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CS 303 - Homework #2

Question 1

a)

$$\begin{array}{r|l} 546 & 2 \\ \underline{546} & 273 \\ 0 & \end{array} \quad \begin{array}{r|l} 273 & 2 \\ \underline{272} & 136 \\ 1 & \end{array} \quad \begin{array}{r|l} 136 & 2 \\ \underline{136} & 68 \\ 0 & \end{array} \quad \begin{array}{r|l} 68 & 2 \\ \underline{68} & 34 \\ 0 & \end{array} \quad \begin{array}{r|l} 34 & 2 \\ \underline{34} & 17 \\ 0 & \end{array}$$

$$\begin{array}{r|l} 17 & 2 \\ \underline{16} & 8 \\ 1 & \end{array} \quad \begin{array}{r|l} 8 & 2 \\ \underline{8} & 4 \\ 0 & \end{array} \quad \begin{array}{r|l} 4 & 2 \\ \underline{4} & 2 \\ 0 & \end{array} \quad \begin{array}{r|l} 2 & 2 \\ \underline{2} & 1 \\ 0 & \end{array}$$

$$(546)_{10} = (1000100010)_2$$

$$(2^9 = 512) + (2^5 = 32) + (2^1 = 2) = 546$$

$$= (\underbrace{0010}_{(2)_{10} \to (2)_{16}}\ \underbrace{0010}_{(2)_{10} \to (2)_{16}}\ \underbrace{0010}_{(2)_{10} \to (2)_{16}})_2$$

$$= (222)_{16}$$

$$(2 \times 16^2) + (2 \times 16^1) + (2 \times 16^0) = (546)_{10}$$ (justification)

$$\begin{array}{r|l} 128 & 2 \\ \underline{128} & 64 \\ 0 & \end{array}$$

$128 \div 2 = 64$, remainder 0
$64 \div 2 = 32$, remainder 0
$32 \div 2 = 16$, remainder 0
$16 \div 2 = 8$, remainder 0
$8 \div 2 = 4$, remainder 0
$4 \div 2 = 2$, remainder 0
$2 \div 2 = 1$, remainder 0

$$(128)_{10} = (10000000)_2$$

$$= (\underbrace{1000}_{} \ \underbrace{0000}_{})_2$$

$$\downarrow \qquad\qquad \downarrow$$

$$(8)_{10} \qquad (0)_{10}$$

$$\downarrow \qquad\qquad \downarrow$$

$$(8)_{16} \qquad (0)_{16}$$

$$= (80)_{16}$$

$(27)_{10} \Rightarrow$

$27 \div 2 = 13$, remainder 1
$13 \div 2 = 6$, remainder 1
$6 \div 2 = 3$, remainder 0
$3 \div 2 = 1$, remainder 1

$$= (11011)_2$$

$(0.375)_{10} \Rightarrow$

$0.375 \times 2 = 0.75 \quad 0$

$0.75 \times 2 = 1.5 \quad 1$

$0.5 \times 2 = 1 \quad 1$

$(0.375)_{10} = (0.011)_2$

$\Rightarrow (27.375)_{10} = (11011.011)_2$

in hexadecimal

$(27)_{10} \Rightarrow 27 \div 16$: $27 - 16 = 11$ (B), quotient 1 $\Rightarrow (1B)_{16}$

$(0.375)_{10} \Rightarrow 0.375 \times 16 = 6 \Rightarrow (0.6)_{16}$

$\Rightarrow (27.375)_{10} = (1B.6)_{16}$

Decimal	Binary	Hexadecimal
546	0010 0010 0010	222
128	1000 0000	80
27.375	11011.011	1B.6

$(-6.375)_{10}$

$= 2\text{'s comp of } (6.375)_{10}$

$(6)_{10} = (0000\ 0110)_2$

$(0.375)_{10} = (.011)_2$

$\Rightarrow (6.375)_{10} = (0000\ 0110.011)_2$

$\Rightarrow 2\text{'s comp} \Rightarrow (1111\ 1001.101)_2$

<u>Justification:</u>

$(-1)\times 2^7 + 1\times 2^6 + 1\times 2^5 + 1\times 2^4 + 1\times 2^3 + 1\times 2^0 + 1\times 2^{-1} + 1\times 2^{-3}$

$= -128 + 64 + 32 + 16 + 8 + 1 + 0.5 + 0.125$

$= -6.375$

Decimal	Signed 8-bit binary in 2's comp. form
-6.375	1111 1001.101

$(0000\ 0011.1101)_2$

$\downarrow$

$1\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 1\times2^{-4}$

$= 2+1+0.5+0.25+0.0625$

$= (3.8125)_{10}$

$(1111\ 1111.\ 1111)_2 = (1111\ 1111\ 1111\ \times2^{-4})_2$

2's comp $\Rightarrow (0000\ 0000\ 0001\ \times2^{-4})_2$

$= (0000\ 0000.\ 0001)_2$

$= 1\times2^{-4} = 0.0625$

$= (0.0625)_{10}$

$(1010\ 0110.0111)_2 = (1010\ 0110\ 0111\ \times2^{-4})_2$

2's comp $\Rightarrow (0101\ 1001\ 1001\ \times2^{-4})_2$

$= (0101\ 1001.\ 1001)_2$

$= 1\times2^6 + 1\times2^4 + 1\times2^3 + 1\times2^0$

$+ 1\times2^{-1} + 1\times2^{-4}$

$= 64+16+8+1+0.5+0.0625$

$= (89.5625)_{10}$

b) $\left(\frac{127}{64}\right)_{10} = (1.984375)_{10}$

$(1)_{10} = (0001)_2$

$(0.984375)_{10} \Rightarrow$

0.984375×2	$=$	1.96875	1
0.96875×2	$=$	1.9375	1
0.9375×2	$=$	1.875	1
0.875×2	$=$	1.75	1
0.75×2	$=$	1.5	1
0.5×2	$=$	1	1

$\Rightarrow (1.984375)_{10} = (0001.111111)_2$

↳ use only 4 bits in the fraction;

$\Rightarrow (0001.1111)_2 = 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 1\times2^{-3} + 1\times2^{-4}$

$= 1 + 0.5 + 0.25 + 0.125 + 0.0625$

$= (1.9375)_{10}$

ERROR $= 1.984375 - 1.9375$

$= (0.046875)_{10}$

How many bits are needed to fully represent in 2's complement binary number system?

$$(\underbrace{00}01.1111\ 11\underbrace{00})_2$$

those two are redundant

those two are redundant

$$\Rightarrow (01.1111\ 11)_2$$

– 8 bits are needed –

Question 2

a)

_ _ _ _ _

with 5 bits, $2^5 = 32$ number can be represented.

the smallest value;

$10000 = (-1) \cdot 2^4 = -16$

the largest value;

$0\,1111 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 2 + 1 = 15$

therefore;

$-16, \ldots, 15$ is the range of numbers which can be represented in 5 bits b)

$14 + 9 = (23)_{10}$

$\downarrow \qquad \searrow$

01110 + 01001

$$\begin{array}{r} \overset{1}{0}1110 \\ + \ 01001 \\ \hline 10111 \end{array} = 2^4 + 2^2 + 2^1 + 2^0 = 16 + 4 + 2 + 1$$

$= (23)_{10}$

(∴ no overflow)

$14 + (-9) = (5)_{10}$

$\downarrow \qquad \searrow$

01110 $\qquad$ 10111

$(9)_{10} = 01001$

$(-9)_{10} = 10\ 111$

$$\begin{array}{r} \overset{1\,1\,1}{0}1110 \\ + \ 10111 \\ \hline \cancel{1}00101 \end{array} = 1 \times 2^2 + 1 \times 2^0 = (5)_{10}$$

(- yes overflow)
- no problem

$(-5)_{10} + (-9)_{10} = (-14)_{10}$

$(5)_{10} = 00110$

$(-5)_{10} = 11010$

$(-9)_{10} = 10111$

$$\begin{array}{r} 11010 \\ +\ 10111 \\ \hline \cancel{1}10001 \end{array}$$

$= 16+1 = (17)_{10}$

(-yes owerflow)

(-yes problem)

$(-7)_{10} + (-10)_{10} = (-17)_{10}$

$(7)_{10} = 00111$

$(-7)_{10} = 11001$

$(10)_{10} = 01010$

$(-10)_{10} = 10110$

$$\begin{array}{r} 11001 \\ +\ 10110 \\ \hline \cancel{1}01111 \end{array}$$

$= 8+4+2+1 = (15)_{10}$

(-yes owerflow)

(-yes problem)

Question 3

a) $f(x,y,z,t) = (x \oplus y)(z+t)$

x	y	z	t	$(x \oplus y)$	$(z+t)$	$(x \oplus y)(z+t)$
0	0	0	0	0	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	1	0	0
0	1	0	1	1	1	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	0	1	0

$$F(x,y,z,t) = m_5 + m_6 + m_7 + m_9 + m_{10} + m_{11}$$

$$= x'yz't + x'yzt' + x'yzt + xy'z't + xy'zt' + xy'zt$$

b)

$xy \backslash zt$	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	0	0	0	0
10	0	1	1	1

z (columns 11, 10); y (rows 01, 11); x (rows 11, 10); t (columns 01, 11)

$$F(x,y,z,t) = x'yt + x'yz + xy't + xy'z$$

$$= x'y(t+z) + xy'(t+z)$$

$$= (t+z)\cdot(x'y+xy')$$

c)

$xy \backslash zt$	00 (t')	01 (t)	11 (t)	10 (t')
y' 00	0	0	0	0
y 01	0	1	1	1
y 11	X	X	X	X
y' 10	0	1	1	1

x' (rows 00, 01); x (rows 11, 10); z' (columns 00, 01); z (columns 11, 10)

$$F(x,y,z,t) = yt + yz + xt + xz$$

Question 4

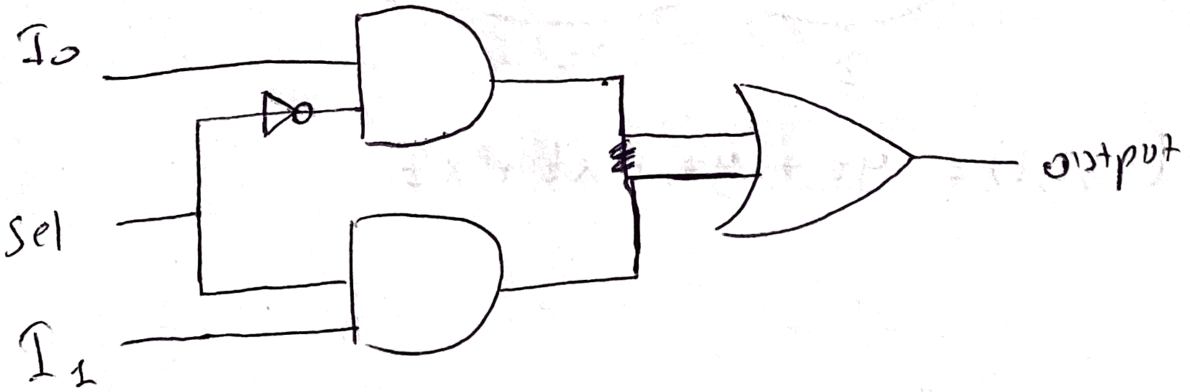
	I_0	I_1	sel	output
	0	0	0	0
	0	0	1	0
	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	1
	1	0	1	0
m_6	1	1	0	1
m_7	1	1	1	1

I_1' $\quad$ I_1

	$I_0 \backslash I_1$ sel	00	01	11	10
I_0'	0	0	0	1	0
I_0	1	1	0	1	1

sel' $\quad$ sel $\quad$ sel'

$$F(I_0, I_1, sel) = I_0\, sel' + I_1\, sel$$



Question 5/

A		B			A<B	A>B
A1	A0	B1	B0	EQ	L	G
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	0	1
1	1	1	1	1	0	0

!! Note that, EQ can be obtained by combining L and G outputs.

$$EQ = L' \cdot G' = (L+G)'$$

$$EQ = F(A_0, A_1, B_0, B_1) = A_0' A_1' B_0' B_1' + A_0' A_1 B_0' B_1 + A_1' A_0 B_1' B_0 + A_1 A_0 B_1 B_0$$

L:

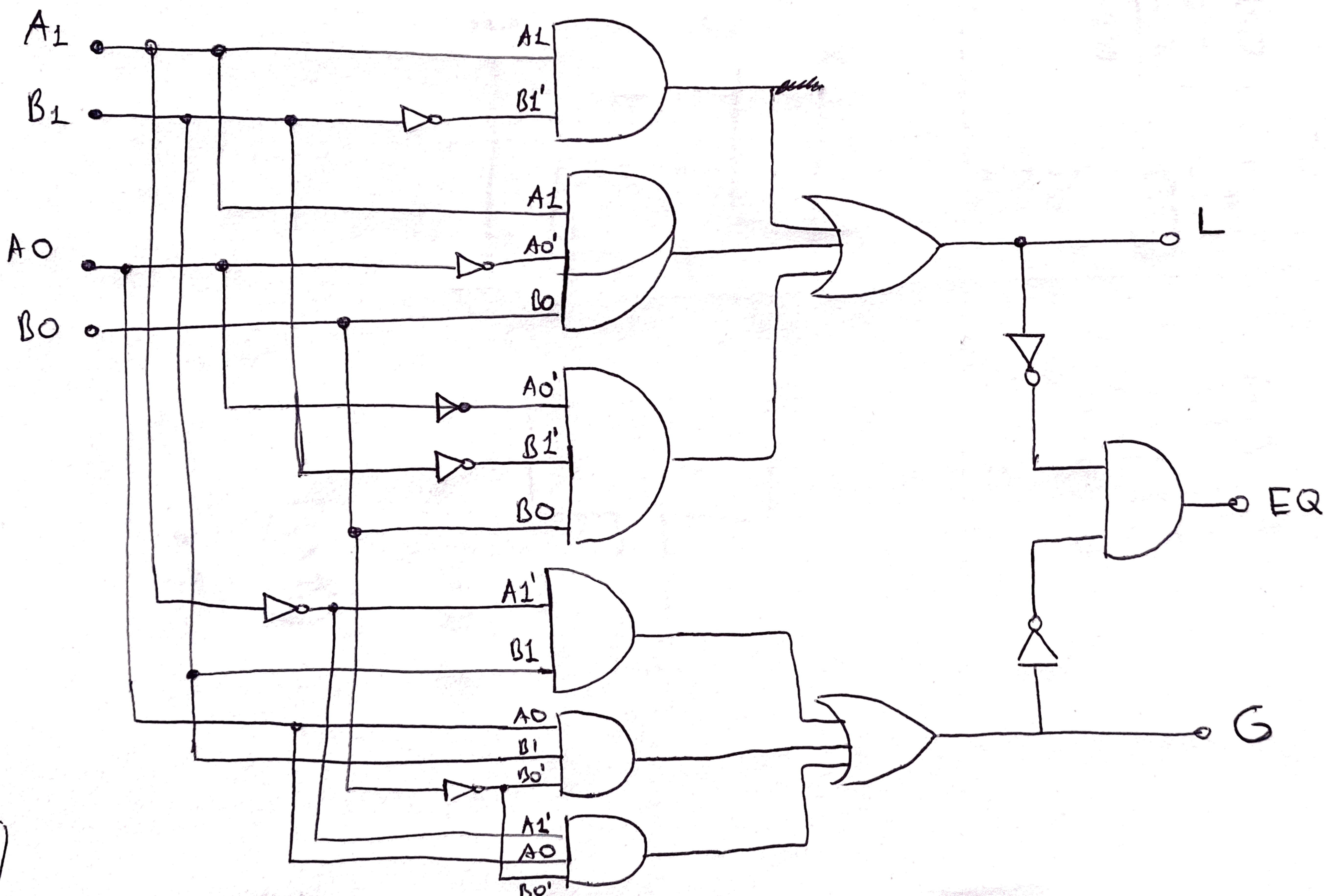
A_1A_0 \ B_1B_0	00 (B_1')	01 (B_1')	11 (B_1)	10 (B_1)	
A_1' 00	0	1	0	0	A_0'
A_1' 01	0	0	0	0	A_0
A_1 11	1	1	0	0	A_0
A_1 10	1	1	1	0	A_0'
	B_0'	B_0	B_0	B_0'	

$$L = F(A_1, A_0, B_1, B_0) = A_1 B_1' + A_1 A_0' B_0 + A_0' B_1' B_0$$

G:

A_1A_0 \ B_1B_0	00 (B_1')	01 (B_1')	11 (B_1)	10 (B_1)	
A_1' 00	0	0	1	1	A_0'
A_1' 01	1	0	1	1	A_0
A_1 11	0	0	0	1	A_0
A_1 10	0	0	0	0	A_0'
	B_0'	B_0	B_0	B_0'	

$$G = F(A_1, A_0, B_1, B_0) = A_1' B_1 + A_0 B_1 B_0' + A_1' A_0 B_0'$$


A1
B1
A0
B0
A1
B1'
A1
A0'
B0
A0'
B1'
B0
A1'
B1
A0
B1
B0'
A1'
A0
B0'
L
EQ
G