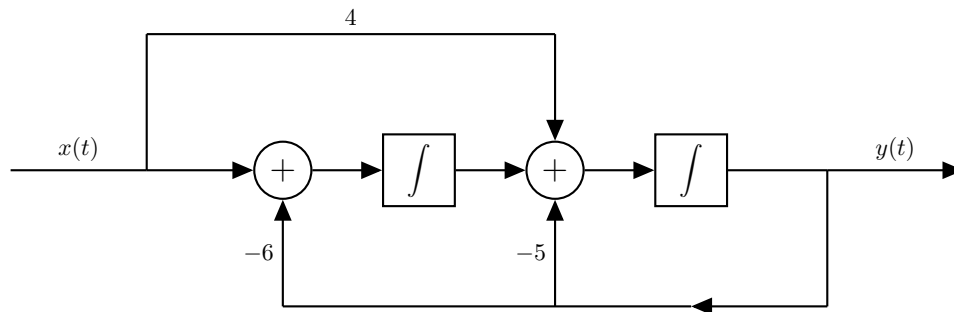




Regulations:

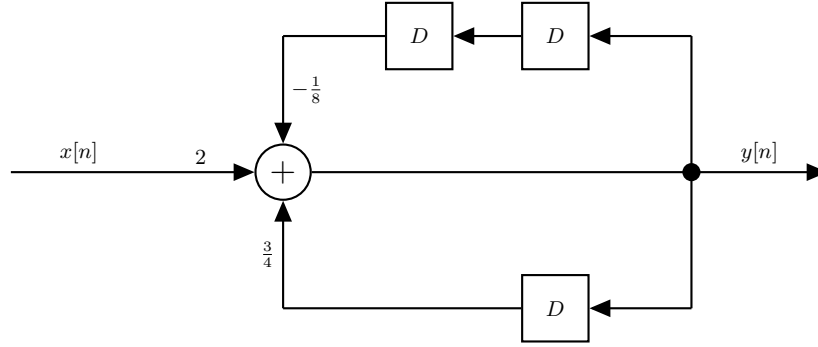
- **Grouping:** You are allowed to work in pairs.
- **Submission:** We provide a latex template for your solutions. Use that template and create a hw4.tar.gz file that includes hw4.tex and all other related files. Tar.gz file should not contain any directories and should create a hw4.pdf file with the following commands, otherwise you will get zero;
`tar xvzf hw4.tar.gz`
`pdflatex hw4.tex`
Submit hw4.tar.gz to the COW page of the course.
- **Deadline:** 23:55, 13 June, 2021 (Sunday).
- **Late Submission:** Not allowed.

1. (20 pts) Consider an LTI system given by the following block diagram:



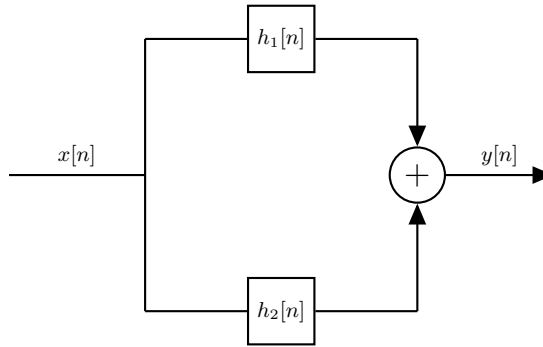
- (a) (5 pts) Find the differential equation which represents this system.
- (b) (5 pts) Find the frequency response of this system.
- (c) (5 pts) Find the impulse response of this system from its frequency response.
- (d) (5 pts) Find the output $y(t)$ for the input $x(t) = \frac{1}{4}e^{-t/4}u(t)$ using the frequency response.
2. (20 pts) Consider the following LTI system defined by the frequency response below:
- $$H(j\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6}$$
- (a) (5 pts) Find the differential equation which represents this system.
- (b) (5 pts) Find the impulse response of this system.
- (c) (5 pts) Find $Y(j\omega)$ when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.
- (d) (5 pts) Find the output $y(t)$ using the result you found in part c.
3. (15 pts) Solve the following questions using the appropriate properties of the Fourier Transform. (In part a give the derivation and in parts b, c use the properties.)
- (a) (5 pts) Derive the Fourier transform of the signal $e^{-|t|}$.
- (b) (5 pts) Find the Fourier transform of $te^{-|t|}$.
- (c) (5 pts) Using the result of part b find the Fourier transform of $\frac{4t}{(1+t^2)^2}$.

4. (20 pts) Consider an LTI system given by the following block diagram:



where D is the unit-delay operator.

- (5 pts) Find the difference equation which represents this system.
 - (5 pts) Find the frequency response of this system.
 - (5 pts) Find the impulse response of this system from its frequency response.
 - (5 pts) Find the output $y[n]$ for the input $x[n] = (\frac{1}{4})^n u[n]$ using the frequency response.
5. (10 pts) Suppose that two discrete-time LTI systems with the impulse responses $h_1[n]$ and $h_2[n]$ are connected in parallel.



We have the following information about this combined system:

- The frequency response of the combined system is $H(e^{j\omega}) = \frac{5e^{-j\omega} - 12}{e^{-2j\omega} - 7e^{-j\omega} + 12}$.
- The impulse response of the first system is $h_1[n] = (\frac{1}{3})^n u[n]$.

Find $h_2[n]$, the impulse response of the second system.

6. (15 pts) Consider a discrete-time LTI system represented by the following frequency response:

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}}$$

- (5 pts) Find the difference equation which represents this system.
- (5 pts) Find a block diagram representation of this system using unit delay operators and adders.
- (5 pts) Find the impulse response of this system.