Question: Consider an LTI system given by the following block diagram: ...

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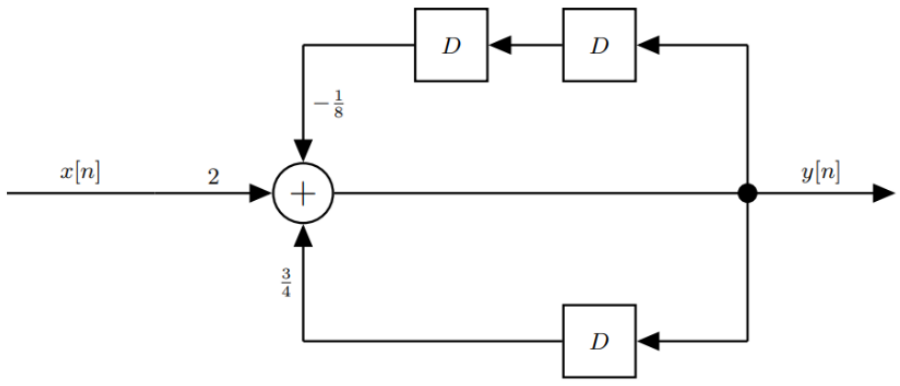

where D is the unit-delay operator.

(a) Find the difference equation which represents this system.

(b) Find the frequency response of this system.

(c) Find the impulse response of this system from its frequency response.

(d) Find the output $y[n]$ for the input $x[n] = (\frac{1}{4})^n u[n]$ using the frequency response.

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Expert Answer

Vasumathi answered this
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a) Difference equation

$$2x(n) + \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) = y(n)$$

b) Frequency Response Apply DTFT,

$$2X(e^{j\omega}) + \frac{3}{4}e^{-j\omega}Y(e^{j\omega}) - \frac{1}{8}e^{-2j\omega}Y(e^{j\omega}) = Y(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-2j\omega}\right] = 2X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-2j\omega}}$$

c) Impulse Response

Apply Inverse DTFT on $H(e^{j\omega}) = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$

$$H(e^{j\omega}) = \frac{A}{(1-\frac{1}{2}e^{-j\omega})} + \frac{B}{(1-\frac{1}{4}e^{-j\omega})}$$

$$A = H(e^{j\omega})(1-\frac{1}{2}e^{-j\omega})\Big|_{e^{-j\omega}=2}$$

$$= \frac{2}{(1-\frac{1}{4}e^{-j\omega})}\Big|_{e^{-j\omega}=2} = \frac{2}{1-\frac{1}{4}(2)} = \frac{2}{1/2} = \underline{\underline{4}}$$

$$B = H(e^{j\omega})(1-\frac{1}{4}e^{-j\omega})\Big|_{e^{-j\omega}=4}$$

$$= \frac{2}{1-\frac{1}{2}e^{-j\omega}}\Big|_{e^{-j\omega}=4} = \frac{2}{1-\frac{4}{2}(4)} = \underline{\underline{-2}}$$

$$H(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

$\Downarrow$ Inv DTFT

$$h(n) = 4\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{4}\right)^n u(n)$$

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$x(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$; $y(e^{j\omega}) = x(e^{j\omega})\,H(e^{j\omega})$

$= \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2}$

Apply partial fraction expansion,

$$y(e^{j\omega}) = \frac{A}{(1-\frac{1}{2}e^{-j\omega})} + \frac{B_0}{(1-\frac{1}{4}e^{-j\omega})} + \frac{B_1}{(1-\frac{1}{4}e^{-j\omega})^2}$$

$$B_1 = y(e^{j\omega})\left(1-\tfrac{1}{4}e^{-j\omega}\right)^2 \Big|_{e^{j\omega}=4}$$

$$= \frac{2}{1-\frac{1}{2}e^{-j\omega}}\Big|_{e^{-j\omega}=4} = \frac{2}{1-\frac{1}{2}(4)} = -2$$

$$B_0 = \frac{d}{d\omega}\left(y(e^{j\omega})\left(1-\tfrac{1}{4}e^{-j\omega}\right)^2\right)\Big|_{e^{j\omega}=4}$$

$$= \frac{d}{d\omega}\left(\frac{2}{1-\frac{1}{2}e^{-j\omega}}\right)\Big|_{e^{-j\omega}=4} = \frac{-2\left(-\frac{1}{2}e^{-j\omega}\right)}{(1-\frac{1}{2}e^{-j\omega})^2}\Big|_{e^{j\omega}=4}$$

$$A = y(e^{j\omega})\left(1-\tfrac{1}{2}e^{-j\omega}\right)\Big|_{e^{-j\omega}=2}$$

$$= \frac{2}{(1-\frac{1}{4}e^{-j\omega})^2}\Big|_{e^{-j\omega}=2} = \frac{2}{(1-\frac{1}{4}(2))^2} = \frac{2}{1/4} = 8$$

$$Y(e^{j\omega}) = \frac{8}{1-\frac{1}{2}e^{-j\omega}} + \frac{4}{1-\frac{1}{4}e^{-j\omega}} + \frac{(-2)}{(1-\frac{1}{4}e^{-j\omega})^2}$$

$\Downarrow$ Inv DFT

$$y(n) = 8\left(\tfrac{1}{2}\right)^n u(n) + \left(\tfrac{1}{4}\right)^n u(n) + (-2)(n+1)\left(\tfrac{1}{4}\right)^n u(n)$$

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Q: 1. (20 pts) Consider an LTI system given by the following block diagram: y(t) r(t) (a) (5 pts) Find the differential equation which represents this system. (b) (15 pts) Find the output y(t), when the input x(t) = (e-4e-2t)a(t). Assume that the system is initially at rest.

A: See answer 100% (1 rating)

Q: I need help with this signals and systems analysis problem. Thanks!

A: See answer 100% (1 rating)

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