Jacobian matrix - helps to convert angular velocity of joints to end effector velocity

Overview of jacobian

joint velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J_{6\times n} \begin{bmatrix} \dot{q}_{v_1} \\ \dot{q}_{v_2} \\ \cdots \\ \dot{q}_{v_n} \end{bmatrix}_{n\times 1}$$

tip velocity

$$x_1 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta + L_2 \sin(\theta_1 + \theta_2)$$

By differentiating

$$\dot{x} = -L_1 \dot{\theta}_1 \sin\theta - L_2 (\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_1 + \theta_2)$$
$$\dot{y} = L_1 \dot{\theta}_1 \cos\theta + L_2' (\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_1 + \theta_2)$$

$j_1(\theta)$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos\theta_1 + L_1 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$j_2(\theta)$

As long as $J_1(\theta)$ & $J_2(\theta)$ is not collinear
any possible velocity can be generated

$\Downarrow$

Singularties

Boundary singularity
$\downarrow$ a.k.a
workspace boundary

Internal singularity
$\downarrow$ a.k.a
joint space singularity
(also for inverse kinematics)

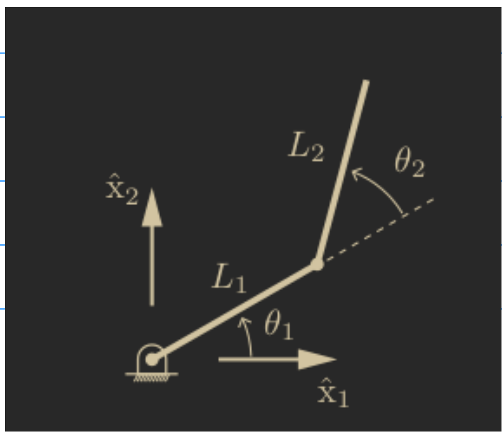
For 2-Dof

$$j(\theta) = \begin{bmatrix} -L_1 \sin(\theta_1) - L_2 \sin(\theta_1+\theta_2) & -L_2 \sin\theta_2 \\ L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2 & L_2 \cos\theta_2 \end{bmatrix}$$

Singularity is found using $|j(\theta)|$

$$\Rightarrow L_1 L_2 \sin(\theta_2) = 0$$

X



Static torque → Study of non-accelerating bodies

Dynamic torque → body with some acceleration

$$\delta \vec{x} = J(\vec{\theta})\,\delta\vec{\theta}$$

∵ Work done = force × displacement

$$\vec{f}^{T}\,\delta\vec{x} = \vec{f}^{T} J(\vec{\theta})\,\delta\vec{\theta}$$

$$\tau = J^{T}(\vec{\theta})\,\vec{f}$$

$\dot{x}$