

Number with different bases

decimal base 10	Binary base 2	octal base 8	Hexadecimal base 16
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Base (or) radix

In general a number expressed in base r system has co-efficients multiplied by powers of r:

$$a_0r^0 + a_1r^1 + \dots + a_2r^2 + a_3r^3 + a_4r^4 + \dots + a_nr^n$$

The coefficients of range 0 in value from 0 to r-1.

Ex. decimal $(30)_{10}$ binary $(1011)_2$ octal $(23)_8$

Hexadecimal $(5A)_{16}$ base 5 $(4021)_5$

① Convert 41 into binary & octal & hexadecimal

$$\begin{array}{r} 2 \overline{)41} \\ 2 \overline{)20-1} \\ 2 \overline{)10-0} \\ 2 \overline{)5-0} \\ 2 \overline{)2-1} \\ \hline 1-0 \end{array}$$

$(41)_{10} = (101001)_2$
 $= \underline{\underline{101001}}$

$$8 \overline{)41} = (51)_8 \quad \div 5 \quad 1 = (51)_8$$

$$16 \overline{)41} = (29)_{16} \quad \div 16 \quad 5 = (29)_{16}$$

② Convert $(0.6875)_{10}$ to binary

$$0.6875 \times 2 = 1.3750 \quad \downarrow$$

$$0.3750 \times 2 = 0.7500 \quad \downarrow$$

$$0.75 \times 2 = 1.50 \quad \downarrow$$

$$0.50 \times 2 = 1.00 \quad \downarrow$$

$$(0.6875)_{10} = (0.1011)_2 = (0.1011)_2$$

③ Convert $(0.513)_{10}$ to octal

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$

$$(0.513)_{10} = (0.406517)_8$$

(4) $(41.6875)_{10}$ ————— decimal separate + fractional separate

$$(41.6875)_{10} = (\underline{10100} \cdot \underline{1011})_2$$

⑤ $(01001 \cdot 1011)_2$ convert to decimal

$$\begin{array}{r}
 101001 \\
 \times 2^0 = 1 \times 1 = 1 \\
 \times 2^1 = 0 \times 2 = 0 \\
 \times 2^2 = 0 \times 4 = 0 \\
 \times 2^3 = 1 \times 8 = 8 \\
 \times 2^4 = 0 \times 16 = 0 \\
 \times 2^5 = 1 \times 32 = 32 \\
 \hline
 & 41
 \end{array}$$

0.1011

$$\begin{array}{r}
 1 \times 2^{-1} = 1 \times 0.5 = 0.5 \\
 0 \times 2^{-2} = 0 \times 0.25 = 0.0 \\
 1 \times 2^{-3} = 1 \times 0.125 = 0.125 \\
 1 \times 2^{-4} = 1 \times 0.0625 = 0.0625 \\
 & = 0.6875
 \end{array}$$

Binary Arithmetic

A B Sum Carry

$$0 \quad 0 \quad 0 \quad 0$$

$$0 \quad 1 \quad 1 \quad 0$$

$$0$$

① Add 28 & 15 in binary

$$\begin{array}{r} 2 \\ \underline{14 - 0} \\ 2 \\ \underline{7 - 0} \\ 2 \\ \underline{3 - 1} \\ 1 - 1 \end{array}$$

$$\begin{array}{r} 2 \\ \underline{15 - } \\ 2 \\ \underline{7 - } \\ 2 \\ \underline{3 - } \\ 1 - 1 \end{array}$$

$$(28)_{10} = 11100$$

$$(15)_{10} = 111$$

$$11100 + 111 = 101011$$

② Binary Subtraction

Rules A B Diff Borrow

0	0	0	0	0
0	1	1	1	1
1	0	1	0	0
1	1	0	0	

Subtract $(0101)_2$ from $(1011)_2$

1011 — decimal 11

- 0101 — decimal 5

$$\underline{0110} \text{ — decimal 6}$$

Binary Multiplication

e.g., 011×110

$$\begin{array}{r} 011 \\ \times 110 \\ \hline 000 \\ 011 \\ \hline 10010 \end{array}$$

Ex. 2

$$1110 \times 1010$$

$$= 10001100$$

Binary division

Rules

$$0 \div 1 = 0$$

$$1 \div 1 = 1.$$

Ex. 1 \Rightarrow Divide $(11011011)_2$ by $(110)_2$.

$$\begin{array}{r} 100100 \\ \hline 110 \overline{)11011011} \\ 110 \\ \hline 000110 \\ 110 \\ \hline 00011 \\ 0 \end{array}$$

Ex 2: Divide $(110101101)_2$ by $(101)_2$

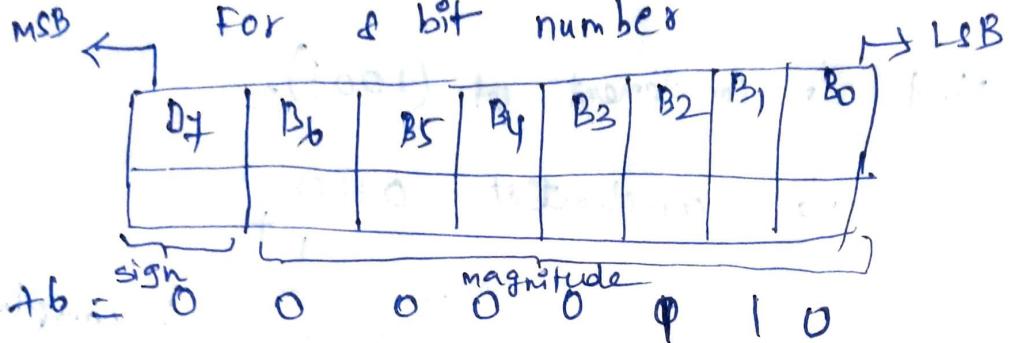
$$\begin{array}{r} 1010101 \\ \hline 101 \overline{)110101101} \\ 101 \\ \hline 0110 \\ 101 \\ \hline 011 \\ 101 \\ \hline 01001 \\ 101 \\ \hline 100 \\ 0 \end{array}$$

Signed Binary numbers

Unsigned numbers represent only magnitude

Signed numbers \Rightarrow MSB represents the sign
MSB = 0 \Rightarrow +ve number & MSB = 1 \Rightarrow -ve number

For a bit number



$$+6 = 00000110$$

$$+24 = 00011000$$

$$-64 = 11000000$$

8 bit binary numbers decimal range 0 to 255

Incase signed 8 bit binary ... +127 to -128

$$\text{Maximum positive number} = +127 = 01111111$$

$$\text{Maximum negative number} = -128 = 10000000$$

(-128 represented as 10000000)

i's complement representation

change all 1's to zero & All 0's to 1.

① Find i's complement of (1101)₂ \Rightarrow (0010)₂

② $(1011100)_2 \Rightarrow (01000110)_2$

2's Complement Representation

2's complement = 1's complement + 1

2's complement form is used to represent -ve no.

- ① Find 2's complement of $(1001)_2$.

$$\begin{array}{r} \text{1's complement: } 0110 \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 0111 \\ \hline \end{array} \quad \text{2's complement}$$

- ② Find 2's complement of $(1010\ 0011)_2$.

1010 0011 number

$$\begin{array}{r} 0101\ 1100 \quad \text{1's complement} \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 0101\ 1101 \Rightarrow 2's \text{ complement} \\ \hline \end{array}$$

1's complement subtraction

- (i) Subtraction of smaller Number from larger num

Method

1. Determine the 1's complement of smallest no.
2. Add the 1's complement to the larger no
3. Remove the carry & add it to the result

This is called end-around carry.

EX

- ① subtract $(00101)_2$ from 111001_2

$$\begin{array}{r} 111001 \\ - 00101 \\ \hline 110100 \end{array} \quad \begin{array}{r} 111001 \\ + 010100 \\ \hline 100110 \end{array} \quad (+)$$

1's comp
end-around carry

$\begin{array}{r} 100110 \\ + 001110 \\ \hline 001110 \end{array}$ - Final answer

Subtraction of Larger Number from Small number

1. Determine the 1's complement of larger number.
2. Add the 1's complement of to the smaller number.
3. Answer is 1's complement form. To get the answer in true form take the 1's complement & assign negative sign to the answer.

Ex Subtract 111001_2 from 101011_2 using 1's complement method.

$$1's \text{ complement of } 101011 = 000110$$

$$\begin{array}{r} 110001 \\ - 001110 \\ \hline 100011 \end{array} \leftarrow \begin{array}{l} \text{Answer in 1's} \\ \text{comp form} \end{array}$$

Advantages of 1's complement subtraction

- This is accomplished with an binary adder
- ∴ This is useful in arithmetic logic circuits
- 1's complement of a number is easily obtained by inverting each bit in the number.

2's complement subtraction

① Subtraction of smaller Number from larger Number

Method

1. Determine the 2's complement of a smaller number

- Add the 2's complement to the larger number.
- Discard the carry.

Ex subtract $(10101)_2$ from $(11100)_2$ using 2's Complement method.

$$\begin{array}{r} \text{no: } 11100 \\ \text{2's comp: } 01010 \\ \hline \text{discard } ① 00110 \end{array}$$

$$\begin{array}{r} 010100 \\ 1+ \\ \hline 010101 \end{array}$$

00110 - Final answer.

Subtraction of Larger Number from smaller Number

Method:

- Determine the 2's comp of larger no
- Add the 2's comp to the smaller no
- Answer in 2's comp form. To get the answer in the true form take the 2's complement, assign negative sign to the answer.

Ex subtract $(11100)_2$ from $(10101)_2$ using 2's complement method.

$$\begin{array}{r} \text{no: } 10101 \\ \text{2's comp: } 00011 \\ \hline 110010 \end{array}$$

$$\begin{array}{r} 000110 \\ 1+ \\ \hline 000111 \end{array}$$

- Ans 2's Comp form

$$\begin{array}{r} 110010 \\ 1+ (-) \\ \hline 110001 \end{array}$$

$= 001110$ ←
Answer 1's comp 001110

Gate level minimization

Binary Codes

The digital data is represented, stored & transmitted as groups of binary digits (bits). The group of bits, also known as binary code represented as both numbers & letters of the alphabets as well as many special characters & control functions.

Classification of Binary codes

① Weighted codes:

Each digit position of the number represented a specific weight

Ex:- 567 \Rightarrow weight of 5 is 100, of 6 is 10 & of 7 is 1.

8421, 2421, 5211 are weighted codes.

② Non-weighted codes

- are not assigned with any weight to each digit position.

Ex: Excess-3 & gray codes are non-weighted code

③ Reflexive codes

A code is said to be reflexive when the code for 9's complement must be found. (Reflexivity)

2421, 5211 & excess 3 codes are reflexive 8421 is not

④ Sequential codes:

succeeding code is one binary number greater than

Introduction

its preceding code.

8421 & excess 3 are sequential.

⑤ Alphanumeric codes

Codes which contain both numbers & alphabetic characters are called alphanumeric codes.

Most commonly used codes are:

ASCII - American standard code for Information Interchange

EBCDIC - Extended Binary coded decimal Interchange

Hollerith code.

⑥ Error detecting & correcting Codes

When the digital information in the binary form is transmitted from one circuit or system to another

circuit or system an error may occur. This means a signal corresponding to 0 may change to 1 or vice versa due to presence of noise. To maintain the data Integrity between transmitter & Receiver

extra bit or more than one bit are added in the data. These extra bits allow the detection & sometimes correction of error in the data. The data along with the extra bit/bits forms the code.

- codes which allow only error detection are called error detecting codes & codes which allow error detection & correction are called error detecting & correcting codes.

Binary coded decimal (BCD) (8421)

- most common code 8421 BCD (4bit)
- called 8421 BCD because the weights associated with 4 bits are 8421 from left to right.

Decimal digit	BCD code				8 4 2 1	7 4 2 1
	8	4	2	1		
0	0	0	0	0	0 0 0 0	0 0 0 0
1	0	0	0	1	0 0 0 1	0 0 0 1
2	0	0	1	0	0 0 1 0	0 0 1 0
3	0	0	1	1	0 0 1 1	0 0 1 1
4	0	1	0	0	0 1 0 0	0 1 0 0
5	0	1	0	1	1 0 1 1	0 1 0 1
6	0	1	1	0	1 1 0 0	0 1 1 0
7	0	1	1	1	1 1 0 1	1 0 0 0
8	1	0	0	0	1 1 1 0	1 0 0 1
9	1	0	0	1	1 1 1 1	1 0 1 0

Excess-3 code

- Excess 3 code modified form of BCD number.
- The Excess 3 code can be derived from the natural BCD code by adding 3 to each coded number.

Decimal digit Excess 3

0	0 0 1 1	6	1 0 0 1
1	0 1 0 0	7	1 0 1 0
2	0 1 0 1	8	1 0 1 1
3	0 1 1 0	9	1 1 0 0
4	0 1 1 1		
5	1 0 0 0		

In excess 3 code we get 9's complement of a number by just complementing each bit. Due to this excess 3 code is called self complementary.

Gray code

- Gray code is a special case of unit distance code.
- In unit distance code bit patterns for two consecutive numbers differ in only one bit position.

These codes are also called cyclic codes.

Decimal code	Gray code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000

Gray code any two adjacent code groups differ only in one bit position. The gray code is also called reflected code.

Binary to Gray conversion

Ex: Convert 1011011 in binary to equivalent graycode

Binary $\rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 1$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
Gray 1 1 1 0 0 1 1 0

Gray to binary

convert gray code 101011 into its binary equivalent.

Gray code : 1 0 1 0 1 1
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
Binary 1 1 1 0 0 1 0

Gate Level minimization:

Usually literals (Boolean variables) & terms are arranged in one of the two standard forms of equations (i) Sum of Product form (SOP)
(ii) Product of sum form (POS)

Sum of product :- (SOP)
(i) $ABC + A\bar{B}\bar{C}$

(ii) $\bar{P}Q + Q\bar{R} + P\bar{A}R$

Product of sum : (POS) (i) $(A+B)(B+\bar{C})$

(ii) $(x+y)(y+z)(z+x)$

Canonical Logic Forms

(4)

- sum of products is canonical (standard) sum products if every product term involves every literal or its complement.

$$SOP = AB + BC + AC$$

Canonical (standard) SOP = $A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$

$$POS = (A+B+C)(A+\bar{B}+C)$$

Example: convert the given expression in canonical SOP.

$$Y = AC + AB + BC$$

$$= AC(B+\bar{B}) + AB(C+\bar{C}) + BC(A+\bar{A})$$

$$= ABC + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC$$

$$= ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

(2) Convert the given expression in canonical SOP form.

$$Y = A + AB + ABC$$

$$= A(B+\bar{B})(C+\bar{C}) + AB(C+\bar{C}) + ABC$$

$$= (AB + A\bar{B})(C+\bar{C}) + ABC + A\bar{B}\bar{C} + A\bar{B}C$$

$$= ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C$$

$$= ABC + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$$

(3) Convert the given expression in canonical POS form

$$Y = (A+B)(B+C)(A+C)$$

$$= (A+\bar{B}+\bar{C}, \bar{C}) (B+\bar{C}, A, \bar{A}) (A+\bar{C}, B, \bar{B})$$

$$= (A+\bar{B}+\bar{C})(A+\bar{B}\bar{C})(A+\bar{B}C)(A+\bar{B}\bar{C})(A+\bar{B}C)$$

$$= (A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+C)(A+\bar{B}+C)$$

$$\begin{aligned}
 ④ \quad Y &= A(A+B)(A+B+C) \\
 &= (A+B, \bar{B}+C, \bar{C})(A+B+C)(A+B+C, \bar{C}) \\
 &= (\cancel{A+B})(A+\bar{B}) + C, \bar{C} \\
 &= (A+B, \bar{B}+C)(A+B, \bar{B}+C, \bar{C}) \frac{(A+B+C)}{(A+B+C)(A+B+C)} \\
 &= (A+B+C)(A+B+C)(A+B+C)(A+\bar{B}+C) \\
 &\quad (A+B+C)(A+B+C)(A+B+C) \\
 &= (A+B+C)(A+\bar{B}+C)(A+B+C)(A+\bar{B}+C)
 \end{aligned}$$

Minterms & Max terms

Each individual term in canonical SOP form is called minterm & each individual term in canonical POS form is called maxterm.

For n variable logical function there are 2^n minterms & equal number of maxterms. For 3 variables/literals
 $m_{int} = maxterm = 2^3 = 8$.

Variables			Minterms	Maxterms
A	B	C	m_0	M_0
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = M_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}BC = M_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = M_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = M_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = M_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = M_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = M_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

With these shorthand notations logical function can be represented as.

$$1. \quad Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C}$$

$$= m_0 + m_1 + m_3 + m_6 = \sum m(0, 1, 3, 6)$$

→ denotes sum of products

$$2. \quad Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

$$= M_1 + M_3 + M_6 = \prod M(1, 3, 6)$$

→ Product of sum.

Karnaugh Map Simplification

- For simplification of boolean expression by boolean algebra we need better understanding of boolean laws, rules & theorems.

- Time consuming process

- map method gives us a systematic approach for simplifying a Boolean expression. The map method first proposed by Veitch and modified by Karnaugh hence it is known as the Veitch diagram or.

Karnaugh map. [Kmap]

2 Variables
(2 cells)

A	B	0	1
0	m_0	m_1	
1	m_2	m_3	

3 Variables (4 cells)

A	B	00	01	11	10
0	m_0	m_1	m_3	m_2	
1	m_4	m_5	m_7	m_6	

4 Variables (16 cells)

AB	CD		00	01	11	10
	00	01	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6		
11	m_{12}	m_{13}	m_{15}	m_{14}		
10	m_8	m_9	m_{11}	m_{10}		

2 Variable K MAP

$$f = \sum m(0, 2, 3)$$

SOP

①

	B	0	1
0	A	$\bar{A}B$	$\bar{A}B$
1	$\bar{A}B$	AB	AB

	B	0	1
0	1	0	0
1	1	1	1

$$= f = \bar{B} + A = A + \bar{B}$$

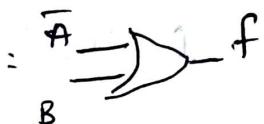
②

$$f = \bar{A}\bar{B} + \bar{A}B + AB \text{ using mapping.}$$

$$f = m_0 + m_1 + m_3 = \sum m(0, 1, 3)$$

	B	0	1
0	1	1	1
1	0	1	1

$$f = \bar{A} + B.$$



③

Mapping Pos form.

$$f = (A+B)(\bar{A}+B)(\bar{A}+\bar{B})$$

$$f = \pi(0, 2, 3)$$

	B	0	1
0	0	01	01
1	D	D	D

$$\bar{f} = \bar{B} + A \Rightarrow \bar{A} \cdot B$$

$$f = \bar{A} + \bar{B}.$$

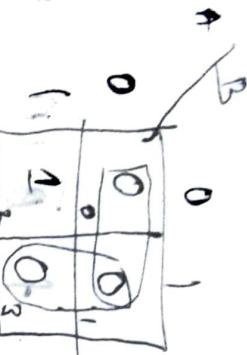
$$= \bar{A} \cdot \bar{B}$$

$$= \bar{A} \cdot B \Rightarrow D$$

Reduce the following expression $f = (A+B)(A+\overline{B})(\overline{A}+C)$

$$= \overline{ABC} + \overline{AC} + \overline{BC}$$

5 variable
product



$$\Rightarrow f = \overline{A} + B$$

$$f = A\overline{B}$$

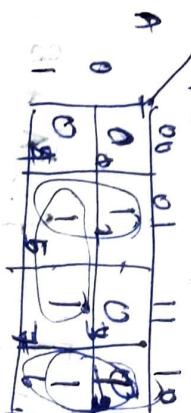
Reduce

(4)

Sum of products

Map the expression

$$f = \overline{A}\overline{B}C + A\overline{B}C + \overline{A}B\overline{C}$$



(5)

Reduce the expression $f = \sum m(0, 2, 3, 4, 5, 6)$

Implement it in A&I logic.

$$f = \sum m(0, 2, 3, 4, 5, 6)$$

$$f_{min} = \overline{I} + \overline{II} + \overline{III}$$

$$\Rightarrow \overline{C} + \overline{AB} + A\overline{B}$$

A

BC

0	00	01	11	10
1	10	11	01	00
1	01	00	11	10
0	11	01	00	11

A

B

C

f

5 Variables 10s

$$F = \overline{NM}(2,3,7,8,9,10,11,12,16,17,18,19,20,21,23,24)$$

Ex.

$$A=0$$

		DE	00	01	11	10
		BC	00	01	11	10
A	B	00	0	0	0	0
		01	0	0	0	0
A	B	11	0	0	0	0
		10	0	0	0	0

$$A=1$$

		DE	00	01	11	10
		BC	00	01	11	10
A	B	00	0	0	0	0
		01	0	0	0	0
A	B	11	0	0	0	0
		10	0	0	0	0

$$m_{2,3,19,18,11,10,24,24} \rightarrow \bar{CD} \Rightarrow (C + \bar{D})$$

$$m_{1,7,20,21} \rightarrow A\bar{B}\bar{D} = (\bar{A} + B + D)$$

$$m_{8,9,11,10} \rightarrow \bar{A}B\bar{C} = (A + \bar{B} + C)$$

$$m_{1,7,10,10} \rightarrow A\bar{B}\bar{D}\bar{E} = (\bar{B} + \bar{D} + \bar{E})$$

$$m_{8,12} \rightarrow \bar{A}\bar{B}\bar{D}\bar{E} = (A + \bar{B} + D + E)$$

$$f = (C + \bar{D})(\bar{A} + B + D)(A + \bar{B} + C)(B + \bar{D} + \bar{E})$$

$$(A + \bar{B} + D + E)$$

Logic diagram

Ex. ① minimize the expression using Kmap.

$$Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

101	001	101	100	011	000
-----	-----	-----	-----	-----	-----

step ① : plot the Kmap according to the given expression.

		BC	00	01	11	10
		A	0	1	1	3
I	II	0	1	1	1	0
		1	1	1	0	0

1. octet 1's
2. quad 1's
3. pair of 1's
4. isolated 1's

step ② no octet & group quad I's

step ③ Group pair of 1's

step ④ no Isolated ones

step ⑤ All 1's grouped

step ⑥ final expression

$$Y = I + II$$

$$= \bar{B} + \bar{A}C$$

$$\boxed{\therefore Y = \bar{B} + \bar{A}C}$$

Ex:2 minimize the expression

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + AB\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$$

0100	0101	1100	1101	1001	+ \bar{A}\bar{B}C\bar{D}
------	------	------	------	------	--------------------------

① plot the kmap

		CD	00	01	11	10
		A	0	0	0	1
I	II	0	0	0	0	1
		1	1	1	0	0

② quad 1's grouped

③ pair 1's grouped

④ Isolated 1's grouped

$$Y = I + II + III$$

$$Y = \bar{B}\bar{C} + A\bar{C}D + \bar{A}\bar{B}\bar{C}D$$

		CD	00	01	11	10
		A	0	0	0	1
I	II	0	0	0	0	1
		1	1	1	0	0

② Reduce the following function using Karnaugh map technique & Implement using basic gates.

$$f(A, B, C, D) = \overline{A} \overline{B} D + A \overline{B} \overline{C} \overline{D} + \overline{A} B D + A B$$

Note: The given function is not in the standard SOP form. So it's converted into SOP form

$$\begin{aligned} f(A, B, C, D) &= \overline{A} \overline{B} D (C + \overline{C}) + A B \overline{C} \overline{D} + \overline{A} B D (C + \overline{C}) \\ &= \overline{A} \overline{B} C D + \overline{A} \overline{B} \overline{C} \overline{D} + A B \overline{C} \overline{D} + \overline{A} B C D \\ &\quad + \overline{A} B \overline{C} D + A B C \overline{D} \end{aligned}$$

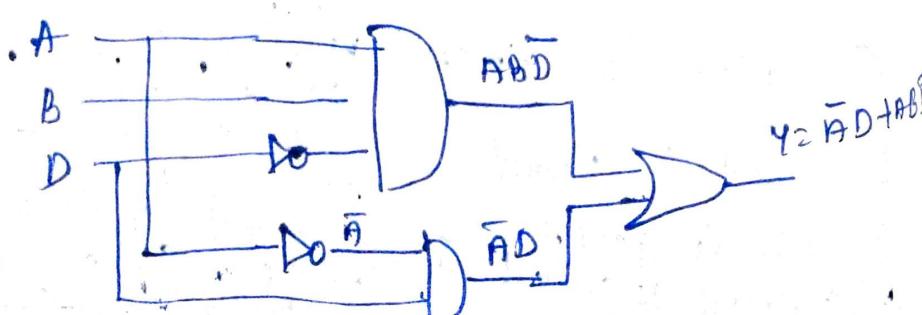
② Plot the Kmap

		CD		Z			
		00	01	11	10	00	01
AB	00	0	1	1	0	0	0
	01	0	1	1	0	0	0
11	1	0	0	0	1	1	1
10	0	0	0	0	0	0	0

$$Y = \underline{\text{I}} + \underline{\text{II}}$$

$$= \overline{A} D + A B \overline{D}$$

Implementation



Reduce the following function, using K-map technique

$$f(A, B, C, D) = \sum m(0, 1, 4, 8, 9, 10)$$

① plot the Kmap

		CD		I	I	
		00	01	11	10	
AB		00	1 1	0 3	0 2	
II	II	01	1 4	0 5	0 7	0 6
III	III	11	1 12	1 13	1 15	1 14
IV	IV	10	1 8	1 9	1 11	1 10

$$Y = I + II + III$$

$$f(A, B, C, D) = \overline{B} \overline{C} + \overline{A} \overline{C} \overline{D} + A \overline{B} \overline{D}$$

Don't care conditions

$$f_C \cup f_A$$

$$f(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + \sum d(6, 14)$$

		CD		00	01	11	10
		00	01	X	1	1	X
AB		00	X				
01	01	01					
11	11	11					
10	10	10					

		CD		00	01	11	10
		00	01	1	1	1	1
AB		00	X				
01	01	01					
11	11	11					
10	10	10					

$$y = \overline{A} \overline{B} + CD$$

$$\textcircled{2} \quad f(A, B, C) = \sum m(0, 1, 3, 7) + \sum d(2, 5)$$

A	B	C	Y	
0	00	01	11	10
1	01	1	1	X
1	0	X	1	0

A	B	C	Y	
0	00	01	11	10
1	1	1	1	1

\textcircled{1}

$$f(A, B, C) = \bar{A} + C$$

\textcircled{ii}

\textcircled{2} Find the reduced SOP form of following function.

$$F(w, x, y, z) = \sum m(0, 1, 8, 9, 10, 12) + \sum d(2, 5, 13)$$

w x	y	z		
00	00	01	11	10
1	00	01	11	X

w x	y	z		
00	00	01	11	10
01	0	(X)	1	0

w x	y	z		
00	00	01	11	10
11	1	X	0	0

w x	y	z		
00	00	01	11	10
10	1	1	0	1

$$Y_2 = \sum + \sum + \sum$$

$$= \overline{BD} + \overline{AC} + \overline{BCD} = \overline{xz} + \overline{w}\overline{y} + \overline{w}xz$$

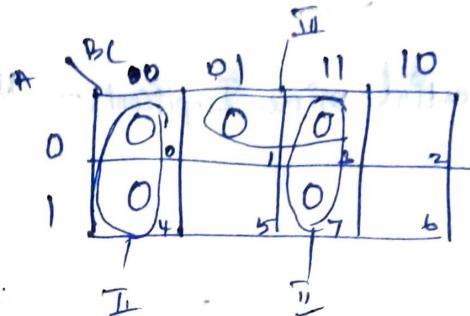
Simplification of Product of Sums (POS) Expressions

$$\textcircled{1} \quad Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(A+B+C)$$

M_1 M_3 M_7 M_4 M_0

i) Plot the K-map

ii) Group the 0's



$$\bar{Y} = \underline{\text{I}} + \underline{\text{II}} + \underline{\text{III}}$$

$$= \bar{B}\bar{C} + BC + \bar{A}C$$

$$Y = \overline{\bar{B}\bar{C} + BC + \bar{A}C}$$

$$\text{DeMorgan's law} \quad Y = (\overline{B}\overline{C}) \cdot (\overline{B}\overline{C}) \cdot (\overline{A}\overline{C})$$

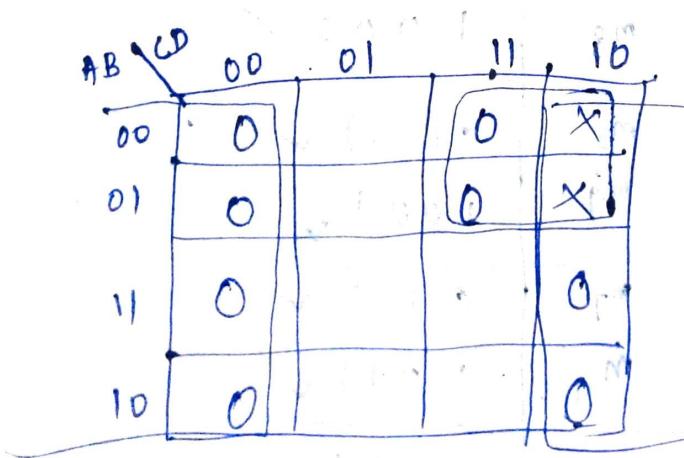
$$= (\overline{B} + \overline{C}) \cdot (\overline{B} + \overline{C}) \cdot (A + \overline{C})$$

$$= (\overline{B} + \overline{C}) \cdot (\overline{B} + \overline{C}) \cdot (A + \overline{C})$$

$$\textcircled{2} \quad f(A, B, C, D) = \pi M(0, 2, 3, 8, 9, 12, 13, 15)$$

$$f = (A+\bar{B}+\bar{D})(A+B+\bar{C})(\bar{A}+C)(A+B+D)$$

$$\textcircled{2} \quad f(A, B, C, D) = \pi M(0, 3, 4, 7, 8, 10, 12, 14) + \sum d(2, 6)$$



$$\begin{aligned} f &= \overline{D} + \overline{A}\overline{C} \\ &= D \cdot (\overline{A} + \overline{C}) \\ f &= (A + \overline{C})D \end{aligned}$$