Sardar Patel Institute of Technology, Mumbai
Department of Electronics and Telecommunication Engineering
T.E. Sem-VI (2020-2021)
ETL61-Discrete Time Signal Processing Lab

Lab - 3: Discrete Fourier Transform

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I. Aim

The aim of this experiment is to implement an algorithm to perform Discrete Fourier Transform on the given input signal.

A. *Objectives*

1. Develop a function to perform DFT of N point Signal.
2. Conclude the effect of zero padding on magnitude spectrum.

B. *Input Specifications*

1. Length of Signal N and Signal values

C. *Problem Definition*

1. Take four-point sequence x[n]. Find X[k] using DFT.
2. Find x[n] of X[k] obtained in part-1 using IDFT.
3. Plot all the input and output signal.
4. Append the sequence x[n] by four zeros at the end position. Find DFT and Plot the spectrum. Observe and compare the spectrum with original DFT signal. Give your conclusion.
5. Append the sequence x[n] by four zeros at the alternate position. Find DFT and Plot the spectrum. Observe and compare the spectrum with original DFT signal.

II. Introduction

The Discrete Fourier transform (DFT) converts a finite sequence of equally-spaced samples of a function into a same-length sequence of equally-spaced samples of the discrete-time Fourier transform (DTFT), which is a complex-valued function of frequency. The interval at which the DTFT is sampled is the reciprocal of the duration of the input sequence. An inverse DFT is a Fourier series, using the DTFT samples as coefficients of complex sinusoids at the corresponding DTFT frequencies.

III. Discrete Fourier Transform

The DFT is said to be a frequency domain representation of the original input sequence. Signals such as sound waves, radio signals, temperature readings, images etc. which are sampled over a finite time interval can be converted into their frequency domain representations for analysis.

A. *Equation for the Discrete Fourier Transform*

The equation for the Discrete Fourier Transform is given by :

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}$$

B. *Equation for Inverse Discrete Fourier Transform*

The equation for the Inverse Discrete Fourier Transform is given by :

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{i\frac{2\pi}{N}kn}$$

C. *Properties*

The Discrete Fourier Transform exhibits the following properties :

1. Linearity
2. Time and frequency reversal
3. Orthogonality
4. Periodicity
5. Shift Theorem
6. Convolutional duality

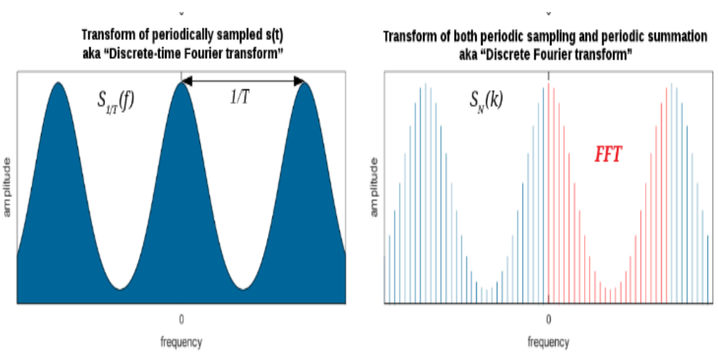


Fig. 1 : Sampling of DTFT to calculate DFT

IV. Code and Observations

```
% Discrete Time Signal Processing Lab 3
% Discrete Fourier  Transform
% MATLAB version R2018a
% Date : 09-02-2021

clc;
clear;
close all;

x = [1+2j,3+4j,-1-2j,5-2j];
N = length(x);

% Plotting the input signal
stem(abs(x),'g','LineWidth',3);
ax = gca();
title('Plot of magnitude of
x(n)','FontSize',15);
xlabel('n','FontSize',15);
ylabel('|x|','FontSize',15);
set(ax,'xlim',[0 N+1],'fontsize',15);
grid on;
figure();

stem(angle(x),'b','LineWidth',3);
ax = gca();
title('Plot of phase of
x(n)','FontSize',15);
xlabel('n','FontSize',15);
ylabel('\angle x','FontSize',15);
set(ax,'xlim',[0 N+1],'fontsize',15);
grid on;
figure();

% Calculating the DFT of the input
signal
X = DFT(x,N);
disp('DFT of input signal : ');
disp(X);

% Plotting the DFT of input signal
stem(abs(X),'r','LineWidth',3);
ax = gca();
title('Plot of magnitude of
X(k)','FontSize',15);
xlabel('n','FontSize',15);
ylabel('|X|','FontSize',15);
set(ax,'xlim',[0 N+1],'fontsize',15);
grid on;
figure();

stem(angle(X),'b','LineWidth',3);
ax = gca();
title('Plot of phase of
X(k)','FontSize',15);
xlabel('n','FontSize',15);
ylabel('\angle X','FontSize',15);
set(ax,'xlim',[0 N+1],'fontsize',15);
grid on;
figure();

% Calculating the IDFT of the spectrum
x_ = IDFT(X,N);
disp('Retrieved signal after IDFT : ');
disp(x_);

% Adding zeros at alternate indices and
calculating DFT
x_alt=reshape([x;zeros(size(x))],[],1);
X = DFT(x_alt,length(x_alt));
disp('DFT of x with alternate zeros: ');
disp(X);

% Plotting the DFT of input signal
stem(abs(X),'r','LineWidth',3);
ax = gca();
title('Plot of magnitude of X(k) with
alternate zeros','FontSize',15);
xlabel('n','FontSize',15);
ylabel('|X|','FontSize',15);
set(ax,'xlim',[0
length(X)+1],'fontsize',15);
grid on;
figure();

stem(angle(X),'b','LineWidth',3);
ax = gca();
title('Plot of phase of X(k) with
alternate zeros','FontSize',15);
xlabel('n','FontSize',15);
ylabel('\angle X','FontSize',15);
set(ax,'xlim',[0
length(X)+1],'fontsize',15);
grid on;
figure();

% Appending zeros to the input signal
and calculating DFT
zeros_n = 4;
x_ = [x, zeros(1,zeros_n)];
X = DFT(x_,length(x_));
disp('DFT of x with zeros appended at
the end : ');
disp(X);

% Plotting the DFT of input signal
stem(abs(X),'r','LineWidth',3);
ax = gca();
title('Plot of magnitude of X(k) with
zeros appended','FontSize',15);
xlabel('n','FontSize',15);
ylabel('|X|','FontSize',15);
set(ax,'xlim',[0
length(X)+1],'fontsize',15);
grid on;
figure();

stem(angle(X),'b','LineWidth',3);
ax = gca();
title('Plot of phase of X(k) with zeros
appended','FontSize',15);
xlabel('n','FontSize',15);
ylabel('\angle X','FontSize',15);
```

```
set(ax,'xlim',[0
length(X)+1],'fontsize',15);
grid on;

% Function to implement Discrete Fourier
Transform
function X = DFT(x,N)
    X = zeros(1,N);
    for k=0:N-1
        for j=0:N-1
            X(k+1) = X(k+1)+
x(j+1)*(cos(2*pi*k*j/N)-
sin(2*pi*k*j/N)*1i);
        end
    end
end

% Function to implement Inverse Discrete
Fourier Transform
function x = IDFT(X,N)
    x = zeros(1,N);
    for k=0:N-1
        for j=0:N-1
            x(k+1) = x(k+1)+
X(j+1)*(cos(2*pi*k*j/N)+sin(2*pi*k*j/N)*
1i);
        end
        x(k+1) = x(k+1)/N;
    end
end
```

V. Observations

A. Output Graphs

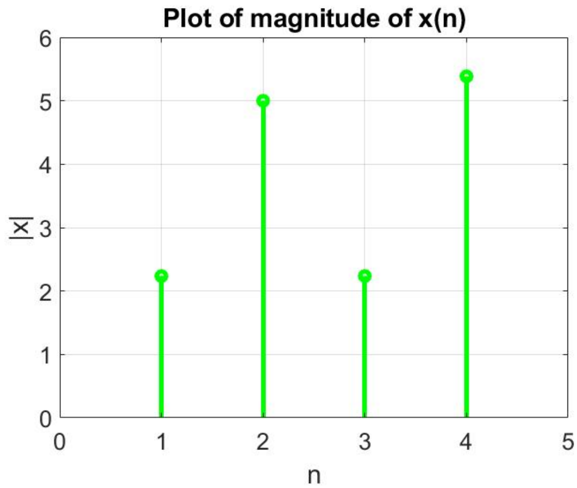


Fig. 2 : Plot of magnitude of x(n)

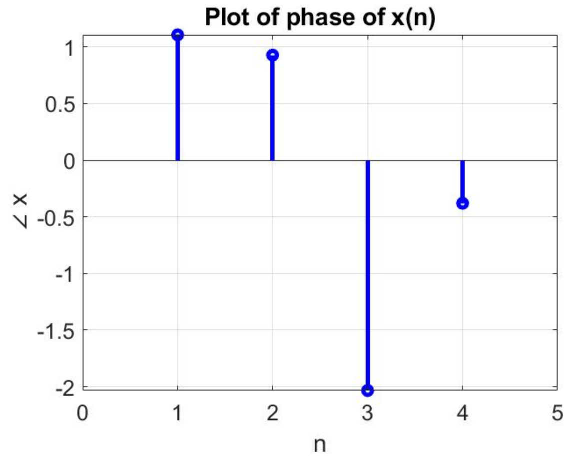


Fig. 3 : Plot of phase of x(n)

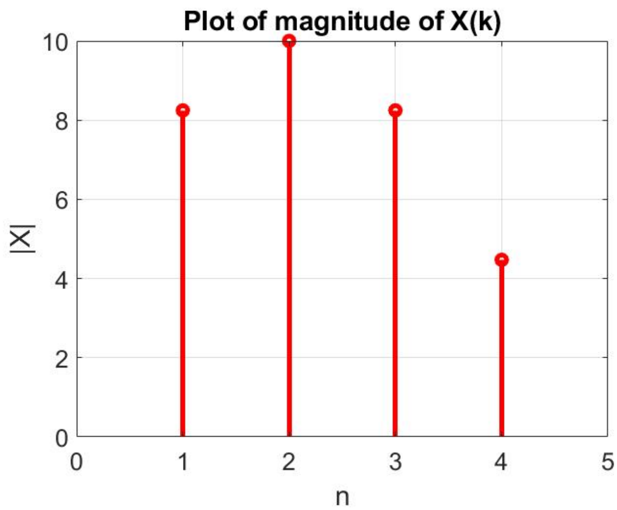


Fig. 4 : Plot of magnitude of X(k)

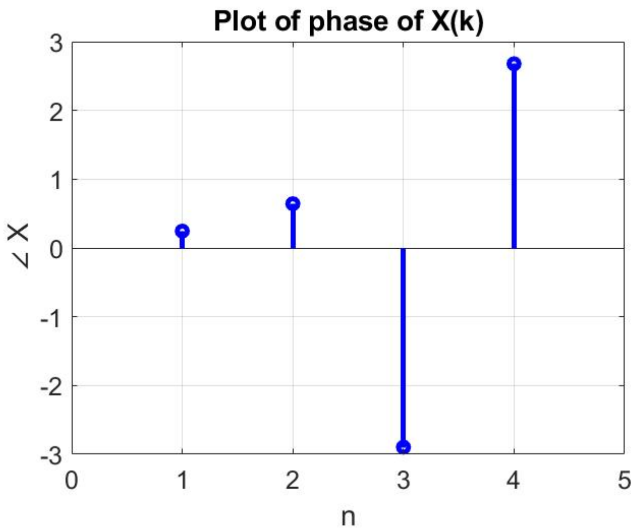


Fig. 5 : Plot of phase of X(k)

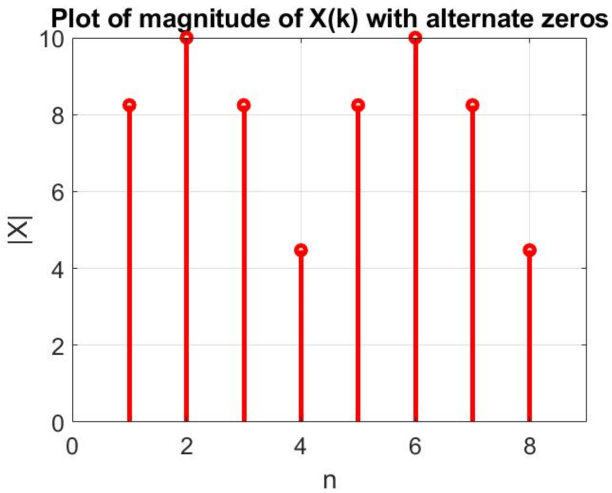


Fig. 6 : Plot of magnitude of X(k) with alternate zeros appended to x(n)

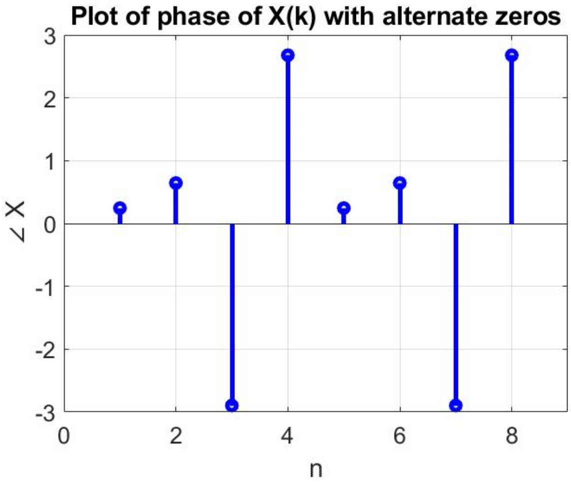


Fig. 7 : Plot of phase of X(k) with alternate zeros appended to x(n)

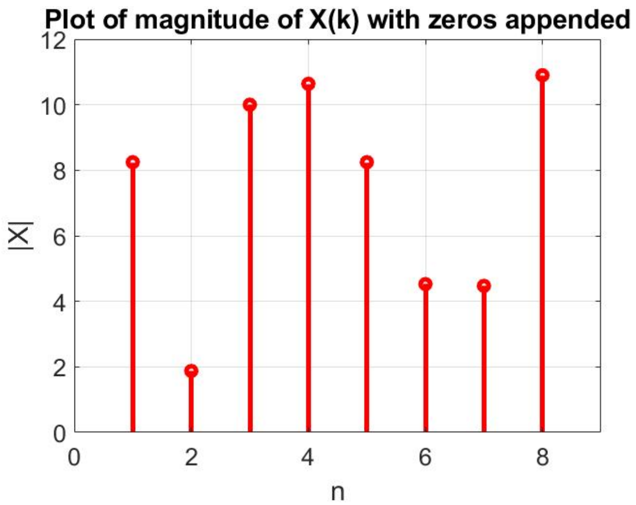


Fig. 8 : Plot of magnitude of X(k) with zeros appended at the end to x(n)

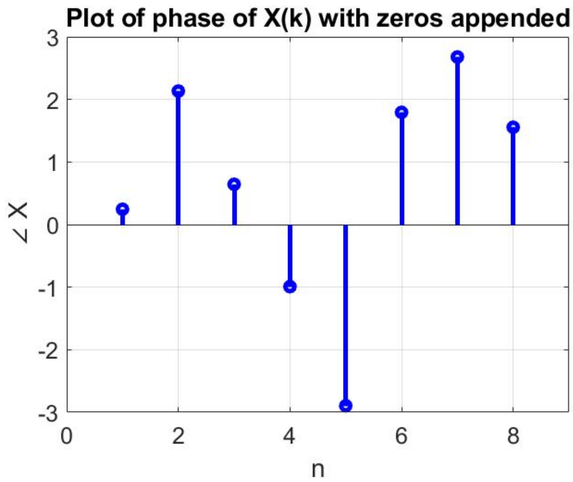


Fig. 9 : Plot of phase of X(k) with zeros appended at the end to x(n)

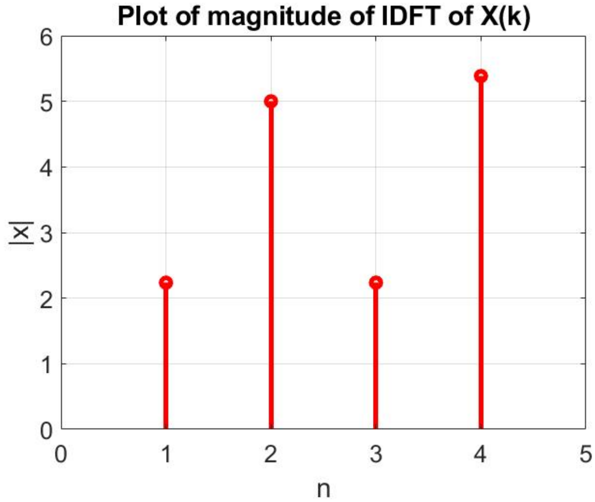


Fig. 10 : Plot of magnitude of IDFT of X(k)

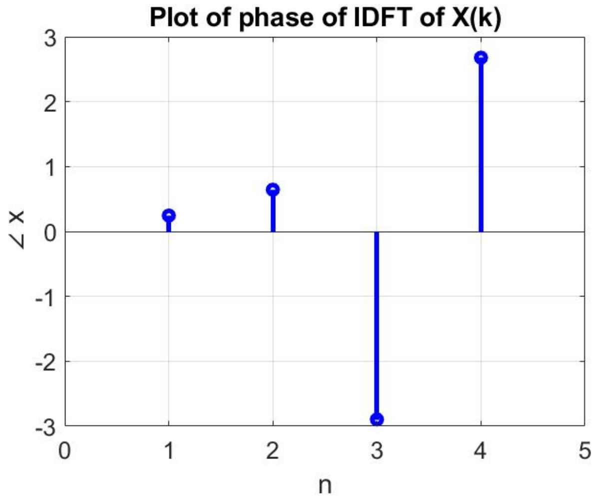
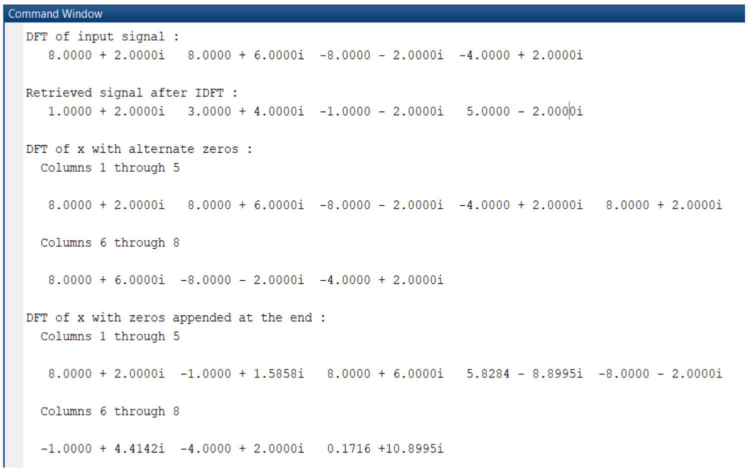


Fig. 11 : Plot of phase of IDFT of X(k)



```
Command Window
DFT of input signal :
   8.0000 + 2.0000i   8.0000 + 6.0000i  -8.0000 - 2.0000i  -4.0000 + 2.0000i

Retrieved signal after IDFT :
   1.0000 + 2.0000i   3.0000 + 4.0000i  -1.0000 - 2.0000i   5.0000 - 2.0000i

DFT of x with alternate zeros :
  Columns 1 through 5

   8.0000 + 2.0000i   8.0000 + 6.0000i  -8.0000 - 2.0000i  -4.0000 + 2.0000i   8.0000 + 2.0000i

  Columns 6 through 8

   8.0000 + 6.0000i  -8.0000 - 2.0000i  -4.0000 + 2.0000i

DFT of x with zeros appended at the end :
  Columns 1 through 5

   8.0000 + 2.0000i  -1.0000 + 1.5858i   8.0000 + 6.0000i   5.8284 - 8.8995i  -8.0000 - 2.0000i

  Columns 6 through 8

  -1.0000 + 4.4142i  -4.0000 + 2.0000i   0.1716 +10.8995i
```

Fig. 12 : Console Output

B. Results

From the output graphs, we can observe that :

1. The DFT of the input signal gives the sampled version of its frequency response which is useful in digital applications.
2. Adding alternate zeros to the original signal causes the original DFT sequence to be repeated.
3. Appending zeros to the end of the input signal causes the DFT sequence to have the original values at even indices and have new values at odd indices.
4. The plots of the original signal and retrieved signal after IDFT are the same.

VI. CONCLUSION

From the experiment conducted, it can be concluded that :

1. DFT is an important mathematical tool used in signal processing which gives the frequency domain representation of the input signal and can be used by digital systems.
2. The DFT of an input signal can be calculated by using two for loops in MATLAB to implement the formula.
3. Appending zeros at the end or at alternate locations to the input signal does add new information while improving the resolution of the frequency spectrum.