CENG 384 - Signals and Systems for Computer Engineers
20202
Written Assignment 1 Solutions

April 19, 2021

1.

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \tag{1}$$
$$= \lim_{n\to 0}(1+n)^{\frac{1}{n}} \tag{2}$$

$$\frac{d}{dt}(e^t) = \lim_{\Delta t\to 0}\frac{e^{t+\Delta t}-e^t}{\Delta t} \tag{3}$$
$$= \lim_{\Delta t\to 0}\frac{e^t e^{\Delta t}-e^t}{\Delta t} \tag{4}$$
$$= e^t \lim_{\Delta t\to 0}\frac{e^{\Delta t}-1}{\Delta t} \tag{5}$$
$$\tag{6}$$

Change of variable:

$$n = e^{\Delta t}-1 \tag{7}$$
$$n+1 = e^{\Delta t} \tag{8}$$
$$\ln(n+1) = \Delta t \tag{9}$$
$$\Delta t \to 0 \implies n \to 0 \tag{10}$$

So we get:

$$e^t \lim_{\Delta t\to 0}\frac{e^{\Delta t}-1}{\Delta t} = e^t \lim_{n\to 0}\frac{n}{\ln(n+1)} \tag{11}$$
$$= e^t \lim_{n\to 0}\frac{\frac{1}{n}n}{\frac{1}{n}\ln(n+1)} \tag{12}$$
$$= e^t \lim_{n\to 0}\frac{1}{\ln((1+n)^{\frac{1}{n}})} \tag{13}$$
$$= e^t \frac{1}{\ln(\lim_{n\to 0}(1+n)^{\frac{1}{n}})} \tag{14}$$
$$= e^t \frac{1}{\ln(e)} \tag{15}$$
$$= e^t \frac{1}{1} \tag{16}$$
$$= e^t \tag{17}$$

2. (a) $z = x + yj$ and $z - 3 = j - 2\bar{z}$, find $|z|^2$ and plot z on the complex plane.

$$x - 3 + yj = -2x + (2y+1)j \tag{18}$$
$$x - 3 = -2x \tag{19}$$
$$x = 1 \tag{20}$$
$$2y + 1 = y \tag{21}$$
$$y = -1 \tag{22}$$

$$|z|^2 = z\bar{z} = (1-j)(1+j) = 2 \tag{23}$$

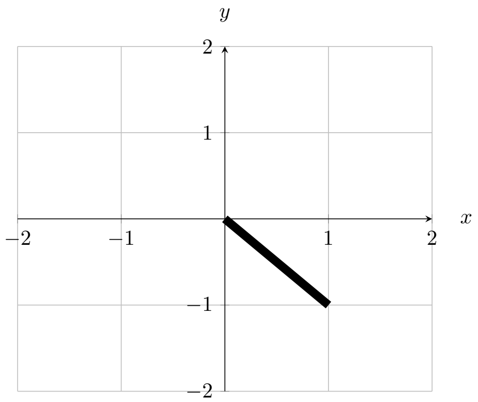

Figure 1: z on the complex plane.

(b) $z = re^{j\theta}$ and $z^4 = -81$

$$z^4 = r^4 e^{4j\theta} = 81e^{j\pi} \tag{24}$$
$$r = 3 \tag{25}$$

$$4\theta = \pi + 2\pi k, k \in Z \tag{26}$$
$$k = -2, \theta = \frac{-3\pi}{4} = -135^\circ \tag{27}$$
$$k = -1, \theta = \frac{-\pi}{4} = -45^\circ \tag{28}$$
$$k = 0, \theta = \frac{\pi}{4} = 45^\circ \tag{29}$$
$$k = 1, \theta = \frac{3\pi}{4} = 135^\circ \tag{30}$$
$$z_1 = 3e^{-j\frac{3\pi}{4}} \tag{31}$$
$$z_2 = 3e^{-j\frac{\pi}{4}} \tag{32}$$
$$z_3 = 3e^{j\frac{\pi}{4}} \tag{33}$$
$$z_4 = 3e^{j\frac{3\pi}{4}} \tag{34}$$

(c) $z_1 = (\frac{1}{2} + \frac{1}{2}j) = \frac{1}{\sqrt{2}}e^{j\frac{\pi}{4}}$
$z_2 = 1 - j = \sqrt{2}e^{-j\frac{\pi}{4}}$
$z_3 = 1 - \sqrt{3}j = 2e^{-j\frac{\pi}{3}}$
$z = \frac{z_1 z_2}{z_3} = \frac{1}{2}e^{j\frac{\pi}{3}}$
$|z| = \frac{1}{2}$, $<\!)z = \frac{\pi}{3} rad = 60^\circ$

(d) $-\frac{1}{j} = j = e^{j\frac{\pi}{2}}$ so
$z = 3e^{j\frac{\pi}{2}}e^{j\frac{\pi}{2}} = 3e^{j\pi}$

3. i. time scale: expand by 2
   ii. time shift: shift by 6 to the left
   iii. finally scale the signal by 2

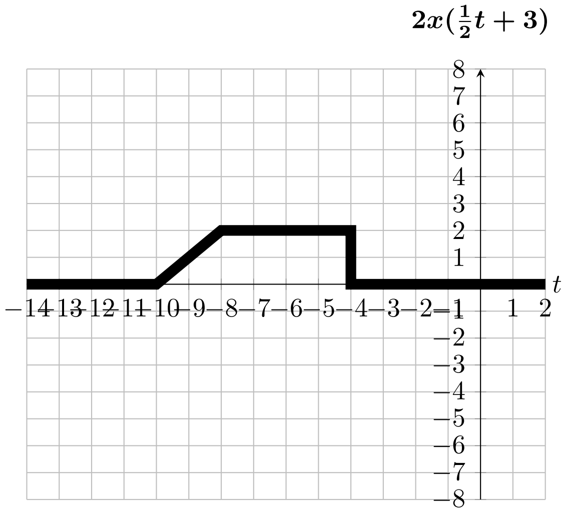

Figure 2: t vs. $y(t) = 2x(\frac{1}{2}t+3)$.

4. (a) $x[-n]$ is the reflection of $x[n]$ about y-axis.
   $x[2n+1]$: we first shrink $x[n]$ by 2 and then shift to the left by 1/2 and take the values of integer n values.
   At the end we sum $x[-n]$ and $x[2n+1]$.

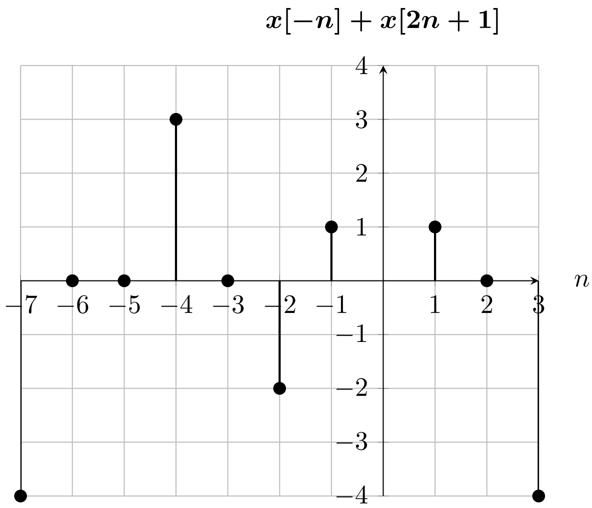

Figure 3: n vs. $x[-n] + x[2n+1]$.

   (b) $x[-n] + x[2n+1] = -4\delta[n+7] + 3\delta[n+4] - 2\delta[n+2] + \delta[n+1] + \delta[n-1] - 4\delta[n-3]$

5. (a) $W_0 = 7\pi \implies T_0 = \frac{2\pi}{7\pi} \implies T_0 = \frac{2}{7}$
   (b) $W_0 = 4 \implies N_0 = \frac{2\pi}{W_0}m$. There are no integer values of m that makes N_0 an integer. Therefore this signal is not periodic.
   (c) For $x_1, W_0 = \frac{7\pi}{5} \implies N_0 = \frac{2\pi}{W_0}m = \frac{10m}{7} \implies$ When $m = 7, N_0 = 10$
   For $x_2, W_0 = \frac{5\pi}{2} \implies N_0 = \frac{2\pi}{W_0}m = \frac{4m}{5} \implies$ When $m = 5, N_0 = 4$
   LCM is 20.

6. (a) Since it is not symmetric about y-axis, it is not even. Also it is not symmetric about origin, so it is not odd. Therefore the signal is neither even nor odd.

   (b) $Ev\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$ and $Odd\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$

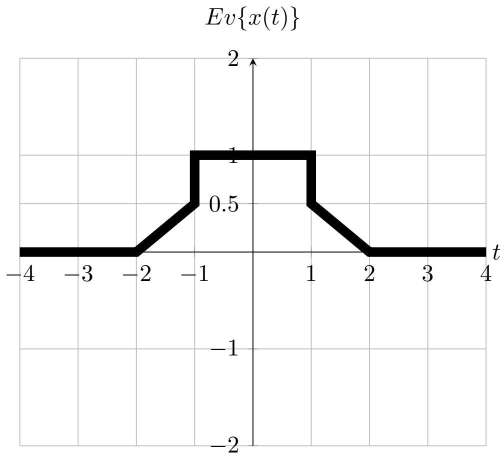

Figure 4: t vs. $Ev\{x(t)\}$.

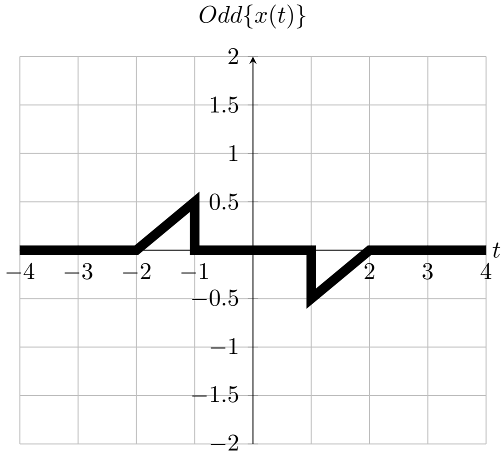

Figure 5: t vs. $Odd\{x(t)\}$.

7. (a) $x(t) = -3u(t-2) + 5u(t-3) - 3u(t-5)$

   (b) $\frac{dx(t)}{dt} = -3\delta(t-2) + 5\delta(t-3) - 3\delta(t-5)$

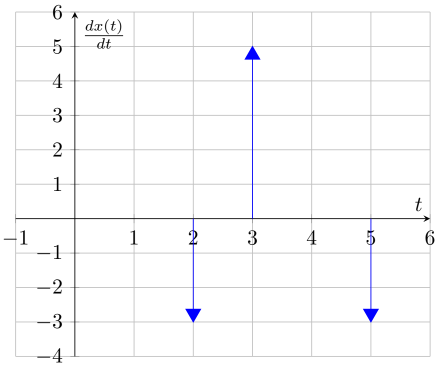

Figure 6: t vs. $\frac{dx(t)}{dt}$.

8. (a) $y[n] = x[3n-5]$
   i. **Memory** Has memory, $y[2] = x[1]$.
   ii. **Stability** Stable, all bounded inputs result in bounded outputs.
   iii. **Causality** Not causal $y[4] = x[7]$, output depends on future input.
   iv. **Linearity** Linear, superposition holds.
   v. **Invertibility** Not invertible, $x[n] = y[\frac{n+5}{3}]$ which is not defined for all n values.
   vi. **Time Invariance** Time varying, $x[3n-3n_0-5] \neq x[3n-n_0-5]$.

   (b) $y(t) = x(3t-5)$
   i. **Memory** Has memory, $y(2) = x(1)$.
   ii. **Stability** Stable, $-B < x(t) < B \implies -B < x(3t-5) < B$.
   iii. **Causality** Not causal, $y(4) = x(7)$, output depends on future input.
   iv. **Linearity** Linear, superposition holds.
   v. **Invertibility** Invertible, $x(t) = y(\frac{t+5}{3})$.
   vi. **Time Invariance** Time varying, $x(3t-3t_0-5) \neq x(3t-t_0-5)$.

   (c) $y(t) = tx(t-1)$
   i. **Memory** Has memory, the output is dependent on the input at a different time. Ex: $y(1) = x(0)$.
   ii. **Stability** Not Stable, assume that the input is constant. In such a case $y(t)$ depends on t which is unbounded.
   iii. **Causality** Causal, output does not depend on future input values.
   iv. **Linearity** Linear, superposition holds.
   v. **Invertibility** Not invertible, $x(t) = \frac{y(t+1)}{t+1}$, not defined when $t = -1$.
   vi. **Time Invariance** Time varying, $tx(t-t_0-1) \neq (t-t_0)x(t-t_0-1)$.

   (d) $y[n] = \sum_{k=1}^{\infty} x[n-k]$
   i. **Memory** Has memory, output depends on the past values of input.
   ii. **Stability** Not stable, system response grows without bound in response to small inputs.
   iii. **Causality** Causal, output does not depend on future input values.
   iv. **Linearity** Linear, superposition holds.
   v. **Invertibility** Invertible, $x[n] = y[n+1] - y[n]$.
   vi. **Time Invariance** Time invariant, a time shift in input results in an identical time shift in output.