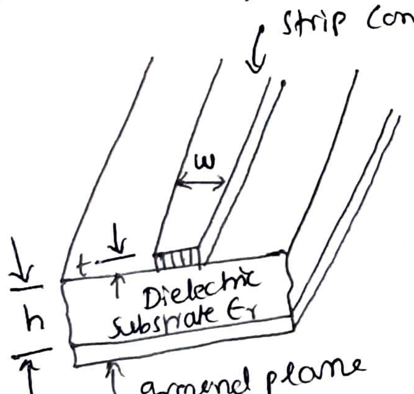
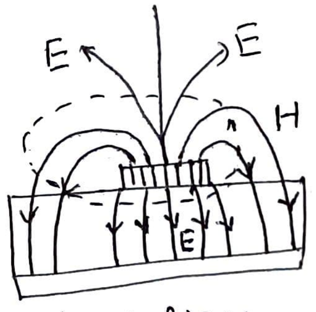
Module - 3

Strip lines

In recent years with the introduction of monolithic microwave integrated circuits (MMICs) microstrip lines and coplanar strip lines have been used extensively, because they provide one free and accessible surface on which solid state devices can be placed. Modes on microstrip lines are only quasi-transverse electric and magnetic (TEM). Radiation loss in microstrip lines is a problem, particularly at such discontinuities as short circuit posts, corners etc. The use of thin, high-dielectric materials considerably reduces the radiation loss of the open strip. A microstrip line has better interconnection features and easier fabrication.



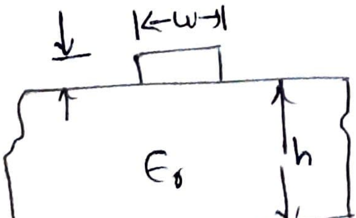
ⓐ Schematic diagram



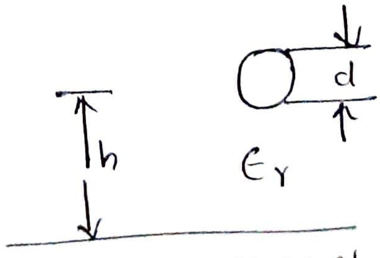
ⓑ field lines

Characteristic Impedance of microstrip lines:

microstrip lines are used to interconnect high speed logic circuits in digital computers. The cross sections of microstrip line and wire-over-ground line are shown below:



ⓐ cross section of microstrip line



ⓐ cross sections of a wire-over-ground line

The characteristic impedance of a microstripline is a function of strip-line width, the strip line thickness, the distance between the line and the ground plane, and the homogeneous dielectric constant of the board material.

The characteristic impedance of a wire-over-ground transmission line is given by:

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{4h}{d}, \quad \text{for } h >> d$$

ϵ_r - dielectric constant of the ambient medium

h - the hight from the center of the wire to the ground plane

d - diameter of the wire.

Effective dielectric constant ϵ_{re}: For a homogeneous dielectric medium, the propagation-delay time per unit length is:

$$T_d = \sqrt{\mu \epsilon}$$

where μ is the permeability of the medium and ϵ is the permittivity of the medium

In free space, propagation delay time is:

$$T_{df} = \sqrt{\mu_0 \epsilon_0} = 3.333 \text{ ns/m or } 1.016 \text{ ns/ft}$$

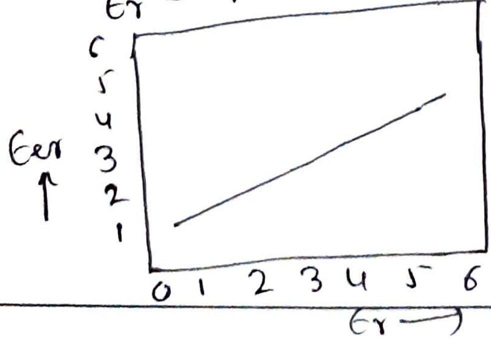
generally $\mu_r = 1 \quad \therefore T_d = 1.106\sqrt{\epsilon_r}$ ns/ft

The effective dielectric constant for a microstrip line can be related to the relative dielectric constant of the board material.

$$\epsilon_{re} = 0.475\epsilon_r + 0.67$$

ϵ_{re} - effective ~~die~~ relative dielectric constant for a microstrip line

ϵ_r - relative dielectric constant of board material.



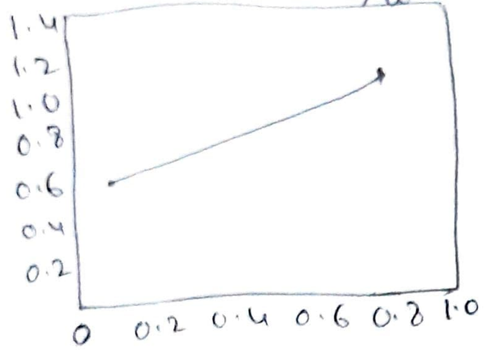
Transformation of a rectangular conductor into an equivalent Circular Conductor

The transformation equation is:

$$d = 0.67w\left(0.8 + \frac{t}{w}\right)$$

d - diameter of the wire over ground
w - width of the microstrip line
t - thickness of the microstrip line

$$0.1 < t/w < 0.8$$



characteristic impedance equation:

$$Z_0 = \frac{60}{\sqrt{\epsilon_{re}}} \ln \frac{4h}{d}$$

$$= \frac{60}{\sqrt{0.475\epsilon_r + 0.67}} \ln \left[\frac{4h}{0.67w\left(0.8+\frac{t}{w}\right)}\right]$$

$$= \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left[\frac{5.98h}{0.8w + t}\right] \quad \text{for } h < 0.8w$$

ϵ_r – relative permittivity of board material
h - height from the microstrip line to the ground
w - width of the microstrip line
t - thickness of the microstrip line

velocity of propagation is: $v = \frac{c}{\sqrt{\epsilon_{re}}} = \frac{3 \times 10^8}{\sqrt{\epsilon_{re}}}$ m/s

The characteristic impedance for a wide microstrip line was derived ~~by Ass~~ as:

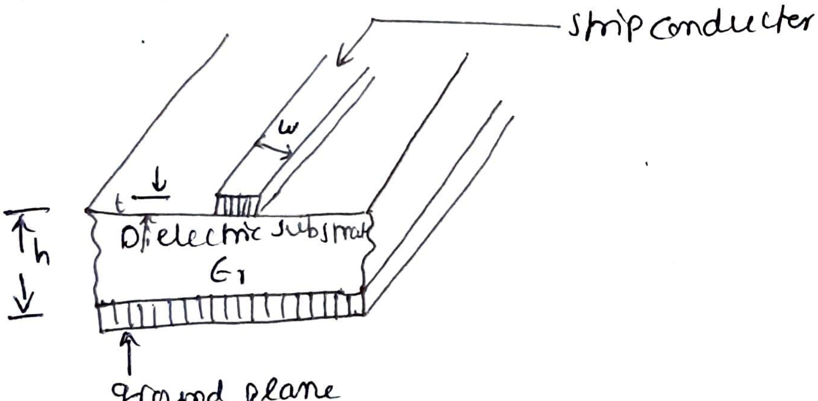
$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}}\frac{h}{w} \quad \text{for } (w>>h)$$

Eg: A certain microstrip line has $\epsilon_r = 5.23$, $h = 7$ mils, $t = 2.8$ mils, $w = 10$ mils Calculate the characteristic impedance Z_0 of the line

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right] = 45.78\,\Omega$$

Losses in microstrip lines:

Microstrip transmission lines consisting of a conductive ribbon attached to a dielectric sheet with conductive backing are widely used in both microwave and computer technology.



For a nonmagnetic dielectric substrate, two types of losses occur in the dorminant microstrip mode: i) dielectric loss in the substrate and ii) ohmic skin loss in the strip conductor and the ground plane. The sum of these two losses may be expressed as losses per unit length in terms of an attenuation factor α. Power carried by a wave traveling in the positive z-direction is given by,

$$P = \frac{1}{2}VI^* = \frac{1}{2}\left(V_+ e^{-\alpha z} I_+ e^{-\alpha z}\right) = \frac{1}{2}\frac{|V_+|^2}{Z_0} e^{-2\alpha z} = P_0 e^{-2\alpha z}$$

where $P_0 = |V_+|^2/(2Z_0)$ is the power at $z = 0$

The attenuation constant α can be expressed as

$$\alpha = \frac{-dP/dz}{2P(z)} = \alpha_d + \alpha_c$$

where α_d is the dielectric attenuation constant and α_c is the ohmic attenuation constant.

$$-\frac{dP(z)}{dz} = -\frac{d}{dz}\left(\frac{1}{2}VI^*\right)$$

$$= \frac{1}{2}\left[-\frac{dV}{dz}\right]I^* + \frac{1}{2}\left[-\frac{dI^*}{dz}\right]V$$

$$= \frac{1}{2}(RI)I^* + \frac{1}{2}\sigma V^*V$$

$$= \frac{1}{2}|I|^2R + \frac{1}{2}|V|^2\sigma = P_c + P_d$$

where σ is the conductivity of the dielectric substrate board.

$$\alpha_d + \alpha_c = \frac{P_c + P_d}{2P(z)} \Rightarrow \alpha_d = \frac{P_d}{2P(z)} \text{ NP/cm}$$

$$\alpha_c = \frac{P_c}{2P(z)} \text{ NP/cm}$$

Dielectric losses: when conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. The dielectric attenuation constant is given by,

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \text{ NP/cm}$$

where σ is the conductivity of the dielectric substrate board in $\mho$/cm
This can be expressed in terms of dielectric loss tangent as:

$$\tan\theta = \frac{\sigma}{\omega\epsilon}$$

∴ the dielectric attenuation constant is expressed by,

$$\alpha_d = \frac{\omega}{2}\sqrt{\mu\epsilon}\tan\theta \text{ NP/cm}$$

$$= 1.634 \times 10^3 \frac{q_\epsilon}{\sqrt{\epsilon_{re}}} \text{ dB/cm}$$

1 NP = 8.686 dB

&

q denotes the dielectric filling factor, defined by:

$$q = \frac{\epsilon_{re} - 1}{\epsilon_r - 1}$$

attenuation constant / wavelength can be expressed as:

$$\alpha_d = 27.3 \left(\frac{q\epsilon_r}{\epsilon_{re}} \right) \frac{\tan\theta}{\lambda_g} \; dB/\lambda_g$$

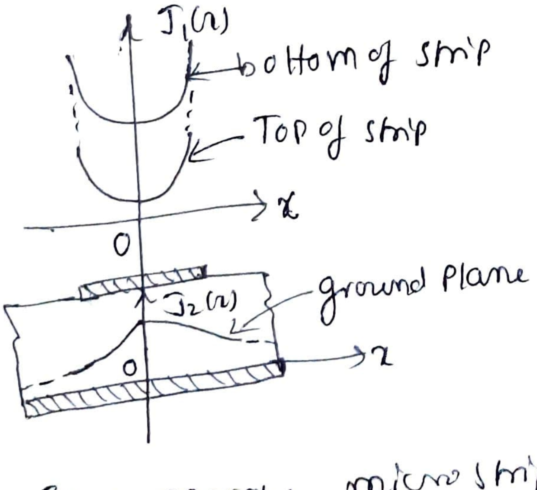
where $\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{re}}}$ and λ_0 is the wavelength in free space, or

$$\lambda_g = \frac{c}{f\sqrt{\epsilon_{re}}} \quad c\text{ - velocity of light in vaccum}$$

If $\tan\theta$ is independent of frequency, dielectric attenuation / λ is also independent of frequency.

<u>ohmic losses</u>: The current density in the conductors of a micro strip line is concentrated in a sheet that is approximately a skin depth thick inside the conductor surface and exposed to the electric field. The conducting attenuation constant of a wide microstrip line is given by

$$\alpha_c \approx \frac{8.686 R_s}{Z_0 W} dB/cm, \text{ for } \frac{w}{h} > 1$$



where $R_s = \sqrt{\frac{\pi f \mu}{\sigma}}$ is skin resistance in Ω/square

$R_s = \frac{1}{\sigma\delta}$ is Ω/square

$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ is skin depth in cm

For a narrow microstrip line with $w/h > 1$, the following formulas are derived.

where α_d - dielectric attenuation constant &

α_c - ohmic attenuation constant

$$\frac{\alpha_c Z_0 h}{R_s} = \frac{8.68}{2\pi}\left[1-\left(\frac{w'}{4h}\right)^2\right]\left[1+\frac{h}{w'}+\frac{h}{\pi w'}\left(\ln\frac{4\pi w}{t}+\frac{t}{w}\right)\right]$$

for $\frac{w}{h} \leq \frac{1}{2\pi}$

$$\frac{\alpha_c Z_0 h}{R_s} = \frac{8.68}{2\pi}\left[1-\left(\frac{w'}{4h}\right)^2\right]\left[1+\frac{h}{w'}+\frac{h}{w'}\left(\ln\frac{2h}{t}-\frac{t}{h}\right)\right]$$

for $\frac{1}{2\pi} < \frac{w}{h} \leq 2$

and

$$\frac{\alpha_c Z_0 h}{R_s} = \frac{8.68}{\left\{\frac{w'}{h}+\frac{2}{\pi}\ln\left[2\pi e\left(\frac{w'}{2h}+0.94\right)\right]\right\}^2}\left[\frac{w'}{h}+\frac{w'/\pi h}{\frac{w'}{2h}+0.94}\right]$$

$$\times\left[1+\frac{h}{w'}+\frac{h}{\pi w'}\left(\ln\frac{2h}{t}-\frac{t}{h}\right)\right] \quad \text{for } 2 \leq \frac{w}{h}$$

where α_c is expressed in dB/cm and

$e = 2.718$, $w' = w + \Delta w$, $\Delta w = \frac{t}{\pi}\left(\ln\frac{4\pi w}{t}+1\right)$ for $\frac{2t}{h} < \frac{w}{h} \leq \frac{\pi}{2}$

$$\Delta w = \frac{t}{\pi}\left[\ln\frac{2h}{t}+1\right] \quad \text{for } \frac{w}{h} \geq \frac{\pi}{2}$$

Radiation losses: This loss depends on the substrate's thickness and dielectric constant, as well as geometry. The ratio of radiated power to total dissipated power for an open-circuited microstrip line is:

$$\frac{P_{rad}}{P_t} = 240\pi^2\left(\frac{h}{\lambda_0}\right)^2\frac{F(\epsilon_{re})}{Z_0}$$

where $F(\epsilon_{re})$ is given by

$$F(\epsilon_{re}) = \frac{\epsilon_{re}+1}{\epsilon_{re}} - \frac{\epsilon_{re}-1}{2\epsilon_{re}\sqrt{\epsilon_{re}}} \ln \frac{\sqrt{\epsilon_{re}}+1}{\sqrt{\epsilon_{re}}-1}$$

where ϵ_{re} is the effective dielectric constant and $\lambda_0 = \frac{c}{f}$ is the free space wave length.

The radiation factor decreases with increasing substrate dielectric constant. $\therefore \frac{P_{rad}}{P_t} = \frac{R_r}{Z_0}$

where R_r is the radiation resistance of an open circuited microstrip and is given by,

$$R_r = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 F(\epsilon_{re})$$

For lower dielectric constant substrates, radiation is significant at higher impedance levels. For higher dielectric constant substrates, radiation becomes significant until very low impedance levels are reached.

Quality factor Q of microstrip lines: The quality factor of a microstrip line is very high, but it is limited by the radiation losses of the substrates and with low dielectric constant.

$$\alpha_c = \frac{8.686 R_s}{Z_0 W} \text{ dB/cm}, \quad Z_0 = \frac{h}{W}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}} \frac{h}{W} \ \Omega$$

The wavelength in the microstrip line is:

$$\lambda_g = \frac{30}{f\sqrt{\epsilon_r}} \text{ cm}, \text{ where } f \text{ is the frequency in GHz}$$

Q_c is related to the conductor attenuation constant by,

$$Q_c = \frac{27.3}{\alpha_c}$$

Q_c of a wide microstrip line is expressed as

$$Q_c = 39.5\left(\frac{h}{R_s}\right) f_{GHz}$$

Here h is measured in cm and $R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 2\pi\sqrt{\frac{f_{GHz}}{\sigma}}$ Ω/square

Finally, quality factor Q_c of a wide microstrip line is:

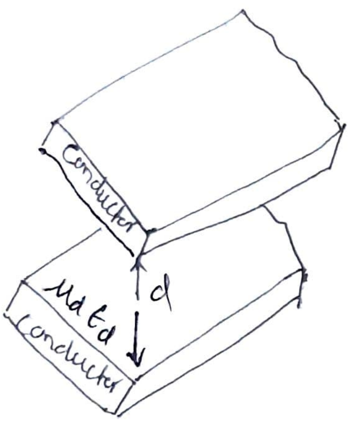
$Q_c = 0.63h\sqrt{\sigma f_{GHz}}$ σ is conductivity in ℧/m

$Q_d = \frac{27.3}{\alpha_d}$, $Q_d \rightarrow$ quality factor, α_d is in dB/λ_g

$$Q_d = \frac{\lambda_0}{\sqrt{\epsilon_{re}}\tan\theta} \approx \frac{1}{\tan\theta}$$

Parallel strip lines:

A parallel strip line consists of two perfectly parallel strips separated by a perfect dielectric slab of uniform thickness.



Distributed parameters: A parallel strip line is similar to a two conductor transmission line, so it can support a quasi-TEM mode. The inductance along the two conducting strips can be written as, $L = \mu_c d/w$ H/m.

Capacitance $C = \frac{\epsilon_d w}{d}$ F/m

The series resistance of both strips is given by

$$R = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\ \Omega/m$$

where $R_s = \sqrt{\pi f \mu_c/\sigma_c}$ is the conductor surface resistance in Ω/square and σ_c is the conductor conductivity in ℧/m.

The shunt conductance of the strip line is:

$$G = \frac{\sigma_d w}{d} \ \mho/m$$, where σ_d is the conductivity of the dielectric substrate.

Characteristic impedance:

The characteristic impedance of a lossless parallel strip line is:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w}\sqrt{\frac{\mu_d}{\epsilon_d}} = \frac{377}{\sqrt{\epsilon_{rd}}}\frac{d}{w}, \text{ for } w >> d$$

The phase velocity along a parallel strip line is:

$$v_p = \frac{w}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_d}} = \frac{c}{\sqrt{\epsilon_{rd}}} \ m/s \text{ for } \mu_c = \mu_0$$

The characteristic impedance of a lossy parallel strip line at microwave frequencies ($R << wL$ and $G << wC$) can be approximated as:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\epsilon_{rd}}}\frac{d}{w} \quad \text{for } w >> d$$

Attenuation losses: The propagation constant of a parallel strip line at microwave frequencies can be expressed by,

$$\gamma = \sqrt{(R+jwL)(G+jwC)} \quad \text{for } R << wL \text{ and } G << wC$$

$$= \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + jw\sqrt{LC}$$

attenuation constant $\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$ Np/m

phase constant $\beta = w\sqrt{LC}$ rad/m

$$\alpha_c = \frac{1}{2}R\sqrt{\frac{C}{L}} = \frac{1}{d}\sqrt{\frac{\pi f \epsilon_d}{\sigma_c}} \ Np/m$$

$$\alpha_d = \frac{1}{2}G\sqrt{\frac{L}{C}} = \frac{188\sigma_d}{\sqrt{\epsilon_{rd}}} \ Np/m$$

Coplanar strip lines:

A coplanar strip line consists of two conducting strips on one substrate surface with one strip grounded. Its two strips are on the same substrate surface for convenient connections. They eliminate the difficulties involved in connecting the shunt elements between the hot and ground strips.

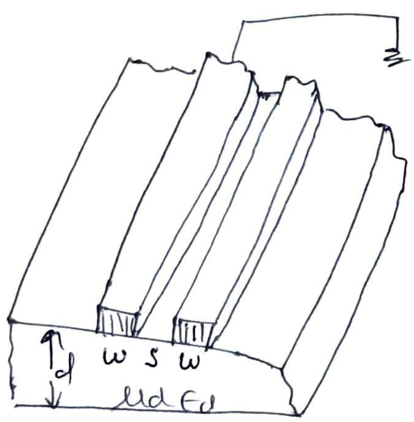
The characteristic impedance of a coplanar strip line is:

$$Z_0 = \frac{2 P_{avg}}{I_0^2}$$

, where I_0 is the peak current in one strip and P_{avg} is the average power flowing in the positive z direction.

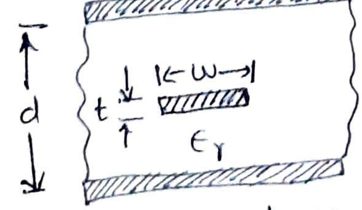
The average flowing power can be expressed as:

$$P_{avg} = \frac{1}{2} Re \iint (E \times H^*) \cdot u_z \, dx \, dy$$



Shielded strip Lines:

A partially shielded strip line has its strip conductor embedded in a dielectric medium, and its top and bottom ground planes have no connection, as shown below:



partially shielded strip line

The characteristic impedance for a wide strip ($w/d >> 0.35$) is

$$Z_0 = \frac{94.15}{\sqrt{\epsilon_r}} \left(\frac{w}{d} K + \frac{C_f}{8.854 \epsilon_r} \right)^{-1}$$

where $K = \dfrac{1}{1 - t/d}$, t = the strip thickness

d = the distance between the two ground planes

$$C_f = \frac{8.854 \epsilon_r}{\pi} \left[2K \ln(K+1) - (K-1) \ln(K^2 - 1) \right] \text{ pF/m}$$

Ex:

A lossless parallel strip line has a conducting strip width w. The substrate dielectric separating the two conducting strips has a relative dielectric constant ϵ_{rd} of 6 and a thickness d of 4 mm.

Calculate i) The required width w of the conducting strip in order to have a characteristic impedance of 50 Ω.

ii) The strip-line capacitance iii) The strip line inductance

iv) The phase velocity of the wave in the parallel strip line.

$$w = \frac{377}{\sqrt{\epsilon_{rd}}} \frac{d}{Z_0} = \frac{377}{\sqrt{6}} \cdot \frac{4 * 10^{-3}}{50} = 12.31 * 10^{-3} \text{ m}$$

The stripline capacitance is:

$$C = \frac{\epsilon_d w}{d} = \frac{8.854 * 10^{-12} * 6 * 12.31 * 10^{-3}}{4 * 10^{-3}} = 163.50 \text{ pF/m}$$

The strip-line inductance is:

$$L = \frac{\mu_c d}{w} = \frac{4\pi * 10^{-7} * 4 * 10^{-3}}{12.31 * 10^{-3}} = 0.41 \mu H/m$$

The phase velocity is:

$$V_p = \frac{c}{\sqrt{\epsilon_{rd}}} = \frac{3 * 10^8}{\sqrt{6}} = 1.22 * 10^8 \text{ m/s}$$