

## CWPT

- Bias
- Variance
- Underfitting
- Overfitting
- Bias-variance tradeoff.

model is not learning the reason behind the o/p for the ip data point

Dots  
train testing

identification  $\rightarrow$  test set accuracy << training accuracy

Variance: for an unknown ip  $\approx$  Ans.

reason: - less number of sample data

error  $\rightarrow$  variance =

model  
Training data  
over fit

Predict (training data set)  $\rightarrow$  high

But unknown data set  
 $\downarrow$   
accuracy low

underfitting: not learn too much.

- training set accuracy is less

- test set accuracy is also less.

- error introduced because it underfit is called bias.

- reason:

- few numbers of parameters.

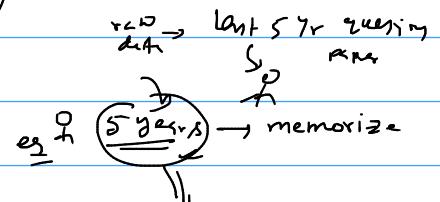
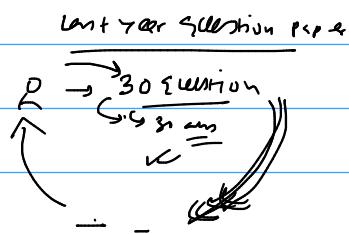
- less training iteration

underfit  $\uparrow$  overfit  $\downarrow$   
Bias-variance trade off

H/W Q, write 4 diff of bias vs variance

Q2: write 4 diff b/w Underfit and overfitting of a model.

Q3: What do you mean by bias-variance trade off



$$\begin{aligned} P_1 & \quad \theta_1 \approx \text{Ans}_1 \\ P_2 & \quad \theta_1 \approx \text{Ans}_2 \\ P_3 & \quad \theta_1 \approx \text{Ans}_3 \end{aligned} \quad \left. \begin{array}{l} \text{Ans}_1 \\ \text{Ans}_2 \\ \text{Ans}_3 \end{array} \right\} \text{Very}$$



## Ch 10 KNN algorithm

K-nearest neighbour algorithm

?

KNN algorithm  $\leftarrow$  Supervised classification algorithm

BASIC KNN

Discrete o/p variable

Real/Continuous  
o/p value  
variable

weighted KNN

Discrete  
o/p variable

Real/continuous  
o/p value  
variable

### BASIC KNN (Discrete)

e.g.: Height weight target-distance nearest neighbor

$K=3$

given

unknown  $\rightarrow 157$

Height	weight	target
150	50	M
155	55	M
160	60	L
161	59	L
158	65	L

Soln

2.34	1
6.71	3
6.40	2
11.05	.

among  $K=3$   
nearest neighbor  
 $M \rightarrow 1$

$L = 2$

Soln  
157 54 L

note:

Distance: Euclidean distance

$$\begin{array}{l} x \\ \bar{x}_i \\ n_2 \\ n_3 \\ \vdots \\ n_n \end{array} \quad \begin{array}{l} y \\ \bar{y}_i \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{array}$$

$$E(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$

## Basic KNN (Continuous/Real data)

Euclidean distance

<u>Q8</u>	<u>height</u>	<u>weight</u>	<u>target</u>	<u>Distance</u>	<u>nearest-neighbour</u>
$K=3$	150	50	1.5	8.06	
	155	55	1.2	2.24	1
	160	60	1.8	6.71	3
	161	59	2.1	6.40	2
	158	65	1.7	11.05	
	157	54	?		

$$\text{So, O/P for } (157, 54) = \frac{1.2 + 1.8 + 2.1}{3} \\ = 1.7$$

- How to decide value of  $K$

Ans: Approach 1  $K = \sqrt{n}$

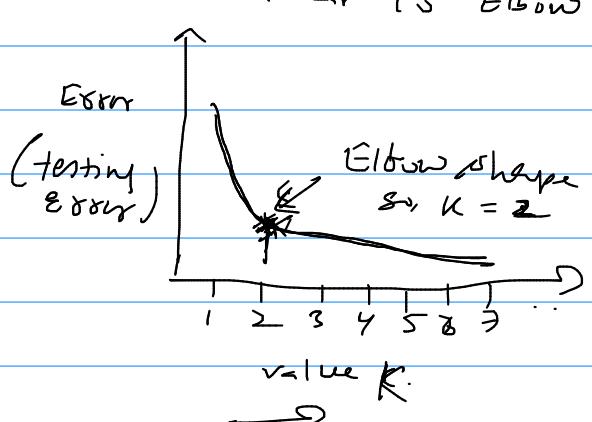
Sample size

Approach 2  $K \leftarrow$  odd number (to avoid any tie)

Approach 3 : Elbow method.

<u>X</u>	<u>Y</u>	<u>Target</u>
$x_1$	$y_1$	M
$x_2$	$y_2$	L
$x_3$	$y_3$	M
:	:	.
$x_{50}$	$y_{50}$	M
<u>Training data</u>		
$x_{51}$	$y_{51}$	L
$x_{52}$	$y_{52}$	L
:	:	M
$x_{60}$	$y_{60}$	L
<u>Testing data</u>		

- Consider odd  $K$   
near 1.5 Elbow shape



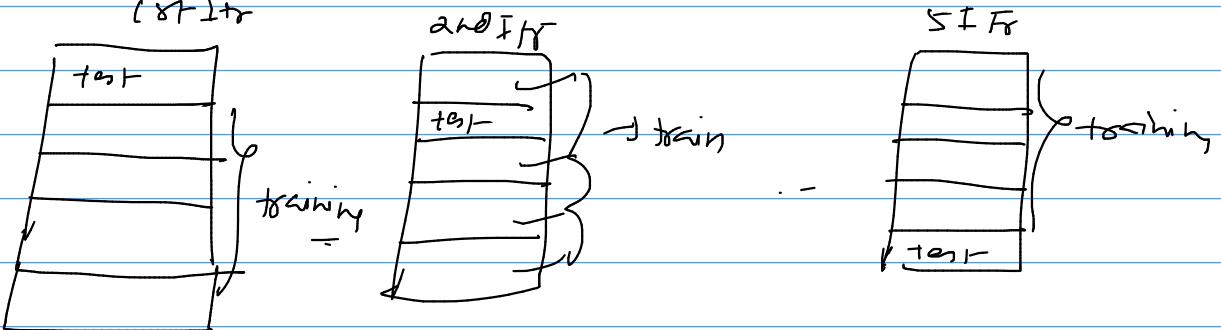
$K=2$

① - find O/P label for any testing data points

② - find test accuracy. (By comparing given O/P labels for test data and model predicted O/P for test data)

$$\text{Error} = (100 - \text{Accuracy}) \%$$

Approach: use 5-fold Cross validation



$$\text{test accuracy} = \frac{\sum_{i=1}^n (\text{test accuracy}_{\text{It}_i})}{5}$$

$$\text{Error} = (100 - \text{test accuracy}) \%$$

- plot the elbow curve.

✓

- Basic KNN algorithm for discrete valued target function is given below -

Training algorithm:

- For each training Example  $\langle x, f(x) \rangle$ , add the Example to the list training Examples.

Classification algorithm:

- Given a query instance  $x_q$  to be classified.
- Let  $x_1, x_2, \dots, x_K$  denote the  $K$  instances from training Example that are nearest to  $x_q$ .
- Return  $f(x_q) \leftarrow \arg \max_{v \in V} \sum_{i=1}^K \delta(v, f(x_i))$

where  $\delta(a, b) = 1$  if  $a = b$  otherwise  $\delta(a, b) = 0$ .  
 $V$ : set of output/target levels.

Data of -

height	weight	target	dis. neighbor	sort
150	50	M	8.06	
155	55	M	2.4	1
160	60	L	6.71	2
161	59	L	6.40	3
158	65	L	11.05	
157	54	? L		

Basic KNN Algorithm

Supervised algorithm.

Classification algorithm.

two type  
Output in discrete level  
Output in continuous level.

$d(x_i) = \sqrt{\sum_{j=1}^n (x_i - x_j)^2}$

$f(x_q) = \arg \max_{v \in V} \sum_{i=1}^K \delta(v, f(x_i))$

$f(\text{medium}) = f(\text{medium}, \text{medium}) + f(v \in \text{set of target values})$

$f(\text{large}) = f(f(\text{large}, \text{large}) + f(\text{large}, \text{medium}) + f(\text{medium}, \text{large}))$

$f(\text{small}) = f(f(\text{small}, \text{small}) + f(\text{small}, \text{medium}) + f(\text{medium}, \text{small}))$

height	weight	target	dist.	height
150	50	Medium	8.06	
155	55	Medium	2.4	1
160	60	Large	6.71	2
161	59	Large	6.40	3
158	65	Large	11.05	

$K$ : number of nearest neighbor considered

$x_i \rightarrow$  datapoint

$f(x_i) \rightarrow$  target of  $x_i$ .

$x_1: (155, 55)$   
 $x_2: (161, 59)$   
 $x_3: (160, 60)$

predicted

$f(x_q): 0/r \delta x_q$

$V$ : set of target or  
 $v \Rightarrow M, L$

$$\begin{cases} \delta(M, M) + \delta(M, L) + \delta(M, L) \\ \delta(L, M) + \delta(L, L) + \delta(L, L) \end{cases} \text{ so, } v \Rightarrow M, L$$



Basic KNN when output is continuous/real values

Training Algorithm

- For each training Example  $\langle x, f(x) \rangle$ , add the example to the list training examples

Classification Algorithm

- Given a query instance  $x_q$  to be classified
- let  $x_1, x_2, \dots, x_K$  denote the  $K$ -nearest instances of  $x_q$ .
- Return  $f(x_q) \leftarrow \frac{\sum_{i=1}^K f(x_i)}{K}$

Basic KNN when output level is continuous.

Height	Weight	target	Distanz	Nearer	Counter, $K=3$
150	50	1.5	8.06		
155	55	1.2	2.34	1	
160	60	1.8	6.71	3	
161	59	2.1	6.40	2	
158	65	1.7	11.05		
157	54	1.7			

$$f(157, 54) = \frac{1.2 + 2.1 + 1.8}{3} = \frac{5.1}{3} = 1.7$$

$$f(\langle 153, 54 \rangle) =$$

## Discrete o/p

Distance Weighted KNN algorithm

- Supervised algorithm
- Classification algorithm
- Output types
  - Discrete
  - Real valued

for discrete output value

$$\hat{f}(x_2) \leftarrow \underset{v \in V}{\operatorname{argmax}} \sum_{i=1}^K w_i S(v, f(x_i))$$

where,  $w_i = \frac{1}{d(x_2, x_i)^2}$

Height	Weight	target	Distance	Weight	Count
150	50	1.5	8.06	1	1
155	55	1.2	2.24	1	1
160	60	1.8	6.71	3	3
161	59	2.1	6.40	2	2
158	65	1.7	11.05		
157	54	1.7			

$\hat{f}(157, 54) = \frac{0.45 \times f(M, M) + 0.15 \times f(M, L) + 0.16 \times f(L, M) + 0.16 \times f(L, L)}{0.45 + 0.15 + 0.16}$

$= \frac{0.45 \times 1 + 0.15 \times 1.8 + 0.16 \times 2.1}{0.45 + 0.15 + 0.16}$

$= 1.507$

## Distance Weighted KNN for real valued output

$$\hat{f}(x_2) \leftarrow \frac{\sum_{i=1}^K w_i f(x_i)}{\sum_{i=1}^K w_i}$$

where,  $w_i = \frac{1}{d(x_2, x_i)^2}$

→ continuous

## Basic KNN when output level is continuous

Height	Weight	target	Distance	Weight	Count
150	50	1.5	8.06	1	1
155	55	1.2	2.24	1	1
160	60	1.8	6.71	3	3
161	59	2.1	6.40	2	2
158	65	1.7	11.05		
157	54	1.7			

$\hat{f}(157, 54) = \frac{1.2 + 1.8 + 2.1}{3} = 1.7$

$\hat{f}(157, 54) = \frac{0.45 \times 1.2 + 0.15 \times 1.8 + 0.16 \times 2.1}{0.45 + 0.15 + 0.16}$

$= 1.507$

$$\sqrt{(157 - 150)^2 + (54 - 50)^2}$$

$= \sqrt{49 + 16} = \sqrt{65}$



✓ Note :

(x)

- KNN algorithm is based on distance. So, if the  $\vec{i/p}$  are in different scale then it might produce incorrect result -

↓ So it make all inputs are in same scale

- Z-normalization =

or

- min-max standardization ↙

(in exams for standardization you can prefer either of this two method).

↓ then apply KNN algorithm.