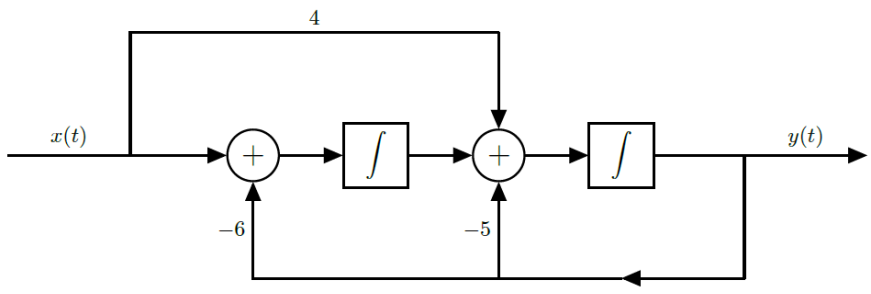
Question: (20 pts) This is an LTI system: 4 a(t) g(t) + s + us -6 -5 (a) Firstly ...

(20 pts) This is an LTI system:



(a) Firstly determine the differential equation that represents the system above

(b) Determine the frequency response of the system.

(c) Determine the Impulse response of this system from its frequency response.

(d) (5 pts) Find the output $y(t)$ for the input $x(t) = \frac{1}{4}e^{-t/4}u(t)$ using the frequency response.

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Expert Answer

Anonymous answered this
192 answers

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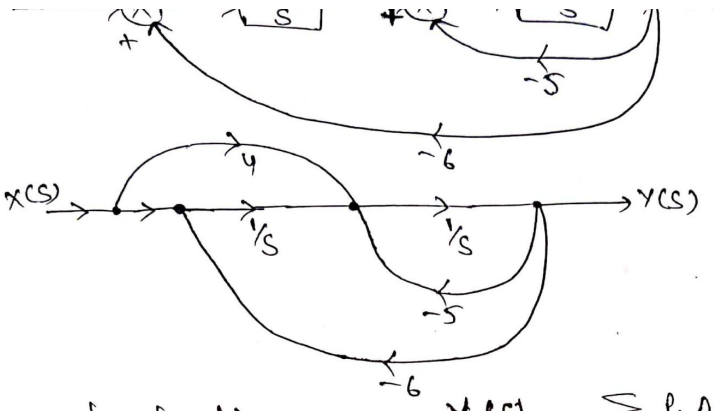

transfer function $(G(S)) = \frac{Y(S)}{X(S)} = \frac{\sum_K P_K \Delta_K}{\Delta}$

K = no. of forward path, P_K = the Kth forward path gain

$\Delta = 1 - (\Sigma$ loop gains$) + (\Sigma$ non. touching loop gain taken two at a time$) - \cdots$

$\Delta_1 = 1 \quad P_1 = \frac{4}{S}$

$\Delta_2 = 1 \quad P_2 = \frac{1}{S^2}$

L = loop gain

$L_1 = \frac{-5}{S}$, $L_2 = \frac{-6}{S^2}$

$$G(S) = \frac{Y(S)}{X(S)} = \frac{\frac{4}{S} + \frac{1}{S^2}}{1 + \frac{5}{S} + \frac{6}{S^2}}$$

$$\frac{Y(S)}{X(S)} = \frac{4S+1}{S^2+5S+6} = \frac{-7}{S+2} + \frac{11}{S+3}$$

differential equation

$$Y(S)(S^2+5S+6) = (4S+1)X(S)$$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 4\frac{dx(t)}{dt} + x(t)$$

(b) $$G(j\omega) = \frac{j4\omega + 1}{(j\omega)^2 + j5\omega + 6}$$

(c) apply inverse laplace transform

$$g(t) = -7e^{-2t}u(t) + 11e^{-3t}u(t).$$

(d) $x(t) = \frac{1}{4}e^{-t/4}u(t)$

$$X(S) = \frac{1}{4} \times \frac{1}{S+\frac{1}{4}}$$

$$Y(S) = \frac{-7}{4(S+2)(S+\frac{1}{4})} + \frac{11}{4(S+3)(S+\frac{1}{4})}$$

$$Y(S) = \frac{1}{S+2} - \frac{1}{S+3}$$

apply inverse laplace transform

$$y(t) = (e^{-2t} - e^{-3t})u(t).$$