Spring 2021 Homework 1

Regulations:

• Grouping: You are allowed to work in pairs.

• Drawing Plots: Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.

• **Submission:** We provide a latex template for your solutions. Use that template and create a hw1.tar.gz file that includes hw1.tex and all other related files. Tar.gz file should not contain any directories and should create a hw1.pdf file with the following commands, otherwise you will get zero;

 $tar\ xvzf\ hw1.tar.gz$

pdflatex hw1.tex

Submit hw1.tar.gz to the odtuclass page of the course.

• Deadline: 23:55, 11 April, 2021 (Sunday).

• Late Submission: Not allowed.

1. (10 points) Prove the following equality, showing your steps in detail.

$$\frac{de^t}{dt} = e^t$$

- 2. (16 pts) Solve the following, showing your solution in detail.
 - (a) (4 pts) Given a complex number in Cartesian coordinate system, z=x+jy and $z-3=j-2\bar{z}$,
 - i. find $|z|^2$ and
 - ii. find and plot z on the complex plane.
 - (b) (4 pts) Given $z = re^{i\theta}$ and $z^4 = -81$, find z in polar form.
 - (c) (4 pts) Find the magnitude and angle of $z = \frac{(\frac{1}{2} + \frac{1}{2}j)(1-j)}{1-\sqrt{3}j}$.
 - (d) (4 pts) Write z in polar form where $z = -\frac{3}{i}e^{j\pi/2}$.
- 3. (10 pts) Given the x(t) signal in Figure 1, draw the signal $y(t) = 2x(\frac{1}{2}t + 3)$.

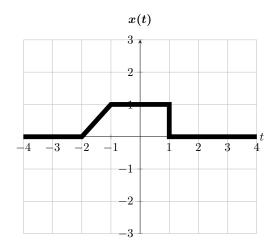


Figure 1: t vs. x(t).

- 4. (10 pts) Given the x[n] signal in Figure 2,
 - (a) (6 pts) Draw x[-n] + x[2n+1].
 - (b) (4 pts) Express x[-n] + x[2n+1] in terms of the unit impulse function.

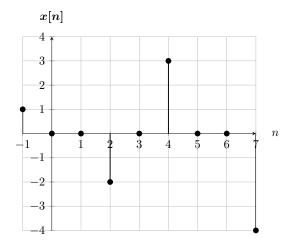


Figure 2: n vs. x[n].

5. (9 pts) Determine whether the following signals are periodic and if periodic find the fundamental period.

(a) (3 pts)
$$x(t) = 3\cos(7\pi t - \frac{4\pi}{5})$$

(b) (3 pts)
$$x[n] = \sin[4n - \frac{\pi}{2}]$$

(c) (3 pts)
$$x[n] = 2\cos\left[\frac{7\pi}{5}n\right] + 7\sin\left[\frac{5\pi}{2}n - \frac{\pi}{3}\right]$$

6. (15 pts) Consider the signal in Figure 1.

- (a) (5 pts) Show that the signal is neither even nor odd.
- (b) (10 pts) Find the even and odd decompositions of the signal and draw these parts.

7. (12 pts) Given the x(t) signal in Figure 3,

- (a) (5 pts) Express x(t) in terms of the unit step function.
- (b) (7 pts) Find and draw $\frac{dx(t)}{dt}$.

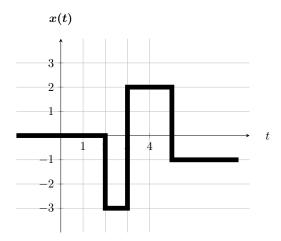


Figure 3: t vs. x(t).

8. (18 pts) Analyze whether the following systems have these properties: memory, stability, causality, linearity, invertibility, time-invariance. Provide your answer in detail.

(a) (3 pts)
$$y[n] = x[3n - 5]$$

(b) (3 pts)
$$y(t) = x(3t - 5)$$

(c) (6 pts)
$$y(t) = tx(t-1)$$

(d) (6 pts)
$$y[n] = \sum_{k=1}^{\infty} x[n-k]$$