CENG 384 - Signals and Systems for Computer Engineers
20202
Written Assignment 4 Solutions

June 14, 2021

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1. (a)

$$\begin{aligned}
\int_{-\infty}^{t} x(\tau) - \int_{-\infty}^{t} 6y(\tau) + 4x(t) - 5y(t) &= y'(t) \\
x(t) - 6y(t) + 4x'(t) - 5y'(t) &= y''(t) \\
4x'(t) + x(t) &= y''(t) + 5y'(t) + 6y(t)
\end{aligned}$$

(b)

$$\begin{aligned}
4j\omega X(j\omega) + X(j\omega) &= (j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) \\
(4j\omega + 1)X(j\omega) &= ((j\omega)^2 + 5j\omega + 6)Y(j\omega) \\
H(j\omega) &= \frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6}
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6} &= \frac{B}{j\omega + 3} + \frac{A}{j\omega + 2} \\
Aj\omega + 3A + Bj\omega + 2B &= 4j\omega + 1 \\
A + B = 4 &\qquad 3A + 2B = 1 \\
A = -7 &\qquad B = 11 \\
H(j\omega) &= \frac{11}{j\omega + 3} - \frac{7}{j\omega + 2} \\
h(t) &= (11e^{-3t} - 7e^{-2t})u(t)
\end{aligned}$$

(d)

$$\begin{aligned}
Y(j\omega) &= X(j\omega) \cdot H(j\omega) \\
&= \frac{1}{4} \cdot \frac{1}{\frac{1}{4} + j\omega} \cdot \frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6} \\
&= \frac{1}{(j\omega + 2)(j\omega + 3)} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 3}
\end{aligned}$$

$Aj\omega + 3A + Bj\omega + 2B = 1$
$A + B = 0 \quad 3A + 2B = 1 \quad \Rightarrow \quad A = 1 \quad B = -1$

$$\begin{aligned}
Y(j\omega) &= \frac{1}{j\omega + 2} - \frac{1}{j\omega + 3} \\
y(t) &= (e^{-2t} - e^{-3t})u(t)
\end{aligned}$$

2. (a)

$$H(j\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6}$$
$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6}$$

$$((j\omega)^2 + 5j\omega + 6)Y(j\omega) = (j\omega + 4)X(j\omega)$$
$$y''(t) + 5y'(t) + 6y(t) = x'(t) + 4x(t)$$

(b)

$$H(j\omega) = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 3}$$

$Aj\omega + 3A + Bj\omega + 2B = j\omega + 4$
$A + B = 1 \quad 3A + 2B = 4 \quad \Rightarrow \quad A = 2 \quad B = -1$

$$H(j\omega) = \frac{2}{j\omega + 2} - \frac{1}{j\omega + 3}$$
$$h(t) = (2e^{-2t} - e^{-3t})u(t)$$

(c)

$$X(j\omega) = \frac{1}{j\omega + 4} - \frac{1}{(j\omega + 4)^2}$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) = \frac{1}{(j\omega + 2)(j\omega + 4)}$$

(d)

$$Y(j\omega) = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 4}$$

$Aj\omega + 4A + Bj\omega + 2B = 1$
$A + B = 0 \quad 4A + 2B = 1 \quad \Rightarrow \quad A = \frac{1}{2} \quad B = -\frac{1}{2}$

$$Y(j\omega) = \frac{1/2}{j\omega + 2} - \frac{1/2}{j\omega + 4}$$
$$y(t) = \frac{1}{2}(e^{-2t} - e^{-4t})u(t)$$

3. (a)

$$x(t) = e^{-|t|}$$
$$X(j\omega) = \int_{-\infty}^{\infty} e^{-|t|}e^{-j\omega t}dt = \int_{-\infty}^{0} e^{t}e^{-j\omega t}dt + \int_{0}^{\infty} e^{-t}e^{-j\omega t}dt$$
$$= \frac{1}{1 - j\omega} + \frac{1}{1 + j\omega}$$
$$= \frac{2}{1 + \omega^2}$$

(b) Using the "differentiation in frequency" property, we get the following:

$$te^{-|t|} \overset{\text{FT}}{\longleftrightarrow} j\frac{d}{d\omega}\left(\frac{2}{1 + \omega^2}\right) = \frac{-4j\omega}{(1 + \omega^2)^2}$$

(c) The duality property states that if

$$g(t) \overset{\text{FT}}{\longleftrightarrow} G(j\omega)$$

then

$$G(t) \overset{\text{FT}}{\longleftrightarrow} 2\pi g(-j\omega).$$

So first replace ω with t in the result found in part b:

$$\frac{-4jt}{(1 + t^2)^2} \overset{\text{FT}}{\longleftrightarrow} -2\pi\omega e^{-|\omega|}$$

Now multiply both sides by j, and get

$$\frac{4t}{(1 + t^2)^2} \overset{\text{FT}}{\longleftrightarrow} -2\pi j\omega e^{-|\omega|}$$

4. (a)

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

(b)

$$Y(e^{j\omega})\left(1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-2j\omega}\right) = 2X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$$

(c) By partial fraction we get

$$H(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

Take IFT and get

$$h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

(d)

$$x[n] = \left(\frac{1}{4}\right)^n u[n] \overset{\text{FT}}{\longleftrightarrow} X(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}) = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2}$$

By partial fraction we get

$$Y(e^{j\omega}) = \frac{-4}{1-\frac{1}{4}e^{-j\omega}} - \frac{2}{(1-\frac{1}{4}e^{-j\omega})^2} + \frac{8}{1-\frac{1}{2}e^{-j\omega}}$$

Take IFT and get

$$y[n] = -4\left(\frac{1}{4}\right)^n u[n] - 2(n+1)\left(\frac{1}{4}\right)^n u[n] + 8\left(\frac{1}{2}\right)^n u[n]$$

5.

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

$$h_1[n] = \left(\frac{1}{3}\right)^n u[n] \overset{\text{FT}}{\longleftrightarrow} H_1(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$\begin{aligned} H_2(e^{j\omega}) &= H(e^{j\omega}) - H_1(e^{j\omega}) \\ &= \frac{-8}{4-e^{-j\omega}} = \frac{-2}{1-\frac{1}{4}e^{-j\omega}} \end{aligned}$$

Take IFT

$$h_2[n] = -2\left(\frac{1}{4}\right)^n u[n]$$

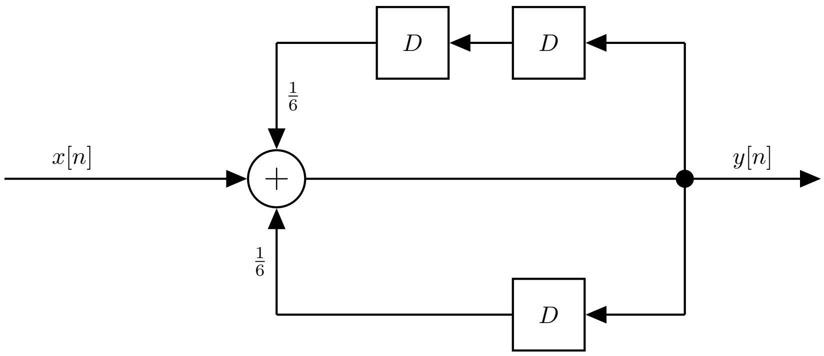
6. (a)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}}$$

$$Y(e^{j\omega})\left(1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}\right) = X(e^{j\omega})$$

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

(b) The block diagram is below:



(c)

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}}$$
$$= \frac{1}{(1 - \frac{1}{2}e^{-j\omega})(1 + \frac{1}{3}e^{-j\omega})}$$

By partial fraction we get

$$H(e^{j\omega}) = \frac{3/5}{1 - \frac{1}{2}e^{-j\omega}} + \frac{2/5}{1 + \frac{1}{3}e^{-j\omega}}$$

Take IFT and get

$$h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n]$$