Question: This is an discrete time LTI system represented by the followin...

This is an discrete time LTI system represented by the following frequency response:

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}}$$

(a) Determine the difference equation which represents this system.

(b) Find a block diagram representation of this system using unit delay operators and adders.

(c) Find the impulse response of this system.

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Expert Answer

Anonymous answered this
245 answers

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$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-2j\omega}}$$

(a) To determine the difference equation, represent $H(e^{j\omega})$ as $\frac{Y(e^{j\omega})}{X(e^{j\omega})}$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-j2\omega}}.$$

which yields

$$Y(e^{j\omega}) - \frac{1}{6}e^{-j\omega}Y(e^{j\omega}) - \frac{1}{6}e^{-j2\omega}Y(e^{j\omega}) = X(e^{j\omega}).$$

Apply Inverse DTFT (IDTFT) on both sides.

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n).$$

So, the difference equation is

$$\boxed{y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)}$$

(b) Block diagram representation:

The difference equation is given by

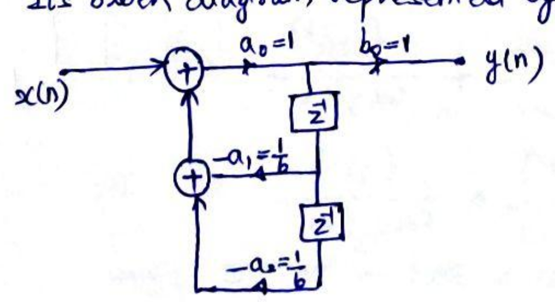
$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$$

It is compared with Linear constant coefficient difference equation, then

$$a_0y(n) + a_1y(n-1) + a_2y(n-2) + a_3y(n-3) + \cdots + a_Ny(n-N)$$
$$= b_0x(n) + b_1x(n-1) + b_2x(n-2) + \cdots + b_Mx(n-M)$$

Here $a_0=1$, $a_1 = -\frac{1}{6}$, $a_2 = -\frac{1}{6}$, $b_0 = 1$.

It's block diagram represented by direct form(II) realization as



(c) impulse response $h(n)$

Given $H(e^{j\omega}) = \frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-j2\omega}}$

$$H(e^{j\omega}) \Rightarrow \frac{6}{6-e^{-j\omega}-e^{-j2\omega}}$$

$$H(e^{j\omega}) \Rightarrow \frac{6}{6-3e^{-j\omega}+2e^{-j\omega}-e^{-j2\omega}}$$

$$H(e^{j\omega}) \Rightarrow \frac{6}{3(2-e^{-j\omega})+e^{-j\omega}(2-e^{-j\omega})}$$

$$H(e^{j\omega}) \Rightarrow \frac{6}{(3+e^{-j\omega})(2-e^{-j\omega})}$$

Now apply partial fractions

$$\frac{6}{(3+e^{-j\omega})(2-e^{-j\omega})} = \frac{A}{3+e^{-j\omega}} + \frac{B}{2-e^{-j\omega}}$$

$$A = \left.\frac{6}{2-e^{-j\omega}}\right|_{e^{-j\omega}=-3} \Rightarrow \frac{6}{2+3} \Rightarrow \frac{6}{5}.$$

$$B = \left.\frac{6}{3+e^{-j\omega}}\right|_{e^{-j\omega}=2} \Rightarrow \frac{6}{3+2} \Rightarrow \frac{6}{5}$$

$$\therefore H(e^{j\omega}) = \frac{(6/5)}{3+e^{-j\omega}} + \frac{(6/5)}{2-e^{-j\omega}}.$$

$$H(e^{j\omega}) \Rightarrow \frac{(6/5)}{3\left(1+\frac{1}{3}e^{-j\omega}\right)} + \frac{(6/5)}{2\left(1-\frac{1}{2}e^{-j\omega}\right)}$$

$$H(e^{j\omega}) \Rightarrow \frac{(2/5)}{1+\frac{1}{3}e^{-j\omega}} + \frac{(3/5)}{1-\frac{1}{2}e^{-j\omega}}$$

Apply IDTFT on both sides

$$h(n) = \left(\frac{2}{5}\right)\cdot\left(-\frac{1}{3}\right)^n u(n) + \left(\frac{3}{5}\right)\cdot\left(\frac{1}{2}\right)^n u(n),$$

So, Impulse response $h(n)$ is given by

$$\boxed{h(n) = \left(\frac{2}{5}\right)\left(-\frac{1}{3}\right)^n u(n) + \frac{3}{5}\left(\frac{1}{2}\right)^n u(n)}$$

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Q: (20 pts) This is an LTI system: 4 a(t) g(t) + s + us -6 -5 (a) Firstly determine the differential equation that represents the system above (b) Determine the frequency response of the system. (C) Determine the Impulse response of this system from its frequency response. (d) (5 pts) Find the output y(t) for the input ä(t) = (e=t/4u(t) using the frequency response.

A: See answer

Q: (20 pts) Consider the following LTI system defined by the frequency response below: jw +4 H(jw) -W2 + 5jw + 6 (a) (5 pts) Find the differential equation which represents this system. (b) (5 pts) Find the impulse response of this system. (c) (5 pts) Find Y (jw) when the input is x(t) = e-4tu(t) – te-4tu(t). (d) (5 pts) Find the output y(t) using the result you found in part c.

A: See answer

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