



Regulations:

- **Grouping:** You are allowed to work in pairs.
- **Submission:** We provide a latex template for your solutions. Use that template and create a hw3.tar.gz file that includes hw3.tex and all other related files. Tar.gz file should not contain any directories and should create a hw3.pdf file with the following commands, otherwise you will get zero;
`tar xvzf hw3.tar.gz`
`pdflatex hw3.tex`
Submit hw3.tar.gz to the odtuclass page of the course.
- **Deadline:** 23:55, 28 May, 2021 (Friday).
- **Late Submission:** Not allowed.

1. (15 pts) Find and plot the spectral coefficients of the Fourier series representation for the following signals.

- (a) (5 pts) $x(t) = \frac{1}{2} + \cos \omega_0 t$
- (b) (5 pts) $y(t) = \frac{3}{2} + 2 \sin \omega_0 t$
- (c) (5 pts) $z(t) = x(t) + y(t) + \cos(2\omega_0 t + \frac{\pi}{4})$

2. (15 pts) Consider the continuous time signal, $x(t)$ given in Figure 1.

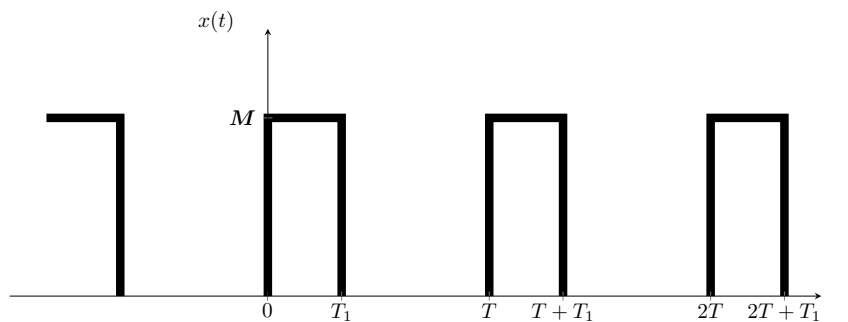


Figure 1: t vs. $x(t)$.

Find and plot the coefficients, A_0 , A_k and B_k of the trigonometric Fourier representation of signal $x(t)$, given below:

$$x(t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos k\omega_0 t + B_k \sin k\omega_0 t.$$

3. (16 pts) Consider the following signal $x(t)$:

$$x(t) = 1 + \frac{1}{2} \cos 2\pi t + \cos 4\pi t + \frac{2}{3} \cos 6\pi t$$

- (a) (4 pts) Plot the signal, $x(t)$.
 - (b) (4 pts) Find and plot the spectral coefficients of the Fourier series representation for $x(t)$.
 - (c) (4 pts) Compare the plots of the time domain signal $x(t)$ and its Fourier series representation, $\{a_k\}$ obtained in part a and part b. Explain clearly the behavior (period, frequency) of the signal $x(t)$ in terms of its Fourier series representation.
 - (d) (4 pts) Suppose, now, that the signal in part a is fed as an input to an LTI system and produces the output $y(t)$. The impulse response of the system is $h(t) = e^{-2t}u(t)$. Find and plot the Fourier series coefficients of $y(t)$. (Hint: Find the relationship of the spectral coefficients of input and output pair in terms of eigenvalues of the LTI system).
4. (10 pts) Given that the continuous time periodic signal $x(t)$ has a_k as Fourier series coefficients and its fundamental period is T , determine the Fourier series coefficients of the following signals.

- (a) (5 pts) $\frac{1}{3}x(t-3) - \frac{2}{7}x(-t)$
- (b) (5 pts) $\frac{d^3 x(t)}{dt^3}$

5. (16 pts) Consider the following discrete time signals:

$$x[n] = \sin \frac{\pi}{2}n$$

$$y[n] = 1 + \cos \frac{\pi}{2}n$$

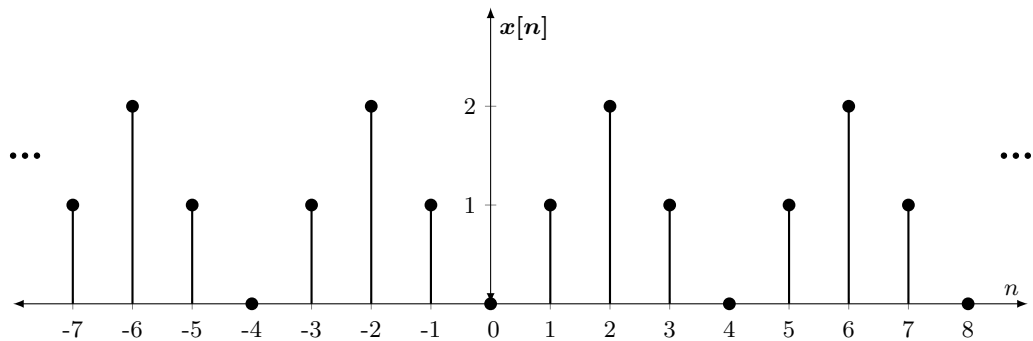
- (4 pts) Find the Fourier series coefficients of $x[n]$.
- (4 pts) Find the Fourier series coefficients of $y[n]$.
- (4 pts) Using the multiplication property of the discrete time Fourier series, find the Fourier series coefficients of $x[n] \times y[n]$.
- (4 pts) Using Fourier analysis equation (direct evaluation), find the Fourier series coefficients of $x[n] \times y[n]$. Compare the result with the result of part c.

6. (8 pts) For the specified Fourier series coefficient below, find the periodic signal $x[n]$.

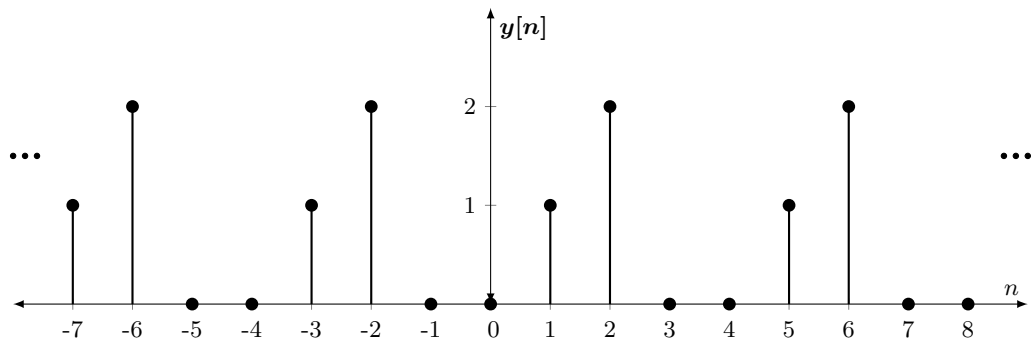
$$a_k = \cos \frac{k\pi}{6} + \sin \frac{5k\pi}{6}$$

7. (20 pts)

- (8 pts) Find and plot the spectral coefficients of the Fourier series representation for the following discrete time signal, $x[n]$:



- Consider the following discrete time signal, $y[n]$:



- (4 pts) Define $y[n]$ in terms of $x[n]$.
- (8 pts) Find and plot the spectral coefficients of Fourier series for $y[n]$.