CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 2

Karaman, Arda
e2237568@ceng.metu.edu.tr

Fulser, Göktürk
e2237386@ceng.metu.edu.tr

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1. (a)

$$\int x(t) - 5 \cdot y(t) - (\int 6 \cdot y(t)dt) \cdot dt = y(t) \tag{1}$$

$$y''(t) + 5 \cdot y'(t) + 6 \cdot y(t) = x'(t) \tag{2}$$

(b) The complete solution consists two parts; a homogeneous and a particular solution.

$$y(t) = y_h(t) + y_p(t) \tag{3}$$

The system is initially at rest and with the homogeneous solution;

$$y_h(t) = Ke^{\alpha t} \tag{4}$$

$$(\alpha^2 + 5\alpha + 6) \cdot Ke^{\alpha t} = 0 \tag{5}$$

$$(\alpha + 2) \cdot (\alpha + 3) = 0 \tag{6}$$

$$\alpha_{1,2} = -2, -3 \tag{7}$$

Therefore there are 2 solutions for the homogeneous part of the system. For the particular solution:

$$x(t) = (e^{-t} + e^{-4t})u(t) \tag{8}$$

$$y_p(t) = \alpha e^{-t} + \beta e^{-4t} \tag{9}$$

$$\alpha e^{-t} + 16\beta e^{-4t} + 5(-\alpha e^{-t} - 16\beta e^{-4t}) + 6(\alpha e^{-t} + 16\beta e^{-4t}) = (e^{-t} + e^{-4t})' \tag{10}$$

$$2\alpha e^{-t} + 2\beta e^{-4t} = -e^{-t} - 4 \cdot e^{-4t} \tag{11}$$

$$\alpha = -\frac{1}{2}, \beta = -2 \tag{12}$$

Summing up the y(t):

$$y(t) = y_h(t) + y_p(t) \tag{13}$$

$$y(t) = Ke^{-2t} - \frac{1}{2}e^{-t} - 2 \cdot e^{-4t} \tag{14}$$

$$\tag{15}$$

Since the system is initially at rest;

$$y(0) = 0 \tag{16}$$

$$0 = K - \frac{1}{2} - 2 \tag{17}$$

$$\frac{5}{2} = K \tag{18}$$

Therefore the final result is,

$$y(t) = (-\frac{1}{2}e^{-t} - 2 \cdot e^{-4t} + \frac{5}{2} \cdot e^{-2t})u(t) \tag{19}$$

2. (a) x(n) can be written as :

$$x(n) = \delta(n) + \delta(n-1)$$
$$y(n) = \delta(n-1)$$

x' can be written w.r.t. x(n) as

$$x' = x(n) - x(n-2)$$

By the linearity theorem:

$$x(n) \longrightarrow y(n)$$
$$x(n-2) \longrightarrow y(n-2)$$

Therefore,

$$\begin{aligned} y'(n) &= y(n) - y(n-2) \\ &= \delta(n-1) - \delta(n-3) \end{aligned}$$

(b)

$$\begin{aligned} Y(Z) &= X(Z) \cdot H(Z) \\ H(Z) &= \frac{Y(Z)}{X(Z)} \\ x(n) = \delta(n) + \delta(n+1) \Longrightarrow X(Z) = 1 + Z^{-1} &= \frac{1+Z}{Z} \\ y(n) = \delta(n-1) \Longrightarrow y(Z) = Z^{-1} &= \frac{1}{Z} \\ H(Z) = \frac{\frac{1}{Z}}{\frac{Z+1}{Z}} = \frac{1}{Z+1} &= \frac{Z^{-1}}{1+Z^{-1}} \end{aligned}$$

The impulse response,

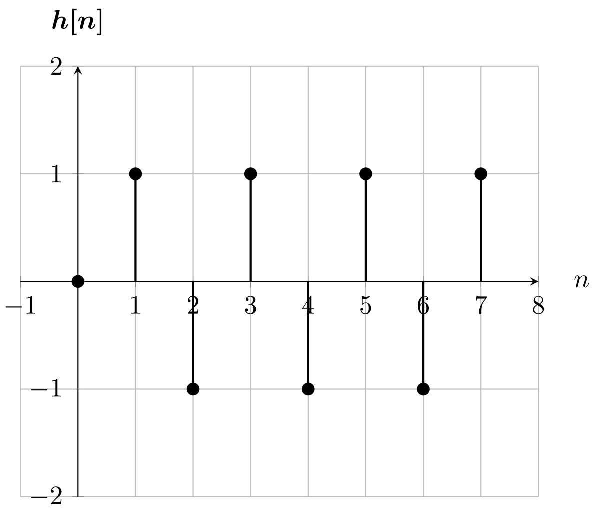


Figure 1: n vs. $h[n]$.

(c) In the last part we figured out that,

$$H(Z) = \frac{1}{Z+1}$$
$$\frac{Y(Z)}{X(Z)} = \frac{Z^{-1}}{1+Z^{-1}}$$

So that,

$$(1+Z^{-1}) \cdot Y(Z) = Z^{-1} \cdot X(Z) \Longrightarrow X(Z) + Z^{-1} \cdot Y(Z) = Z^{-1} \cdot X(Z)$$

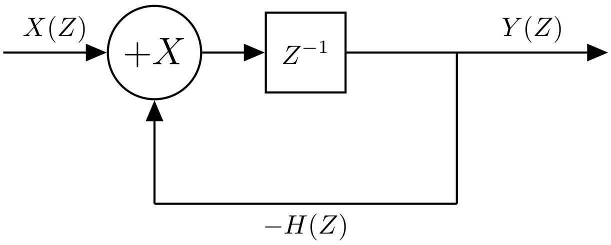
And by taking the inverse z-transform of the equation above,

$$y(n) + y(n-1) = x(n-1) \quad (20)$$

(d) We have already got the equation,

$$Y(Z) = \frac{Z^{-1}}{1+Z^{-1}} \cdot X(Z) \tag{21}$$

from the last part. So the block diagram of the total system is,



3. (a) $x[n] = \delta[n-3] + 2\delta[n+1]$,
$h[n] = \delta[n-1] + 3\delta[n+2]$,
To use $h[n]$ or $x[n]$ we have to divide one of them. Therefore;
We divide $h[n]$ into two sub-parts using the distribution property of convolution operator.

$$(h[n] = h_a[n] + h_b[n] \tag{22}$$

$$h_a[n] = 3\delta[n+2], h_b[n] = \delta[n-1] \tag{23}$$

So $y[n]$ becomes $y[n] = x[n] * h_a[n] + x[n] * h_b[n]$
$y_a[n] = (\delta[n-3] + 2\delta[n+1]) * 3\delta[n+2] = 3\delta[n-1] + 6\delta[n+3]$
$y_b[n] = (\delta[n-3] + 2\delta[n+1]) * \delta[n-1] = \delta[n-4] + 2\delta[n]$
So $y[n] = 3\delta[n-1] + 6\delta[n+3] + \delta[n-4] + 2\delta[n]$.

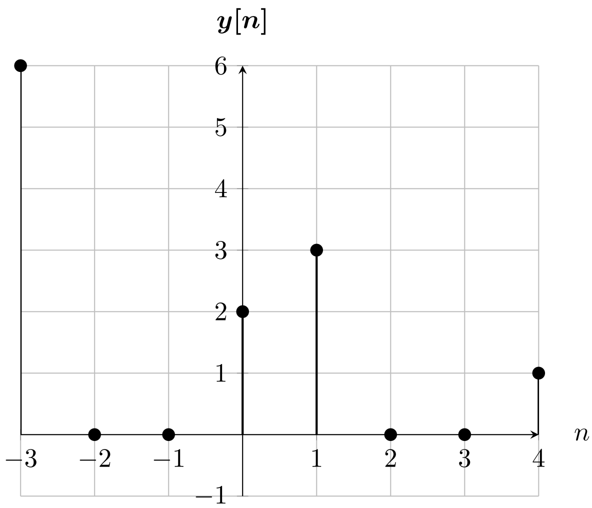


Figure 2: n vs. $y[n]$.

(b) $h[n] = u[n-1] - u[n-3]$,
We can write $h[n]$ in unit impulse form as follows:

$$h[n] = \delta[n-1] + \delta[n-2] \tag{24}$$
$$x[n] = u[n+3] + u[n] \tag{25}$$

We can write $x[n]$ in unit impulse form as follows:

$$x[n] = \delta[n+3] + \delta[n+2] + \delta[n+1] \tag{26}$$

$h[n] = h_a[n] + h_b[n]$ by distribution property
So $h_a[n] = \delta[n-1], h_b[n] = 3\delta[n-2]$ *and* $y[n] = x[n] * h_a[n] + x[n] * h_b[n]$

$$y_a[n] = (\delta[n+3] + \delta[n+2] + \delta[n+1]) * \delta[n-1] = \delta[n+2] + \delta[n+1] + \delta[n].$$
$$y_b[n] = (\delta[n+3] + \delta[n+2] + \delta[n+1]) * \delta[n-2] = \delta[n+1] + \delta[n] + \delta[n-1].$$

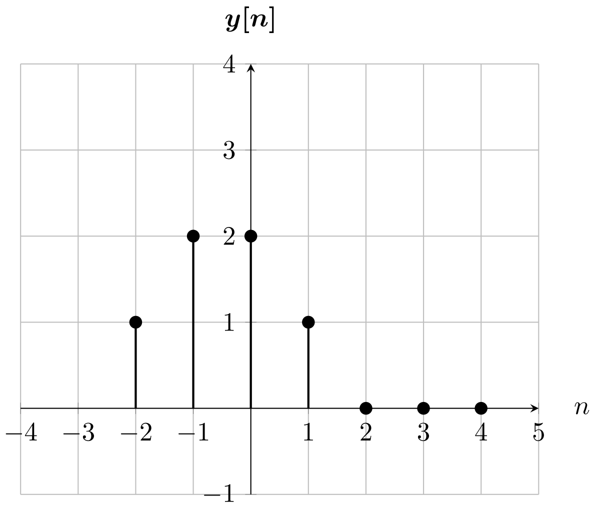


Figure 3: n vs. $y[n]$. from question 3b

So $y[n] = \delta[n+2] + 2\delta[n+1] + 2\delta[n] + \delta[n-1]$.

The figure is above.

4. (a) $h(t) = e^{-3t}u(t),\ x(t) = e^{-2t}u(t)$
   $y(t) = x(t) * h(t)$
   $y(t) = \int_0^t (e^{-2\tau}e^{-3(t-\tau)}d\tau) = \int_0^t (e^{-2\tau}e^{-3t}e^{3\tau}d\tau)$
   $y(t) = e^{-3t}\int_0^t (e^{\tau}d\tau) = e^{-3t}(e^t - 1)$
   So $y(t) = (e^{-2t} - e^{-3t})u(t)$

   (b) $h(t) = e^{-2t}u(t),\ x(t) = u(t) - u(t-2)$
   $y(t) = x(t) * h(t)$
   $y(t) = \int_0^t x(t)h(t-\tau)d\tau$
   $y(t) = u(t)\int_0^t (e^{-2(t-\tau)}d\tau) - u(t-2)\int_0^t (e^{-2(t-\tau)}d\tau)$
   $y(t) = u(t)(e^{-2t}(e^{2t}-1)) - u(t-2)(e^{-2t}(e^{2t}-1))$
   So $y(t) = (1 - e^{-2t}) \cdot (u(t) - u(t-2))$

5. (a)

$$s[n] = nu[n] \quad (27)$$
$$h[n] = nu[n] - (n-1)u[n-1] = u[n-1] \quad (28)$$

   (b)

$$y[n] - y[n-1] = x[n] * (h[n] - h[n-1]) \quad (29)$$
$$x[n-1] = \delta[n] - 2\delta[n-1] + \delta[n-2] = x[n] * \delta[n-1] \quad (30)$$
$$x[n] = \delta[n+1] - 2\delta[n] + \delta[n-1] \quad (31)$$

   (c)

$$y[n] = y[n+1] - x[n] \quad (32)$$

6. $s(t) = \frac{1}{2}t^2u(t)$.
   $h(t) = \frac{ds(t)}{dt} = tu(t), x(t) = e^{-t}u(t)$.
   $y(t) = h(t) * x(t)$
   $h(t-\tau) = (t-\tau)u(t-\tau)$
   $y(t) = \int_0^t e^{-\tau}(t-\tau)d\tau$
   $y(t) = t\int_0^t e^{-\tau}d\tau - \int_0^t e^{-\tau}\tau d\tau$
   $y(t) = t(1-e^{-t}) + te^{-t} + e^{-t} - 1$
   So $y(t) = t + e^{-t} - 1$.

7. (a) $x(t) = u(t)$
   $y(t) = u(t-3) - u(t-5)$
   $h(t) = \delta(t-3) - \delta(t-5)$

   The figure is below.

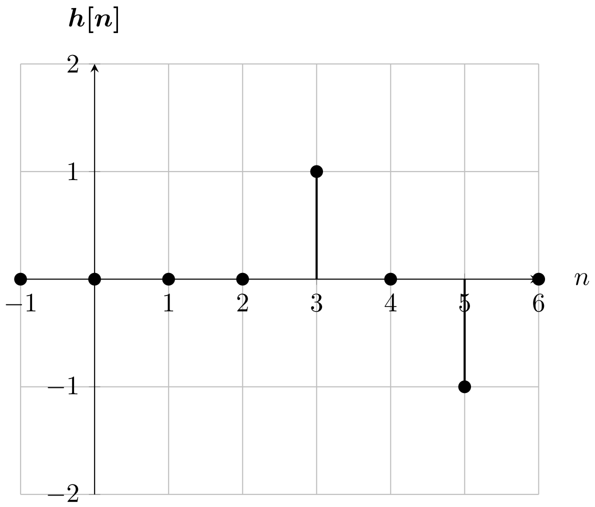


Figure 4: n vs. $h[n]$(continuous impulse response) from question 7a

(b) $x(t) = e^{-3t}u(t)$
$x(t-\tau) = e^{-3(t-\tau)}$
$y(t) = \int_0^t x(t-\tau)h(\tau)d\tau$
$y(t) = \int_0^t e^{-3(t-\tau)}d\tau$
$y(t) = e^{-3t}(e^{3t} - 1)$
So $y(t) = (1 - e^{-3t})(\delta(t-3) - \delta(t-5))$

(c)