Student Information

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Answer 1

a)

Probability density function (pdf) is the derivative of the cdf, $f(x) = F'(x)$. The notation for the uniform distribution is X U(a, b) where $a =$ the lowest value of x and $b =$ the highest value of x. The probability density function of the uniform distribution is

$$f(x) = \frac{1}{b-a} \quad a \leq x \leq b \tag{1}$$

b)

The equation to find mean for uniform distribution between a and b is

$$Mean = \frac{a+b}{2} \tag{2}$$

Using this equation;

$$Mean = \frac{60+180}{2} = 120. \tag{3}$$

The equation to find variance for uniform distribution between a and b is

$$Variance = \frac{(b-a)^2}{12} \tag{4}$$

Using this equation;

$$Variance = \frac{(180-60)^2}{12} = 1200. \tag{5}$$

The equation to find standart deviaton for uniform distribution between a and b is

$$Standard\ Deviation = \sqrt{Variance} \tag{6}$$

Using this equation;

$$Standard\ Deviation = \sqrt{\frac{(180-60)^2}{12}} = 34.641 \tag{7}$$

c)

The equation to find probability between x_1 and $x2$ for uniform distribution between a and b is

$$P(x_1 < x < x_2) = \frac{(x_2 - x_1)}{(b - a)} \tag{8}$$

$$P(90 < x < 120) = \frac{(120 - 90)}{(180 - 60)} = 0.25 \tag{9}$$

d)

Since he always takes more than 120 minutes to finish any CENG222 homework, we can think the limits a=120 b = 180.

$$P(x_1 < x < x_2) = \frac{(x_2 - x_1)}{(b - a)} \tag{10}$$

$$P(150 < x < 180) = \frac{(180 - 150)}{(180 - 120)} = 0.5 \tag{11}$$

Answer 2

The normal distribution can be used as an approximation to the binomial distribution, under certain circumstances, namely:
If $X \approx B(n,p)$ and if n is large or p is close to $\frac{1}{2}$, then X is approximately $N(n \cdot p, n \cdot p \cdot q)$. $(whereq = 1-p)$. In this example $n = 500$ $p = 0.02q = 0.98$, so we can use binomial approximation.

a)

The equation to find mean for normal distribution is

$$Mean = \mu = n \cdot p. \tag{12}$$

Using this equation;

$$Mean = 500 \cdot 0.02 = 10. \tag{13}$$

The equation to find standard deviation for normal distribution is

$$Standard\ Deviation = \sigma = \sqrt{n \cdot p \cdot q}. \tag{14}$$

Using this equation;

$$Standard\ Deviation = \sqrt{500 \cdot 0.02 \cdot 0.98} = 3.1305. \tag{15}$$

b)

We need to find $P(N < 8)$.
The equation to find standard normal random variable(Z) for normal distribution is

$$.Z = \frac{N - \mu}{\sigma}. \tag{16}$$

We need to use continuity correction to avoid getting "0" as a probability. Using this equation;

$$Z = \frac{7.5 - 10}{3.1305} = -0.7986 \tag{17}$$

$$P(N < 7.5) = P(Z < -0.7986) \tag{18}$$

Using the table A4 from the book "PROBABILITY AND STATISTICS FOR COMPUTER SCIENTISTS SECOND EDITION" we find

$$P(N < 8) = \Phi(-0.79) = 0.2148 \tag{19}$$

Using the "Octave Online" website with code $stdnormalcdf(-0.7986)$

$$P(N < 8) = \Phi(-0.7986) = 0.2123 \tag{20}$$

c)

We need to find $P(N > 15)$.
We need to use continuity correction to avoid getting "0" as a probability. Using the equation in the previous question

$$Z = \frac{15.5 - 10}{3.1305} = 1.757. \tag{21}$$

$$P(N > 15.5) = 1 - P(Z > 1.757) \tag{22}$$

Using the table A4 from the book "PROBABILITY AND STATISTICS FOR COMPUTER SCIENTISTS SECOND EDITION" we find

$$P(N > 15) = 1 - \Phi(1.76) = 1 - 0.9608 = 0.0392 \tag{23}$$

Using the "Octave Online" website with code $1 - stdnormalcdf(1.757)$

$$P(N > 15) = 1 - \Phi(1.757) = 0.039459 \tag{24}$$

d)

We need to use continuity correction to avoid getting "0" as a probability. We need to find $P(6.5 < N < 14.5)$.
Using the equation in the previous question

$$Z = \frac{6.5 - 10}{3.1305} = -1.118 \tag{25}$$

$$P(6.5 < N) = P(Z < -1.118) \tag{26}$$

$$Z = \frac{14.5 - 10}{3.1305} = 1.438. \tag{27}$$

$$P(N < 14.5) = P(Z < 1.438) \tag{28}$$

$$P(6.5 < N < 14.5) = P(-1.118 < Z < 1.438) \tag{29}$$

Using the table A4 from the book "PROBABILITY AND STATISTICS FOR COMPUTER SCIENTISTS SECOND EDITION" we find

$$P(N > 6.5) = \Phi(-1.12) = 0.1314 \tag{30}$$

$$P(N < 14.5) = \Phi(1.44) = 0.9251 \tag{31}$$

$$P(6.5 < N < 14.5) = \Phi(1.44) - \Phi(-1.12) = 0.7937 \tag{32}$$

Using the "Octave Online" website with code "$stdnormalcdf(1.438) - stdnormalcdf(-1.118)$"

$$P(6.5 < N < 14.5) = \Phi(1.438) - \Phi(-1.118) = 0.7930 \tag{33}$$

Answer 3

a)

We need to use exponential distribution to solve this question.

$$P(X > 1) = e^{-\lambda t} \tag{34}$$

$$P(X > 1) = e^{-1} = 0.3678 \tag{35}$$

b)

We need to find $P(X > 2|X > 1)$.

$$P(X > 1) = e^{-1} = 0.3678 \tag{36}$$

$$P(X > 2) = e^{-2} = 0,1353 \tag{37}$$

$$P(X > 2|X > 1) = \frac{e^{-2}}{e^{-1}} = e^{-1} = 0.3678 \tag{38}$$