Multivariate Time Series Analysis of EUR/TRY Exchange Rate and BIST Index

Seminar Paper

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1. Introduction

Multivariate time series analysis is a statistical method used to analyse and model the joint dynamics of multiple time series variables. Unlike univariate time series analysis, which focuses on a single variable evolving over time, multivariate time series analysis involves examining the relationships and interactions among several variables that are observed at the same points in time.

Economists often work with datasets that include various interconnected time series, such as data on GDP growth, inflation rates, and unemployment. Figuring out how these variables evolve together is vital for building accurate economic models and predicting future trends.

Time series analysis uses various techniques for forecasting. An example of time series analysis is Autoregressive Moving Average (ARMA) models. The ARMA model combines the properties of Autoregressive (AR) and Moving Average(MA) in order to describe the dynamic structure of univariate data.

Although the chance of using ARMA models is low for the return series in finance, the concept of ARMA models is highly relevant in volatility modelling. The ARMA model enables economists to uncover the cause-and-effect relationship by capturing past values of each economic variable and their interdependencies and provides a versatile framework for understanding the evolving relationships of economic indicators.

As we explore time series analysis in econometrics, we also look into the relevance of models like Autoregressive Conditional Heteroscedasticity (ARCH) and Generalised Autoregressive Conditional Heteroscedasticity (GARCH). These models, commonly used in financial contexts, prove valuable for understanding volatility and time-varying variances in economic time series.

This study will focus on the connection between the foreign exchange market and the stock market. This connection has drawn the attention of many researchers. Numerous empirical studies demonstrate complex and occasionally contradictory results. In this study, I will delve into the impacts of fluctuating exchange rates in TRY/EUR on Turkey's stock market, Borsa ISTANBUL (BIST100).

When examining the studies on a global scale. In the early stages of their research, Franck and Young (1972) did not find a significant correlation between stock prices and foreign exchange rates. Aggarwal (1981) noted a stronger positive relationship in the short term than in the long term, utilising a simple regression method. In contrast, Soenen and Hennigar (1988) recognized a negative association.

When we shift focus on the studies conducted in Turkey, the findings from Savaş and Can's study in 2011 suggest that both Euro-Dollar Parity and the Real Effective Exchange Rate Index have a positive impact on the BIST100, accounting for 77.5%. Moreover, based on the results of the Granger Causality Test, a causality was identified from the BIST100 to the Euro-Dollar Parity and Real Effective Exchange Rate. The cointegration test results conducted by Ayvaz in 2006 demonstrate the existence of a stable, long-term relationship between the foreign exchange rate and BIST100, when monthly returns are used.

2. Theoretical part

In the theoretical examination of multivariate time series analysis, I will delve into basic ideas that serve as the fundamental knowledge in econometrics. For instance, I will highlight the importance of stationarity, white noise, and autoregressive models. I will explore necessary tools for explaining the complex relationships and interdependencies among economic variables. This theoretical basis offers the necessary framework for the analysis of the relationship between the Borsa Istanbul 100 stock index and the exchange rate between the Euro and the Turkish lira.

2.1 Stationarity

Stationarity is a crucial concept in time series analysis. It refers to the statistical properties of a time series remaining constant over time. In simpler terms, it implies that the mean, variance and autocorrelation of the data do not show significant changes in different time points.

2.2 White Noise

A time series X_t is called a white noise if it's a sequence of independent and identically distributed (iid) random variables with a finite mean and variance. Specifically, when X_t is normally distributed with a mean of 0 and a variance of σ^2, it is called a Gaussian white noise.
For a white noise series, all the autocorrelation functions (ACFs) are 0. In practical terms, if all sample ACFs are close to 0, it means the series is a white noise series.

2.3 Simple Autoregressive Model

An AR (AutoRegressive) model is a type of time series forecasting model that predicts future values based on past observations. In an AR model, the predicted value at a given time point is a linear combination of its own past values. The model assumes that the relationship between the current value and its past values can be used to make accurate predictions. The term "autoregressive" signifies that the model relies on its own past values for making predictions. The order of the model (AR order) indicates how many past values are considered in the prediction. The formulation can be shown as the following:

$$X_t = a_0 + a_1 X_{t-1} + \varepsilon_t$$

The ε_t is assumed to be a white noise with mean 0 and variance σ^2. This form of the model is the same as the well-known simple linear regression model, where X_t is the dependent variable and X_{t-1} is the independent variable. In the time series literature, the above model is an AR model of order 1 AR(1).

2.4 Multivariate Time Series

Assume that for each t $Y_t = (Y_{1,t}, \ldots, Y_{d,t})'$ which stands for a set of d-dimensional random values. These values represent quantities measured at time t, for instance returns on d equities. Thus, Y_1 and Y_2... are called a d-dimensional time series.

For multivariate time series, the definition of stationarity is the same as the univariate time series. If $Y_1, \ldots, Y_n$ and $Y_{1+m}, \ldots, Y_{n+m}$ have the same distributions for every n and m, a multivariate time series is stationary.

2.5 Cross Correlation Function

Assume that Y_j and Y_i are component vectors of stationary multivariate time series. We can write the cross-correlation function as the following:

$$\mathrm{P}Y_j, Y_i(h) = Corr\{Y_j(t), Y_i(t-h)\}$$

This is formula gives the correlation between Y_j at time point t and Y_i at time point $(t-h)$. h is called the lag as with the autocorrelation. The CCF is not symmetric in the lag variable h like ACF. Meaning that, $\mathrm{P}Y_j, Y_i(h) \neq \mathrm{P}Y_j, Y_i(-h)$ and as a direct consequence of definition we have $\mathrm{P}Y_j, Y_i(h) = \mathrm{P}Y_i, Y_j(-h)$.

A multivariate time series, denoted as $Y_1, Y_2, \ldots$ is considered weakly stationary when the average and covariance matrix of Y_t are finite and do not depend on t.

Cross-correlations in this context provide insights into how individual series within the multivariate time series might be influencing each other or influenced by a common factor. It's crucial to note that cross-correlations reveal statistical associations, not causal relationships. Although the correlations suggest connections, establishing a cause-and-effect relationship requires additional knowledge.

2.6 Multivariate White Noise

Y_1 and Y_2 d-dimensional multivariate time series is a weak white noise process $WN(\mu; \Sigma)$ if:

1. $E(Y_t) = \mu$ constant and finite for all t,
2. $COV(Y_t) = \sum$ constant and finite for all t,
3. For all $t \neq s$, all components of Y_t are uncorrelated with all components of Y_s

If the covariance matrix Σ is not diagonal, it implies that there may be simultaneous correlations between components of the multivariate time series. However, these correlations are specific to the same time point. At different time points, the components are assumed to be independent $Corr(Y_{j,t}, Y_{i,s}) = 0 \; if \; t \neq s$, maintaining the characteristics of white noise.

2.7 Multivariate ACF Plots and the Multivariate Ljung-Box Test

For each individual multivariate time series $(Y_1, Y_2, \ldots, Y_d)$ there are d-marginal ACFs denoted as $(\mathrm{P}Y_1(h), \mathrm{P}Y_2(h), \ldots, \mathrm{P}Y_d(h)$. These capture the autocorrelation at different lags (h) for each individual series. Additionally, the Cross-Correlation Function (CCF) includes d(d − 1)/2 Cross-Correlation Functions for all unordered pairs of univariate series. The consideration of unordered pairs is sufficient due to the symmetry property, where $\mathrm{P}Y_j, Y_i(h) = \mathrm{P}Y_i, Y_j(-h)$. This approach efficiently captures both the autocorrelation within each series and the cross-correlation between different series, providing a comprehensive understanding of temporal dependencies in the multivariate time series.

Let ρ(h) represent the cross-correlation matrix for a d-dimensional multivariate time series at lag h. The null hypothesis in the multivariate Ljung–Box test is:

$$H_0\colon \rho(1) = \rho(2) = \cdots = \rho(K) = 0,$$

with K typically set to 5 or 10. If the test rejects this hypothesis, it suggests that at least one of the cross-correlations from lag 1 to K is not zero. If, indeed, all these lagged cross-correlations are zero, there is only a 5% chance of mistakenly concluding otherwise (at a 0.05 significance level).

2.8 Multivariate ARMA Process

A d-dimensional multivariate time series $(Y_1, Y_2, \ldots)$ is considered an ARMA(p,q) process with mean μ if, for d x d matrices $\alpha_1, \ldots, \alpha_p \; and \; ß_1, \ldots, ßq$, the following relation holds:

$$Y_t - \mu = \alpha_1 (Y_{t-1} - \mu) + \ldots + \alpha_p (Y_{t-p} - \mu) + \varepsilon_t + ß_1 \varepsilon_{t-1} + \ldots + ß_q \varepsilon_{t-q}$$

where $\varepsilon_1, \ldots, \varepsilon_n$ is a multivariate weak white noise (WN(0,Σ)) process. Multivariate AR processes (in the case q=0) are alternatively known as vector autoregressive (VAR) processes and find widespread application in practical settings.

2.9 Multivariate Autoregressive (Vector Autoregressive) Model

In the world of analysing multiple time series, the Vector Autoregressive (VAR) model stands out as a robust tool for understanding how various variables interact and evolve together. Unlike models that focus on just one variable at a time, VAR extends the idea of autoregression to capture the joint evolution of several variables observed simultaneously.
At its core, the VAR model involves a set of simultaneous equations, where each equation represents how one variable behaves based on its past values and the past values of all other variables in the system. Mathematically, a VAR(p) model of order 'p' for 'k' variables can be expressed like this:

$$Y_t = \alpha + A_1 Y_{t-1} + A_2 Y_{t-2} + \dots + A_p Y_{t-p} + \varepsilon_t$$

Here:

- Y_t is a is a k-dimensional vector of endogenous variables at time *'t'*
- α is a constant vector
- $A_1, A_2, \dots, A_p$ are matrices capturing the lagged effects
- ε_t is a vector of white noise disturbances.

2.10 Lag Order Selection:

In the context of Vector Autoregressive (VAR) modelling, selecting the appropriate lag order (p) is significant. The lag order determines the number of past observations considered in forecasting the current values of multiple variables. The selection process involves utilizing criteria such as the Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC), which find a balance between model fit and complexity. A lower AIC or BIC indicates a better model. This trade-off is crucial to prevent overfitting or underfitting and to ensure the model captures the essential dynamics of the underlying multivariate time series.

2.11 Forecasting with VAR:

One significant advantage of the VAR model is its ability to forecast. By estimating coefficients from historical data, the VAR model lets us predict future values of all variables in the system. This becomes particularly useful when dealing with interconnected economic indicators, offering a comprehensive view of how changes in one variable may affect the entire system over time.

2.12 Granger Causality Test:

The Granger causality test evaluates whether past values of one variable can predict the future values of another. The p-value below a significance level means rejecting the null hypothesis shows that past values of one time series help predict another time series, indicating a potential causal relationship between them.

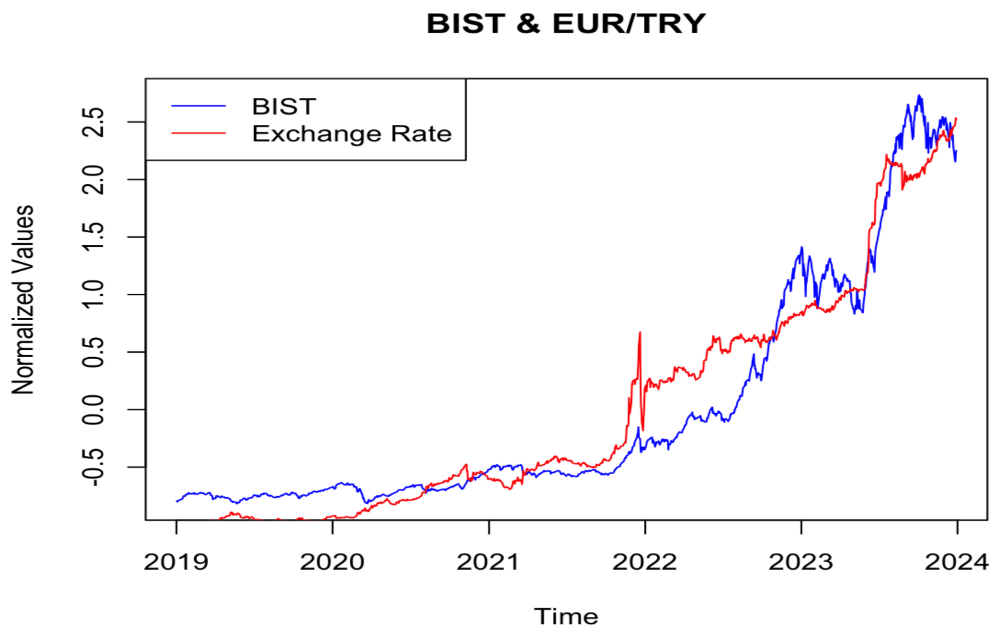
In the upcoming sections of this paper, I will apply the VAR model to the specific context of analysing the relationship between the foreign exchange market and the stock market, using TRY/EUR exchange rates and Borsa Istanbul (BIST100) as key variables. Through this application, my goal is to unravel the complex dynamics of these economic indicators, offering valuable insights for both academic study and practical applications.

3. Practical Application

The dataset utilized for this analysis has been obtained from Yahoo Finance, which is a financial platform known for its comprehensive and up-to-date financial information. This dataset focuses on critical economic indicators, specifically daily close values of the Borsa Istanbul 100 (BIST100) stock index and the Euro-to-Turkish Lira (EUR/TRY) exchange rate. The BIST100 index is a significant benchmark for the Turkish stock market, reflecting the performance of the top 100 companies listed on the Borsa Istanbul Stock Exchange. Simultaneously, the EUR/TRY exchange rate is a vital variable, indicating the relative value of the Euro against the Turkish Lira in the foreign exchange market.

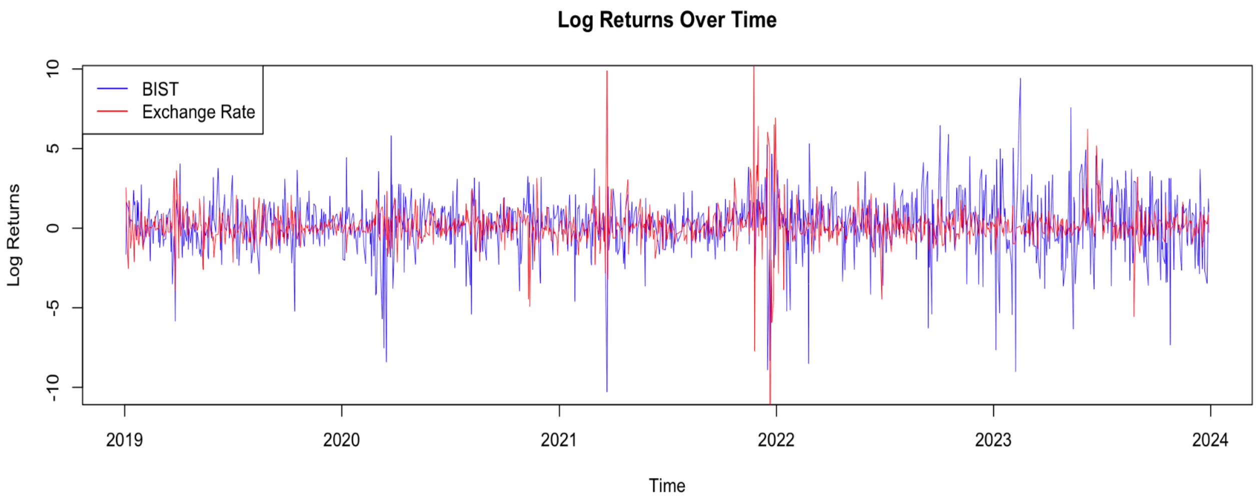
The data covers the years between 2019 and 2024, it consists of 3 variables: “Date”, “BistIndex” and “ExchangeRate” and 1247 observations capturing daily fluctuations in these economic indicators. Through a comprehensive analysis of this dataset, the goal is to unveil intricate patterns, dependencies, and dynamics between stock market performance and the foreign exchange rate.

I have normalized the values of two key variables, the BIST Index and the EUR/TRY exchange rate, to present their behavior and relationship in a line graph. By placing them on a common scale, we can easily observe their patterns and trends in a comparative manner. This graphical representation allows us to explore potential correlations between the BIST index and the EUR/TRY exchange rate. The similarities in their movements suggest a relationship, and further analysis may uncover insights into how changes in one variable correspond to changes in the other. This visualization serves as a valuable tool for understanding their dynamics and potential interdependence.



3.1 Descriptive Statistics of Log Returns:

I first examine the descriptive statistics of the log returns of the BIST Index and EUR/TRY exchange rate in order to obtain a better understanding of the underlying dynamics. The log returns give information on the volatility and distribution of the variables as well as the percentage change in the variables over time. I computed important statistical measures, such as the lowest and maximum values, mean, median, standard deviation, skewness, and kurtosis, for the EUR/TRY exchange rate log returns and the BIST Index. The shape, dispersion, and core tendency of the data are described by these descriptive statistics. For the purpose of creating a Vector Autoregressive (VAR) model to examine the correlation between various economic indicators, it is essential to comprehend the properties of log returns.

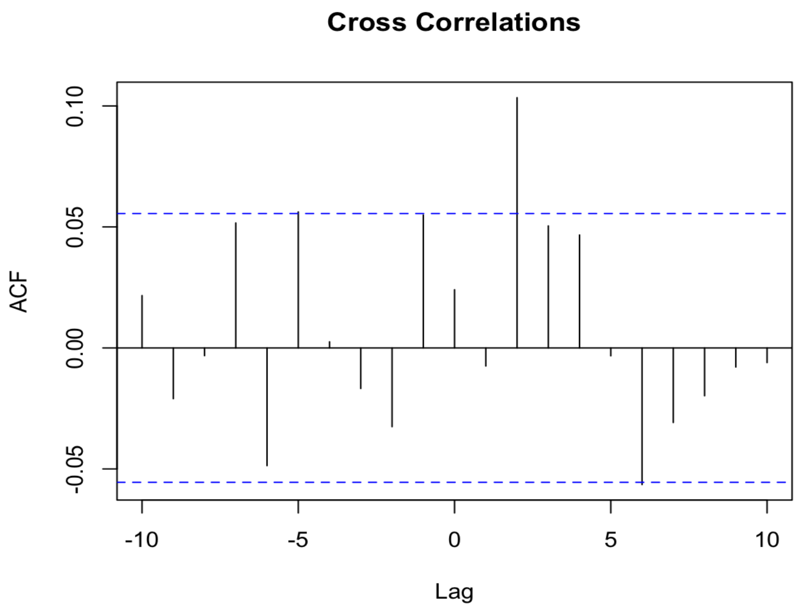


Variables	*BIST*	*Exchange Rate*
Min	-10.3	-19.62
Max	9.42	11.77
Mean	0.17	0.13
Median	0.25	0.09
Standard Deviation	1.8	1.3
Skewness	-0.77	-2.08
Kurtosis	4.88	56.62

There is a positive mean log return for both variables (0.17 and 0.13) we can infer an average tendency towards positive returns. The standard deviation (1.8 and 1.3) shows the extent of their fluctuations, with higher values indicating more significant variability. Additionally, there is negative skewness for the BIST index and exchange rate (-0.77 and -2.08) suggests a slightly leftward skewness, indicating the presence of more extreme negative values. The exchange rate's high kurtosis (56.62) suggests the possibility of heavy tails and outliers.

3.2 Lag Order Estimation:

In order to determine the appropriate order of lagged observations to include in the VAR model, Cross-Correlation Function is used. In the CCF plot below, a peak on the second lag is observed. It indicates a significant correlation between the variables at the second time lag. This might imply that exchange rate movements influence the stock market with a two-day delay.



In Var modelling we can reach AIC scores. In order to have BIC scores we have to use the following formula: $BIC=AIC+(\log(n)-2)\cdot p$.

Lag Order	1	2	3	4	5	6	7	8	9	10
AIC	1.697	1.690	1.695	1.689	1.687	1.684	1.686	1.691	1.692	1.697
BIC	6.0497	12.051	18.121	24.086	30.071	36.024	42.071	48.214	54.285	60.502

The suggested lag order for the VAR model is 6 based on the AIC values, and for the BIC, it offers a lag order of 1. Goodness of fit and model complexity are balanced by the AIC and BIC model selection criteria. When compared to BIC, the AIC chooses models that are more complicated.

3.3 VAR Model

To simplify the study and look for the daily returns, I will use the lag order of 1, as recommended by the BIC score. The following are the approximated equations for lag order 1. Based on the Bayesian Information Criterion (BIC), indicated balance between model fit and complexity, this decision was made. The associations between the variables' current values and their lag values, as well as constant terms, are shown in the equations, offering insights into the system's short-term dynamics.

3.3.1 VAR Model with Lag Order 1

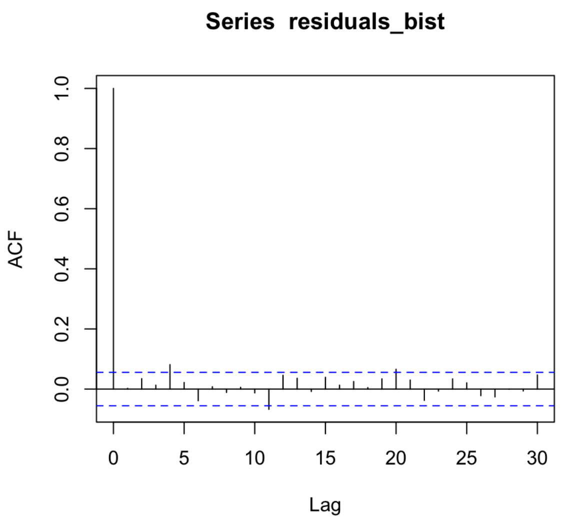
Estimated Model For Bist Log Returns:

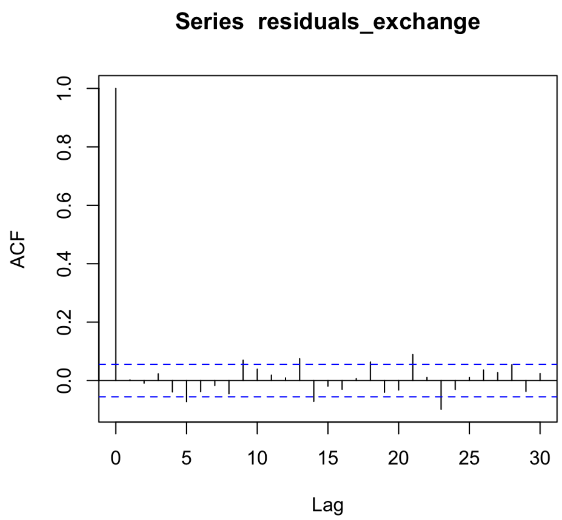
$$BistLr_t = 0.00283622 * BistLr_{t-1} + 0.07586992 * ExchangeLr_{t-1} + 0.16159631$$

Estimated Model For Exchange Rate Log Returns:

$$ExchangeLr_t = -0.006994592 * ExchangeLr_{t-1} + 0.090586065 * BistLr_{t-1} + 0.121870756$$

3.3.1.1 Model Check for Independence





Plots show that some lags exceed the confidence intervals, suggesting possible data dependencies that the current model might not adequately account for. I then evaluated the residuals' independence using the Ljung-Box test. The BIST index residuals' p-value is 0.924, which implies independence and the acceptance of the null hypothesis. On the other hand, the Exchange Rate has a p-value of 0.934, which results in the null hypothesis being accepted, suggesting independence.

3.3.2 VAR Model with Lag Order 5

Because the plots revealed that some lags exceeded the confidence intervals, suggesting possible data dependencies that the current model might not adequately explain, I decided to evaluate the residuals' independence using the Ljung-Box test with a lag order of 5, intending to capture returns of the entire week.

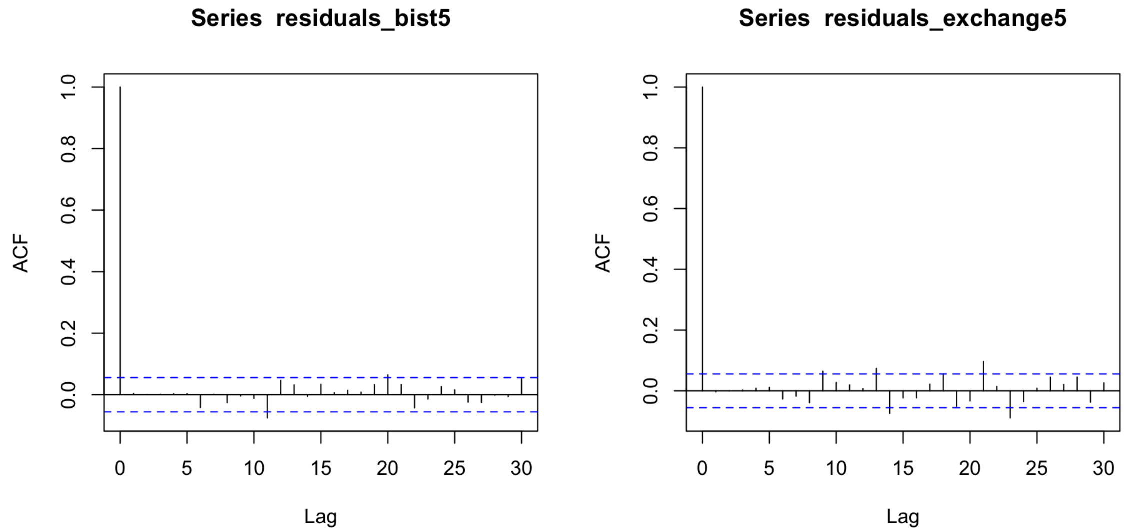
Estimated Model For Bist Log Returns:

$$\begin{aligned}
BistLr_t = \;& 0.000500240 * BistLr_{t-1} + 0.075871242 * ExchangeLr_{t-1} \\
& + 0.032887473 * BistLr_{t-2} - 0.067569949 * ExchangeLr_{t-2} \\
& + 0.014242828 * BistLr_{t-3} - 0.022513813 * ExchangeLr_{t-3} \\
& + 0.085434809 * BistLr_{t-4} - 0.005520909 * ExchangeLr_{t-4} \\
& - 0.021344686 * BistLr_{t-5} - 0.076929009 * ExchangeLr_{t-5} + 0.135229010
\end{aligned}$$

Estimated Model For Exchange Rate Log Returns:

$$\begin{aligned}
ExchangeLr_t = & - 0.007522027 * BistLr_{t-1} + 0.085715020 * ExchangeLr_{t-1} \\
& + 0.069780967 * BistLr_{t-2} - 0.011982028 * ExchangeLr_{t-2} \\
& + 0.030104261 * BistLr_{t-3} + 0.018024631 * ExchangeLr_{t-3} \\
& + 0.033517556 * BistLr_{t-4} - 0.040676252 * ExchangeLr_{t-4} \\
& - 0.006993589 * BistLr_{t-5} - 0.069418856 * ExchangeLr_{t-5} + 0.114807951
\end{aligned}$$

3.3.2.1 Model Check for Independence

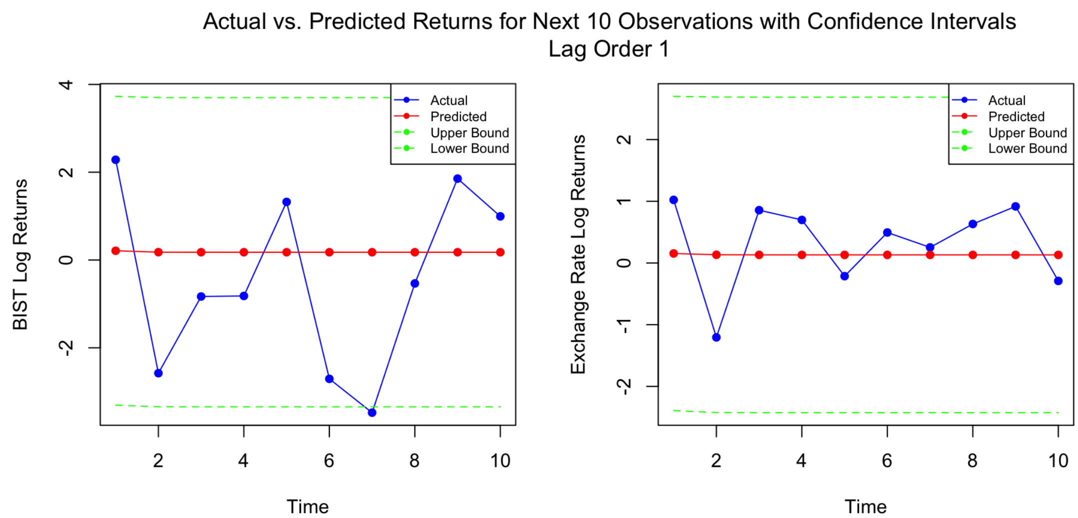


For Bist residuals, the ACF plots indicate a more independent model although certain lags exceed the confidence interval. In order to check the independence, the Ljung-Box is used again. The BIST and Exchange Rate p-values of 1 and 0.9981 obtained from the test indicate that the residuals are independent.

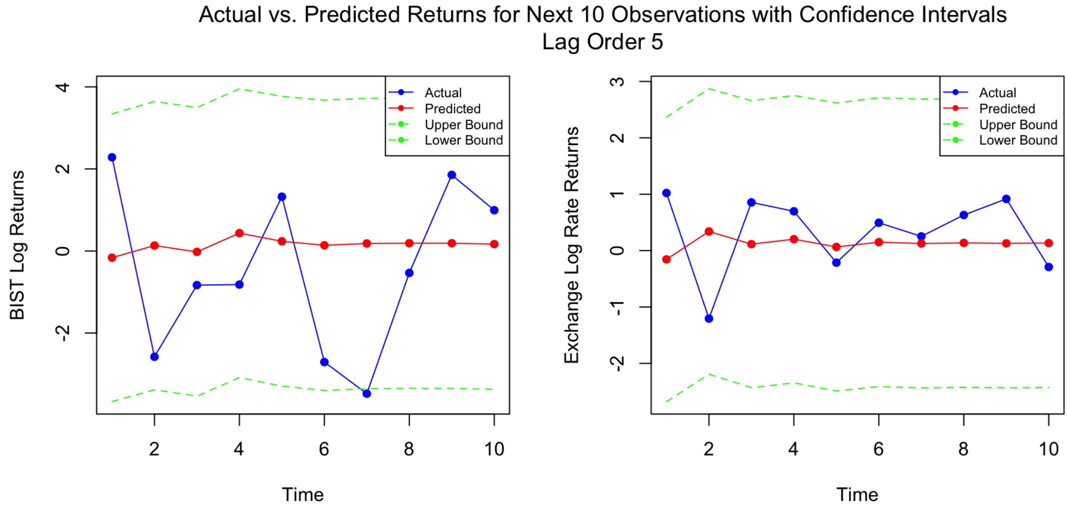
3.4 Forecasting

Two distinct Vector Autoregressive (VAR) models with lag order 1 and 5 are used to predict the future values of the EUR/TRY exchange rate and the BIST100 stock index. The forecasting process included generating predictions for 10 time points in the future, allowing for an assessment of the models' accuracy and performance. Visualizations of the forecasted values with the actual values helped to understand how well the VAR models captured the patterns.

3.4.1 Lag Order 1



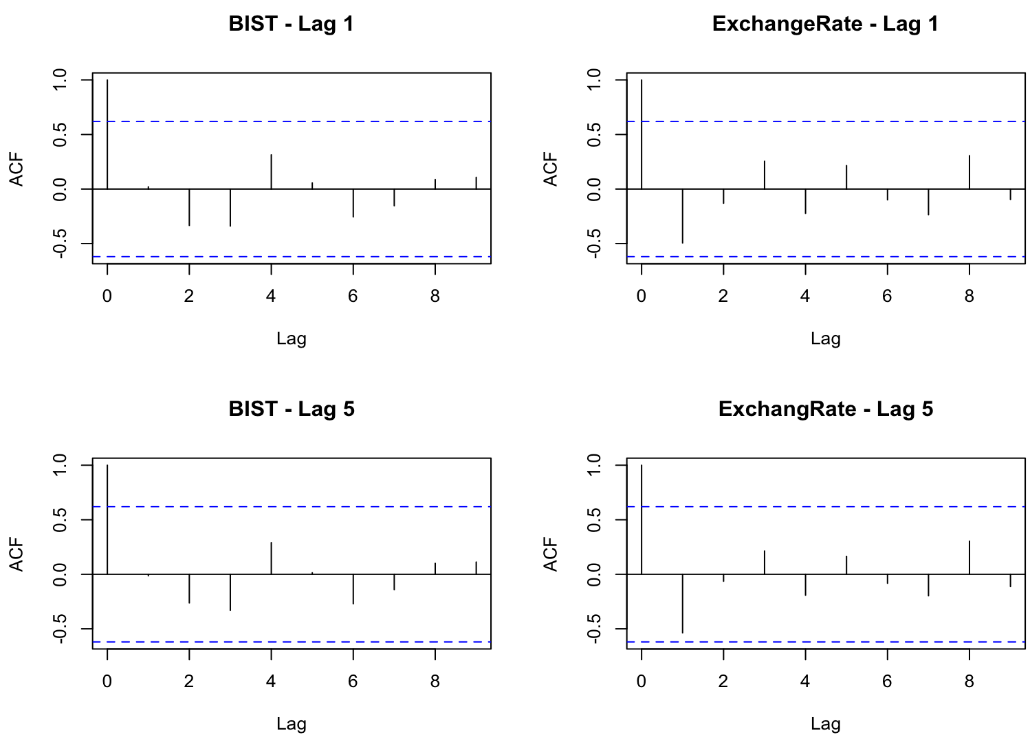
3.4.2 Lag Order 5



In the graphs above, it is clear that the predictions for the Exchange Rate Log Returns perform better than Bist Log Returns for both lag orders 1 and 5. Notably, the predictions for Exchange Rate Log Returns at lag order are closer to the actual values.

3.4.3 ACF Plots for Residuals

In order to assess the independence of the residuals from the VAR (Vector Autoregressive) models for both BIST log returns and Exchange Rate log returns, I used the Autocorrelation Function (ACF) plots and Ljung-Box tests. The ACF Plots indicate no autocorrelation at chosen lags.



3.4.5 Ljung-Box Test

The Ljung-Box test results for the estimated log returns at different lag orders are summarized in the table below:

Variable	Lag Order	p-value	Significance
BIST	1	0.9404786	Not Significant
Exchange Rate	1	0.0710651	Not Significant
BIST	5	0.4687587	Not Significant
Exchange Rate	5	0.2991121	Not Significant

The p-values exceed the confidence interval of 0.05. This indicates that, across various lag orders, we fail to reject the null hypothesis of independence. Therefore, the residuals show a lack of significant autocorrelation, supporting the validity of the Vector Autoregressive (VAR) models.

3.5 Granger Causality

In the Granger causality tests, I assessed the causal relationships between BIST log returns and Exchange Rate log returns. The tests were conducted for lag orders of 1 and 5.

Response Variable	Causal Variable	Lag Order	p-value	Significance
BIST	Exchange Rate	1	0.05334	Not Significant
Exchange Rate	BIST	1	0.7804	Not Significant
BIST	Exchange Rate	5	0.0821	Not Significant
Exchange Rate	BIST	5	0.00286	Significant

Granger causality tests show that there are no other significant causal links, with one exception of BIST log returns that significantly impact Exchange Rate log returns at lag order 5. This implies that over time, changes in BIST log returns could result in changes in Exchange Rate log returns.

4. Conclusion

In conclusion, multivariate time series analysis was used in this work to examine the relationship between the Euro-to-Turkish Lira (EUR/TRY) exchange rate and the Borsa Istanbul 100 (BIST100) stock index. The complex relationships within the financial dataset can be understood by using a range of models and tests that were developed using a comprehensive study of the theoretical basis and practical application of econometrics.

Fundamental concepts like stationarity, white noise, autoregressive models, multivariate time series, cross-correlation functions, and Granger causality tests were discussed theoretically. The basis for understanding the dynamic interactions between economic variables that have been seen across time has been provided by these theoretical foundations.

In the study's practical section, the financial dataset was analyzed according to the Vector Autoregressive (VAR) model, which took into account the log returns of the exchange rate and the BIST index. The study presented the trade-off between model fit and complexity using metrics like AIC and BIC, emphasizing the importance of lag order selection. The short-term dynamics between the exchange rate and the BIST index were explained by the computed VAR models.

Comprehensive checks for residual independence were made using Ljung-Box tests and Autocorrelation Function (ACF) plots throughout the analysis. The validity of the VAR models was supported by the Ljung-Box tests, which consistently showed the independence of residuals, even though several lags in the ACF plots exceeded confidence ranges.

A significant causal relationship between BIST log returns and Exchange Rate log returns at lag order 5 was found by the Granger causality tests. However, the tests failed to show statistical significance for other lag orders and causal relationships.

In conclusion, this research expands our knowledge of the complex relationships there are between exchange rates and stock market performance. The findings show the importance of independence checks, model fit, and lag order when doing multivariate time series analysis. The results demonstrate the complexity of causal links, emphasizing the need for more investigation and model improvement.

This study demonstrates the use of time series analysis to detect patterns, dependencies, and causal relationships in complex economic systems, which is significant in the broader field of econometrics. With its strong framework for predicting and decision-making, econometrics is a valuable tool for both academics and practitioners wanting to understand the fluctuations of financial markets.

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