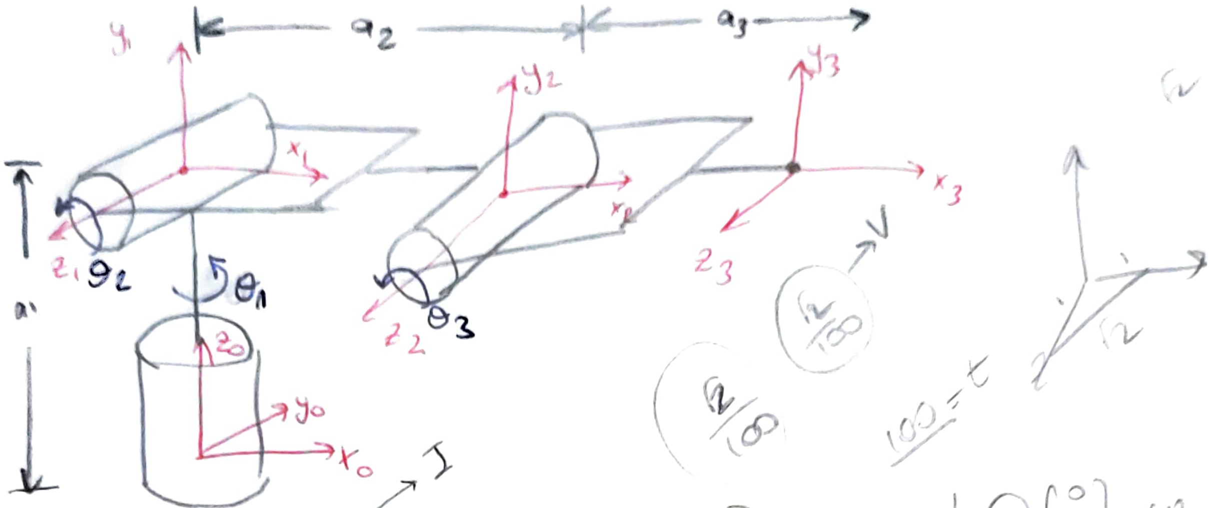
Göksel Tokur - 150116049

Intro. to Robotics
Project #2

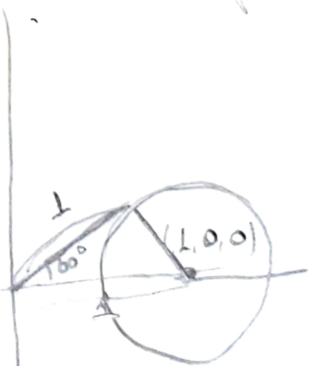


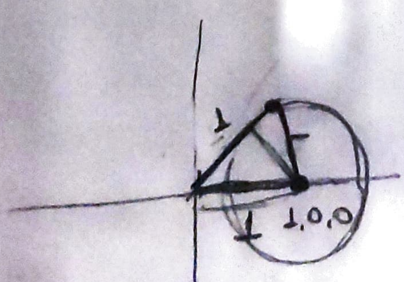
$$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} = \begin{bmatrix} \underbrace{R_0^0}_{I} \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - \cancel{d_0^0}) & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_1^0) & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_2^0) \\ \underbrace{R_0^0}_{I} \begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix}$$

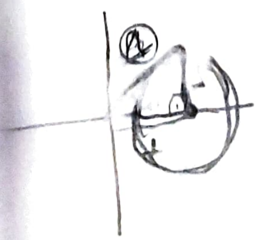
$$H_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

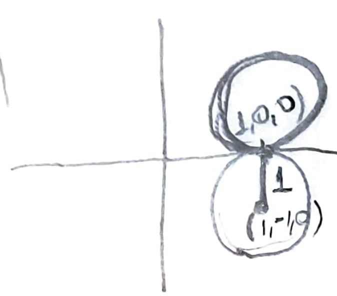
$$H_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

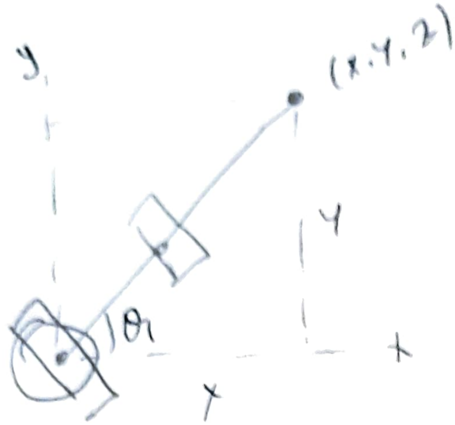
$$H_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



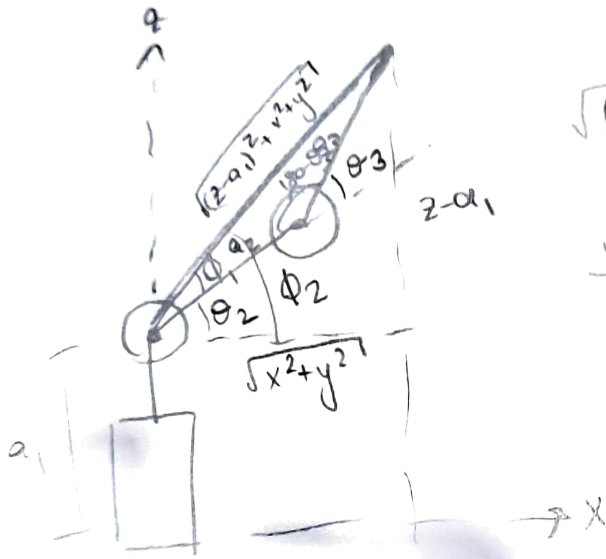








$$\boxed{\theta_1 = \arctan\left(\frac{y}{x}\right)}$$



$$\sqrt{(z-a_1)^2 + x^2+y^2} = a_2^2 + a_3^2 - 2a_2a_3\cos(180-$$

$$\frac{\sqrt{(z-a_1)^2+x^2+y^2} - a_2^2 - a_3^2}{-2a_2a_3} = \cos(180-\theta_3)$$

$$180-\theta_3 = \arccos\left(\quad\right)$$

$$180 - \arccos\left(\quad\right) = \theta_3$$

$$\theta_2 = \phi_2 - \phi_1$$

$$\phi_2 = \arctan\left(\frac{z-a_1}{\sqrt{x^2+y^2}}\right)$$

$$a_3^2 = (z-a_1)^2+x^2+y^2+a_2^2 - 2\sqrt{(z-a_1)^2+x^2+y^2}\,a_2\cos\phi_1$$

$$\frac{a_3^2-(z-a_1)^2-x^2-y^2-a_2^2}{-2\sqrt{(z-a_1)^2+x^2+y^2}\,a_2} = \cos\phi_1$$

$$\arccos\left(\quad\right) = \phi_1$$

$$\begin{bmatrix}\dot x\\ \dot y\\ \dot z\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = J \begin{bmatrix}\dot q_1\\ \dot q_2\\ \dot q_3\end{bmatrix}$$

Calc. the end effector velocities given the joint velocities

$A^{-1}A = I$

$$J^{-1}\begin{bmatrix}\dot x\\ \dot y\\ \dot z\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \cancel{J^{-1}J}\begin{bmatrix}\dot q_1\\ \dot q_2\\ \vdots\\ \dot q_n\end{bmatrix}$$

$$\begin{bmatrix}\dot q_1\\ \dot q_2\\ \vdots\\ \dot q_n\end{bmatrix} = J^{-1}\begin{bmatrix}\dot x\\ \dot y\\ \dot z\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

$$\begin{bmatrix}\dot x\\ \dot y\\ \dot z\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}\square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\\ \dot\theta_3\end{bmatrix}$$

$$\begin{bmatrix}\dot x\\ \dot y\\ \dot z\end{bmatrix} = \begin{bmatrix}J_{11} & J_{12} & J_{13}\\ J_{21} & J_{22} & J_{23}\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\\ \dot\theta_3\end{bmatrix}$$

$A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$ then $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

$$\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\\ \dot\theta_3\end{bmatrix} = J^{-1}\begin{bmatrix}\dot x\\ \dot y\\ \dot z\end{bmatrix}$$

$\theta_1 = 0$ $\theta_2 = 0$ $\theta_3 = -90$

$(x_1, y_1) = \langle 1, 0, 0\rangle$

$\theta_1 = 90$ $\theta_2 = 0$ $\theta_3 = -90$

$(x_2, y_2) = \langle 0, 0, -1\rangle$