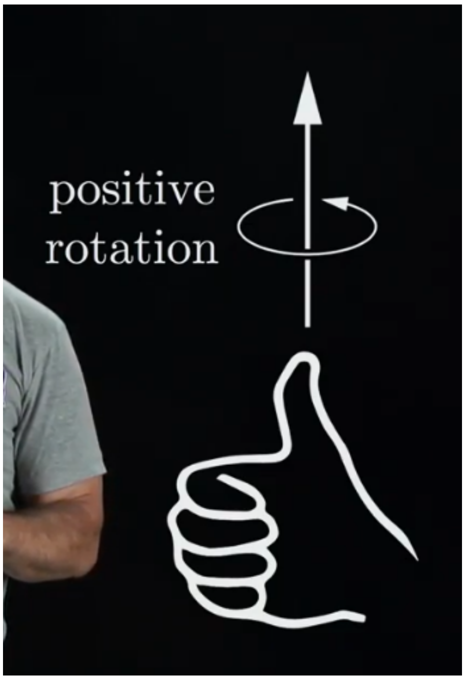
1 Intro - lightboard

Everything follows right hand rule



Figuur 1:

2 Foundation of mobile robotics ch-2

2.1 Degrees of freedom rigid body

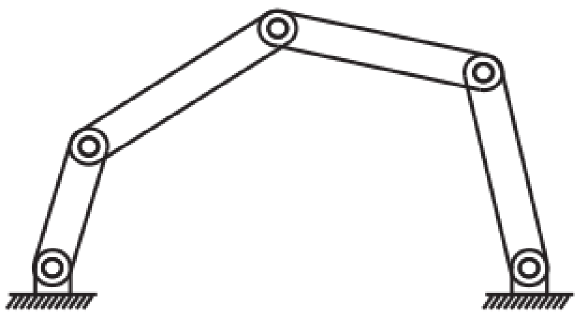
- **Configuration** - A specific position of the position of all points of robot
- **C-space** - The space of all configuration
- **degrees of freedom** - dimension of c space
- Rigid body has 6 degrees of freedom
- dof - $\sum(freedompoints) - no.of.constrains$

points	dof	no.of.constrains	constrans
point A	2	0	-
point B	2	1	d_{ab}
Point C	2	2	$d_{ac}, d_b c$

- $dof = m(N-1) - \sum_{i=1}^{J} c_i$
- also $c_i + f_i = m$

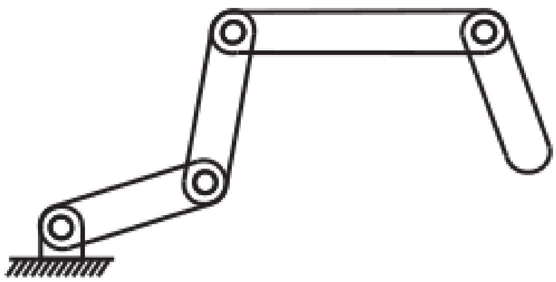
2.1.1 Grublers formula

- N = no of links
- J = no of joints
- m = dof of rigid body (3 for planar 6 for spatial)
- f_i = No of freedom of joint
- c_i = No of constrains by joint
- $$\begin{aligned} Dof &= m(N-1) - \sum_{i=1}^{j} c_i \\ &= m(N-1) \sum_{i=1}^{j} m - f_i && \because m = f_i + c_i \\ &= m(N-1) - J.m \sum_{i=1}^{j} f_i && \because \sum_{i=0}^{j} \mathbf{c} = \mathbf{J}.\mathbf{c} \\ &= m(N-1-J) + \sum_{i=1}^{j} f_i \end{aligned}$$
- Grubler formula is valid only for joints with independant constrain
- Closed chain mechanism - both base and end point is connected to ground



Figuur 2:

- Open chain mechanism only base is attached to ground



Figuur 3:

2.2 Configuration space topology

- sphere and plance has same dimension (x,y and latitude,longitude) but diffrent hape
- shape of the c-space is **topology**
- **Topologically equivalent -** if two spaces can be deformed into each other without any removal or addition
- $\mathbb{E}^1$ - euclidean line
- c space can be expressed as **cartesian product** of two are more space of lower dimension
- c- space of some common robots (X^n where n is the dimension)
  - Rigid body in plane $= \mathbb{R}^2 \times \mathbb{S}^1$
  - PR robot arm $= \mathbb{R}^1 \times \mathbb{S}^1$
  - planar rigid body with PR arm $=$

$$\begin{aligned} \text{for arm} &= \mathbb{S}^1 \times \mathbb{S}^1 = T^2 \qquad \qquad \ldots \text{T} = \text{torus} \\ \text{for mobile base} &= \mathbb{R}^2 \times \mathbb{S}^1 \\ \therefore c - space &= \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{T}^2 \\ &= \mathbb{R}^2 \times T^3 \end{aligned}$$

2.3 Configure space Representation

To perform computations we need numerical represenstation

2.3.1 Explicit representation

- choice of n co-ordinates for to represent an n-dimensional space
- for sphere,longitude and latitude
- causes singularaties (at northpole,sudden shift in values)
- to avoid this use co-ordinate chart(split parts like atlas)

2.3.2 Implicit representation

- represensting n n-dimensional space as if it is embedded in euclidean space
- disadvantage as has more variable to consider
- easy to define closed loops

2.4 Configuration and velocity constrain

For a 4 bar linkage,the closed loop can be expressed by the following equations

$$
\begin{aligned}
L_1 \cos\theta_1 + L_2\cos(\theta_1+\theta_2) + \ldots + L_4\cos(\theta_1+\ldots+\theta_4) &= 0 \\
L_1 \sin\theta_1 + L_2\sin(\theta_1+\theta_2) + \ldots + L_4\sin(\theta_1+\ldots+\theta_4) &= 0 \quad \because \text{sum of all vectors in loop is zero} \\
\theta_1+\theta_2+\theta_3+\theta_4+2\pi &= 0 \quad \because \text{sum of all angles is } 2\pi
\end{aligned}
$$

These equations are called **loop closure equations**.For general robots c-space can be implicitly represented by column vector $\theta = \begin{bmatrix}\theta_1 & \ldots & \theta_n\end{bmatrix}^T \in \mathbb{R}^n$

$$
g(\theta) = \begin{bmatrix} g_1(\theta_1,\ldots,theta_n) \\ \vdots \\ {}_k(\theta_1,\ldots,\theta_k) \end{bmatrix} = 0 \tag{1}
$$

By diffrentiating the above equation [1]

$$
\begin{bmatrix}
\frac{\partial g_1}{\partial \theta_1}(\theta)\dot{\theta}_1 + \cdots + \frac{\partial g_1}{\partial \theta_n}(\theta)\dot{\theta}_n \\
\vdots \\
\frac{\partial g_k}{\partial \theta_1}(\theta)\dot{\theta}_1 + \cdots + \frac{\partial g_k}{\partial \theta_n}(\theta)\dot{\theta}_n
\end{bmatrix} = 0.
$$

Figuur 4:

this can be expressed as matrix multiplying a column vector

[1]find how this happened?

$$
\begin{bmatrix}
\dfrac{\partial g_1}{\partial \theta_1}(\theta) & \cdots & \dfrac{\partial g_1}{\partial \theta_n}(\theta) \\
\vdots & \ddots & \vdots \\
\dfrac{\partial g_k}{\partial \theta_1}(\theta) & \cdots & \dfrac{\partial g_k}{\partial \theta_n}(\theta)
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\vdots \\
\dot{\theta}_n
\end{bmatrix}
= 0,
$$

Figuur 5: