Question: (20 pts) Consider the following LTI system defined by the freq...

(20 pts) Consider the following LTI system defined by the frequency response below:

$$H(j\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6}$$

(a) (5 pts) Find the differential equation which represents this system.

(b) (5 pts) Find the impulse response of this system.

(c) (5 pts) Find $Y(j\omega)$ when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.

(d) (5 pts) Find the output $y(t)$ using the result you found in part c.

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Expert Answer

Manvendra Srivastava answered this
212 answers

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Given: $H(j\omega) = \dfrac{j\omega + 4}{-\omega^2 + 5j\omega + 6}$

Convert it into s-domain — we get —

$$H(s) = \frac{s+4}{s^2+5s+6} \quad \text{— (i)}$$

$$\boxed{H(s) = \frac{s+4}{(s+3)(s+2)}} \quad \text{— (ii)}$$

$\because$ $H(s) = \dfrac{Y(s)}{R(s)} \rightarrow$ o/p, $\rightarrow$ i/p

Transfer fn

(a): $\dfrac{Y(s)}{R(s)} = \dfrac{s+4}{s^2+5s+6}$

$Y(s)s^2 + 5sY(s) + 6Y(s) = sR(s) + 4R(s)$

$$\boxed{\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = \frac{dr}{dt} + 4r(t)}$$

↓ differential eqn represent the system.

(b): Impulse response. use eqn: (2)

$H(s) = \dfrac{Y(s)}{R(s)} = \dfrac{s+4}{(s+3)(s+2)}$ ∴ $r(t) = \delta(t)$ i/p

$R(s) = 1$

$\therefore Y(s) = H(s)$

$\therefore Y(s) = \dfrac{A}{s+3} + \dfrac{B}{s+2}$

$A\big|_{s=-3} = -1$; $B\big|_{s=-2} = 2$

Hence $Y(s) = \dfrac{2}{s+2} - \dfrac{1}{s+3}$

Hence

$$\boxed{y(t) = 2e^{-2t}u(t) - e^{-3t}u(t)}$$

(3): if i/p: $r(t) = x(t) = (e^{-4t} - te^{-4t})u(t)$

↓

then $R(s) = X(s) = \left(\dfrac{1}{s+4} - \dfrac{1}{(s+4)^2}\right)$

$$\boxed{R(s) = -\frac{(s+3)}{(s+4)^2}} \quad \text{— (iii)}$$

$\because$ from eqn (2):

$H(s) = \dfrac{Y(s)}{R(s)} = \dfrac{Y(s)}{Y(s)} = \dfrac{(s+4)}{(s+2)(s+3)}$

Use eqn (3)

$\therefore$ $Y(s) = \dfrac{s+4}{(s+2)(s+3)} \times \dfrac{s+3}{(s+4)^2} = \dfrac{1}{(s+4)(s+2)}$ — (4)

$\therefore$ $Y(j\omega)\big|_{s=j\omega} = \dfrac{1}{(4+j\omega)(2+j\omega)} = \dfrac{1}{(8-\omega^2)+6j\omega}$ Ans

(4): from eqn 4: $Y(s) = \dfrac{1}{(s+4)(s+2)} = \dfrac{A}{s+4} + \dfrac{B}{s+2}$

$A\big|_{s=-4} = -\frac{1}{2}$; $B\big|_{s=-2} = \frac{1}{2}$

$\therefore Y(s) = \dfrac{-1/2}{s+4} + 1/2\big|_{s+2}$

$\therefore$ Ans :- $y(t) = \left(-\frac{1}{2}e^{-4t} + \frac{1}{2}e^{-2t}\right)u(t)$ Ans

Questions viewed by other students

Q: This is an discrete time LTI system represented by the following frequency response: 1 H(ejw) 1 7 be iw -2 jw (a) Determine the difference equation which represents this system. (b) Find a block diagram representation of this system using unit delay operators and adders. (c) Find the impulse response of this system.

A: See answer

Q: (20 pts) This is a LTI system defined by the frequency response below: H(jw) = jw +4 -w2 + 5jw + 6 (a) Firstly determine the differential equation that represents the system above. (b) Determine the impulse response of the system. (c) Find Y (jw) when the input is æ(t) = e-4tu(t) – te-4tu(t). d) Find the output y(t)

A: See answer