. It does not have to be very accurate but we are going to assume that it is unbiased. It is unbiased in the sense that Yelp's algorithm works independent of the score of the review. In terms of its accuracy, our formal assumption is that the proportion of fake reviews is strictly smaller among the reviews Yelp publishes than among the reviews that Yelp censors off. We consider this to be a modest assumption whose validity can be qualitatively evaluated.

3.2 Data

As the popularity of review platforms has grown, so have the concerns about the trustworthiness of the reviews because the credibility of the platforms can be undermined by businesses leaving fake reviews for themselves or for their competitors. There is more than enough evidence that this type of cheating is present in the industry. For instance, the New York Times recently reported on the case of businesses hiring workers on Mechanical Turk – an Amazon-owned crowd-sourcing marketplace – to post fake 5-star Yelp reviews on their behalf for as little as 25 cents per review. In 2004, Amazon.ca mistakenly revealed the identities of "anonymous" reviewers, briefly unmasking considerable self-reviewing by book authors. The fake 5-star ratings were not made public though.

To get around the obstacle of not observing fake reviews, we begin by exploiting a unique Yelp feature: Yelp is the only major review site I know of that allows access to censored reviews that it has classied as illegitimate using a set of machine learning techniques and manual observation. Filtered reviews do not count while calculating a business' average star-rating. Also, they are not published on the main listing of a business. However, if a visitor scrolls all the way down to the bottom of a business' site on Yelp, there is a section called '... other reviews not recommended by Yelp. If the visitor clicks on that link, s/he is directed to the reviews that the Yelp algorithm has labeled as fake. Among those reviews, not all of them are 5 stars. There are also reviews with 1-star and something between. We can attribute the first feature to competitors writing bad reviews to deflate the rating of the business. We can attribute the second feature either to the algorithm making a Type I error or the fake reviewers being very sneaky and leaving mediocre reviews once in a while to avoid from getting caught. We are not interested in finding out the incentives for the different starts of ratings though. What is important for us is that most of the reviews that are labeled as fake by Yelp are 5-stars so the algorithm has a fair amount of accuracy.

Overall, roughly 16% of restaurant reviews are filtered by Yelp. While Yelp's goal is to filter fake reviews, the ltering algorithm is imperfect as we just mentioned. Therefore, there are both false positives (Type I error, i.e., filtered reviews that are not fake) and false negatives (Type II error, i.e., fake reviews that were not filtered). Those errors in the classication aects our interpretation of filtered reviews in two important ways. First, the rate of fake reviews on Yelp could potentially be higher or lower than the 16% that are filtered. Second, the existence of false positives implies that perfectly honest restaurants may sometimes have their reviews filtered. Similarly, there may be restaurants with no filtered reviews that have successfully committed review fraud. Hence, we do not use filtered reviews to identify specic businesses that committed review fraud. Instead, our main focus is proving the economic incentives to commit review fraud. Moreover, we provide further empirical support for using filtered reviews as proxy for low effort level by the business and analyze their impact on the future real reviews.

Yelp contains more than 70 million reviews of restaurants, barbers, mechanics, and other services, and has a market capitalization of roughly four billion dollars. For our analysis, we only use feedback given to restaurants. Yelp provides a public data set that contains 3GB of review information by millions of users. However, the data includes only the published reviews, not the filtered ones. In order to get abundance of filtered reviews, we use Selenium in automating our web scraping process (The Python codes can be found in Appendix). As a result, we scrape the published and filtered reviews of randomly selected 1073 restaurants all over the US. The total number of reviews that we are analyzing is 662,995. Our data set has 4 columns, the first of which shows the date and time a review was written, the second of which is how many stars were given, the third of which is if a review was labeled as fake or honest by the Yelp algorithm, and finally the fourth of which is the restaurant identity. We add the restaurant identities to our data set because we are going to be controlling for restaurant fixed effects.

Our main analysis takes filtered reviews as a proxy for fake reviews. However, one might be concerned that we are reverse engineering Yelp's algorithm rather than analyzing fraud. To support our interpretation, and to provide further insight into the economics of review fraud, we collect and analyze a second data set consisting of businesses that were caught in the act of soliciting fake reviews. This second data set derives from a series of sting operations that Yelp began performing in October 2012. The goal of these stings was to uncover businesses attempting to buy fake reviews.[1]

[1] https://blog.yelp.com/2018/08/consumer-alerts-buyer-beware

Yelp performed the stings by looking for fake review solicitations on classied ads boards like Craigslist – the sting did not rely on the filter in any way. By responding to these solicitations, Yelp was able to expose the identities of the businesses that were attempting to commit review fraud. Businesses which Yelp determined to be buying fake reviews received a notice on their Yelp pages. The notice, known as a consumer alert, lasts for at least 90 days, and may be renewed if a business does not seize its efforts to commit review fraud. Yelp continues to perform sporadic sting operations to the present day.

During March 2014, Luca & Zervas [34] collected data from all US businesses listed on Yelp. They identied 126 businesses – none of which are Boston restaurants – that had received a consumer alert over the prior 90 days, and collected their entire review histories. In total, the data set of businesses that received consumer alerts contains 2,233 published and 8,246 filtered reviews.

To cross-validate the accuracy of Yelp's filter, Luca & Zervas [34] analyze the rate of filtered reviews among businesses that were caught in the sting (which did not affect the filter). They hypothesize that if filtered reviews are a reasonable proxy for fake reviews, then businesses caught in the sting should also have higher rates of filtered reviews. Their sting data set supports this hypothesis: among businesses that received a consumer alert the average fraction of reviews that are filtered is 79%, compared to 19% for an average restaurant. This suggests that Yelp's filtering algorithm is doing a reasonable job of identifying review fraud and provides support for our use of filtered reviews as a proxy for review fraud. This also shows that on average almost a fifth of the reviews written on Yelp is determined to be so dubious by Yelp's internal filter and is banned to the censored reviews page that regulars users do not bother to see. This shocking result proves the validity of our analysis.

3.3 Empirical Strategy

In this section, we introduce our empirical strategy for identifying the economic incentives behind Yelp review fraud. Ideally, if we could recognize fake reviews, we would estimate the following regression model:

$$y^*_{it} = x^{'}_{it}\beta + b_i + \tau_t + \epsilon_{it} \tag{3.1}$$

where y^*_{it} is the star rating of a fake review business i received during period t, $x^{'}_{it}$ is a vector of time varying covariates measuring a business' economic incentives to engage in review fraud such as the average star rating at time t, β are the structural parameters of interest, b_i is business fixed effect, τ_t is time fixed effect, and the ϵ_{it} is an error term.

As is often the case in studies of gaming and corruption (e.g., see Mayzlin et al. (2014), Duggan and Levitt (2002), and references therein) we do not directly observe y^*_{it}, and hence we cannot estimate the parameters of this model. To proceed, we assume that Yelp's filter possesses some positive predictive power in distinguishing fake reviews from genuine ones. Is this a credible assumption to make? We have already argued that it is. While Yelp is secretive about how its review filter works, it states that "the filter sometimes aects perfectly legitimate reviews and misses some fake ones, too," but "does a good job given the sheer volume of reviews and the difficulty of its task." The analysis by Luca & Zervas [34] of businesses that were caught committing review fraud provides further empirical evidence supporting our use of filtered reviews as a proxy for review fraud. In addition, we suggest a subjective test to assess the assumption's validity: for any business, one can qualitatively check whether the fraction of suspicious-looking reviews is larger among the reviews Yelp publishes, rather than among the ones it filters.

Formally, we assume that $Pr[Filtered|\neg Fake] = a_0$, and $Pr[Filtered|Fake] = a_0 + a_1$, for constants $a_0 \in [0, 1]$, and $a_1 \in (0, 1 - a_0]$, i.e., that the probability a fake review is filtered is strictly greater than the probability a genuine review is filtered. Letting y^*_{it} be the star rating the fake review to business i at time t, we can write the following equation:

$$E[y^f_{it}] = \frac{(a_0 + a_1)pE[y^*_{it}]}{2a_o + a_1} + \frac{a_0(1-p)E[y^0_{it}]}{2a_0 + a_1}$$

where y^f_{it} is the star rating of a filtered review to business i at time t, p is the prior probability of a review being fake, and y^0_{it} is the star rating of a genuine review to business i at time t. This leads to the following condition:

$$y^f_{it} = \frac{a_0 y_{it}}{2a_o + a_1} + \frac{a_1 p y^*_{it}}{2a_0 + a_1} + u_{it} \tag{3.2}$$

where y_{it} is the star rating of any review no matter it is fake or not to business i at time t and $u_i t$ is an error term.

By substituting condition 1 into condition 2, we get the following model:

$$y^f_{it} = b_i + \tau_t + \frac{a_1 p}{2a_0 + a_1} x^*_{it} \beta + u_{it} \tag{3.3}$$

where b_i is the business-fixed effect. Our vector of independent variables x_{it} contains the average star rating of business i at time t. In addition, we add some control variables such as review length, the number of prior reviews that the reviewer has written and whether the reviewer has a profile picture or not.

We can estimate the model in (3.3) by using the 'within estimator' which wipes out the fixed effects. To be more specific, the transformation looks like as follows:

$$
\begin{aligned}
y_{it}^f + y^f - y_i^f - y_t^f &= b_i + b - b_i - b \\
&+ \tau_t + \tau - \tau - \tau_t \\
&+ \frac{a_1 p}{2a_0 + a_1}(x_{it}^* + x^* - x_i^* - x_t^*)\beta \\
&+ u_{it} + u - u_i - u_t \qquad (3.4) \\
y_{\ddot{it}} &= \frac{a_1 p}{2a_0 + a_1}(x_{\ddot{it}})\beta + u_{\ddot{it}} \qquad (3.5)
\end{aligned}
$$

where every removal of a subscript is made by averaging over all the values that subscript takes.

While we can identify the reduced-form parameter $\frac{a_1 p}{2a_0+a_1}\beta$, we cannot separately identify the structural parameter of interest, β. Therefore, we can only test for the presence of fraud through the estimates of the reduced-form parameter. Furthermore, since $\frac{a_1 p}{2a_0+a_1} < 1$, this estimate will be lower than the structural parameter, β.

3.4 Results

First, we present a summary statistics histogram. The vertical axis is for the portion of a star-rating. On Yelp, a star rating can be any integer from 1 to 5. As Figure 3.1 demonstrates the distribution of star-ratings for real reviews. The numbers increase with the star-rating except that the number of 1-star ratings is slightly higher than the number of 2-star ratings. The number of 1-star and 5-star ratings combined forms approximately 50% of the total number of reviews.

Figure 3.2 displays the distribution of fake reviews star-ratings. The main difference between fake ratings and real ratings is that fake ratings are more extreme. This is intuitive because fake reviews are written either to deflate or inflate the overall

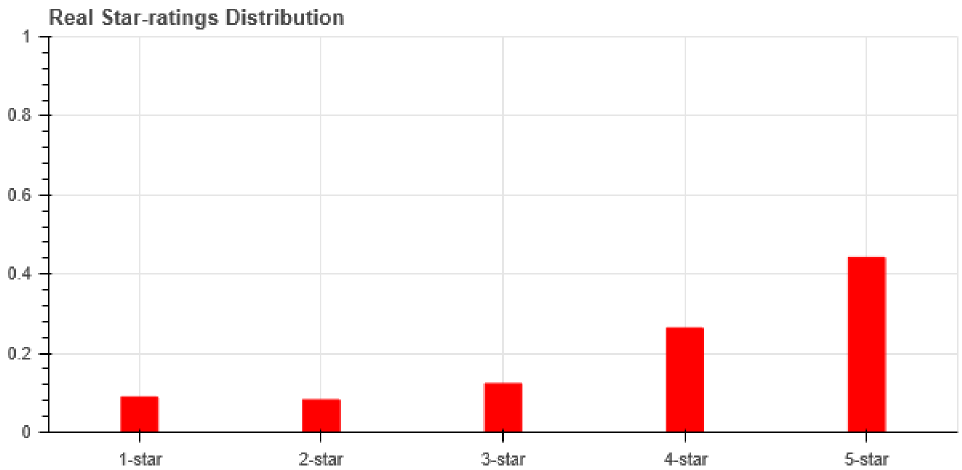

Figure 3.1. 5-star and 1-star reviews together make up around 50% of the total number of real reviews.

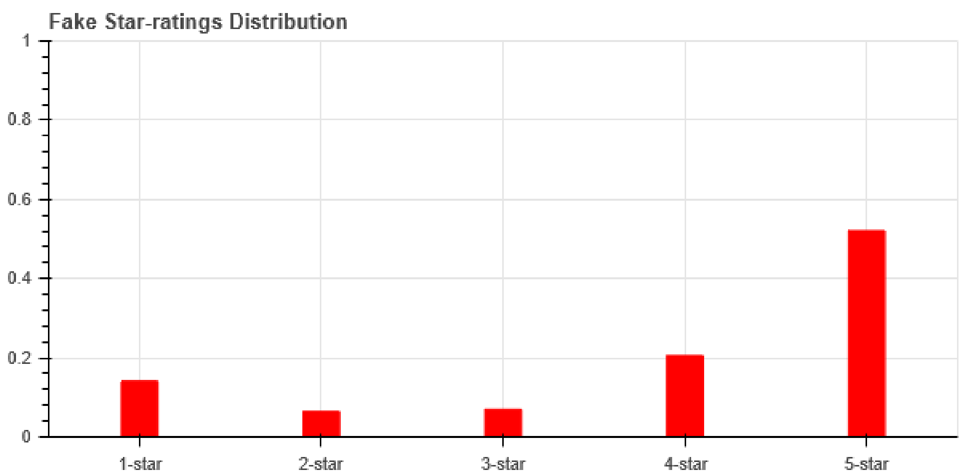

Figure 3.2. 5-star and 1-star reviews together make up around 75% of total reviews.

rating of a business. That's why they tend to be either 1-star or 5-star. Figure 2.6 implies that the number of 1-star and 5-star ratings combined make up about 75% of the total number of fake reviews. This is a significant increase over real reviews. As we stated earlier, we scraped 662996 reviews. Figure 3.1 was drawn with 581407 real reviews and Figure 3.2 was drawn with the remaining 81588 fake reviews. In other words, our sample indicates that 12% of reviews on Yelp are detected to be fake by Yelp's internal algorithm. Since our data is large enough, a t-test confirms the differences between the numbers of 1-star and 5-star ratings among the real reviews and the fake reviews.

Our first fixed-effects model finds the relation between reputation and the amount of fake reviews. Our equilibrium finds that as the reputation of a restaurant increases, the probability of exerting high effort decreases except for the move from the lowest reputation to the next. We measure reputation of a restaurant at a given time by its average Yelp rating that is the mean of all real review ratings given for the restaurant. We also add business fixed effects to our set of independent variables. As a result, we find that as average star-rating of a restaurant increases by 1 unit, the ratio of the number of fake reviews to the total number of reviews increases by 0.0156 unit, and this coefficient is significant. This result may be counter-intuitive at first. One may think that a restaurant that is becoming more reputable does not need fake reviews. However, as our equilibrium agrees, as a restaurant becomes more reputable, it tends to drop quality. Hence, it starts receiving low ratings. In order to offset them, it tries to pump fake reviews. Table 3.1 is only displaying the impact of the reputation on

	coeff	p-value	conf_lower	conf_higher
intercept	1.424058e+07	0.786186	-8.865128e+07	1.171324e+08
avg-star	1.564404e-02	0.000000	1.489664e-02	1.639143e-02

Table 3.1. Impact of Reputation on (Number of Fake Reviews)/(Total Number of Reviews) ratio

the portion of fake reviews among the total reviews. There are also restaurant-fixed effects in this model that we do not include in the figure. We measure the reputation for a restaurant i at time t by the average star-rating of the restaurant i at time t. Our OLS results show that if the average star-rating increases by 1 unit, the ratio of the number of fake reviews to the total number of reviews increases by 1.56 unit. It is true that our dependent variable is in the range of 0 to 1, so it can never change by 1.56 unit. However, we should take into account that the average-star rating changes only slightly after some reviews have been accumulated for a restaurant. Therefore, the change of 1.56 unit in the fake reviews portion upon a 1-unit change in the reputation is a legitimate number. Later on, we will run the same regression where the reputation will be measured by the average of most recent star-ratings.

Table 3.2 presents how the average-star rating of fake reviews changes with the reputation which is still measured by the average star-rating of the real reviews. If the average star-rating of real reviews increases by 1 unit, the average star-rating of fake reviews increases by 1.17. Note that this is an underestimated coefficient as we demonstrated in the previous section. With the p-value being very close to 0, we are confident that the average star-rating of fake reviews changes faster than the average star-rating of fake reviews. This is an indication that the more reputable firms tend to produce more fake reviews. Next, we run similar regression updating the way

	coeff	p-value	conf_lower	conf_higher
intercept	-0.679148	1.880614e-10	-0.888064	-0.470233
avg-star	1.167158	0.000000e+00	1.113457	1.220858

Table 3.2. Impact of Reputation on Average Fake Reviews Rating

we measure the reputation. In Yelp, the overall rating of a business is calculated over the entire set of reviews. This undermines the recent reviews and we think that customers would be more interested in getting information about more recent performance of a business and it may be too costly for some of them to read the recent reviews one by one even if they are available. Some review platforms such as eBay measures the reputation by more recent reviews (e.g. from reviews written in the last 6 months for some products). Moreover, in order to see if our equilibrium dynamics are taking place in real world, it is helpful to analyze the impact of recent real reviews on the upcoming fake reviews. We have already established the fact that the average fake review star-rating changes faster than the average real review star-rating, and we suggest to explain the difference between the different paces by our equilibrium characteristics. That is, on top of the real change in the ratings of a restaurant, there is a fake change generated by the fake reviews. Suppose that a restaurant's average rating is rising from 3 to 4. Then, even if there was no fake rating at all, the average rating of the filtered reviews would also increase from 3 to 4 because we assumed that Yelp's internal algorithm is unbiased. However, Figure 2.8 suggests that the average rating of filtered reviews would rise from 3 to 4.17 because the coefficient of the structured model is 1.17. Hence, the average star-rating of the fake reviews would rise to even beyond 4.17. The reason why the real and the fake changes are in the same direction is explained by the reputation exploitation phase of our equilibrium. Specifically, as a restaurant becomes more reputable, it drops quality, which is attempted to be offset by pumping fake reviews with high ratings. As a result, the fake star-rating increases.

Table 3.3 lays out the impact of the average star-rating of the most recent 10 real reviews on the next fake review's star-rating. We see that 1 unit increase in the reputation increases the fake reputation by 0.2. The business and time fixed effects are taken care of by using within estimator.

	coeff	p-value	conf_lower	conf_higher
intercept	3.098340	0.000000e+00	3.013426	3.183255
avg-star	0.194935	5.661986e-69	0.173190	0.216680

Table 3.3. Impact of the most recent 10 real reviews on the next fake review

We now conduct a couple of more rigorous analysis with additional controls to strengthen our conclusion that a better reputation is in line with a larger number of fake reviews produced by the business owner, in contrast to what Luca & Zervas [34] suggested.

Our empirical strategy is slightly different than before though it has the same flavors. We start with a review-level equation and end up with only time and business level terms. Our initial equation is

$$y_{it}^{*} = X_{it}\beta + b_i + \tau_t + \epsilon_{it}. \tag{3.6}$$

This time we have also time fixed effects even though we doubt that there is any. Removing the time-fixed effect parameter does not change our results but we add it for the sake of comprehensiveness. In equation 3.6, y_{it}^{*} is the number of fake reviews, X_{it} is a covariate matrix, b_i is business effect, and τ_t is time effect. Since, we do not know if a review is fake or real, we cannot observe y_{it}^{*}. Thus, we make use of Yelp's filtering algorithm as before. We make exactly the same assumptions that $Pr[Filtered|\neg Fake] = a_0$ and $Pr[Filtered|Fake] = a_0 + a_1$. Hence, we can write

$$\begin{aligned} y_{itk} &= \alpha_0(1 - y_{itk}^{*}) + (\alpha_0 + \alpha_1)y_{itk}^{*} + u_{itk} \\ \sum_{k=1}^{n_{it}} y_{itk} &= \sum_{k=1}^{n_{it}} \left[\alpha_0(1 - y_{itk}^{*}) + (\alpha_0 + \alpha_1)y_{itk}^{*} + u_{itk}\right] \\ y_{it} &= \alpha_0 n_{it} + \alpha_1 y_{it}^{*} + u_{it} \end{aligned} \tag{3.7}$$

where y_{it} is the number of censored reviews for business i at time t, n_{it} is the total number of reviews, and u_{it} is the residual term.

By substituting equation 3.6 into 3.7, we get

$$y_{it} = \alpha_0 n_{it} + \alpha_1(X_{it}\beta + b_i + \tau_t + \epsilon_{it}) + u_{it} \tag{3.8}$$

We can identify $\alpha_1\beta$ even though our parameter of interest is β. However, α_1 is a positive number smaller than 1 and we are only interested in the sign of β this time.

As before, we use within estimator as shown below in order to reach our final equation to be estimated.

$$
\begin{aligned}
y_{it} + \bar{y} - \bar{y}_t - \bar{y}_i = {} & \alpha_0(n_{it} + \bar{n} - \bar{n}_t - \bar{n}_i) \\
& + \alpha_1\beta(X_{it} + \bar{X} - \bar{X}_t - \bar{X}_i) \\
& + \epsilon_{it} + \bar{\epsilon} - \bar{\epsilon}_t - \bar{\epsilon}_i \\
& + u_{it} + \bar{u} - \bar{u}_t - \bar{u}_i \\
\ddot{y}_{it} = {} & \alpha_0\ddot{n}_{it} + \alpha_1\beta\ddot{X}_{it} + \ddot{\epsilon}_{it} \qquad (3.9)
\end{aligned}
$$

where $\bar{y} = \sum_i \sum_t y_{it}$, $\bar{y}_t = \sum_i y_{it}$, $\bar{y}_i = \sum_t y_{it}$ and so on.

First, we look at the effect of reputation on the incentive to produce fake reviews. Since fake reviews may come from the competitors as well as the business itself, we only consider the 5-star fake reviews at the next period. At the end of the day, our equilibrium in Chapter 2 suggests a chain reaction as follows: better reputation $\Rightarrow$ reputation exploitation $\Rightarrow$ lower effort $\Rightarrow$ more fake reviews and worse (real) reputation. In this chain of reactions, there has to be a step where time switched from t to $t+1$. According to our analysis, this switch occurs while the impact of reputation is reflected to the incentive to write fake reviews. Reasonably, business owners look past and evaluate the reviews written for them and take an action accordingly in the period they are in. Each period is one month in our analysis. For instance, if a period was considered a week, we would not see the same results. This means that the time span in the past that business owners take into account while determining their current action is not as short as 1 week.

Table 3.4 shows that a better reputation that is indicated by a higher rating in the most recent month is in line with a higher incentive to produce fake reviews next month. This finding supports our previous non-trivial results. As opposed to the prior literature, our model shows that as restaurants become more reputable, they have even more incentive to keep up their reputation because of the reduced effort, that the prior literature ignores. Our final table confirms the relation at the

OLS Regression Results

Dep. Variable:	Fake 5 stars (t+1)	R-squared (uncentered):	0.725
Model:	OLS	Adj. R-squared (uncentered):	0.725
Method:	Least Squares	F-statistic:	1.477e+04
Date:	Wed, 01 Jan 2020	Prob (F-statistic):	0.00
Time:	10:38:52	Log-Likelihood:	-1.3908e+05
No. Observations:	33672	AIC:	2.782e+05
Df Residuals:	33666	BIC:	2.782e+05
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Total Reviews	0.1727	0.003	50.768	0.000	0.166	0.179
Rating	0.4706	0.179	2.628	0.009	0.120	0.822
Friends	2.444e-06	6.25e-07	3.912	0.000	1.22e-06	3.67e-06
Review Length	-0.0019	1.82e-05	-104.544	0.000	-0.002	-0.002
Review Count	-0.0004	1.51e-05	-24.790	0.000	-0.000	-0.000
Photo	-0.0667	0.012	-5.745	0.000	-0.089	-0.044

Omnibus:	30553.751	Durbin-Watson:	0.057
Prob(Omnibus):	0.000	Jarque-Bera (JB):	11373300.825
Skew:	3.548	Prob(JB):	0.00
Kurtosis:	92.756	Cond. No.	1.25e+06

Table 3.4. Rating at t on the incentive to create fake reviews at t+1

right-end in the chain reaction. That is, a larger of fake reviews leads to a worse reputation that is observed through a lower number of 5-star real reviews. Table 3.5 finds the negative impact that the number of fake 5-star reviews has on the number of real 5-star reviews in the same period. This is explained by our model as follows: increasing number of fake reviews is an indicator of low effort, and therefore leads to more negative real reviews and less positive reviews.

```
                            OLS Regression Results
=======================================================================================
Dep. Variable:           Real 5-stars   R-squared (uncentered):                   0.487
Model:                            OLS   Adj. R-squared (uncentered):              0.487
Method:                 Least Squares   F-statistic:                              5213.
Date:                Wed, 01 Jan 2020   Prob (F-statistic):                        0.00
Time:                        10:41:24   Log-Likelihood:                         -60745.
No. Observations:               33004   AIC:                                  1.215e+05
Df Residuals:                   32998   BIC:                                  1.216e+05
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Total Reviews     0.0206      0.002     11.932      0.000       0.017       0.024
Fake 5-stars     -0.0347      0.005     -7.303      0.000      -0.044      -0.025
Review Length     0.0001   2.65e-05      3.984      0.000    5.37e-05       0.000
Friends        6.698e-06   4.78e-07     14.010      0.000    5.76e-06    7.63e-06
Review Count     -0.0002   1.36e-05    -16.386      0.000      -0.000      -0.000
Photo             0.3065      0.003    100.217      0.000       0.301       0.313
==============================================================================
Omnibus:                     9230.628   Durbin-Watson:                   1.537
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           651068.962
Skew:                          -0.429   Prob(JB):                         0.00
Kurtosis:                      24.742   Cond. No.                     1.34e+04
==============================================================================
```

Table 3.5. Number of fake 5-star reviews at t on number of real 5-star reviews at t

Appendix A

The Proofs of The Theorems in Chapter 2

A.1 Proof of Theorem 2.3.1

A.1.1 Constructing the transition rule:

The structure of the transition rule changes depending on the size of the jump. Let us assume that the size is 2 for visualization purpose:

$$a_1 = H : \begin{bmatrix} 1-\gamma & 0 & \gamma & 0 & \dots & & \\ P^*(2,1,H) & P^*(2,2,H) & P^*(2,3,H) & 0 & \dots & & \\ & \ddots & \ddots & \ddots & & & \\ \dots & 0 & P^*(s,s-1,H) & P^*(s,s,H) & P^*(s,s+1,H) & 0 & \dots \\ & & & \ddots & \ddots & \ddots & \\ & & \dots & 0 & P^*(K-1,K-2,H) & P^*(K-1,K-1,H) & P^*(K-1,K,H) \\ & & & \dots & 0 & P^*(K,K-1,H) & P^*(K,K,H) \end{bmatrix}$$

and

$$a_1 = L : \begin{bmatrix} 1-\gamma & 0 & \gamma & 0 & \dots & & \\ P^*(2,1,L) & P^*(2,2,L) & 0 & \dots & & & \\ & \ddots & \ddots & \ddots & & & \\ \dots & 0 & P^*(s,s-1,L) & P^*(s,s,L) & 0 & \dots & \\ & & & \ddots & \ddots & \ddots & \\ & & \dots & 0 & P^*(K-1,K-2,L) & P^*(K-1,K-1,L) & 0 \\ & & & \dots & 0 & P^*(K,K-1,L) & P^*(K,K,L) \end{bmatrix}$$

where

$$\begin{aligned} \gamma &= \frac{p-\alpha_s(\eta-c)}{2\eta} - \frac{(K-1)(1-\delta)}{2\delta} \\ P^*(s,s-1,a_1) &= \alpha_s\frac{\eta-c}{\eta} + \frac{(K-s)(1-\delta)}{\delta} \quad a_1 \in \{H, L\} \\ P^*(s,s,1) &= (1-\alpha_s)\frac{\eta-c}{\eta} - \frac{(K-s)(1-\delta)}{\delta} \quad s \in \{2,3,...,K-1\} \\ P^*(s,s,0) &= P^*(s,s,1) + \frac{c}{\eta} \\ P^*(s,s+1,1) &= \frac{c}{\eta} \end{aligned}$$

The transition rule has a parameter η. Hence, we denote it by $P^*(\eta)$.

The transition rule we constructed for the example has the same form where $\eta = 1.5$. We take it for granted now that η takes values between c and 2c. We will show later why this is true.

A.1.2 The conditions implied by the transition rule:

The vectors of steady state probabilities for the normal and the commitment type, π_n^* and π_c^* induced by $P^*(\eta)$ and σ_1^* can be found as shown below:

$$\pi_\omega^* = \pi_\omega^* P_\omega^* \;\; \omega \in \Omega \tag{A.1}$$

where P_n^* and P_c^* are the K x K transition matrices for the normal and the commitment types of player 1. Their each element can be found as shown below:

$$
\begin{aligned}
&P_n^*(i,j) = \sigma_1^*(i)P^*(i,j,H) + (1-\sigma_1^*(i))P^*(i,j,L) \;\; i,j \in S \\
&P_c^*(i,j) = P^*(i,j,H) \;\; i,j \in S
\end{aligned}
$$

The set $(A.1)$ of equations implies the following conditions:

$$
\begin{aligned}
\pi_c^*(1)\gamma &= \pi_c^*(2)P_c^*(2,1) \\
\pi_n^*(1)\gamma &= \pi_n^*(2)P_n^*(2,1) \\
\Rightarrow \lim_{K(1-\delta)\to 0} \frac{\pi_c^*(2)}{\pi_c^*(1)} &= \lim_{K(1-\delta)\to 0} \frac{\pi_n^*(2)}{\pi_n^*(1)} = \frac{p-\alpha_s(\eta-c)}{2\alpha_s(\eta-c)} \qquad \text{(A.2)} \\
\pi_c^*(3) &= \pi_c^*(2)\frac{\frac{\eta-(\eta-c)(1-\alpha_s)}{\eta}+\epsilon_2}{\frac{\eta-c}{\eta}\alpha_s+\epsilon_3} \qquad \text{(A.3)} \\
\pi_n^*(3) &= \pi_n^*(2)\frac{\frac{\eta-(\eta-c)(1-\alpha_s)-(1-\sigma_1^*(2))c}{\eta}+\epsilon_2}{\frac{\eta-c}{\eta}\alpha_s+\epsilon_3} \qquad \text{(A.4)} \\
\Rightarrow \lim_{K(1-\delta)\to 0} \frac{\pi_c^*(3)}{\pi_c^*(2)} &= \frac{c+\alpha_s(\eta-c)}{\alpha_s(\eta-c)} > 1 \quad \text{and} \\
\lim_{K(1-\delta)\to 0} \frac{\pi_n^*(3)}{\pi_n^*(2)} &= \frac{c+\alpha_s(\eta-c)-(1-\sigma_1^*(2))c}{\alpha_s(\eta-c)} > 1 \\
\pi_c^*(4)\Big(\alpha_s\frac{\eta-c}{\eta}+\epsilon_4\Big) &= \pi_c^*(3)\Big(1-(1-\alpha_s)\frac{\eta-c}{\eta}+\epsilon_3\Big) \qquad \text{(A.5)} \\
&\quad -\pi_c^*(2)\frac{c}{\eta} - \pi_c^*(1)\Big(\frac{p-\alpha_s(\eta-c)}{2\eta c}-\epsilon_1/2\Big) \\
\pi_n^*(4)\Big(\alpha_s\frac{\eta-c}{\eta}+\epsilon_4\Big) &= \pi_n^*(3)\Big(1-(1-\alpha_s)\frac{\eta-c}{\eta}-(1-\sigma_1^*(3))\frac{c}{\eta}+\epsilon_3\Big) \qquad \text{(A.6)} \\
&\quad -\pi_n^*(2)\frac{c\sigma_1^*(2)}{\eta} - \pi_n^*(1)\Big(\frac{p-\alpha_s(\eta-c)}{2\eta c}-\epsilon_1/2\Big) \\
\pi_c^*(s+1)\Big(\alpha_s\frac{\eta-c}{\eta}+\epsilon_{s+1}\Big) &= \pi_c^*(s)\Big(1-(1-\alpha_s)\frac{\eta-c}{\eta}+\epsilon_s\Big) \qquad \text{(A.7)} \\
&\quad -\pi_c^*(s-1)\frac{c}{\eta}
\end{aligned}
$$

$$\pi_n^*(s+1)\Big(\alpha_s\frac{\eta-c}{\eta}+\epsilon_{s+1}\Big)=\pi_n^*(s)\Big(1-(1-\alpha_s)\frac{\eta-c}{\eta}-(1-\sigma_1^*(s))\frac{c}{\eta}+\epsilon_s\Big) \quad \text{(A.8)}$$

$$-\pi_n^*(s-1)\frac{c\sigma_1^*(s-1)}{\eta} \qquad s\in\{4,...,K-1\}$$

$$\pi_n^*(K)\alpha_s(\eta-c)=\pi_n^*(K-1)c\sigma_1^*(K-1) \quad \text{(A.9)}$$

$$\pi_c^*(K)\alpha_s(\eta-c)=\pi_c^*(K-1)c \quad \text{(A.10)}$$

where $\epsilon_s=\frac{(K-s)(1-\delta)}{\delta}$.

A.1.3 Equilibrium strategies:

The strategies are as follows:

$$\sigma_1^*(s)=\begin{cases}0 & \text{if } s\in\{1,K\}\\ \frac{\alpha_s-\lambda^*(s)}{1-\lambda^*(s)} & \text{otherwise}\end{cases}$$

$$\sigma_2^*(s)=\begin{cases}0 & \text{if } s=1\\ 1 & \text{otherwise}\end{cases}$$

A.1.4 Existence of optimal η:

We pin down η by the condition $\lambda^*(K)=\alpha_s$. Hence, we show that there exists such an $\eta\in(c,2c)$.

Consider when η is c. Equations (A.3), (A.5) and (2.7) imply that $\pi_c^*(s+1)/\pi_c^*(s)$ converges to infinity as ϵ_s goes to 0. Therefore, $\pi_c^*(K)$ converges to 1.

Consider a positive number smaller than α_s, say $\alpha_s/2$. Let us define N^0 to be $N^0=\{s\in S\setminus K:\sigma_1^*(s)>\alpha_s/2\}$ and $N^1=\{s\in S\setminus K:\sigma_1^*(s)\leq\alpha_s/2\}$. If s is in N^0, we will show that $\pi_n^*(s+1)/\pi_n^*(s)$ grows infinitely for some large values of K and δ. Combining $(A.4)$ and $(A.6)$, we can write the same relation between any adjacent scores s and $s+1$ for $s>2$. Also referring to $(A.4)$ and $(A.6)$ we can say that if s is in N^0, $\pi_n^*(s+1)/\pi_n^*(s)$ goes to infinity as η approaches c. Therefore, $\sum_{s\in N^0}\pi_n^*(s)$ converges to 0.

If s is in N^1, then there exists a $b>0$ such that $\pi_n^*(s)<\pi_c^*(s)/b$. Therefore, $\sum_{s\in N^1}\pi_n^*(s)<\sum_{s\in N^1}\pi_c^*(s)/b$.
Since for every $s<K$, s is either in N^0 or N^1, we conclude $\pi_n^*(K)$ goes to 1.

When η is 2c, the probability of downgrade and the probability of upgrade at any $s>1$ would be equal to each other in the limit of ϵ_s going to 0 for every s if $\sigma_1^*(s)$ was α_s for every middle score. Given that $\sigma_1^*(s)$ is always less than α_s, the upward drift for the normal type fades away. However, there is always a strict upward drift for the commitment type. Therefore, the posterior belief at score K approaches to 1 as η goes to 2c.

We have shown that for small values of η, the prior is not updated at score K, and for high values of K, the posterior at score K converges to infinity. Since the posterior at score K is continuous with respect to η, by intermediate value theorem, there has to be an η such that $\lambda^*(K)$ is exactly α_s.

A.1.5 $\lim_{K(1-\delta)\to 0,\delta\to 1}\pi_n^*(1)$=0:

First, we prove that the probability of being at one of a lower portion of the scores for the commitment type gets arbitrarily close to 0 for some large values of K and δ. By using this result, we then show that the probability of visiting the lowest score even for the normal type is arbitrarily close to 0.

We can state the first result more formally as follows.

Lemma A.1.1. *For every $\eta\in(c,2c)$ and $\rho>0$, there exist a $\bar{K}$ such that for all $K>\bar{K}$, there exists a $\bar{\delta}$ such that for all $\delta>\bar{\delta}$ and $s<K/2$, $\pi_c^*(s)<\rho$.*

Proof. We prove the lemma by showing that there is always an upward drift ($\pi_c^*(s+1) > \pi_c^*(s)$) for the committed type. This leads to steady-states of lower scores approaching to 0 as the total number of scores and the discount factor increase. The equations $(A.2)$ and $(A.5)$ imply that for some large values of δ,

$$\frac{\pi_c^*(s+1)}{\pi_c^*(s)} \approx \frac{c}{\alpha_s(\eta - c)} > 1 \tag{A.11}$$

for $s > 2$. For s equal to 1 and 2, the equations (A.2) and (A.3) imply

$$\frac{\pi_c^*(3)}{\pi_c^*(2)} \approx \frac{c + \alpha_s(\eta - c)}{\alpha_s(\eta - c)} > 1 \tag{A.12}$$

and

$$\frac{\pi_c^*(2)}{\pi_c^*(1)} \approx \frac{p - \alpha_s(\eta - c)}{2\alpha_s(\eta - c)} > b \tag{A.13}$$

where b is a positive number. Let us define $\epsilon = \frac{c-\alpha_s(\eta-c)}{\alpha_s(\eta-c)}$ and $s^o = \max\{s \in S|s < K/2\}$. Then, there exist a K and a δ such that we can write $\pi_c^*(s^o) < \pi_c^*(K)/(1+\epsilon)^{K-s^o}$. Hence, $\pi_c^*(s^o)$ and $\pi_c^*(s)$ for all $s < s^o$ approaches to 0. ☐

Now, using Lemma A.1.1, we show that the probability of visiting score 1 for the normal type of player 1 is also arbitrarily close to 0 for some large values of K and δ.

Lemma A.1.2. *For every $\eta \in (c, 2c)$ and $\rho > 0$, if $\pi_n^*(s) > \rho$ for some $s \in \{s \in S|1 < s < K/2\}$; then there exists a $\bar{K}$ such that for all $K > \bar{K}$, there exists a $\bar{\delta}$ and a $\rho' > 0$ such that for all $\delta > \bar{\delta}$, $\sigma_1^*(s) > \alpha_s - \rho'$.*

Proof. For any $\eta \in (c, 2c)$ and $\rho > 0$, consider the $\bar{K}$ and the correspondence $\bar{\delta}(K)$, satisfying Lemma 2.10.1. For each $K > \bar{K}$ and $\delta > \bar{\delta}(K)$, $\pi_c^*(s)$ for $s < K/2$ is arbitrarily close to 0. Since $\pi_n^*(s) > \rho$ for a $\rho > 0$, the Bayesian update implies that the posterior probability is arbitrarily close to 0, which in turn implies that $\sigma_1^*(s)$ is arbitrarily close to α_s. □

Lemma A.1.3. *For every $\eta \in (c, 2c)$ and K, there exist a $\rho > 0$ and $\bar{\delta}$ such that for all $\delta > \bar{\delta}$, $(\alpha_s - \rho)P^*(s, s+1, H) + (1 - \alpha_s + \rho)P^*(s, s+1, L) \geq P^*(s, s-1, a_1)$ for $s \in \{2, 3, ..., K-1\}$.*

Proof. Since K is finite, it is enough to show

$$\alpha_s P^*(s, s+1, H) + (1 - \alpha_s)P^*(s, s+1, L) > P^*(s, s-1, a_1)$$

which can be simplified as

$$\alpha_s \frac{c}{\eta} > \left(\alpha_s \frac{\eta - c}{\eta} + \epsilon_s\right)$$

Since η is fixed at a value less than $2c$, there exist a small enough ϵ_s that makes the inequality satisfied. Such small ϵ_s can be obtained by high enough δ. □

Lemma A.1.4. *For every $\eta \in (c, 2c)$ and $\rho > 0$, there exist a $\bar{K}$ such that for every $K > \bar{K}$, there exists a $\bar{\delta}$ such that for all $\delta > \bar{\delta}$; $\pi_n^*(1) < \rho$.*

Proof. For a fixed $\eta \in (c, 2c)$, suppose, by contradiction that there exists a $p > 0$ such that for every K and δ, $\pi_n^*(1) > p$.

Followed by $(A.2)$, $\pi_n^*(2) > pb$ where b is a positive constant. Then, by Lemma A.1.1, for every $p' > 0$ there exist some large values of K and δ such that $\sigma_1^*(2) > \alpha_s - p'$. Lemma A.1.3 in turn implies that the probability of upgrade is higher than the probability of downgrade for score s. Also supported by equation (A.4), we can say that $\pi_n^*(3)$ is greater than pb for some large values of K and δ, followed by Lemma A.1.2 implying that $\sigma_1^*(3)$ being arbitrarily close to α_s, and Lemma A.1.3 and equation (A.6) together implying $\pi_n^*(4) > \pi_n^*(3)$.

The law of motion with $\pi_n^*(s)$ together with the initial probability of $\pi_n^*(1)$ implies a bound, typically less than K, on the first state where the upward drift becomes downward. Let us call that score s^o. It is important to note that s^o is independent of K as long as K and δ are large enough.

In order for the upward drift to end, a necessary condition is that there exists a $\beta > 0$ such that $\sigma_1^*(s^o) < \alpha_s - \beta$. However, this is a contradiction. For any fixed s^0 and $\beta > 0$, if we continue increasing K, $\pi_c^*(s)$ for $s < s^0$ gets even closer to 0 and thus $\sigma_1^*(s)$ for $s < s^0$ gets even closer to α_s. Therefore, there exists a large enough K after which there exist high enough values of δ such that $\sigma_1^*(s^o) > \alpha_s - \beta$. $\square$

A.1.6 Early periods:

We take care of the early rounds by choosing the initial distribution to reflect the steady states:

$$\pi_0^*(w, s) = \begin{cases} \pi_c^*(s) & \text{if } w = c \\ \\ \pi_n^*(s) & \text{if } w = n \end{cases}$$

In Theorem 2.6.1, we get rid of the assumption that the review platform knows the type of player 1. We chose to present our main result in two theorems in the hope of clarity. The goal of Theorem 2.3.1 is to give the reader the essence of our method.

A.1.7 Calculating the payoff:

In order to achieve the highest payoff to the normal player 1, we do not only need trade happening, but we also need her to exert as low effort as possible. In other words, the normal type of player 1 should not build any reputation more than she needs. At any middle score, player 2 is indifferent, so there is no unnecessary reputation at middle scores. The transition rule implies that at score K, the normal player 1 plays L and player 2 plays B. In order for the equilibrium to work, the posterior belief at K must be no less than α_s. Since we do not want any unnecessary reputation, it should not be greater than α_s either. Therefore, we pick an η such that $\lambda^*(K)$ is exactly α_s[1]. So far, we have proved the existence of such η. Also, we have shown that the probability of visiting the only state in which trade does not occur, which is score 1, is almost 0 even for the normal type of player 1.

The rest of the work is only showing the closed form expression of the payoff for each score. The value of a score of s can be determined by the following recursive equation:

$$\begin{aligned} V(s) = (1-\delta)U_1(\sigma_1^*(s), \sigma_2^*(s)) + \delta[\sigma_1^*(s) \sum_{s' \in S} \tau(s, s', H)V(s') \\ + (1-\sigma_1^*(s)) \sum_{s' \in S} \tau(s, s', L)V(s')] \end{aligned}$$

and we find

$$V(s) = p - \alpha_s(\eta - c) - \frac{(K-s)(1-\delta)\eta}{\delta}$$

As $K(1-\delta)$ goes to 0, the values approach $p - \alpha_s(\eta - c)$. In the limit, the values are also equal to the adjusted Stackelberg paoyff, that is $\frac{p-\alpha_s-\lambda}{1-\lambda}$. Therefore, η must be asymptotically equal to $c + \frac{(\alpha_s-\lambda)c}{(1-\lambda)\alpha_s}$. Note that the functional form of η confirms that it is always between c and 2c. $\square$

Before we close up, we show how the beliefs are distributed because we need the following result in the proof of Theorem 2.6.1.

[1] It is important to emphasize that all we need is for the transition probabilities such that the buyer is indifferent at K because the other states will be taken care with randomization.

Lemma A.1.5. *For every $s > 2$; the posterior belief $\lambda^*(s)$ is equal to α_s^{K-s+1}.*

Proof. Remember that we pinned down η by the condition $\lambda^*(K) = \alpha_s$. Within a period, we have the following condition

$$\sigma_1^*(K-1) = \frac{\alpha_s - \lambda^*(K-1)}{1 - \lambda^*(K-1)} \tag{A.14}$$

$$= \alpha_s - (1-\alpha_s)\frac{\lambda \pi_c^*(K-1)}{(1-\lambda)\pi_n^*(K-1)} \tag{A.15}$$

By combining (A.9), (A.10) and (A.15); we find $\sigma_1^*(K-1) = \frac{\alpha_s}{1+\alpha_s}$. Plugging $\sigma_1^*(K-1)$ back in (2.24), we find $\lambda^*(K-1) = \alpha_s^2$. By iterating towards lower scores until $s = 2$, we find $\lambda^*(s) = \alpha_s \lambda^*(s+1)$. □

Lemma A.1.5 highlights a crucial insight of our machinery. What we do in order to prove the lemma is to find a fixed point, that is σ_1^*. Once a vector of strategies is plugged in, the transition rule P^* yields a K dimensional vector of beliefs which, in turn, yields a vector of new strategies. We have shown that σ_1^* is a fixed point of this process. We will use this argument in more detail in the next section.

A.2 Proof of Theorem 2.6.1:

A.2.1 Constructing the platform.

We denote the transition rule function that we proposed in the previous proof as P^*. Let us define P^{**} as the following:

$$P^{**}(K, K-1, H) = P^*(K, K-1, H) - \epsilon$$

$$P^{**}(K, K, H) = P^*(K, K, H) + \epsilon$$

$$P^{**}(i, j, a_1) = P^*(i, j, a_1)$$

Since the normal type will still not be exerting any effort at score K, this update does not change her transition rule. However, it increases the stationary probability of visiting K for the commitment type while decreasing those of the other scores. Having said that, the recursive equation at every score for the normal type stays the same. Despite the fact that there is going to be a small amount of loss; by choosing a small ϵ and large N, K and δ; we can still stay arbitrarily close to the Pareto frontier. Before we finish up this section, it is worthwhile to note that the platform does not violate the time-independence. It is still defined on a finite space.

A.2.2 Equilibrium strategies.

We propose $\sigma_{1,t}^{**}$ and $\sigma_{2,t}^{**}$ as

$$\sigma_{1,t}^{**} = \begin{cases} 0 & \text{if } t < N \\ \frac{\alpha_s - \lambda_t^{**}(s)}{1-\lambda_t^{**}(s)} & \text{if } t \geq N, 1 < s < K \\ 0 & \text{if } t \geq N, s \in \{1, K\} \end{cases}$$

and

$$\sigma_{2,t}^{**} = \begin{cases} 0 & \text{if } t < N \\ 1 & \text{if } t \geq N, s > 1 \\ 0 & \text{if } t \geq N, s = 1 \end{cases}$$

We consider the first N periods as the experimentation stage. Since we focus on the payoff as δ goes to 1, we will ignore the payoff loss in this stage. During experimentation, the normal type and the short-run buyers play the no-trade strategies. We will update the platform to take care of the experimentation stage in the last section of the proof.

From N onward, player 1 plays time-dependent strategies. The reason that they are required is the following. Suppose that we have a stationary equilibrium. For some of the scores, the belief at any time would be lower than its steady-state level. Hence, the strategy of player 1 would not be optimal. This argument concludes that there cannot be a stationary equilibrium that achieves arbitrarily close payoff to $\bar{V}$. As a result, we impose the time-dependent strategies for player 1 from N onward. The drawback of time-dependent strategies is that now we have a heterogeneous transition matrix for the normal type because her transition depends on her strategy which is now distorted with the time-dependence. Therefore, we need to show that the distortion caused by the time-dependent strategies does not cause the belief at the highest score to move away from where it was at N.

A.2.3 Checking if the proposed strategies are optimal:

We need to show that even though the transition rule is distorted after period N with time-dependent strategies, the distorted distribution can be approximated by the stationary distribution which would be reached if there was no distortion.

Now, let us return the argument we pointed out at the end of the previous section. We have shown that σ_1^* is a fixed point of a function that we are about to call f^*. Let us explain f^* step by step. The strategy of the normal type is defined on $[0, \alpha_s]^{K-2} \times \{0\}^2$ which is a compact and convex set. The function f^* takes a K dimensional vector of strategies as its input and by combining the transition rule P^*, Bayesian update and our equilibrium definition (in order); returns a new vector of strategies. Since f^* is continuous, by Brouwer's fixed point theorem, it has a fixed point. In Theorem 2.3.1, we showed that σ_1^* is a fixed point of f^* and σ_1^* is such that $\lambda^*(K) = \alpha_s$. Let us call f^{**} the composite function associated with P^{**}. f^{**} carries the same properties with f^*. Therefore, it has a fixed point as well. Let us call σ_1^{**} a fixed point of f^{**}. By Fudenberg & Tirole [21], we know that the equilibrium set is upper hemicontinuous. Moreover, f^* and f^{**} each return only a vector of singletons, not a correspondence. Therefore; for any sequence of f^n with $f^n \to f^*$ and $\sigma_1^n = f^n(\sigma_1^n)$, it is true that $\sigma_1^n \to \sigma_1^*$. Hence, there is a small tweak ϵ such that $||\sigma_1^* - \sigma_1^{**}|| < \epsilon'$ for any $\epsilon' > 0$. Note that σ_1^{**} is only a vector of stationary strategies, not our equilibrium strategies. However, we are going to show that our equilibrium vector is arbitrarily close to σ_1^{**}.

In addition to the stationary parameters that we have defined, let us call $(\pi_c^{**}(s), \pi_n^{**}(s))$ the stationary distributions associated with P^{**} and $\sigma_1{**}$. Notice that we denote our equilibrium parameters with time subscript and the stationary levels without one. Due to how we tweaked the transition rule, we are going to have a large enough N such that $\lambda_N^{**}(K) > \alpha_s$. We can write

$$\pi_n^{**}(K) = \pi_n^{**}(K-1)P_n^{**}(K-1, K) + \pi_n^{**}(K)P_n^{**}(K, K) \tag{A.16}$$

where $P_n^{**}(i,j) = P^{**}(i,j,H)\sigma_1^{**}(i) + P^{**}(i,j,L)(1-\sigma_1^{**}(i))$. In addition, the transition between two consecutive periods implies

$$\pi_{n,N+1}^{**}(K) = \pi_{n,N}^{**}(K-1)P_{n,N}^{**}(K-1, K) + \pi_{n,N}^{**}(K)P_{n,N}^{**}(K, K) \tag{A.17}$$

where $P_{n,N}^{**}(i,j) = P^{**}(i,j,H)\sigma_{1,N}^{**}(i) + P^{**}(i,j,L)(1-\sigma_{1,N}^{**}(i))$. Combining (A.16) and (A.17), we obtain

$$\pi_{n,N+1}^{**}(K) - \pi_n^{**}(K) = [\pi_{n,N}^{**}(K-1) - \pi_n^{**}(K-1)]P_n^{**}(K-1, K)$$

$$+ [\pi^{**}_{n,N}(K) - \pi^{**}_{n}(K)]P^{**}_{n}(K,K)$$
$$+ \frac{\sigma^{**}_{1,N}(K-1) - \sigma^{**}_{1}(K-1)}{\eta}\pi^{**}_{n,N}(K-1)$$

Let us define $\Delta\sigma^{t}_{1}(s) = \sigma^{**}_{1,t}(s) - \sigma^{**}_{1}(s)$. If we rewrite the above equation, it becomes

$$\pi^{**}_{n,N+1}(K) - \pi^{**}_{n}(K) = [\pi^{**}_{n,N}(K-1) - \pi^{**}_{n}(K-1)]P^{**}_{n}(K-1,K) \tag{A.18}$$
$$+ [\pi^{**}_{n,N}(K) - \pi^{**}_{n}(K)]P^{**}_{n}(K,K)$$
$$+ \frac{\Delta\sigma^{N}_{1}(K-1)}{\eta}\pi^{**}_{n,N}(K-1) \tag{A.19}$$

We know that we can choose a large enough N such that $\pi^{**}_{n,N}$ will be close enough to $\pi^{**}_{n}(K)$ so that λ^{**}_{N} will still be greater than α_s.

Now, we show that $\Delta\sigma^{t}_{1}(K-1) < \Delta\sigma^{N}_{1}(K-1)$ for every $t > N$. This means that the strategies and so the other variables such as the transition probabilities and the beliefs stay around the steady-state levels.

Now, consider $\Delta\sigma^{N+1}_{1}(K-1)$. It can be expressed as

$$\frac{\alpha_s - \lambda^{**}_{N+1}(K-1)}{1 - \lambda^{**}_{N+1}(K-1)} - \frac{\alpha_s - \lambda^{**}(K-1)}{1 - \lambda^{**}(K-1)}$$
$$=(1-\alpha_s)\frac{\lambda}{1-\lambda}\left[\frac{\pi^{**}_{c}(K-1)}{\pi^{**}_{n}(K-1)} - \frac{\pi^{**}_{c,N+1}(K-1)}{\pi^{**}_{n,N+1}(K-1)}\right] \tag{A.20}$$

where

$$\pi^{**}_{n,N+1}(K-1) = \pi^{**}_{n,N}(K-2)P^{**}_{n,N}(K-2,K-1) + \pi^{**}_{n,N}(K-1)P^{**}_{n,N}(K-1,K-1)$$
$$+ \pi^{**}_{n,N}(K)P^{**}_{n,N}(K,K-1) \tag{A.21}$$
$$= \pi^{**}_{n,N}(K-2)P^{**}_{n}(K-2,K-1) + \pi^{**}_{n,N}(K-1)P^{**}_{n}(K-1,K-1)$$
$$+ \pi^{**}_{n,N}(K)P^{**}_{n}(K,K-1)$$

$$+\frac{\Delta\sigma_1^N(K-2)\pi_{n,N}^{**}(K-2)}{\eta}-\frac{\Delta\sigma_1^N(K-1)\pi_{n,N}^{**}(K-1)}{\eta} \tag{A.22}$$

As a result, we can consider $\Delta\sigma_1^{N+1}(K-1)$ as a function of $\Delta\sigma_1^N(K-2)$ and $\Delta\sigma_1^N(K-1)$. Let us call that function g^{N+1}. Note that $g^{N+1}(0,0)=0$ for a small ϵ because when the strategies at time N are at their steady-state levels, so will be the strategies at $N+1$. Moreover, for a small ϵ and large N, we can write:

$$\begin{aligned}
\Delta\sigma_1^{N+1}(K-1) &\approx \frac{\partial g(\Delta\sigma_1^{N+1}(K-1),\Delta\sigma_1^{N+1}(K-2))}{\partial\Delta\sigma_1^N(K-2)}\Delta\sigma_1^N(K-2)\\
&+\frac{\partial g(\Delta\sigma_1^{N+1}(K-1),\Delta\sigma_1^{N+1}(K-2))}{\partial\Delta\sigma_1^N(K-1)}\Delta\sigma_1^N(K-1)\\
&\approx \frac{\alpha_s^2(1+\alpha_s+\alpha_s^2)(\eta-1)}{(1+\alpha_s)^2\eta}\Delta\sigma_1^N(K-2)-\frac{\alpha_s}{(1+\alpha_s)\eta}\Delta\sigma_1^N(K-1) \qquad \text{(A.23)}\\
&<\frac{\alpha_s}{1+\alpha_s}\max\{\Delta\sigma_1^N(K-1),\Delta\sigma_1^N(K-2)\}
\end{aligned}$$

where (A.23) can be written by Lemma A.1.5. Let $\gamma_N=\max_{s\in S}\{|\Delta\sigma_1^N(s)|\}$. Then, the condition above implies that there exists a small enough tweak such that $|\Delta\sigma_1^{N+1}(K-1)|<\gamma_N/2$ for a large N. Moreover, we can write the same condition for every score $s<K$. Consequently, we can say that the deviation from the stationary levels is contracting. This process can be iterated over infinitely many time periods. Hence, the strategies σ_2^{**} of player 2 are still optimal.

Let us explain what is happening intuitively. Suppose that there are only 3 scores: 1, 2 and 3. By tweaking the review platform, we have slightly increased the posterior belief at the score of 3 and decreased the beliefs at 1 and 2. As a result of this, the effort level at 2 now has to be higher. However, the effect on the tweak cannot perpetuate because since the effort level at 2 increased, the probability of being at the score of 2 (for the normal type) will decrease, which results in an increase in the belief. All in all, the parameters stay around their steady-state levels which are arbitrarily close to those in the equilibrium proposed by Theorem 2.3.1. Note that a (K, δ) tuple that works for Theorem 2.3.1 may not work to sustain this equilibrium. We would need even higher values of those in this one.

A.2.4 Experimentation Stage:

We take care of the experimentation stage as the following so that by the time N, the beliefs will be close to their steady-state levels.

$$P^{**}(i, j, a_1) := \begin{cases} P^{**}(i, j, a_1) & \text{if } a_1 = H, t < N \\ \sigma_1^{**}(i) P^{**}(i, j, H) + (1 - \sigma_1^{**}(i)) P^{**}(i, j, L) & \text{if } a_1 = L, t < N \\ P^{**}(i, j, a_1) & \text{if } t \geq N \end{cases}$$

Given the above transition rule, even though the normal type of player 1 plays L until period N, the transition rule announces a score as if she follows σ_1^{**}.

A.3 Proof of Theorem 2.8.1:

Suppose, by contradiction, that there is an equilibrium with $(\sigma_1(s), \sigma_2(s) = (0, 0)$ for every $s \in \{1, 2\}$. Then, $\lambda(2)$ becomes 1 and player 2 strictly prefers buying at score 2. This is contradiction to no-trade at every score. $\square$

Appendix B
The Codes:

B.1 The function to obtain urls for all the 1073 restaurants

```
from selenium import webdriver


def get_links(df_business):
    urls=[]
    driver = webdriver.Chrome()
    ids=[]

    for name,city,id_ in zip(df_business['name'],df_business['city'],
        df_business['id']):
        search_query=name+' '+city+' yelp'
        driver.get("https://www.google.com/search?q="
                   + search_query)
        matched=driver.find_elements_by_xpath('//a[starts-with(@href,
        "https://www.yelp.com")]')
```

```
        try:
            matched[0].click()
            urls.append(driver.current_url)
            ids.append(id_)
        except:
            True

    driver.quit()

    return (urls,ids)
```

B.2 The functions to read review, user and business data from the public data set stored in a csv file

```
import pandas as pd
from tqdm import tqdm
import json

def read_review_data(data_loc):
    #### Reading and converting Reviews data in Json to Dataframe

    data = {'stars': [], 'date': [], 'id': [], 'user_id':[],
            'text_len': []}

    with open(data_loc,encoding="utf8") as f:
```

```
        for line in tqdm(f):
            review = json.loads(line)
            data['stars'].append(review['stars'])
            data['id'].append(review['business_id'])
            data['user_id'].append(review['user_id'])
            data['date'].append(review['date'])
            data['text_len'].append(len(review['text'].split()))

    return pd.DataFrame(data)

def read_business_data(data_loc):
    #### Reading and converting Business data in Json to Dataframe

    data_business={'id': [], 'stars':[], 'category': [],
                   'review_count': []}

    with open(data_loc,encoding="utf8") as f:
        for line in tqdm(f):
            review = json.loads(line)
            data_business['id'].append(review['business_id'])
            data_business['stars'].append(review['stars'])
            data_business['category'].append(review['categories'])
            data_business['review_count'].
            append(review['review_count'])

    return pd.DataFrame(data_business)
```

```
def read_user_data(data_loc):
    #### Reading and converting User data in Json to Dataframe

    data_user={'user_id': [], 'review_count': [], 'friends': [] }

    with open(data_loc,encoding="utf8") as f:
        for line in tqdm(f):
            review = json.loads(line)
            data_user['user_id'].append(review['user_id'])
            data_user['review_count'].append(review['review_count'])
            if review['friends']!='None':
                data_user['friends'].append(len(review['friends']))
            else:
                data_user['friends'].append(0)

    return pd.DataFrame(data_user)
```

B.3 The function to scrape Yelp censored reviews

```
import requests
from bs4 import BeautifulSoup
import time
from random import randint
from datetime import datetime
from sklearn.preprocessing import OneHotEncoder
```

```
import numpy as np

def get_filtered_reviews(url):

    url_filtered=url.replace('biz','not_recommended_reviews')
    soup=BeautifulSoup(requests.get(url_filtered).text,"html.parser")

    timestamps_f=[]
    stars_f=[]
    text_len_f=[]
    friends_f=[]
    reviews_f=[]
    photos_f=[]

    page=int(soup.find('div',attrs={'class':'page-of-pages'})
        .text.split()[-1])
    links=['?not_recommended_start='+str(10*i) for i in range(page)]



    for link in links:
        time.sleep(randint(5,8))
        soup=BeautifulSoup(requests.get(url_filtered+link).text,
                           "html.parser")
        timestamps_f.extend(list(map(lambda x:datetime.strptime(
            x.text.split()[0],'%m/%d/%Y'),
            soup.find_all(lambda x: x.name == 'span'
```

```
            and x.get('class') == ['rating-qualifier']))))
        stars_f.extend(list(map(lambda x:
            float(x.find('img')['alt'][:3]),
            soup.find_all(class_='biz-rating__stars'))))
        text_len_f.extend(list(map(lambda x:len(x.find('p').
            text.split()),
            soup.find_all(class_='review-content'))))
        friends_f.extend(list(map(lambda x:int(x.text.split()[0]),
            soup.find_all(class_='friend-count'))))
        reviews_f.extend(list(map(lambda x:int(x.text.split()[0]),
            soup.find_all(class_='review-count'))))
        photos_f.extend(list(map(lambda x:0 if x['src']==
            'https://s3-media2.fl.yelpcdn.com/assets/srv0/
            yelp_styleguide/514f6997a318/assets/img/
            default_avatars/user_60_square.png'
            else 1,
            soup.find_all('img',attrs={'class':'photo-box-img'}))))

    return list(zip(timestamps_f,stars_f,text_len_f,friends_f,
                    reviews_f,photos_f,[1]*len(stars_f)))

def get_passed_reviews(ids_scraped,df,df_user):

    passed_reviews=[]

    for j,i in enumerate(ids_scraped):
```

```
        tmp=[]
        for z,t in df[df['id']==i].iterrows():
            try:
                tmp.append((datetime.strptime(t['date'][:10],
                    '%Y-%m-%d'),t['stars'],t['text_len'],
                df_user[df_user['user_id']==t['user_id']]
                    ['friends'].values[0],
                df_user[df_user['user_id']==t['user_id']]
                    ['review_count'].values[0],
                    randint(0,1),0))
            except:
                continue
        passed_reviews.append(tmp)
        print(j)

    return passed_reviews

def get_business_features(ids_scraped,df_business):

    features=[]
    categories=[]

    for i in ids_scraped:
        for z,t in df_business[df_business['id']==i].iterrows():
            features.append([t['stars'],t['review_count']])
            categories.append(t['category'].split(','))
```

```
    max_len=len(max(categories,key=lambda x:len(x)))

    categories=OneHotEncoder(sparse=False).fit_transform(
        [c+(max_len-len(c))*['end'] for c in categories])

    return list(zip(features,categories))
```

B.4 The main function to preprocess all data sets, run regressions and show the results

```
from Read_Yelp_Data import read_review_data, read_user_data,
read_business_data
from Scrape_Yelp import get_filtered_reviews, get_passed_reviews,
get_business_features
from Get_Links import get_links
from Results_Table import results_table
import pandas as pd
import dill
import time
from random import randint
from datetime import datetime, timedelta
import numpy as np
from sklearn.preprocessing import OneHotEncoder
from sklearn.linear_model import LinearRegression
```

```
import statsmodels.api as sm


#### Reading and converting Reviews data in Json to Dataframe
df=read_review_data('yelp_academic_dataset_review.json')


#### Reading and converting Business data in Json to Dataframe
df_business=read_business_data('yelp_academic_dataset_business.json')
# Selecting businesses whose category is 'restaurants'
# and review_count>400
df_business=df_business.dropna()
df_business=df_business[df_business.category.str.contains(
    'Restaurants')]
df_business=df_business[(df_business['review_count']>400)
    & (df_business['review_count']<750)]


#### Reading and converting User data in Json to Dataframe


df_user=read_user_data('yelp_academic_dataset_user.json')


#### pickle everything:
dill.dump(df,open('df.pkd','wb'))
dill.dump(df_user,open('df_user.pkd','wb'))
dill.dump(df_business,open('df_business.pkd','wb'))



#### Using Selenium, get the link of every business
```

```
urls,ids=get_links(df_business)
# pickle 'em
dill.dump(urls,open('urls.pkd','wb'))
dill.dump(ids,open('ids.pkd','wb'))



#### calling get_filtered_reviews function to get all the filtered
#### reviews for every restaurant

ids_scraped=[]
filtered_reviews=[]
i=0

for url,id_ in zip(urls,ids):
    time.sleep(randint(5,8))
    print(i)
    i+=1
    try:
        filtered_reviews.append(get_filtered_reviews(url))
        ids_scraped.append(id_)
    except:
        continue

# pickle 'em
dill.dump(filtered_reviews,open('filtered_reviews1.pkd','wb'))
dill.dump(ids_scraped,open('ids_scraped1.pkd','wb'))
```

```
#### get real (passed the Yelp's algorithm) reviews
passed_reviews=get_passed_reviews(ids_scraped,df,df_user)

# pickle 'em
dill.dump(passed_reviews,open('passed_reviews1.pkd','wb'))

#### remove filtered reviews that were written after the max date
#### in our data:
maxdate=datetime.strptime(max(df.date)[:10],'%Y-%m-%d')
for i in range(len(filtered_reviews)):
    filtered_reviews[i]=[j for j in filtered_reviews[i]
                         if j[0]<=maxdate]

#### merge filtered and passed reviews, and sort by date
reviews=[]
for i,j in zip(passed_reviews,filtered_reviews):
    reviews.append(sorted(i+j,key=lambda x:x[0]))

# pickle 'em
dill.dump(reviews,open('new_reviews.pkd','wb'))

######################################################

#### each time period is 1 month:
```

```
X,y=[],[]

for res in reviews:
    date=res[0][0]
    maxdate=res[-1][0]

    fake=[]
    total_reviews=[]
    avg_star=[]
    text_len=[]
    friends=[]
    review_count=[]
    photos=[]

    while date<=maxdate:
        tmp_real=[r for r in res if r[0]<date+timedelta(days=30)
                  and r[6]==0]
        tmp_all=[r for r in res if r[0]<date+timedelta(days=30)]
        if tmp_real:
            avg_star.append(sum([r[1] for r in tmp_real])/len([r[1]
                            for r in tmp_real]))
        else:
            avg_star.append(0)


        fake.append(sum([r[6] for r in tmp_all if r[1]==5]))
```

```
        total_reviews.append(2*len(tmp_all))
        text_len.append(sum([r[2] for r in tmp_all]))
        friends.append(sum([r[3] for r in tmp_all]))
        review_count.append(sum([r[4] for r in tmp_all]))
        photos.append(sum([r[5] for r in tmp_all]))

        date+=timedelta(days=30)

    X.append(list(zip(total_reviews,avg_star,friends,text_len,
                  review_count,photos)))
    y.append(fake)

#### scale by substracting avg for each restaurant (within estimator)
X_scaled=[]
y_scaled=[]
for x,y_ in zip(X,y):
    avg0=sum([rev[0] for rev in x])/len([rev[0] for rev in x])
    avg1=sum([rev[1] for rev in x])/len([rev[1] for rev in x])
    avg2=sum([rev[2] for rev in x])/len([rev[2] for rev in x])
    avg3=sum([rev[3] for rev in x])/len([rev[3] for rev in x])
    avg4=sum([rev[4] for rev in x])/len([rev[4] for rev in x])
    avg5=sum([rev[5] for rev in x])/len([rev[5] for rev in x])

    avg_y=sum(y_)/len(y_)

    X_scaled.extend([[rev[0]-avg0,rev[1]-avg1,rev[2]-avg2,rev[3]-avg3,
```

```
                      rev[4]-avg4,rev[5]-avg5] for rev in x])
    y_scaled.extend([rev-avg_y for rev in y_])


#y=[np.log(Y+1) for Y in y]
est = sm.OLS(pd.DataFrame(y_scaled,columns=['Fake 5 stars (t+1)']),
    pd.DataFrame(X_scaled,columns=['Total Reviews','Rating','Friends',
        'Review Length','Review Count','Photo']))
est=est.fit()
est.summary()


###########################################
X,y=[],[]
#business_features=get_business_features(ids_scraped,df_business)

for i,res in enumerate(reviews):
    date1=res[0][0]
    date2=res[0][0]+timedelta(days=30)
    maxdate=res[-1][0]

    real=[]
    fake=[]
    total_reviews=[]
    text_len=[]
    friends=[]
```

```
    review_count=[]
    photos=[]
#    star=[]
#    bus_rev_count=[]

    while date2+timedelta(days=30)<=maxdate:
        tmp_future=[r for r in res if r[0]>=date2
            and r[0]<date2+timedelta(days=30)]
        tmp_past=[r for r in res if r[0]>=date1 and r[0]<date2]



        real.append(sum([abs(r[6]-1) for r in tmp_future if r[1]==5]))
        fake.append(sum([r[6] for r in tmp_past if r[1]==5]))
        total_reviews.append(len(tmp_past))
        text_len.append(sum([r[2] for r in tmp_future]))
        friends.append(sum([r[3] for r in tmp_future]))
        review_count.append(sum([r[4] for r in tmp_future]))
        photos.append(sum([r[5] for r in tmp_future]))
#        star.append(business_features[i][0])
#        bus_rev_count.append(business_features[i][1])

        date1=date2
        date2=date1+timedelta(days=30)

    X.append(list(zip(total_reviews,fake,text_len,friends,
                      review_count,photos)))
```

```
    y.append(real)

# scale
X_scaled=[]
y_scaled=[]
for x,y_ in zip(X,y):
    avg0=sum([rev[0] for rev in x])/len([rev[0] for rev in x])
    avg1=sum([rev[1] for rev in x])/len([rev[1] for rev in x])
    avg2=sum([rev[2] for rev in x])/len([rev[2] for rev in x])
    avg3=sum([rev[3] for rev in x])/len([rev[3] for rev in x])
    avg4=sum([rev[4] for rev in x])/len([rev[4] for rev in x])
    avg5=sum([rev[5] for rev in x])/len([rev[5] for rev in x])
    avg_y=sum(y_)/len(y_)

    X_scaled.extend([[rev[0]-avg0,rev[1]-avg1,rev[2]-avg2,rev[3]-avg3,
                  rev[4]-avg4,rev[5]-avg5] for rev in x])
    y_scaled.extend([rev-avg_y for rev in y_])

est = sm.OLS(pd.DataFrame(y_scaled,columns=['Real 5-stars']),
    pd.DataFrame(X_scaled,columns=['Total Reviews','Fake 5-stars',
        'Review Length','Friends','Review Count','Photo']))
est=est.fit()
est.summary()

################################################
```

```
results_table(est)

from bokeh.plotting import figure
from bokeh.models import ColumnDataSource
from bokeh.transform import dodge
from bokeh.io import show, output_file, save

stars = ['1-star', '2-star', '3-star', '4-star', '5-star']
dataset = {'stars' : stars,
           'rating': [len([rev for rev in reviews_real if rev==1])/
               len(reviews_real),
                     len([rev for rev in reviews_real if rev==2])/
                         len(reviews_real),
                     len([rev for rev in reviews_real if rev==3])/
                         len(reviews_real),
                     len([rev for rev in reviews_real if rev==4])/
                         len(reviews_real),
                     len([rev for rev in reviews_real if rev==5])/
                         len(reviews_real)]}

source = ColumnDataSource(data=dataset)

p = figure(x_range=stars, y_range=(0, 1), plot_height=300,
           title="Real Star-ratings Distribution",
           toolbar_location=None, tools="")
```

```
p.vbar(x=dodge('stars', 0, range=p.x_range), top='rating', width=0.2,
       source=source, color="red")

show(p)
```

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