Göktuş Korkulu
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CS 303 Homework #1

Q1

0 1 2 3 4 5 6 7 8 9 A B C D E F
10 11 12 13 14 15

a) E3 73 B3 B0 B3 20 68 F7 31 20 F1 31.

1110 0011 / 0111 0011 / 1011 0011 / 1011 0000 /
1011 0011 / 0010 0000 / 0110 1000 / 1111 0111 /
0011 0001 / 0010 0000 / 1111 0001 / 0011 0001

binary to decimal conversion

$(64+32+2+1)_{10}$ / $(64+32+16+2+1)_{10}$ / $(32+16+2+1)_{10}$ / $(32+16)_{10}$

$(32+16+2+1)_{10}$ / $(32)_{10}$ / $(64+32+8)_{10}$ / $(64+32+16+4+2+1)_{10}$

$(32+16+1)_{10}$ / $(32)_{10}$ / $(64+32+16+1)_{10}$ / $(32+16+1)_{10}$

$(99)_{10}$ / $(115)_{10}$ / $(51)_{10}$ / $(48)_{10}$ /

$(51)_{10}$ / $(32)_{10}$ / $(104)_{10}$ / $(119)_{10}$ /

$(49)_{10}$ / $(32)_{10}$ / $(113)_{10}$ / $(49)_{10}$ /

decimal to ascii decoding

'c' / 's' / '3' / '0' / '3' / 'space' / 'h' / 'w' / '1' / 'space' / 'q' / '1'

⇒ "CS303 hw1 q1"

b)

				parity bit
E3:	four 1's	=	even	→ '1'
73:	five 1's	=	odd	→ '0'
B3:	four 1's	=	even	→ '1'
B0:	two 1's	=	even	→ '1'
B3:	four	=	even	→ '1'
10:	one	=	odd	→ '0'
68:	three	=	odd	→ '0'
F7:	six	=	even	→ '1'
31:	three	=	odd	→ '0'
20:	one	=	odd	→ '0'
F1:	four	=	even	→ '1'
31:	three	=	odd	→ '0'

Since the parity makes the entire amount of 1's in the binary number odd, the used parity is odd.

Q2)

	paritled representation
'0' $= (48)_{10} = 32+16 = (0011\ 0000)_2$	1 00110000
'1' $= (49)_{10} = (0011\ 0001)_2$	0 00110001
'2' $= (50)_{10} = (0011\ 0010)_2$	0 00110010
'3' $= (51)_{10} = (0011\ 0011)_2$	1 0011 0011
'4' $= (52)_{10} = (0011\ 0100)_2$	0 0011 0100
'5' $= (53)_{10} = (0011\ 0101)_2$	1 0011 0101
'6' $= (54)_{10} = (0011\ 0110)_2$	1 0011 0110
'7' $= (55)_{10} = (0011\ 1000)_2$	0 0011 1000
'8' $= (56)_{10} = (0011\ 1001)_2$	1 0011 1001
'9' $= (57)_{10} = (0011\ 1010)_3$	1 0011 1010

Q31

a) $(127)_{10} - (1)_{10} = (0111\ 1111)_2 - (0000\ 0001)_2 = (0111\ 1110)_2$

verification

$$0.2^0 + \underset{2}{\underline{1.2^1}} + \underset{4}{\underline{1.2^2}} + \underset{8}{\underline{1.2^3}} + \underset{16}{\underline{1.2^4}} + \underset{32}{\underline{1.2^5}} + \underset{64}{\underline{1.2^6}} + 0.2^7 = 126$$

b) $(30)_{10} + (-71)_{10}$

$(30)_{10} = (16+8+4+2)_{10} = (0001\ 1110)_2$

$(71)_{10} = (64+4+2+1)_{10} = (0100\ 0111)_2$

$(-71)_{10} = (1011\ 1001)_2$ 2's complement

$$(30)_{10} + (-71)_{10} = \begin{array}{r} 0001\ 1110 \\ +\ 1011\ 1001 \\ \hline 1101\ 0111 \end{array} = (1101\ 0111)_2$$

verification

$$(1101\ 0111)_2 = (-1).2^7 + 1.2^6 + 0.2^5 + 1.2^4 + 0.2^3 + 1.2^2 + 1.2^1 + 1.2^0$$
$$= -128 + 64 + 16 + 4 + 2 + 1 = (-41)_{10}$$

c) $$(30)_{10} - (-71)_{10} = (30)_{10} + (71)_{10} = \begin{array}{r} 0001\ 1110 \\ +\ 0100\ 0111 \\ \hline 0110\ 0101 \end{array} = (0110\ 0101)_2$$

verification

$$(0110\ 0101)_2 = 0.2^7 + 1.2^6 + 1.2^5 + 0.2^4 + 0.2^3 + 1.2^2 + 0.2^1 + 1.2^0$$
$$= 64 + 32 + 4 + 1 = (101)_{10}$$

d) $(-60)_{10} - (-127)_{10} = (-60)_{10} + (127)_{10} = (67)_{10}$

$(60)_{10} = (32+16+8+4)_{10} = (0011\ 1100)_2$

$(-60)_{10} = (1100\ 0100)_2$ ← 2's complement

$(127)_{10} = (0111\ 1111)_2$

$(-60)_{10} + (127)_{10} =$

$$\begin{array}{r} 1100\ 0100 \\ +\ 0111\ 1111 \\ \hline *\ 0100\ 0011 \end{array}$$

$= (0100\ 0011)_2$

verification

$(0100\ 0011)_2 = 0.2^7 + 1.2^6 + 0.2^5 + 0.2^4 + 0.2^3 + 0.2^2 + 1.2^1 + 1.2^0$

$= 64 + 2 + 1 = (67)_{10}$

e) $(39.5)_{10} - (41.75)_{10} = (-2.25)_{10}$

$(39.5)_{10} \longrightarrow (39)_{10} \Rightarrow$

39 | 2: 38, 1 — 19 | 2: 18, 1 — 9 | 2: 8, 1 — 4 | 2: 4, 0 — 2 | 2: 2, 0 — 1

$= (0010\ 0111)_2$

$(39.5)_{10} \longrightarrow (0.5)_{10} \Rightarrow 0.5 \times 2 = 1 \quad = (.1000\ 000)_2$

$(41.75)_{10} \longrightarrow (41)_{10} \Rightarrow$

41 | 2: 40, 1 — 20 | 2: 20, 0 — 10 | 2: 10, 0 — 5 | 2: 4, 1 — 2 | 2: 2, 0 — 1

$= (0010\ 1001)_2$

$(41.75)_{10} \longrightarrow (0.75)_{10} \Rightarrow$

$0.75 \times 2 = 1.5 \quad 1$

$0.5 \times 2 = 1 \quad 1$

$= (.1100\ 0000)_2$

$(-41.75)_{10}$: 2's complement of $(0010\ 1001.11)_2$

$= $ 2's comp of $0010\ 100111 \times 2^{-2}$

$= 1101011001 \times 2^{-2}$

$= 11010110.01$

$(39.5)_{10} + (-41.75)_{10} =$

$$\begin{array}{r} 0010\ \ 0111.1\ \ \\ +\ 1101\ \ 0110.01 \\ \hline 1111\ \ 1101.11 \end{array}$$

$= (1111\ 1101.11)_2$

verification

$(1111\ 1101.11)_2 = (-1).2^7 + 1.2^6 + 1.2^5 + 1.2^4 + 1.2^3 + 1.2^2 + 0.2^1 + 1.2^0 + 1.2^{-1} + 1.2^{-2}$

$= -128 + 64 + 32 + 16 + 8 + 4 + 1 + 0.5 + 0.25$

$= (-2.25)_{10}$

f) $(41.84375)_{10} - (80.15625)_{10} = (-38.3125)_{10}$

$(41.84375)_{10} \longrightarrow (41)_{10} = (0010\ 1001)_2$

$\longrightarrow (.84375)_{10}$;

$0.84375 \times 2 = 1.6875$ 1

$0.6875 \times 2 = 1.375$ 1

$0.375 \times 2 = 0.75$ 0

$0.75 \times 2 = 1.5$ 1

$0.5 \times 2 = 1$ 1

$= (.11011)_2$

$\Rightarrow (41.84375)_{10} = (0010\ 1001.11011)_2$

$(80.15625)_{10} \longrightarrow (80)_{10} = 64 + 16 = (0101\ 0000)_2$

$\searrow (.15625)_{10}$;

$0.15625 \times 2 = 0.3125$ (0)

$0.3125 \times 2 = 0.625$ (0)

$0.625 \times 2 = 1.25$ (1)

$0.25 \times 2 = 0.5$ (0)

$0.5 \times 2 = 1$ (1)

$= .00101$

$\Rightarrow (80.15625)_{10} = (0101\ 0000.00101)_2$

$(-80.15625)_{10} = 2\text{'s comp. of } (80.15625)_{10}$

$= 2\text{'s comp of } (0101\ 0000.00101)_2$

$= (1010\ 1111.11011)_2$

$$(41.84375)_{10} + (-80.15625)_{10} = \begin{array}{r} 0010\ 1001.11011 \\ +\ 1010\ 1111.11011 \\ \hline 1101\ 1001.10110 \end{array}$$

$= (1101\ 1001.10110)_2$

verification

$(-1).2^7 + 1.2^6 + 0.2^5 + 1.2^4 + 1.2^3 + 0.2^2 + 0.2^1 + 1.2^0 + 1.2^{-1} + 0.2^{-2} + 1.2^{-3}$
$+ 1.2^{-4} + 0.2^{-5}$

$= -128 + 64 + 16 + 8 + 1 + 0.5 + 0.125 + 0.0625 = (-38.3125)_{10}$

Q4)

Binary number system can be used due to the following method;

→ when the binary is given to us, if we just consider right most six bits only and discard the remaining left bits, we can create a number system that realizes arithmetic modula 64.

e.g.

$(90 \% 64)_{10} \Rightarrow \underbrace{1\,011010}_{\text{take those}} \Rightarrow (011010)_2 = (16)_{10}$

$(-16 \% 64)_{10} \Rightarrow (48)_{10} \Rightarrow (16)_{10} = 0001\ 0000$

$(-16)_{10} = 11\underbrace{11\ 0000}_{\text{take these}}$

$\Rightarrow (11\ 0000)_2 = (48)_{10}$

Q5|

$$\left(\frac{1}{7}\right)_{10} = (0.1428571429)_{10}$$

$$
\left.\begin{array}{llll}
0.1428571429 & \times 2 & = 0.2857142857 & 0 \\
0.2857142857 & \times 2 & = 0.5714285714 & 0 \\
0.5714285714 & \times 2 & = 1.142857143 & 1
\end{array}\right\}\text{pattern}
$$

$$0.142857143 \quad \times 2 \quad = 0.2857142857 \qquad 0$$

$$\Rightarrow \left(\frac{1}{7}\right)_{10} = (.001001001001\ldots)_2$$

but since we only considered left four bits;

$$\left(\frac{1}{7}\right)_{10} \cong (.0010\ 0100)_2$$

and the error;

$$(.0010\ 0100)_2 = 0.2^{-1}+0.2^{-2}+1.2^{-3}+0.2^{-4}+0.2^{-5}+1.2^{-6}+0.2^{-7}+0.2^{-8}$$

$$= 0.125 + 0.015625 = (0.140625)_{10}$$

the error is;

$$0.\overline{142857} - 0.1406250000 = 0.002232\overline{142857}$$