Microwave Tubes

Introduction :

For extremely high frequency applications above 1GHz the inter electrode capacitances and transit time delays of standard electron tube construction become prohibitive. In the microwave tubes commonly known as klystrons, magnetrons and Travelling wave tubes (TWT), which differ from conventional electronic vaccum tubes, in that transit time is utilised for micro wave oscillations or amplification. The principle uses an electron beam on which space charge waves interact with electromagnetic fields in the microwave cavities to transfer energy to the output circuit of cavity, or interact with the electromagnetic fields in a slow-wave structure to give amplification through transfer of energy.

Klystrons and TWTs are linear beam or 'o'- type tubes in which the accelerating electric field is in the same direction as the static magnetic field used to focus the electron beam. magnetrons are crossed field devices (m-type) where the static magnetic field is perpendicular to the electric field.

Klystrons:

There are two basic configurations of klystron tubes, i) reflex klystron used as a low power microwave oscillator ii) multi cavity klystron used as a low power microwave amplifier.

Reflex klystron Oscillator:

The schematic configuration of a reflex klystron is indicat below: It uses only a single re-entrant microwave cavity as resonator. The electron beam emitted from the cathode K is accelerated by the grid G and passes through the cavity anode A to the repeller space between the cavity anode and the repeller electrode.

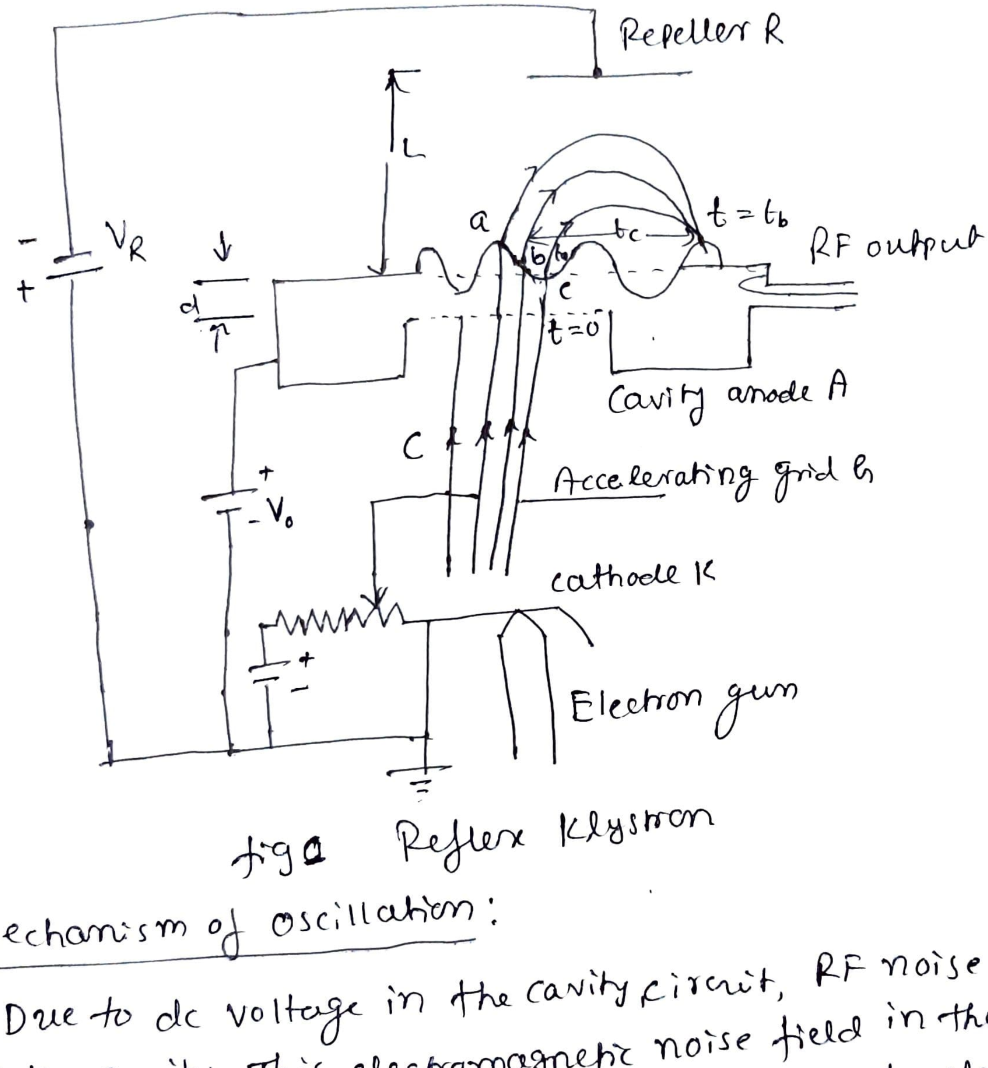


fig a Reflex Klystron

Mechanism of oscillation:

Due to dc voltage in the cavity circuit, RF noise is generated in the cavity. This electromagnetic noise field in the cavity becomes pronounced at cavity resonant frequency. The electrons passing through the cavity gap d experience this RF field and are velocity modulated. The electrons a shown in figure 2 which encountered the positive half cycle of the RF field in the cavity gap d will be accelerated, those reference electrons b which encountered zero RF field will pass with unchanged original velocity, and the electrons c which encountered the negative half cycle will be retarded on entering the repeller space.

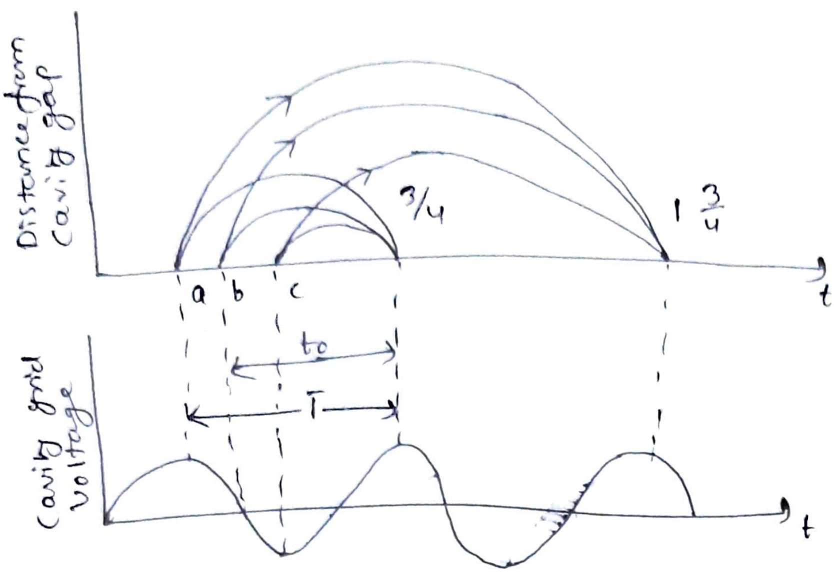


fig. 2 Reflex klystron modes

All these velocity modulated electrons will be repelled back to the cavity by the repeller due to its negative potential. The repeller distance L and the voltages can be adjusted to receive all the velocity modulated electrons at a same time on the positive peak of the cavity RF voltage cycle. Thus the velocity modulated electrons are bunched together and lose their kinetic energy when they encounter the positive cycle of the cavity RF field. This loss of energy is transferred to the cavity to conserve the total power.

If the power delivered by the bunched electrons to the cavity is greater than the power loss in the cavity, the electro magnetic field amplitude at the resonant frequency of the cavity will increase to produce microwave oscillations. The RF power is coupled to the output load by means of a small loop which forms the center conductor of the coaxial line. When the power delivered by the electrons becomes equal to the total power loss in the cavity system, a steady microwave oscillation is generated at resonant frequency of the cavity.

Mode of oscillation:

The bunched electrons in reflex klystron can deliver maximum power to the cavity at any instant, which corresponds to the positive peak of the RF cycle of the cavity oscillations.

If T is the time period at the resonant frequency, t_0 is the time taken by the reference electron to travel in the repeller space between entering the repeller space at b and the returning to the cavity at positive peak voltage on formation of the bunch, then

$$t_0 = (n + 3/4)T = NT$$

where $N = n + 3/4$, $n = 0, 1, 2, \ldots$

By adjusting repeller voltage for a given dimensions of the reflex klystron, the bunching can be made to occur at $N = n + 3/4$ positive half cycle. Accordingly the mode of oscillations is named as $N = 3/4, 1\,3/4, 2\,3/4 \ldots$ for modes $n = 0, 1, 2, \ldots$

The lowest order mode $3/4$ occurs for a maximum value of repeller voltage when the transit time t_0 of the electrons in the repeller space is minimum. Higher modes occur at lower repeller voltages. Since at the highest repeller voltage the acceleration of the bunched electrons on return is maximum the power output of the lowest mode is maximum.

Mode curve:

The output power and frequency can be electronically controlled by varying the repeller voltage, The RF power is given by,

$$P_{RF} = \frac{0.3986\, V_0 I_0 (V_0 + V_R)}{2fL} \sqrt{\frac{e}{2mV_0}}$$

Operation frequency

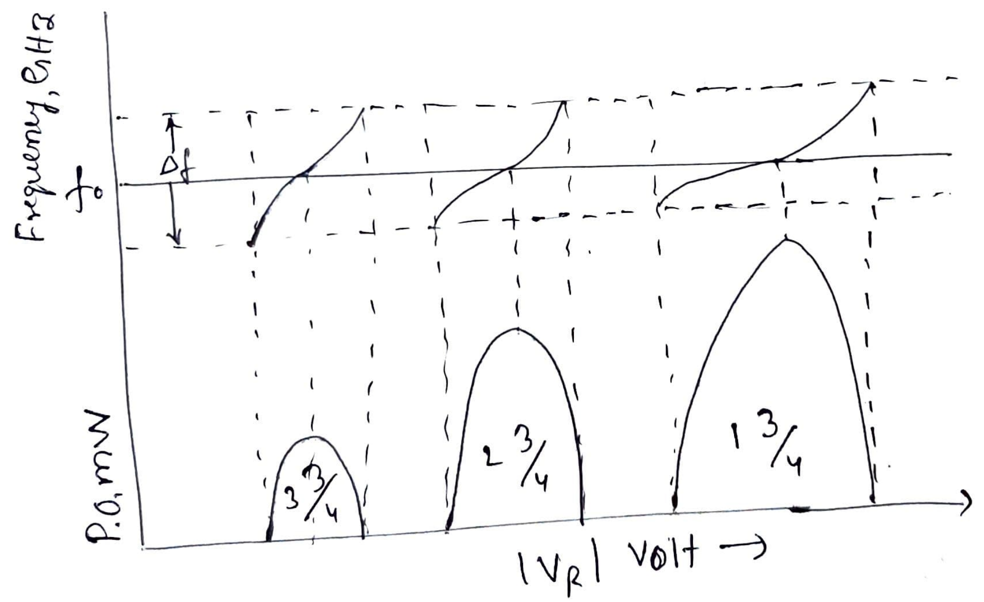
$$f_{MHz} = \frac{(V_0 + V_R)N}{L_{cm}\sqrt{V_0} * 6.74 * 10^{-2}}$$

and

$$|V_R| = \sqrt{\frac{8m}{e}} \left(\frac{fL}{N}\right) \cdot \sqrt{V_0} - V_0$$

$$= 6.74375 * 10^{-6} \frac{fL}{N} \sqrt{V_0} - V_0$$

The variation of output power and frequency with repeller voltage for different modes are shown in fig 3.



Reflex klystron mode curves

problems:

1. A Reflex klystron is to be operated at frequency of 10GHz, with dc beam voltage 300V. Repeller space 0.1cm for $1\frac{3}{4}$ mode. Calculate P_{RFmax} and corresponding repeller voltage for a beam current of 20mA.

$$P_{RFmax} = \frac{0.398 V_0 I_0}{N} = \frac{0.398 * 300 * 20 * 10^{-3}}{1\frac{3}{4}} = 1.365W$$

$$|V_R| = 6.74 * 10^{-6} f_{(Hz)} L_{(m)} \sqrt{V_0}/N - V_0$$

$L = 10^{-3} m, \quad N = 1\frac{3}{4} = 1.75m$

$$\therefore |V_R| = 6.74 * 10^{-6} * 10 * 10^{9} * 10^{-3} * \sqrt{300}/1.75 - 300$$
$$= -367.08V$$

2) A reflex klystron is operated at 5GHz with dc beam voltage 350V, repeller spacing 0.5cms for $N = 3\frac{3}{4}$ mode. Calculate bandwidth over $\Delta V_R = 1V$.

$N = 3\frac{3}{4} = 3.75,$ $\quad \Delta V_R = 6.7438 * 10^{-6} * L_m * \Delta f_{Hz} \sqrt{V_0}/N$

$$1 = 6.74 * 10^{-6} * 0.5/100 * \Delta f_{Hz} * \sqrt{350} * 4/15$$

$$\therefore \Delta f = 5.948 MHz$$

Microwave Frequencies: The term microwave frequencies is generally used for those wavelengths measured in cm, (1 - 300 GHz). However microwave really indicates the wavelengths in the micron ranges. This means microwave frequencies are upto infrared and visible light regions In this revision, microwave frequencies refer to those from 1 GHz upto 10^6 GHz. The IEEE recommended new microwave band designations are indicated in table 1.

Table 1: IEEE microwave frequency Bands

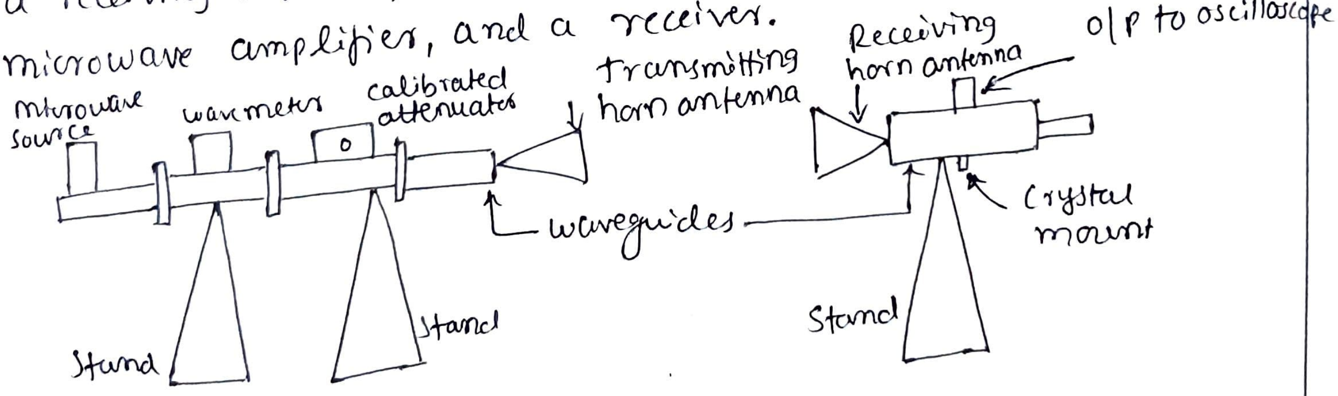
Designation	Frequency range in GHz
HF	0.003 - 0.03
VHF	0.03 - 0.3
UHF	0.3 - 1.0
L band	1.0 - 2.0
S band	2.0 - 4.0
C band	4.0 - 8.0
X band	8.0 - 12.0
Ku band	12.0 - 18.0
K band	18.0 - 27.0
Ka band	27.0 - 40.0
Millimeter	40.0 - 300.0
Submillimeter	> 300.0

Microwave Devices:

Microwave generation and amplification were accompalished by means of velocity modulation theory. Microwave Solid State devices such as tunnel diodes, Gunn diodes, transferred electron devices (TEDs), avalanche transit time devices have been developed to perform microwave generation and amplification. The common characteristic of all microwave solid state devices is the negative resistance that can be used for microwave oscillation and amplification.

Microwave Systems:

A microwave system normally consists of a transmitter sub system, including a microwave oscillator, waveguides, and a transmitting antenna and a receiver subsystem that includes a receiving antenna, transmission line or ~~gui~~ waveguide, a microwave amplifier, and a receiver.

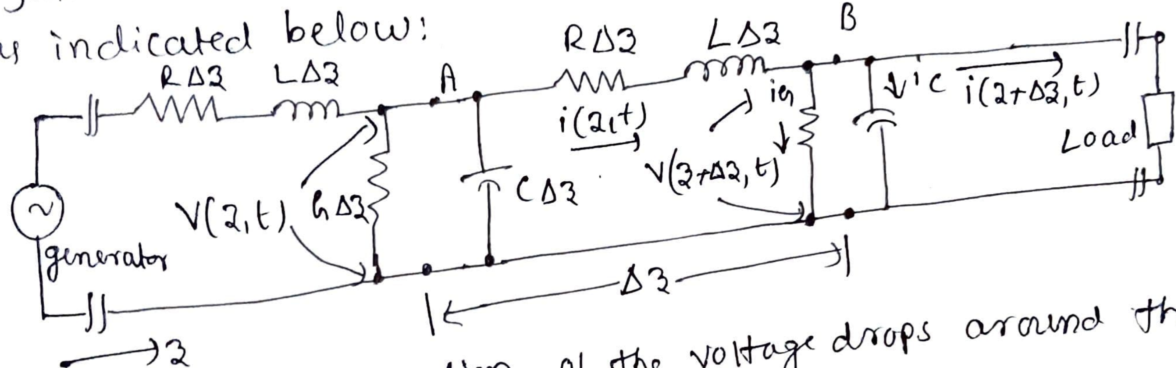


Microwave System

Transmission line equations and Solutions:

Transmission line equations:

A transmission line is a mechanism of guiding electrical energy from one place to another. In communication systems, it is used for conveying RF Power from transmitter to antenna, from antenna to the receiver. In the microwave frequeny range, the circuit constant of a line are distributed throughout the transmission line. Due to this, the voltage drops across each series increment of the line. A small incremental length Δz of a transmission line can be represented by an equivalent symmetrical T-network with constant line parameters R, L, C, G as indicated below:



By KVL, the summation of the voltage drops around the central loop is given by

$$v(z,t) = i(z,t)\,R\Delta z + L\Delta z\,\frac{\partial i(z,t)}{\partial t} + v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z \quad -①$$

replacing $i(z,t)$ by i and $v(z,t)$ by v we get

$$0 = iR\Delta z + L\Delta z\frac{\partial i}{\partial t} + \frac{\partial v}{\partial z}\Delta z \quad - 1a \quad \div \text{ by } \Delta z$$

$$\therefore -\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t} \quad - ②$$

Using KCL the summation of the currents at B in the above figure is expressed as,

$$i(z,t) = v(z+\Delta z,t)\,G\Delta z + C\Delta z\,\frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z,t)$$

$$= \left[v(z,t)+\frac{\partial v(z,t)}{\partial z}\Delta z\right]G\Delta z + C\Delta z\,\frac{\partial}{\partial t}\left[v(z,t)+\frac{\partial v(z,t)}{\partial z}\Delta z\right]$$

$$+\, i(z,t) + \frac{\partial i(z,t)}{\partial z}\Delta z \quad - (3)$$

dropping the subscript (z,t) we get

$$0 = \left[v+\frac{\partial v}{\partial z}\Delta z\right]G\Delta z + C\Delta z\,\frac{\partial}{\partial t}\left[v+\frac{\partial v}{\partial z}\Delta z\right] + \frac{\partial i}{\partial z}\Delta z$$

$\div$ by Δz

$$0 = \left[v+\frac{\partial v}{\partial z}\Delta z\right]G + C\frac{\partial}{\partial t}\left[v+\frac{\partial v}{\partial z}\Delta z\right] + \frac{\partial i}{\partial z}$$

$$-\frac{\partial i}{\partial z} = Gv + \frac{\partial v}{\partial z}\Delta z\,G + C\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t\,\partial z}\Delta z$$

assuming that $\Delta z \to 0$

$$-\frac{\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t} \quad - (4)$$

differentiating eqn (2) we get (w.r.t. z)

$$-\frac{\partial^2 v}{\partial z^2} = R\frac{\partial i}{\partial z} + L\frac{\partial^2 i}{\partial z\,\partial t}$$

and differentiating (4) w.r.t. t we get

$$-\frac{\partial^2 i}{\partial z\,\partial t} = G\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t^2}$$

combining the above equations we get

$$-\frac{\partial^2 v}{\partial z^2} = R\left(-Gv - C\frac{\partial v}{\partial t}\right) + L\left(-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right)$$

$$\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2} \quad - (5)$$

Now considering differentiation of (2) w.r.t. t

$$-\frac{\partial^2 v}{\partial z \partial t} = R\frac{\partial i}{\partial t} + L\frac{\partial^2 i}{\partial t^2}$$

and differentiating (4) w.r.t. z

$$-\frac{\partial^2 i}{\partial z^2} = G\frac{\partial v}{\partial z} + C\frac{\partial^2 v}{\partial z \partial t}$$

$$\therefore -\frac{\partial^2 i}{\partial z^2} = G\left(-Ri - L\frac{\partial i}{\partial t}\right) + C\left(-R\frac{\partial i}{\partial t} - L\frac{\partial^2 i}{\partial t^2}\right)$$

$$\frac{\partial^2 i}{\partial z^2} = RGi + (LG+RC)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2} \quad - (6)$$

The voltage and current on the line are the functions of both position z and time t.

$$v(z,t) = Re\{V(z)e^{j\omega t}\} \quad - (7)$$
$$i(z,t) = Re\{I(z)e^{j\omega t}\} \quad - (8)$$

The factors V(z) and I(z) are complex quantities of the sinusoidal functions of position z on the line and are known as phasors.

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z} \quad - (9)$$
$$I(z) = I_+ e^{-\gamma z} + I_- e^{\gamma z} \quad - (10)$$

where $\gamma = \alpha + j\beta$, is known as propagation constant - (11)

where α is attenuation constant np/m, $\beta \rightarrow$ phase constant in radians/m

For sinusoidal quantities, taking time derivative is equivalent to multiplying by $j\omega$.

using this principle in (2), (4) (5) and (6) and dividing equation by $e^{j\omega t}$ we get;

$$-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t} \quad (12)$$

$$= Ri + Lj\omega i$$

$$= (R + j\omega L)i$$

$$\therefore \frac{dv}{dz} = -(R + j\omega L)I$$

$$= -ZI - (12) \text{ where } Z = R + j\omega L \quad - (13)$$

$$-\frac{\partial I}{\partial z} = Gv + C\frac{\partial v}{\partial t} \quad (4)$$

$$= Gv + C(j\omega)v$$

$$= (G + j\omega C)v$$

$$\frac{dI}{dz} = -(G + j\omega C)V$$

$$= -YV - (14) \text{ where } Y = G + j\omega C \quad - (15)$$

$$\frac{\partial^2 V}{\partial z^2} = RGV + (RC + LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2} \quad (5)$$

$$= RGV + (RC + LG)(j\omega V) + LC(j\omega)^2 V$$

$$= RGV + (RC + LG)j\omega V - LC\omega^2 V$$

or

$$\frac{\partial^2 v}{\partial z^2} = -Z\frac{\partial I}{\partial z} = -(R + j\omega L)\cdot(-)(G + j\omega C)V$$

$$= (R + j\omega L)(G + j\omega C)V$$

$$= \gamma^2 V \quad - (16)$$

$$\text{where } \gamma^2 = ZY = (R + j\omega L)(G + j\omega C) = (\alpha + j\beta)^2 - (17)$$

$$\gamma = \alpha + j\beta$$

consider $\frac{dI}{dz} = -YV$

$$\frac{d^2 I}{dz^2} = -Y\frac{dV}{dz} = -Y(-Z)I = YZI = \gamma^2 I \quad -(18)$$

For a lossless line, $R = G = 0$ and transmission line equations are expressed as,

$$\frac{dV}{dz} = -ZI = -(R+j\omega L)I = -j\omega LI \quad -(19)$$

$$\frac{dI}{dz} = -YV = -(G+j\omega C)V = -j\omega CV - (20)$$

$$\frac{d^2V}{dz^2} = \gamma^2 V = (R+j\omega L)(G+j\omega C)V = -\omega^2 LCV - (21)$$

$$\frac{d^2I}{dz^2} = \gamma^2 I = (R+j\omega L)(G+j\omega C)I = -\omega^2 LCI - (22)$$

Solutions of Transmission line Equations:

consider the equation $\frac{d^2V}{dz^2} = \gamma^2 V$, (16)

The solution for this equation is:

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z} \quad -(23)$$

the term $e^{-j\beta z}$ represents a wave travelling in +ve z direction. βz is called the electrical length of the line and is measured in radian.

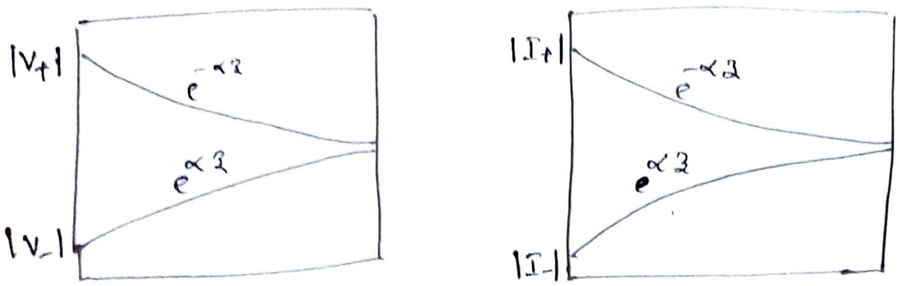
Consider the equation $\frac{d^2I}{dz^2} = \gamma^2 I$ (18)

The solution for this equation is:

$$I = Y_0(V_+ e^{-\gamma z} - V_- e^{\gamma z}) = Y_0\left(V_+ e^{-\alpha z} e^{-j\beta z} - V_- e^{\alpha z} e^{j\beta z}\right) \quad -(24)$$

The characteristic impedance of the line is defined as,

$$Z_0 = \frac{1}{Y_0} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = R_0 \pm jX_0 \quad -(25)$$



voltage wave | current wave

At microwave frequencies it can be seen that

$$R \ll \omega L \text{ and } G \ll \omega C \quad -(26)$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{RG - \omega^2 LC + j\omega RC + j\omega GL}$$

$$= \sqrt{(j\omega)^2 LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\left[\left(1+\frac{R}{2j\omega L}\right)\left(1+\frac{G}{2j\omega C}\right)\right] \quad -(27)$$

$$= j\omega\sqrt{LC}\left[1+\frac{1}{2}\left(\frac{R}{j\omega L}+\frac{G}{j\omega C}\right)+\cancel{\frac{RG}{4(j)^2\omega^2 LC}}^{\,0}\right]$$

$$\alpha + j\beta = j\omega\sqrt{LC} + \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$$

$$\therefore \alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \quad -(28) \qquad \text{and} \quad \beta = \omega\sqrt{LC} \quad -(29)$$

The characteristic impedance is derived as,

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}}\sqrt{\frac{1+\frac{R}{j\omega L}}{1+\frac{G}{j\omega C}}} = \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{-1/2}$$

1) A μw gen at 1.2 GHz supplies power to a μw tx line having the foll parameters.

$R = 0.8\,\Omega/m$, $G = 0.8\,m\mho/m$, $L = 0.01\,\mu H/m$, $C = 0.4\,pF/m$.

Cal a) γ, b) α, c) β, d) Z_0

$$Z = 0.8 + j75.4 = R + j\omega L = 75.4 \angle 89.39^{\circ}$$

$$Y = G + j\omega C = 3.105 \times 10^{-3} \angle 75.07^{\circ}$$

$$\gamma = \alpha + j\beta = \sqrt{ZY} = \underset{\alpha}{0.0654} + j\underset{\beta}{0.48}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = 154.61 + j19.42\,\Omega$$

2) A μw generator operating at 1 GHz supplies power to a cable having line constants $R = 32\,\Omega/km$, $L = 0.02\,\mu H/km$, $C = 0.22\,pF/km$ and $G = 18\,\mu\mho/km$ cal atten const, phase const, char impedance.

$$Z = R + j\omega L = 32 + j125.66 = 129.67 \angle 75.7^{\circ}\,\Omega$$

$$Y = G + j\omega C = (18 + j1380)\mu = 1380.1 \times 10^{-6} \angle 89.25^{\circ}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = 306.5 \angle -6.78^{\circ}\,\Omega = 304.35 - j36.18\,\Omega$$

$$\gamma = \sqrt{ZY} = 0.423 \angle 82.48^{\circ} = 0.0554 + j0.4194 = \alpha + j\beta$$

3) The primary constants of a microwave tx line are: $R = 1.5\,\Omega/m$, $G = 0.2\,m\mho/m$, $L = 2.5\,mH/m$, $C = 0.1\,PF/m$. It is operating at 1.5 GHz. find, $Z_0, \alpha, \beta, \gamma, \lambda, v_p$

$$Z = 1.5 + j23.56 = 23.61 \angle 86.36^{\circ}\,\Omega/m$$

$$Y = G + j\omega C = (0.2 + j0.9425)\,m\mho/m = 0.9635 \times 10^{-3} \angle 78.02\,\mho/m$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = 156.54 \angle 4.17^{\circ}\,\Omega = 156.13 + j11.38\,\Omega$$

$$\gamma = \sqrt{ZY} = 0.1508 \angle 82.49^{\circ} = \underset{\alpha}{0.02} + j\underset{\beta}{0.15}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.15} = 41.88\,m$$

$$v_p = \frac{\omega}{\beta} = 6.283 \times 10^{10}\,m/s.$$

$$Z_0 = \sqrt{\frac{L}{C}}\left(1+\frac{1}{2}\frac{R}{j\omega L}\right)\left(1-\frac{1}{2}\frac{G}{j\omega C}\right) \quad -(30)$$

$$= \sqrt{\frac{L}{C}}\left[1+\frac{1}{2}\left(\cancel{\frac{R}{j\omega L}}^{0}-\cancel{\frac{G}{j\omega C}}^{0}\right)+\frac{1}{4}\cancel{\frac{RG}{\omega^2 LC}}^{0}\right]$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad -30(a)$$

The phase velocity is $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$ $-(31)$

If lossless transmission line is used then,

$$V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3 \times 10^8 m/s \quad -(32)$$

when dielectric of a lossy microwave transmission line is not air, $V_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$ $-(33)$

In general, relative phase velocity factor is expressed as,

$$\text{Velocity factor} = \frac{\text{actual phase velocity}}{\text{Velocity of light in vaccum}}$$

$$V_r = \frac{V_p}{c} = \frac{1}{\sqrt{\mu_r\epsilon_r}} \quad -(34)$$

<u>problems:</u>

1) A transmission line has the following parameters: $R = 2\Omega/m$, $G = 0.5 m\mho/m$, $f = 1GHz$, $L = 8nH/m$, $C = 0.23pF$. Calculate i) the characteristic impedance ii) the propagation constant.

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{2+j2\pi(1\times10^9)\times8\times10^{-9}}{0.5\times10^{-3}+j2\pi(1\times10^9)0.23\times10^{-12}}} = \sqrt{\frac{50.31\angle 87.72°}{15.29\times10^{-4}\angle 70.91°}}$$

$$= 181.39\angle 8.40° = 179.44 + j26.5$$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(50.32\angle 87.72^\circ)(15.29 * 10^{-4}\angle 70.91^\circ)}$

$= \sqrt{769.24 * 10^{-4}\angle 158.63^\circ} = 0.2774\angle 79.31^\circ = 0.051 + j0.273$

2) In a certain microwave ~~tx~~ line the characteristic impedance was measured to be $210\angle 10^\circ\,\Omega$ and propagation constant $0.2\angle 78^\circ$. Determine the primary constants of the line, if the frequency of operation is 1GHz.

Consider $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$ and $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\therefore Z_0\gamma = (R+j\omega L)$

$\therefore R+j\omega L = (210\angle 10^\circ)(0.2\angle 78^\circ) = 42\angle 88^\circ = 1.466 + j41.97$

$\therefore R = 1.466\,\Omega/m, \quad \omega L = 2\pi f L = 41.97$

$\therefore 2\pi * 1 * 10^9 * L = 41.97 \quad \therefore L = 6.68\,nH/m.$

$\dfrac{\gamma}{Z_0} = G+j\omega C \quad \therefore G+j\omega C = \dfrac{0.2\angle 78^\circ}{210\angle 10^\circ} = 9.524 * 10^{-4}\angle 68^\circ$

$= 3.5677 * 10^{-4} + j8.83 * 10^{-4}$

$\therefore G = 3.5677 * 10^{-4}\,\mho/m$

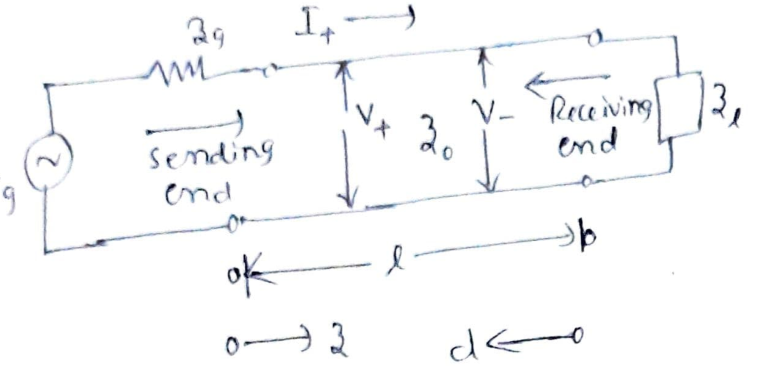
$\omega C = 2\pi f C = 8.83 * 10^{-4}$

$\therefore C = 0.14\,pF/m.$

Reflection coefficient and Transmission coefficient

Reflection coefficient :

Consider a transmission line terminated in an impedance Z_ℓ.



The incident voltage and current waves travelling along the transmission line are given by,

$$V = V_+ e^{-\gamma z} + V_- e^{+\gamma z} \quad -(1)$$

$$I = I_+ e^{-\gamma z} + I_- e^{+\gamma z} \quad -(2)$$

$$\text{where } I = \frac{V_+}{Z_0} e^{-\gamma z} - \frac{V_-}{Z_0} e^{\gamma z} \quad -(3)$$

For a line of length ℓ, voltage and current at the receiving end become,

$$V_\ell = V_+ e^{-\gamma \ell} + V_- e^{\gamma \ell} \quad -(4)$$

$$I_\ell = \frac{1}{Z_0}\left(V_+ e^{-\gamma \ell} - V_- e^{\gamma \ell}\right) - (5)$$

$$\text{load impedance} = \frac{\text{Voltage at the receiving end}}{\text{Current at the receiving end}}$$

$$Z_\ell = \frac{V_\ell}{I_\ell} = Z_0 \, \frac{V_+ e^{-\gamma \ell} + V_- e^{\gamma \ell}}{V_+ e^{-\gamma \ell} - V_- e^{\gamma \ell}} \quad -(6)$$

Reflection Coefficient (Γ) is defined as,

$$\text{Reflection Coefficient} = \frac{\text{reflected voltage or current}}{\text{Incident voltage or current}}$$

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{-I_{ref}}{I_{inc}} \quad -(7)$$

$$= \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{Z_l - Z_0}{Z_l + Z_0} \quad -(8)$$

Reflection coefficient is a complex quantity that can be expressed as, $\Gamma_l = |\Gamma_l| e^{j\theta_l}$ $-(9)$

$|\Gamma_l| \leq 1$, θ_l – angle b/w incident & reflected voltages at the receiving end.

The generalized reflection coefficient is defined as,

$$\Gamma = \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}} \quad -(10)$$

From the figure, let $z = l - d$, the reflection coefficient at some point located a distance d from the receiving end is:

$$\Gamma_d = \frac{V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)}} = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} e^{-2\gamma d} = \Gamma_l e^{-2\gamma d} \quad -(11)$$

The reflection coefficient at any point in terms of reflection coefficient at receiving end is:

$$\Gamma_d = \Gamma_l e^{-2\alpha d} e^{-2j\beta d} = |\Gamma_l| e^{-2\alpha d} e^{j(\theta - 2\beta d)} \quad -(12)$$

Transmission coefficient

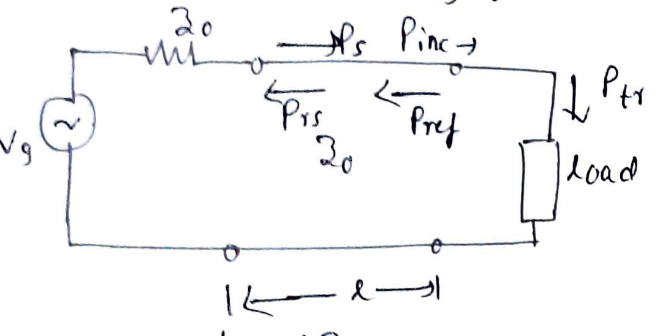
A transmission line terminated in its characteristic impedance Z_0 is called a properly terminated line. Otherwise it is called im properly terminated line.

According to principle of conservation of energy, the incident power minus the reflected power must be equal to the power transmitted to the load. i.e.

$$1 - T_\ell^2 = \frac{Z_0}{Z_\ell} T^2 \quad - (13)$$ where T - transmission coefficient

$$T = \frac{\text{Transmitted voltage or current}}{\text{Incident voltage or current}} = \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{I_{inc}} \quad - (14)$$

consider the below figure!



Here P_{inc} - incident power, P_{ref} - reflected power, $P_{tr} \rightarrow$ transmitted power.

Let the travelling waves at the receiving end be,

$$V_+ e^{-\gamma l} + V_- e^{\gamma l} = V_{tr} e^{-\gamma l} \quad - (15)$$

$$\frac{V_+}{Z_0} e^{-\gamma l} + \frac{V_-}{Z_0} e^{\gamma l} = \frac{V_{tr}}{Z_\ell} e^{-\gamma l} \quad - (16)$$

$$\therefore \frac{Z_\ell V_+ e^{-\gamma l}}{Z_0} - \frac{Z_\ell V_- e^{\gamma l}}{Z_0} = V_{tr} e^{-\gamma l} = V_+ e^{-\gamma l} + V_- e^{\gamma l}$$

$$\therefore \left(\frac{Z_\ell}{Z_0} - 1\right) V_+ e^{-\gamma l} = \left(1 + \frac{Z_\ell}{Z_0}\right) V_- e^{\gamma l}$$

$$\Gamma_l = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{Z_l - Z_0}{Z_l + Z_0} \quad -(17)$$

Consider: $V_+ e^{-\gamma l} + V_- e^{\gamma l} = V_{tr} e^{-\gamma l}$

where $V_- e^{\gamma l} = \Gamma_l e^{-\gamma l} V_+$

$\therefore V_+ \cancel{e^{-\gamma l}} + \Gamma_l V_+ \cancel{e^{-\gamma l}} = V_{tr} \cancel{e^{-\gamma l}}$

$$\therefore V_{tr} = V_+ + \Gamma_l V_+ = (1 + \Gamma_l) V_+ = \left(1 + \frac{Z_l - Z_0}{Z_l + Z_0}\right) V_+$$

$$\frac{V_{tr}}{V_+} = \frac{2 Z_l}{Z_l + Z_0} = T \quad -(18)$$

The power carried by two waves in the side of the incident and reflected waves is:

$$P_{inr} = P_{inc} - P_{ref} = \frac{(V_+ e^{-\alpha l})^2}{2 Z_0} - \frac{(V_- e^{\alpha l})^2}{2 Z_0} \quad -(19)$$

The power carried to the load by the transmitted waves is:

$$P_{tr} = \frac{(V_{tr} e^{-\alpha l})^2}{2 Z_l} \quad -(20)$$

by setting $P_{inr} = P_{tr}$, i.e.

$$\frac{(V_+ e^{-\alpha l})^2}{2 Z_0} - \frac{(V_- e^{\alpha l})^2}{2 Z_0} = \frac{(V_{tr} e^{-\alpha l})^2}{2 Z_l}$$

$$\frac{Z_l}{Z_0}\left[(V_+ e^{-\alpha l})^2 - (V_- e^{\alpha l})^2\right] = (V_{tr} e^{-\alpha l})^2$$

$$\frac{Z_l}{Z_0}\left[1 - \left(\frac{V_- e^{+\alpha l}}{V_+ e^{-\alpha l}}\right)^2\right] = \left(\frac{V_{tr}}{V_+}\right)^2$$

$$\frac{Z_l}{Z_0}(1 - \Gamma_l^2) = T^2 \quad -(21)$$

A certain transmission line has a characteristic impedance of $75 + j0.01\,\Omega$ and is terminated in a load impedance of $70 + j50\,\Omega$. Compute (a) reflection coefficient (b) transmission coefficient.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{70 + j50 - (75 + j0.01)}{70 + j50 + (75 + j0.01)} = \frac{50.24 \angle 95.71^\circ}{153.38 \angle 19.03^\circ} = 0.33 \angle 76.68^\circ$$

$$= 0.08 + j0.32$$

$$T = \frac{2Z_L}{Z_L + Z_0} = \frac{2(70 + j50)}{70 + j50 + (75 + j0.01)} = \frac{172.05 \angle 35.54^\circ}{153.38 \angle 19.03^\circ}$$

$$= 1.12 \angle 16.51^\circ = 1.08 + j0.32$$

Standing wave and Standing wave Ratio:

Standing wave:

The general form of transmission line consist of two waves travelling in opposite directions.

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z}$$

$$= V_+ e^{-\alpha z}\left[\cos(\beta z) - j\sin(\beta z)\right] + V_- e^{\alpha z}\left[\cos(\beta z) + j\sin(\beta z)\right] \quad -(1)$$

$$= \left(V_+ e^{-\alpha z} + V_- e^{\alpha z}\right)\cos(\beta z) - j\left(V_+ e^{-\alpha z} - V_- e^{\alpha z}\right)\sin(\beta z)$$

With no loss, it is assumed that $V_+ e^{-\alpha z}$ and $V_- e^{\alpha z}$ are real.

Then Voltage-wave equation can be expressed as,

$$V_s = V_0 e^{-j\phi} \quad -(2)$$ This is known as equation of voltage standing wave. where,

$$V_0 = \left[\left(V_+ e^{-\alpha z} + V_- e^{\alpha z}\right)^2 \cos^2(\beta z) + \left(V_+ e^{-\alpha z} - V_- e^{\alpha z}\right)^2 \sin^2(\beta z)\right]^{1/2} \quad -(3)$$

which is known as standing wave pattern of the voltage

$$\text{and } \phi = \tan^{-1}\left[\frac{V_+e^{-\alpha z} - V_-e^{\alpha z}}{V_+e^{-\alpha z} + V_-e^{\alpha z}}\tan(\beta z)\right] \quad -(4)$$

which is called the phase pattern of the standing wave.

The maximum amplitude is:

$$V_{max} = V_+e^{-\alpha z} + V_-e^{\alpha z} = V_+e^{-\alpha z}(1+|\Gamma|) \quad -(5)$$

and this occurs at $\beta z = n\pi$, where $n = 0, 1, 2, \ldots$

The minimum amplitude is:

$$V_{min} = V_+e^{-\alpha z} - V_-e^{\alpha z} = V_+e^{-\alpha z}(1-|\Gamma|) \quad -(6)$$

and this occurs at $\beta z = (2n-1)\pi/2$, where $n = 0, 1, 2, \ldots$

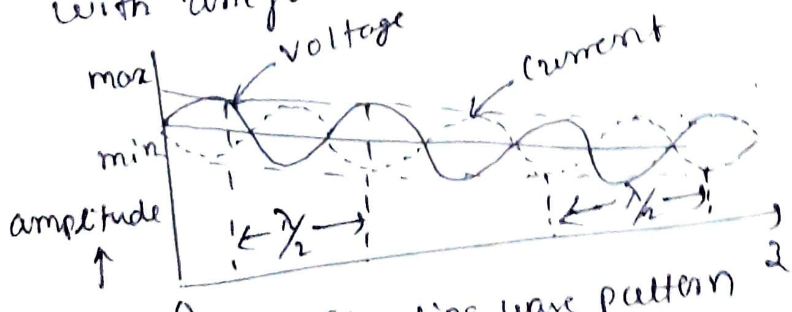
The distance between any two successive maxima or minima is $\lambda/2$, since

$$\beta z = n\pi, \quad z = \frac{n\pi}{\beta} = \frac{n\pi}{2\pi/\lambda} = \frac{n\lambda}{2}, \quad n = 0, \pm1, \pm2, \ldots \quad z_1 = \lambda/2 \quad -(7)$$

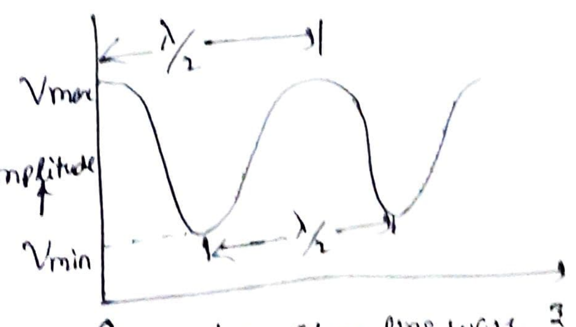
$$I_{max} = I_+e^{-\alpha z} + I_-e^{\alpha z} = I_+e^{-\alpha z}(1+|\Gamma|) \quad -(8)$$

$$I_{min} = I_+e^{-\alpha z} - I_-e^{\alpha z} = I_+e^{-\alpha z}(1-|\Gamma|) \quad -(9)$$

Standing wave patterns of two oppositely travelling waves with unequal amplitude in lossy or lossless line are shown below:



Standing wave pattern in a lossy line



Voltage standing wave pattern in a lossless line when $V_+ \neq 0$ and $V_- = 0$ standing wave pattern becomes

$$V_0 = V_+ e^{-\alpha z} \quad - (10)$$

when $V_+ = 0$ and $V_- \neq 0$ the standing wave pattern becomes

$$V_0 = V_- e^{\alpha z} \quad - (11)$$

when $|V_+ e^{-\alpha z}| = |V_- e^{\alpha z}|$, the standing wave pattern with a zero phase is given by,

$$V_s = 2V_+ e^{-\alpha z} \cos(\beta z) \quad - (12)$$

which is called a pure standing wave.

The equation of a pure standing wave for the current is:

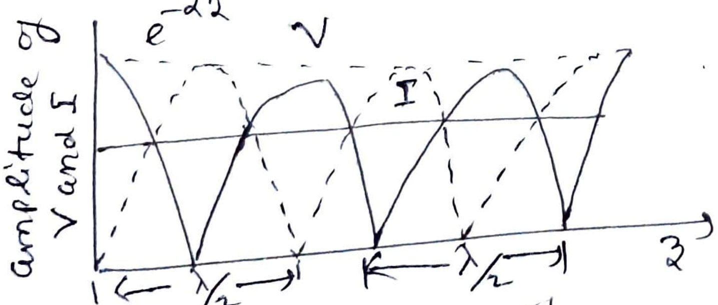
$$I_s = -2jY_0 V_+ e^{-\alpha z} \sin(\beta z) \quad - (13)$$

The equations (12) and (13) show that the voltage and current standing waves are 90° out of phase along the line. The points of zero current are called the current nodes.

$$V_s = Re\left[V_s(z) e^{j\omega t}\right] = 2V_+ e^{-\alpha z} \cos(\beta z) \cos(\omega t) \quad - (14)$$

$$i_s = Re\left[I_s(z) e^{j\omega t}\right] = 2Y_0 V_+ e^{-\alpha z} \sin(\beta z) \sin(\omega t) \quad - (15)$$

The voltage is maximum ~~at~~ when the current is zero and vice versa



pure standing waves of voltage and current

Standing wave Ratio:

Standing waves result from the simultaneous presence of waves travelling in opposite directions on a transmission line. The ratio of the maximum of the standing wave pattern to the minimum is defined as the standing wave ratio, designated by ρ.

$$\text{Standing wave ratio} = \frac{\text{maximum voltage or current}}{\text{minimum voltage or current}}$$

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|} \quad - (16)$$

The standing wave ratio results from the fact that two travelling wave components add in phase at some points and subtract at other points. The distance between two successive maxima or minima is $\lambda/2$. The standing wave ratio of a pure travelling wave is unity and that of a pure standing wave is infinite.

when the S.W.R is unity, there is no reflected wave and the line is called a flat line.

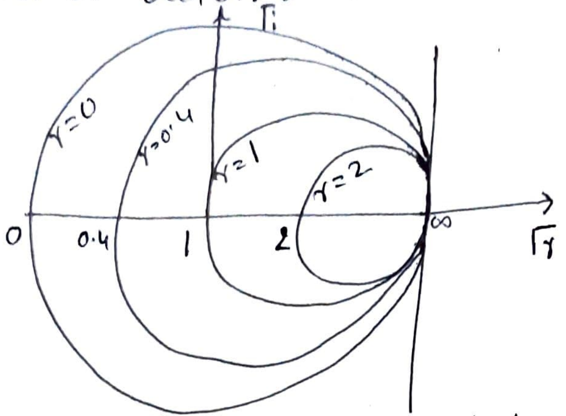
standing wave ~~pattern~~ ratio cannot be defined on a lossy line because the standing wave pattern changes markedly from one position to another.

S.W.R. ρ is related to the reflection coefficient Γ by

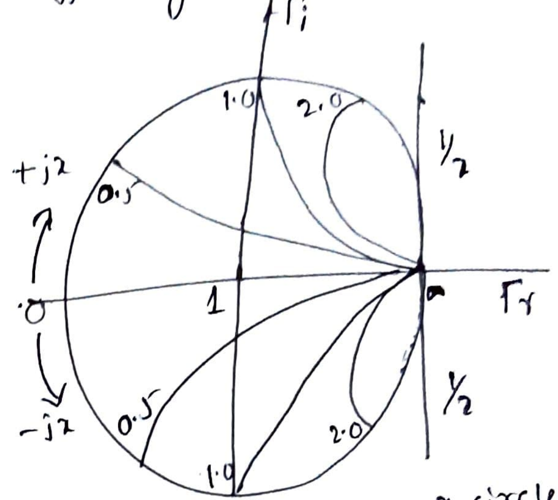
$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} \quad - (17) \quad \text{and} \quad |\Gamma| = \frac{\rho-1}{\rho+1} \quad - (18)$$

SMITH CHART

The smith chart consists of a plot of the normalized impedance or admittance with the angle and magnitude of a generalized complex reflection coefficient in a unity circle. The chart is applicable to the analysis of a lossless line as well as a lossy line. By simple rotation of the chart, the effect of the position on the line can be determined.



Constant resistance r circles



Constant reactance x circles

The characteristics of the smith chart are summarized as follows:

i) The constant r and constant x loci form two families of orthogonal circles in the chart.

ii) The constant r and constant x circles all pass through the point $(\Gamma_r = 1, \Gamma_i = 0)$

iii) The upper half of the diagram represents $\pm jx$.

iv) " lower " " " " $-jx$.

v) For admittance constant r circles become constant g circles, and the constant x circles become constant susceptance b circles.

vi) The distance around the smith chart once is $\lambda/2$.

vii) At a point of $z_{min} = 1/\rho$, there is a V_{min} on the line.

viii) " " " $z_{max} = \rho$ " " a V_{max} " " "

ix) The horizontal radius to the right of the chart center corresponds to V_{max}, I_{min}, z_{max} and ρ (SWR).

x) The horizontal radius to the ~~right~~ left of the chart center corresponds to V_{min}, I_{max}, Z_{min} and $1/e$.

xi) Since the normalized admittance y is $\frac{1}{Z}$ corresponding quantities in the admittance chart are 180° out of phase with those in the impedance chart.

xii) The normalized impedance or admittance is repeated for every half wavelength of distance.

xiii) The distance are given in ~~given~~ wave lengths towards the generator and also toward the load.

Single stub Matching:

Although single lumped inductors or capacitors can match the transmission line, it is more common to use the susceptive properties of short circuited sections of transmission lines. Short circuited sections are prefered to open circuited ones because a good short circuit is easier to obtain a good open circuit.

For a lossless line with $Y_g = Y_0$, maximum power transfer requires $Y_{11} = Y_0$, where Y_{11} - total admittance of the line and stub looking to the right. The stub must be located at that point on the line where the real part of admittance, looking toward the load is Y_0. In a normalized unit Y_{11} must be of the form

$$Y_{11} = Y_d \pm Y_s = 1$$

If the stub has the same characteristic impedance as that of the line. Otherwise $Y_{11} = Y_d \pm Y_s = Y_0$

The stub length is then adjusted so that its susceptance just cancels out the susceptance of the line at the junction.