Question: (20 pts) This is a LTI system defined by the frequency respons...

(20 pts) This is a LTI system defined by the frequency response below:

$$H(j\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6}$$

(a) Firstly determine the differential equation that represents the system above.

(b) Determine the impulse response of the system.

(c) Find $Y(j\omega)$ when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.

d) Find the output $y(t)$

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Expert Answer

Anonymous answered this
274 answers

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Given

$$-\omega^2 + 5j\omega + 6$$

a) $$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6}$$

$$\Rightarrow (j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) = j\omega X(j\omega) + 4X(j\omega)$$

Apply inverse fourier transform

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

i.e. $$\ddot{y} + 5\dot{y} + 6y = \dot{x} + 4x$$

b) $$H(j\omega) = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6}$$

$$= \frac{j\omega + 4}{(j\omega + 2)(j\omega + 3)}$$

$$= \frac{2}{j\omega + 2} - \frac{1}{j\omega + 3}$$

Apply inverse fourier transform.

$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t).$$

$$\therefore h(t) = \left[2e^{-2t} - e^{-3t}\right]u(t)$$

c) $$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$\Rightarrow Y(j\omega) = X(j\omega)H(j\omega)$$

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t).$$

Apply fourier transform.

$$X(j\omega) = \frac{1}{j\omega + 4} - \frac{1}{(j\omega + 4)^2}$$

$$= \frac{j\omega + 4 - 1}{(j\omega + 4)^2}$$

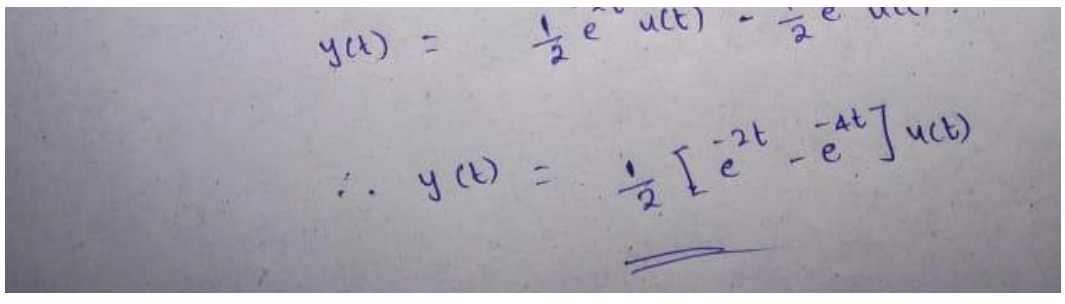
$$= \frac{j\omega + 3}{(j\omega + 4)^2}$$

i.e. $$Y(j\omega) = \frac{j\omega + 3}{(j\omega + 4)^2} \times \frac{j\omega + 4}{(j\omega + 2)(j\omega + 3)}$$

$$Y(j\omega) = \frac{1}{(j\omega + 4)(j\omega + 2)}$$

d) $$Y(j\omega) = \frac{1}{2}\left[\frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}\right]$$

Apply fourier transform



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Q: (20 pts) This is an LTI system: 4 a(t) g(t) + s + us -6 -5 (a) Firstly determine the differential equation that represents the system above (b) Determine the frequency response of the system. (C) Determine the Impulse response of this system from its frequency response. (d) (5 pts) Find the output y(t) for the input ä(t) = {e=t/4u(t) using the frequency response.

A: See answer

Q: This is an discrete time LTI system represented by the following frequency response: 1 H(ejw) 1 7 be iw -2 jw (a) Determine the difference equation which represents this system. (b) Find a block diagram representation of this system using unit delay operators and adders. (c) Find the impulse response of this system.

A: See answer

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