CENG 384 - Signals and Systems for Computer Engineers
20202
Written Assignment 2 Solutions

May 3, 2021

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1. (a)

$$\begin{aligned} x(t) - 6\int_{-\infty}^{t} y(\tau)d\tau - 5y(t) &= y'(t) \\ x'(t) - 6y(t) - 5y'(t) &= y''(t) \\ y''(t) + 5y'(t) + 6y(t) &= x'(t) \end{aligned}$$

(b) char eqn. : $r^2 + 5r + 6 = 0 \quad \Rightarrow \quad r_1 = -3, r_2 = -2 \quad \Rightarrow \quad y_h(t) = A \cdot e^{-3t} + B \cdot e^{-2t}$
$y_p(t) = C \cdot e^{-t} + D \cdot e^{-4t}$
$y_p'(t) = -C \cdot e^{-t} - 4D \cdot e^{-4t}$
$y_p''(t) = C \cdot e^{-t} + 16D \cdot e^{-4t}$
$x'(t) = -e^{-t} - 4e^{-4t}$

$$\begin{aligned} C \cdot e^{-t} + 16D \cdot e^{-4t} - 5C \cdot e^{-t} - 20D \cdot e^{-4t} + 6C \cdot e^{-t} + 6D \cdot e^{-4t} &= -e^{-t} - 4e^{-4t} \\ 2C \cdot e^{-t} + 2D \cdot e^{-4t} &= -e^{-t} - 4e^{-4t} \end{aligned}$$

$2C = -1 \quad \Rightarrow \quad C = -1/2$
$2D = -4 \quad \Rightarrow \quad D = -2$

$$\begin{aligned} y(t) &= y_h(t) + y_p(t) = (A \cdot e^{-3t} + B \cdot e^{-2t} - \frac{1}{2}e^{-t} - 2e^{-4t})u(t) \\ y(0) &= A + B - \frac{1}{2} - 2 = 0 \quad \Rightarrow \quad A + B = \frac{5}{2} \\ y'(t) &= -3A \cdot e^{-3t} - 2B \cdot e^{-2t} + \frac{1}{2}e^{-t} + 8e^{-4t} \\ y'(0) &= -3A - 2B + \frac{1}{2} + 8 = 0 \quad \Rightarrow \quad 3A + 2B = \frac{17}{2} \quad \Rightarrow \quad A = \frac{7}{2}, B = -1 \\ y(t) &= (-\frac{1}{2}e^{-t} - e^{-2t} + \frac{7}{2}e^{-3t} - 2e^{-4t})u(t) \end{aligned}$$

2. (a)

$$x_1[n] = x[n] - x[n-2]$$

Since it is an LTI system,

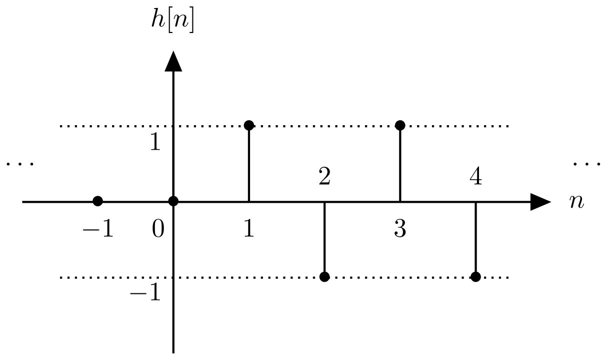
$$y_1[n] = y[n] - y[n-2] = \delta[n-1] - \delta[n-3]$$

(b)

$$\begin{aligned} x[n] * h[n] &= y[n] \\ (\delta[n] + \delta[n-1]) * h[n] &= \delta[n-1] \\ h[n] + h[n-1] &= \delta[n-1] \\ h[n] &= 0 \text{ for } n \leq 0 \\ h[n] &= \delta[n-1] - h[n-1] \\ h[1] &= 1 \\ h[2] &= -1 \\ h[3] &= 1 \\ h[4] &= -1 \\ &\vdots \end{aligned}$$

Therefore,

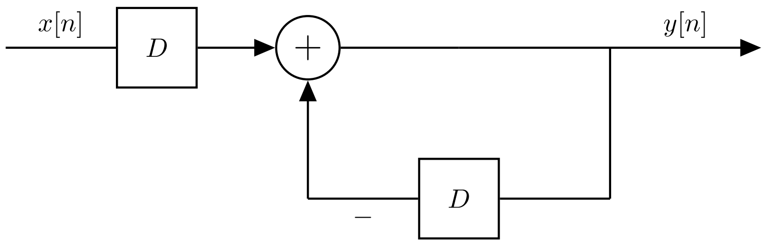
$$h[n] = (-1)^{n-1}u[n-1]$$



(c)

$$\delta[n] \rightarrow \boxed{System} \rightarrow h[n]$$
$$h[n] + h[n-1] = \delta[n-1]$$
$$x[n] \rightarrow \boxed{System} \rightarrow y[n]$$
$$y[n] + y[n-1] = x[n-1]$$

(d) The block diagram:



3. (a)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$\begin{aligned} &= x[-1]h[n+1] + x[3]h[n-3] \\ &= 2(\delta[n] + 3\delta[n+3]) + \delta[n-4] + 3\delta[n-1] \\ &= \delta[n-4] + 3\delta[n-1] + 2\delta[n] + 6\delta[n+3] \end{aligned}$$

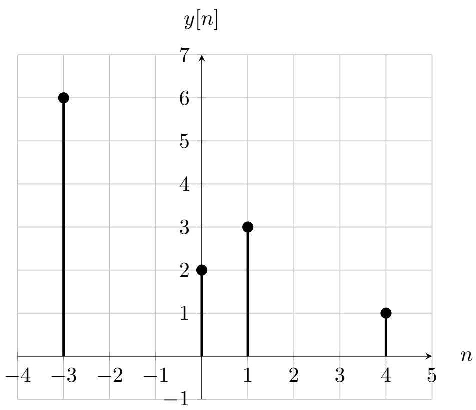


Figure 1: n vs. $y[n]$.

(b)

$$\begin{aligned} x[n] &= \delta[n+3] + \delta[n+2] + \delta[n+1] \\ h[n] &= \delta[n-1] + \delta[n-2] \\ y[n] &= \delta[n+2] + \delta[n+1] + \delta[n] + \delta[n+1] + \delta[n] + \delta[n-1] \\ &= \delta[n+2] + 2\delta[n+1] + 2\delta[n] + \delta[n-1] \end{aligned}$$

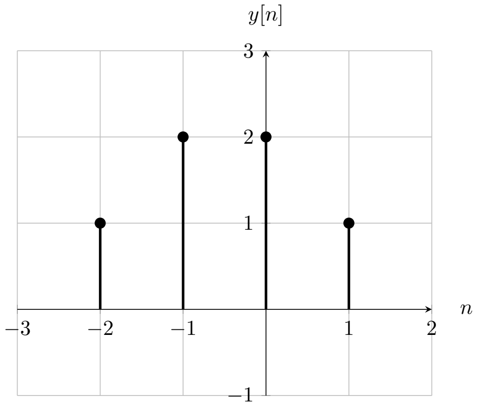


Figure 2: n vs. $y[n]$.

4. (a)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \\
&= \int_0^t e^{-2\tau} e^{-3(t-\tau)} d\tau \\
&= e^{-3t} \int_0^t e^{\tau} d\tau \;=\; e^{-3t}(e^t - 1)u(t) \;=\; (e^{-2t} - e^{-3t})u(t)
\end{aligned}
$$

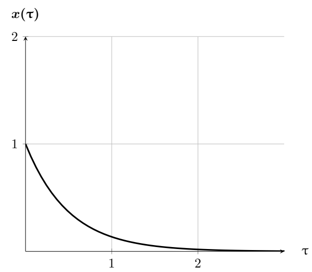


Figure 3: τ vs. $x(\tau)$.

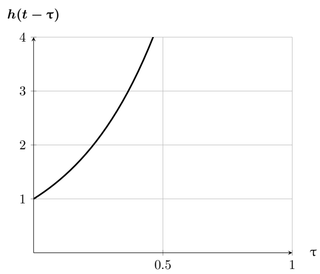


Figure 4: τ vs. $h(t-\tau)$.

(b)

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

For $0 < t < 2$:

$$\int_0^t e^{2(t-\tau)} d\tau \;=\; e^{2t} \int_0^t e^{-2\tau} d\tau \;=\; -\frac{1}{2} e^{2t}(e^{-2t} - 1) \;=\; \frac{1}{2}(e^{2t} - 1)(u(t) - u(t-2))$$

For $t > 2$:

$$\int_0^2 e^{2(t-\tau)} d\tau \;=\; -\frac{1}{2} e^{2t}(e^{-4} - 1) \;=\; \frac{1}{2}(e^{2t} - e^{2t-4})u(t-2)$$

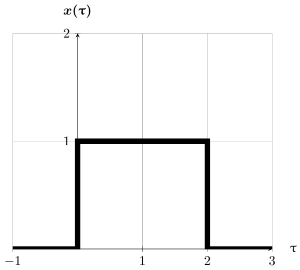


Figure 5: τ vs. $x(\tau)$.

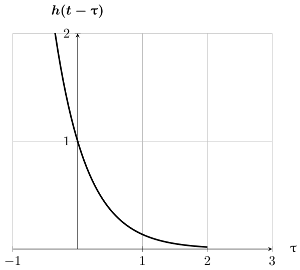


Figure 6: τ vs. $h(t-\tau)$.

5. (a) $h[n]$ is the first-difference of the sample response, $s[n]$. Therefore:

$$\begin{aligned} h[n] &= s[n]-s[n-1] = nu[n]-(n-1)u[n-1] = n(u[n]-u[n-1]) + u[n-1] \\ &= \underbrace{n\delta[n]}_{0} + u[n-1] = u[n-1] \end{aligned}$$

(b) We know that $u[n] * (\delta[n]-\delta[n-1]) = \delta[n]$ (i.e., first difference of the step is the impulse). Now shift the left hand side of the convolution by 1 (Since $h[n]$ is $u[n-1]$):

$$u[n-1] * (\underbrace{\delta[n]-\delta[n-1]}_{x_2}) = \delta[n-1]$$

Now think of what the second part of the convolution should be to get $\delta[n]$ as the result:

$$u[n-1] * (\underbrace{\delta[n+1]-\delta[n]}_{x_1}) = \delta[n]$$

Finally, we will use superposition, time invariance and distributive properties:

$$(\underbrace{x_1 - x_2}_{x[n]}) * h[n] = y[n]$$

$$(\delta[n+1]-\delta[n]-(\delta[n]-\delta[n-1])) * u[n-1] = \delta[n]-\delta[n-1]$$

Therefore,

$$x[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$$

(c)

$$y[n] = u[n-1] * x[n]$$

Since it is an LTI system and convolution has the commutative property:

$$y[n-1] = u[n-2] * x[n]$$

Now consider the superposition, time invariance and distributive properties:

$$y[n]-y[n-1] = (\underbrace{u[n-1]-u[n-2]}_{\delta[n-1]}) * x[n]$$

Therefore our difference equation is the following:

$$y[n]-y[n-1] = x[n-1]$$

6.

$$h(t) = \frac{ds(t)}{dt} = tu(t)$$
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

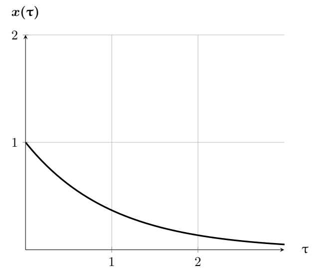


Figure 7: τ vs. $x(\tau)$.

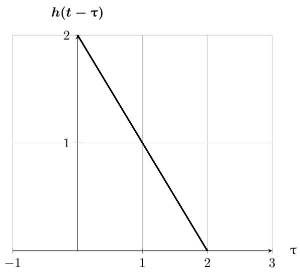


Figure 8: τ vs. $h(t-\tau)$.

$$y(t) = \int_0^t e^{-\tau}(t-\tau)d\tau$$
$$y(t) = \underbrace{t\int_0^t e^{-\tau}d\tau}_{A} + \underbrace{\int_0^t -e^{-\tau}\tau d\tau}_{B}$$

$$\begin{aligned} A &= t\int_0^t e^{-\tau}d\tau \\ &= t\left(-e^{-\tau}\Big|_0^t\right) \\ &= t(-e^{-t} - (-e^0)) \\ &= t(-e^{-t} + 1) \\ &= -te^{-t} + t \end{aligned}$$

$$B = \int_0^t -e^{-\tau}\tau d\tau$$

Integration by parts:

$$u = \tau, v = e^{-\tau}$$
$$du = d\tau, dv = -e^{-\tau}d\tau$$

$$\begin{aligned} B &= \tau e^{-\tau}\Big|_0^t - \int_0^t e^{-\tau}d\tau \\ &= te^{-t} - \left(-e^{-\tau}\Big|_0^t\right) \\ &= te^{-t} - (-e^{-t} - (-e^0)) \\ &= te^{-t} - (-e^{-t} + 1) \\ &= te^{-t} + e^{-t} - 1 \end{aligned}$$

$$
\begin{aligned}
y(t) &= A + B \\
&= -te^{-t} + t + te^{-t} + e^{-t} - 1 \\
&= \cancel{-te^{-t}} + t + \cancel{te^{-t}} + e^{-t} - 1 \\
&= (t + e^{-t} - 1)u(t)
\end{aligned}
$$

7. (a)

$$
\begin{aligned}
h(t) &= u(t) * (\delta(t-3) - \delta(t-5)) \\
&= u(t-3) - u(t-5)
\end{aligned}
$$

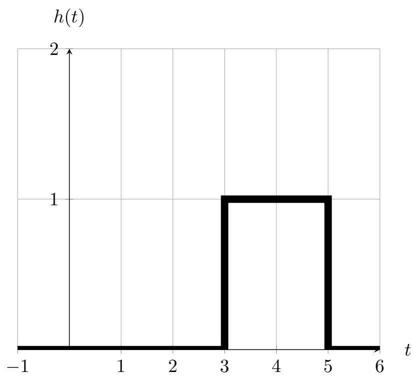


Figure 9: t vs. $h(t)$.

(b)

$$
\begin{aligned}
y(t) &= h(t) * x(t) \\
&= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau
\end{aligned}
$$

For $3 < t < 5$:

$$
\int_3^t e^{-3(t-\tau)}d\tau \;=\; e^{-3t}\int_3^t e^{3\tau}d\tau \;=\; \frac{1}{3}e^{-3t}(e^{3t} - e^{9}) \;=\; \frac{1}{3}(1 - e^{9-3t})(u(t-3) - u(t-5))
$$

For $t > 5$:

$$
\int_3^5 e^{-3(t-\tau)}d\tau \;=\; e^{-3t}\int_3^5 e^{3\tau}d\tau \;=\; \frac{1}{3}e^{-3t}(e^{15} - e^{9})u(t-5)
$$

(c)

$$
h(t) = u(t-3) - u(t-5)
$$

$$
\frac{dh(t)}{dt} = \delta(t-3) - \delta(t-5)
$$

$$
\begin{aligned}
g(t) &= (\delta(t-3) - \delta(t-5)) * x(t) \\
&= x(t-3) - x(t-5) \\
&= e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)
\end{aligned}
$$