Module-3

Controlled Rectifiers.

Introduction:-

→ We know that diode rectifiers provide a fixed output voltage only. To obtain controlled output voltages, phase control thyristors are used instead of diodes.

→ The output voltage of thyristor rectifiers is varied by controlling the delay (or) firing angle of thyristors.

→ A phase control thyristor is turned ON by applying a short pulse to its gate and turned off due to natural or line commutation.

→ These rectifier (phase-controlled rectifiers) are also called as ac to dc converters.

Advantages:-

→ phase controlled rectifiers are simple and less expensive.

→ Efficiency is above 95%.

Applications

Extensively used in industrial application ie in variable speed drivers. portable hand tool drives. Magnet power supplies, Printing press, paper mills,

Based on input supply, phase controlled rectifiers are classified as

1. Single phase converters 2. Three phase converters.

Each type is sub divided into

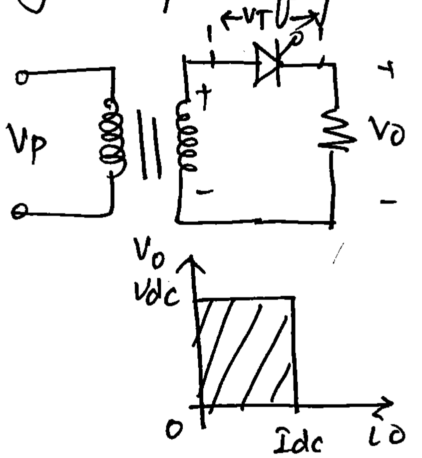
1. Semi converter 2. Full converter 3. Dual converter

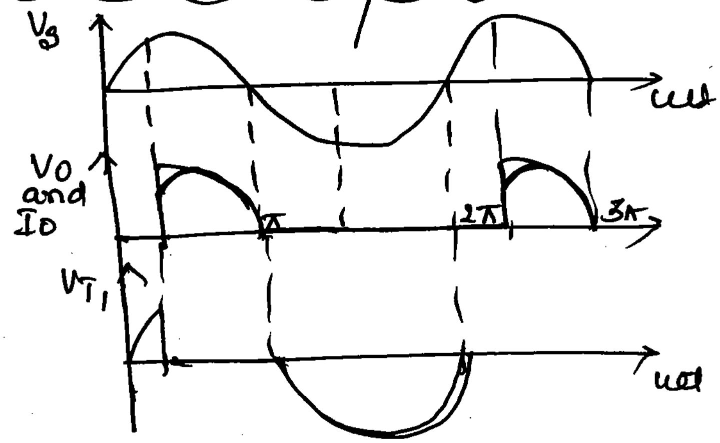
→ Semiconverter is one-quadrant converter and it has one polarity of output voltage & current

→ Full converter is a two-quadrant converter & polarity of its output voltage can be either positive or negative and output current has one polarity.

Dual converters can operate in four quadrants & both the output voltage & current can be either positive or -ve.

Principle of phase controlled converter operation





Let us consider a circuit with resistive load as shown in fig

* During positive half cycle of the input voltage thyristor anode is positive w.r.t cathode and thyristor is said to be forward biased. When thyristor T_1 is fired at $\boxed{\omega t = \alpha}$, T_1 conducts and input voltage appears across the load.

→ When input voltage goes negative i.e at $\boxed{\omega t = \pi}$, thyristor anode becomes negative w.r.t its cathode T_1 is said to be reverse biased & it is turned.

Angle at which thyristor is fired $(\omega t = \alpha)$ is called firing angle or delay angle.

→ figure 1 shows that output voltage and current have one polarity

→ Figure 2 shows the waveforms for input voltage, o/p voltage load current & voltage across T_1.

→ If V_m is the peak input voltage, average output voltage V_{dc} is given by,

$$V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t = \frac{V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi}\left[-\cos\pi + \cos\alpha\right] = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]$$

If $\alpha = 0$, $V_{dc} = \frac{V_m}{2\pi}[1+1] = \frac{V_m}{\pi}$.

$\alpha = \pi$, $V_{dc} = 0$

∴ V_{dc} can be varied from $\frac{V_m}{\pi}$ to 0 by varying α from 0 to π.

→ average output becomes maximum when $\alpha = 0$ and it is represented by V_{dm}

i.e, $\boxed{V_{dm} = \frac{V_m}{\pi}}$

→ Normalizing the output voltage w.r.t V_{dm}, the normalized output voltage is given by

$$= \frac{V_{dc}}{V_{dm}} = \frac{\cancel{V_m}(1+\cos\alpha)}{2\cancel{\pi}\cdot\frac{\cancel{V_m}}{\pi}} = 0.5(1+\cos\alpha)$$

→ The rms output voltage is given by

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2} = \left[\frac{V_m^2}{4\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t)\, d\omega t\right]^{1/2}$$

$$V_{rms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

Problem

If the converter has a purely resistive load of R and the delay angle is $\alpha = \frac{\pi}{2}$, determine a) Rectification efficiency b) form factor c) The ripple factor RF d) The transformer utilization factor TUF e) peak inverse voltage PIV of thyristor T_1

ans $V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{V_m}{2\pi}\left(1+\cos\frac{\pi}{2}\right)$ $\boxed{V_{dc} = 0.1592 V_m}$

$\therefore I_{dc} = \frac{V_{dc}}{R} = 0.1592\frac{V_m}{R}$

when $\alpha = 0$, $V_{dm} = \frac{V_m}{\pi}$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{0.1592 V_m}{V_m/\pi} \Rightarrow \boxed{V_n = 0.5}$$

$$V_{rms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 2(\pi/2)}{2}\right)\right]^{1/2} = 0.3536\, V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = 0.3536\frac{V_m}{R}$$

$P_{dc} = V_{dc} I_{dc} = (0.1592 V_m)^2/R$ $\quad P_{ac} = V_{rms} I_{rms} = (0.3536 V_m)^2/R$

a) Rectifier Efficiency $\eta = \frac{P_{dc}}{P_{ac}} = \frac{(0.1592 V_m)^2/R}{(0.3536 V_m)^2/R} = 20.27\%$

b) Form factor $FF = \frac{V_{rms}}{V_{dc}} = \frac{0.3536 V_m}{0.1592 V_m} = 2.221$ or 222.1%

c) Ripple factor $RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{dc}} = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1}$

$$RF = \sqrt{(FF)^2 - 1} = \sqrt{(2.221)^2 - 1} = 1.983$$

$$\boxed{RF = 198.3\%}$$

d) $TUF = \frac{P_{dc}}{V_{rms} I_{rms}} = \frac{(0.1592 V_m)^2/R}{\left(\frac{V_m}{\sqrt{2}}\right)(0.3536 V_m/R)}$

where $V_s = \frac{V_m}{\sqrt{2}}$

$I_s = I_{rms}$

where I_s, V_s are rms current & voltage of transformer secondary.

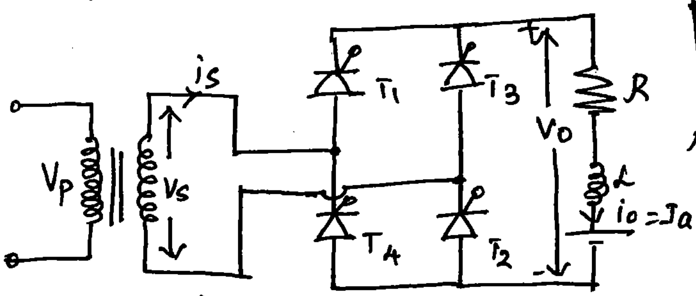
$TUF = 0.1014$

$\frac{1}{TUF} = 9.86$

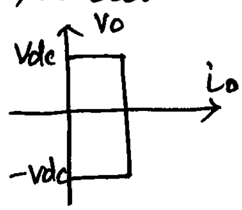
e) peak Inverse Voltage $PIV = V_m$

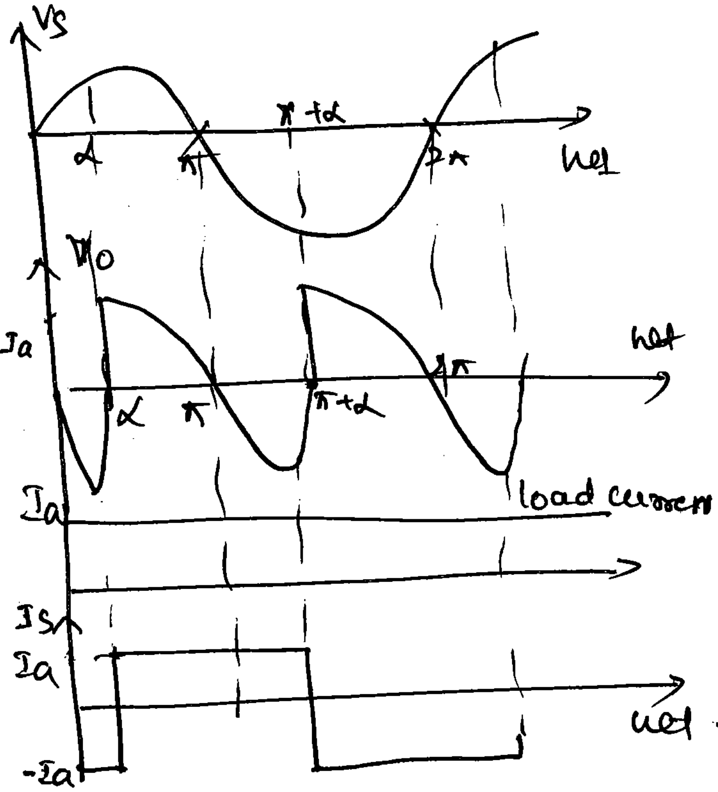
NOTE:- The performance of the Converter is degraded at the lower range of delay angle α.

SINGLE PHASE FULL CONVERTERS:-



a) Circuit





→ Single phase full converter with highly inductive load so that the load current is continuous and ripple free.

→ During positive half cycle, T_1 and T_2 thyristors are forward biased and are triggered simultaneously at $\boxed{\omega t = \alpha}$, load is connected to supply through T_1 & T_2.

Due to inductive load, T_1 and T_2 will conduct beyond $\boxed{\omega t = \pi}$, even though the input voltage is (negativ-ve)

→ During negative half cycle of the input voltage T_3 and T_4 are forward biased and firing of T_3 and T_4 will apply the supply Voltage across T_1 & T_2 as reverse blocking voltage. T_1 & T_2 will be turned off due to line or natural commutation and the load current will be transferred from T_1 & T_2 to T_3 & T_4.

→ During period α to π, input voltage V_s and input current is are positive and the power flows from supply to the load. converter is said to be operated in "rectification mode"

* During period π to $(\pi+\alpha)$ the input voltage V_s is -ve and i_s is +ve, there will be a reverse power flow from the load to the supply. The converter is said to be operated in "Inversion mode"

→ Converter is used in industrial application upto 15KW

→ Depending on the value of α, the average output Voltage could be either positive or negative and it provides two Quadrant operation.

NOTE:- With a purely resistive load, T_1 and T_2 will conduct from α to π and T_3 & T_4 will conduct from $(\alpha+\pi)$ to 2π.

→ average output Voltage is given by

$$V_{dc} = \frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \, d\omega t = \frac{2V_m}{2\pi}\left(-\cos\omega t\right)_{\alpha}^{\pi+\alpha}$$

$$\boxed{V_{dc} = \frac{2V_m}{\pi}\cos\alpha}$$

if $\alpha = 0$, $V_{dc} = 2V_m/\pi$

$\alpha = \pi$, $V_{dc} = \frac{-2V_m}{\pi}$

∴ By varying α from 0 to π, V_{dc} can be varied from $\frac{2V_m}{\pi}$ to $-\frac{2V_m}{\pi}$

Maximum average output Voltage is $\boxed{V_{dm} = \frac{2V_m}{\pi}}$

Normalised average output Voltage is $V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$

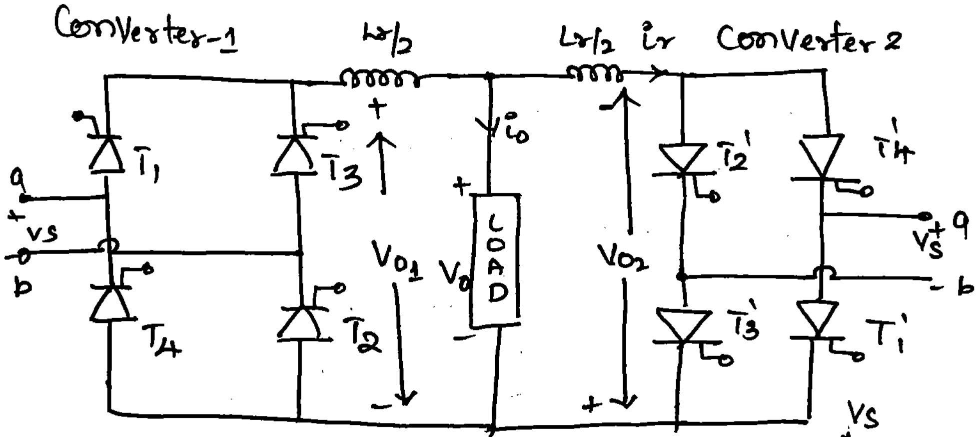
rms Value of the output voltage is given by

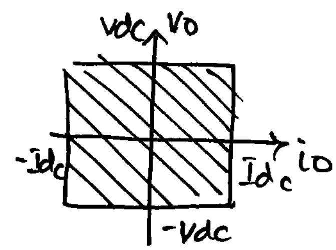
$$V_{rms} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2} = \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi+\alpha}(1-\cos 2\omega t)\, d\omega t\right]^{1/2}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} = V_s}$$

Single phase Dual converters

→ The circuit diagram for 1ϕ dual converter is as shown below.
→ They are used in high-power variable speed drives.





* If α_1, α_2 are delay angles of converters 1 & 2 respectively and the average output voltage is V_{dc1} & V_{dc2}.
* The delay angles are controlled such that one converter operates as a rectifier and other operates in inversion mode.
* The average output voltages

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1 \quad \cdots (1)$$

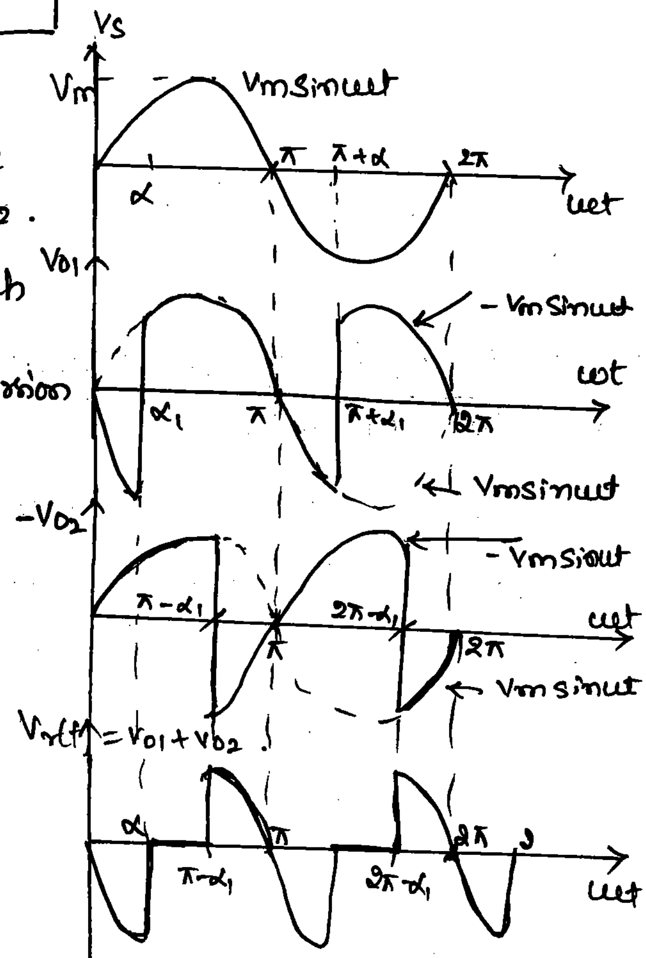
$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2 \quad \cdots (2)$$

Since one converter is rectifying & the other is inverting

$$V_{dc1} = -V_{dc2}$$

$$\therefore \cos\alpha_1 = -\cos\alpha_2 \Rightarrow \cos\alpha_2 = -\cos\alpha_1 = \cos(\pi - \alpha_1)$$

$$\therefore \boxed{\alpha_2 = \pi - \alpha_1}$$



NOTE:- The instantaneous output voltages of two converters are out of phase, there can be an instantaneous voltage difference which results in circulating current between two converters.

NOTE:- Gate Sequence
1. Gate the +ve converter @ $\boxed{\alpha_1 = \alpha}$
2. Gate -ve converter at $\boxed{\alpha_2 = \pi - \alpha}$

→ The Circulating current Cannot flow through the load & is Normally limited by a Circulating Current reactor L_r.

* The Circulating current i_r Can be found as follows.

$$i_r = \frac{1}{\omega L_r}\int_{\pi-\alpha_1}^{\omega t} V_r \, d\omega t = \frac{1}{\omega L_r}\int_{\pi-\alpha_1}^{\omega t} (v_{o1} + v_{o2})\, d\omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{2\pi-\alpha_1}^{\omega t} \sin\omega t \, d\omega t - \int_{2\pi-\alpha_1}^{\omega t} -\sin\omega t \, d(\omega t)\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[2\int_{2\pi-\alpha_1}^{\omega t} \sin\omega t \, d\omega t\right] = \frac{-2V_m}{\omega L_r}\left[-\cos\omega t\right]_{2\pi-\alpha_1}^{\omega t}$$

$$= \frac{-2V_m}{\omega L_r}\left[-\cos\omega t + \cos(2\pi-\alpha_1)\right] = \frac{-2V_m}{\omega L_r}\left[\cos\alpha_1 - \cos\omega t\right]$$

$$\boxed{i_r = \frac{2V_m}{\omega L_r}\left[\cos\omega t - \cos\alpha_1\right]} \qquad \left.\begin{array}{l} i_r > 0 \text{ for } 0 \le \alpha_1 < \pi/2 \\ i_r < 0 \text{ for } \pi/2 < \alpha_1 \le \pi \end{array}\right\}$$

* For $\boxed{\alpha_1 = 0}$, Only the Converter 1 operates; for $\boxed{\alpha_1 = \pi}$, Only the Converter 2 operates.

* For $\boxed{0 \le \alpha_1 \le \pi/2}$ the Converter 1 supplies a positive load Current i_o and the circulating current Can only be positive.

* For $\boxed{\pi/2 < \alpha_1 \le \pi}$ the Converter 2 supplies a negative load current $-i_o$ and thus Only a negative Circulating Current can flow.

NOTE:- At $\boxed{\pi/2 = \alpha_1}$ the Converter1 supplies +ve Circulating during the first half cycle, and the converter 2 Supplies -ve Circulating during Second half cycle.

NOTE:- without circulating current oly one converter operates at a time & carries load current & the other converter is completely block by inhibiting gate pulses.

Advantages of circulating Current:-

- Circulating current maintains Conduction of both Converters Over Whole control range, independent of load.
- Because one Converter operates in rectifying & other in inversion, the power flow in either direction at any time is possible.
- Time response for Changing quadrant operation to another is faster.

(1) A single-phase dual converter is operated from a 120V, 60Hz supply & the load resistance $R = 10\Omega$, the circulating inductance $L_r = 40$mH delay angles are $\alpha_1 = 60^\circ$, $\alpha_2 = 120^\circ$. Calculate the peak circulating current and peak current of converter 1.

ans. Given $\alpha_1 = 60^\circ$, $\alpha_2 = 120^\circ$, $R = 10\Omega$, $L_r = 40$mH, $V_s = 120$, $f = 60$

$\omega = 2\pi f$

$= 2\pi * 60$

$= 377$ rad/s

$V_m = \sqrt{2} * V_s$

$= \sqrt{2} * 120$

$V_m = 169.7$ V

for $\omega t = 2\pi$ & $\alpha_1 = \pi/3$

$$I_{rmax} = \frac{2V_m}{\omega L_r}(1 - \cos\alpha_1)$$

$$= \frac{169.7}{377 * 0.04}$$

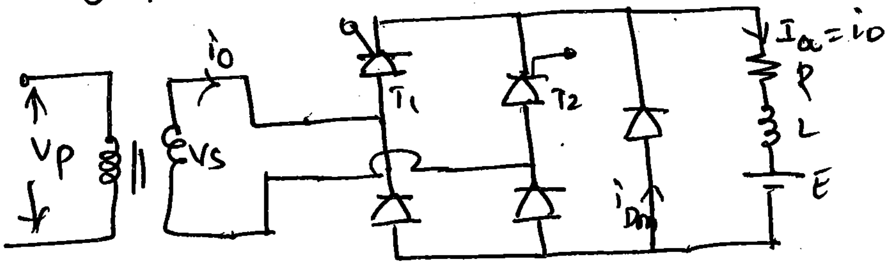
$$\boxed{I_{rmax} = 11.25A}$$

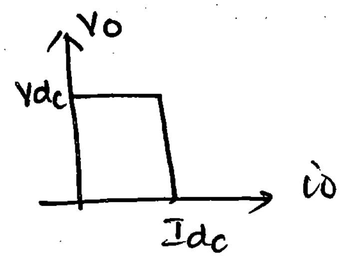
The peak load current is $I_p = \frac{V_m}{R} = \frac{169.7}{10} = 16.97A$.

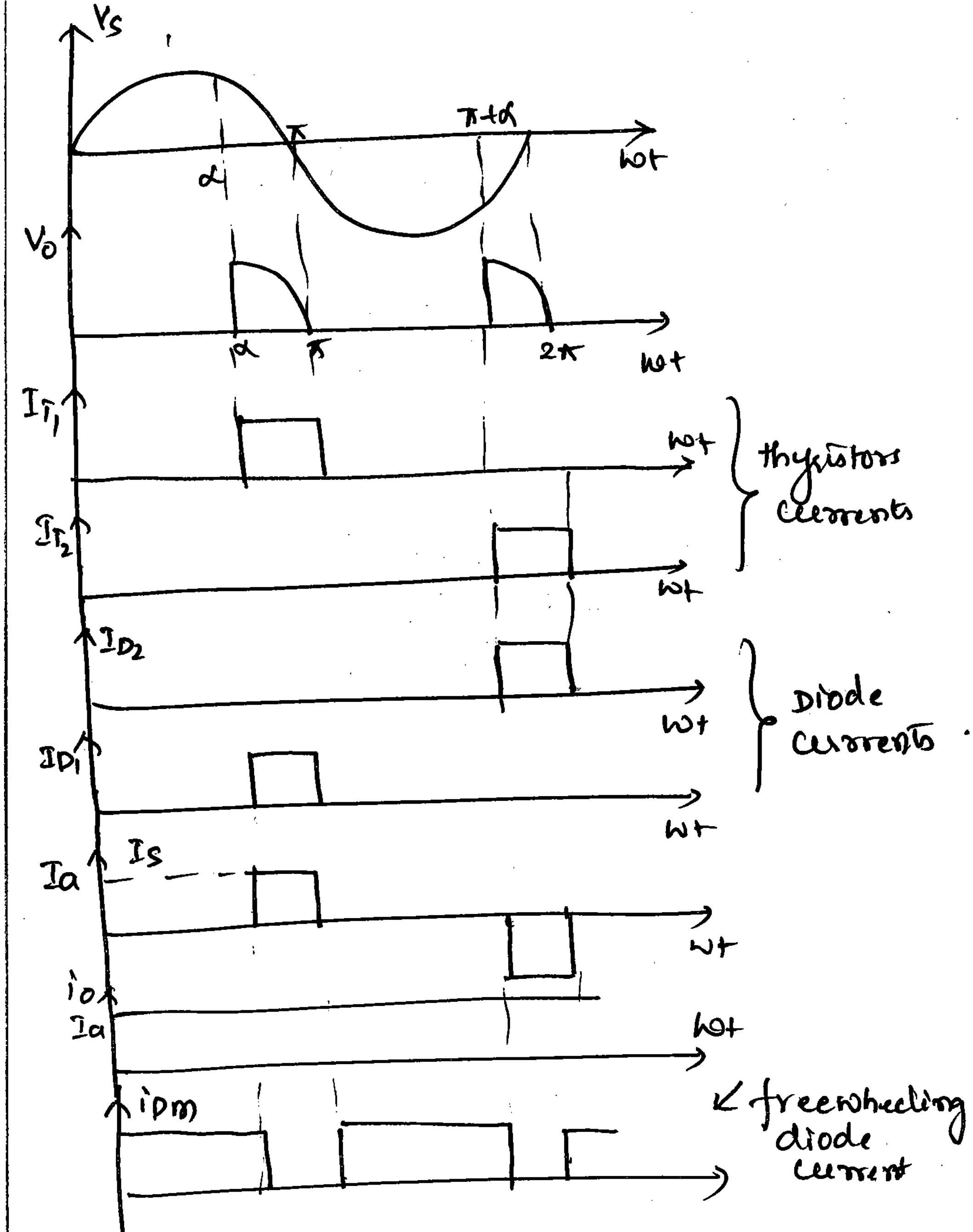
The peak current of converter 1 is $16.97 + 11.25$

$= 28.22A$.

Single phase Semiconverter







The circuit diagram is as shown with highly inductive load

Working

* During +ve half cycle, thyristor T_1 & D_2 diode are forward biased, the load is connected to the input supply through T_1 & D_2 during period $\boxed{\alpha \le \omega t \le \pi}$

* During the period $\boxed{\pi \le \omega t \le \pi+\alpha}$ the input voltage is negative and the free wheeling diode D_m is forward biased D_m conducts to provide continuity of current in the inductive load.
The load current is transferred from T_1 & D_2 to D_m & thyristor T_1 and Diode D_2 are off.

* During -ve half cycle of the input voltage, thyristor T_2 is forward biased, & the firing of thyristor T_2 at $\omega t = \pi+\alpha$, reverses bias D_m. The ~~load~~ D_m is turned off & load is connected the supply through T_2 & D_1

NOTE:- This converter has a better PF due to the free wheeling diode & commonly used in applications upto 15kw, where one quadrant operation is acceptable.

$$V_{dc} = \frac{2}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t$$

$$= \frac{2V_m}{2\pi}\left(-\cos\omega t\right)_{\alpha}^{\pi}$$

$$\boxed{V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha)}$$

and V_{dc} can be varied from $\frac{2V_m}{\pi}$ by varying α from 0 to π.

The maximum average output Voltage $V_{dm} = \frac{2V_m}{\pi}$ and the normalized average o/p voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1+\cos\alpha)$$

The Rms output voltage

$$V_{Rms} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} \frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

(1) Single phase semiconverter is operated from 120V, 60 Hz Supply the load current with an average value of I_a is continuous with negligible ripple content turns ratio of transformers is unity. the delay angle $\alpha = \pi/3$

Calculate a) Harmonic factor of input current b) displacement factor c) Input power factor.

$V_S = 120V$

$I_{o(avg)} = I_a$

$\alpha = \pi/3$

$$HF = \sqrt{\frac{\pi(\pi-\alpha)}{8\cos^2\alpha/2} - 1} = \left\{\frac{\pi(\pi-3)}{8\cos^2(\pi/3)} - 1\right\}^{1/2}$$

$$= 0.3108 \text{ or } 31.8\%.$$

b) Displacement factor

$$DF = \cos(\alpha/2) = \cos\left(\frac{\pi/3}{2}\right) = 0.866$$

(c) Input power factor $\Rightarrow$ $PF = \sqrt{\frac{8}{\pi(\pi-\alpha)}\cos^2\frac{\alpha}{2}}$

$$= \sqrt{\frac{8}{\pi(\pi-\pi/3)}\cos^2\left(\frac{\pi/3}{2}\right)}$$

$$= 0.827 \text{ lagging}$$

Ac Voltage Controllers.

→ If a thyristor switch is connected between ac supply and load, the power flow can be controlled by varying the rms value of ac voltage applied to the load; and this type of power ckt is known as an ac voltage controller.

→ **Applications**:- Industrial heating, on-load transformer connection changing light controls, speed control of polyphase induction motors, ac magnet controls.

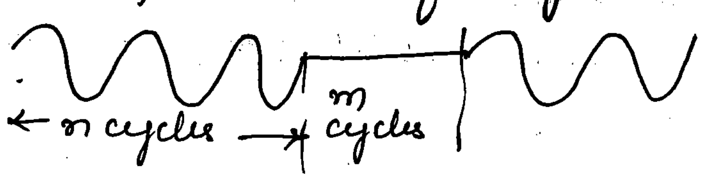
NOTE:- By varying the firing angle "α" the Rms value of the ac output voltage and ac power flow to the load can be controlled.

There are two types of Ac voltage control for power transfer.

1. On-off control 2. phase angle control.

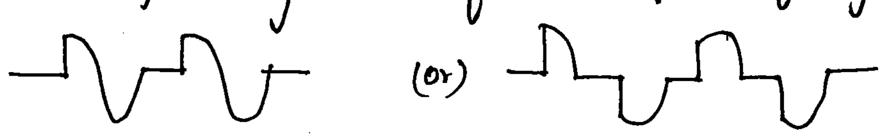
* In ON-off control, thyristor switches connects the load to the ac source for few cycles of input voltage and then disconnected for another few cycles.

v_0 =



* In phase angle control, thyristor switches connect the load to the ac source for a portion of each cycle of input voltage.

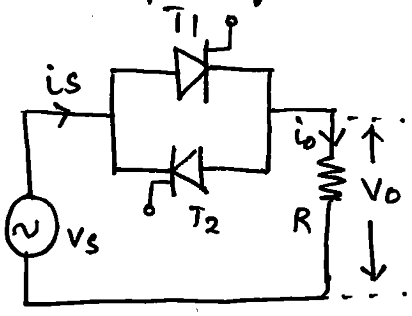
v_0



phase angle controllers can be classified into two types Single phase and 3ϕ (or) three phase controller.

* Each type can either be (1) unidirectional / half wave control
(2) Bidirectional / full wave control.

Principle of ON-OFF control / Integral cycle control / Zero voltage Switching



$V_s = V_m \sin\omega t$

Controller is ON for n cycles
Off for m cycles.

* Two SCR's are connected in antiparallel, ∴ this controller can be operated in ON-off control mode.

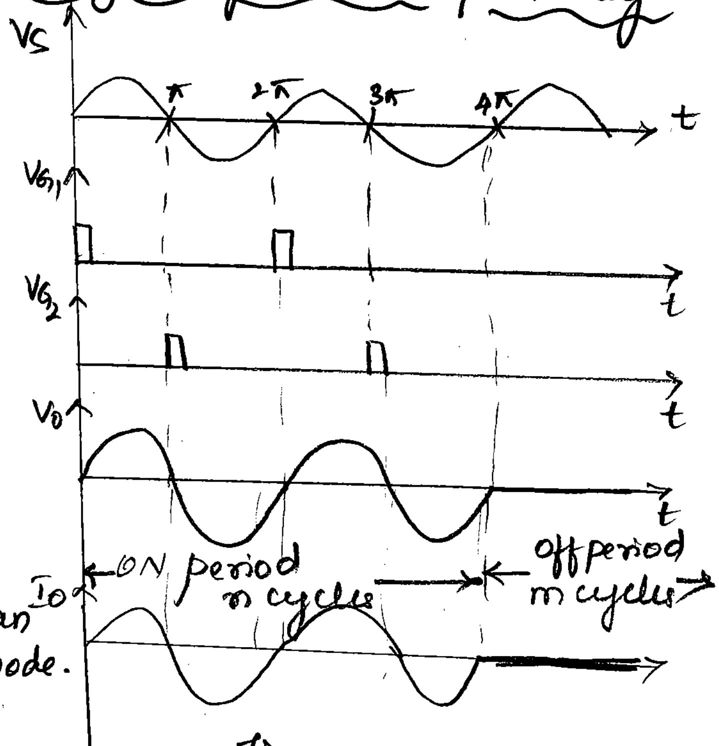


Figure 2.

Working:-

→ During +ve half cycle of the ac input, at instant $\omega t = 0, 2\pi, 4\pi \ldots$ SCR T_1 is triggered at $\boxed{\alpha = 0}$ ∴ SCR T_1 conducts hence positive half cycles appears across the load.

→ During -ve half cycle of the ac input, @ $\omega t = \pi, 3\pi \ldots$ SCR T_2 is triggered at $\boxed{\alpha = 0}$, SCR T_2 conducts & hence -ve half cycles appear across the load.

→ Hence full cycles of the mains is obtained across the load.

→ One should note when gate pulse are removed, both SCRs are off and no output appears across the load.

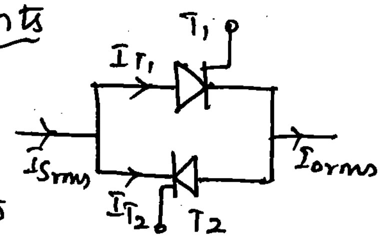
→ SCRs conducts for 'n' number of cycles and they are OFF for "m" number of cycles hence the name ON-OFF control.

* Figure② shows the corresponding input & output waveforms for Resistive load.

Advantages:- SCR's are switched ON at zero crossings ($\alpha = 0$), hence the harmonics due to switching actions are reduced.

Disadvantage:- A supply voltage is applied across the load during ON period and zero during off period due to which the load voltage is not smooth rather it is intermittent. The load has to sustain these variations.

Rms and average Value of thyristor currents



Rms thyristor current ($I_{T(rms)}$)

let I_{T_1} and I_{T_2} be the thyristor Currents

$$\therefore I_{orms}^2 = I_{T_1}^2 + I_{T_2}^2 \quad \cdots (1)$$

and $I_{T_1} = I_{T_2} = I_{T(rms)}$ (Since thyristor currents are equal)

$$\therefore I_{orm}^2 = 2 I_{T(rms)}^2$$

$$\boxed{I_{T(rms)} = \frac{I_{o(rms)}}{\sqrt{2}}}$$

NOTE:-

(1) $I_{T(rms)} = \frac{V_m \sqrt{K}}{2R}$ In terms of V_m.

(2) $I_{T(rms)} = \frac{V_{s(rms)} \sqrt{K}}{\sqrt{2} \cdot R}$ In terms of supply voltage V_s.

(3) $I_{T(rms)} = \frac{V_{o(rms)}}{\sqrt{2} \cdot R}$ In terms of load voltage

Average thyristor Current [$I_{T(avg)}$]

We know that the average current in thyristor T_1 is given by

$$I_{T_1}(avg) = \frac{n}{m+n} * \frac{1}{2\pi} \int_0^{\pi} i(t)\,dt = \frac{n}{m+n} * \frac{1}{2\pi} \int_0^{\pi} I_m \sin \omega t$$

$$I_{T_1}(avg) = \frac{n}{m+n} * \frac{I_m}{2\pi} \left(-\cos \omega t\right)_0^{\pi} = \left(\frac{n}{m+n}\right) \frac{I_m}{\cancel{2}\pi} * \cancel{2} = K \cdot \frac{I_m}{\pi}$$

$$\therefore \boxed{I_{T_1}(avg) = \frac{I_m * K}{\pi}}$$ Where $K = \frac{n}{m+n}$, duty cycle.

|||ly $I_{T_2}(avg) = I_{T}(avg) = \frac{I_m * K}{\pi}$, since both thyristors share the load current equally their average currents will be same.

i.e. $I_{T}(avg) = I_{T_1}(avg) = I_{T_2}(avg)$

$$\therefore \boxed{I_{T}(avg) = \frac{I_m K}{\pi}}$$

Interms of max voltage V_m $\boxed{I_{Tavg} = \frac{V_m K}{\pi R}}$

Interms of supply voltage V_s

$$\boxed{I_{T(avg)} = \frac{\sqrt{2}\, V_{s(rms)} K}{\pi R}}$$

Rms output voltage and current

→ W.K.T $\boxed{V_s = V_m \sin\omega t}$ is the supply voltage and this supply voltage is applied to the load for 'n' cycles out of total $(n+m)$ cycles. Each cycle has period of 2π.

∴ Rms value is given by,

$$V_{o(rms)} = \left[\frac{1}{T}\int_0^T V_s^2 \, d\omega t\right]^{1/2} = \left[\frac{n}{m+n} * \frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$V_{o(rms)} = \left[\frac{n}{m+n}\frac{V_m^2}{2\pi}\int_0^{\pi}\frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2} = \left[\frac{n}{m+n}\frac{V_m^2}{2\pi}\left\{\frac{\omega t}{2}\Big|_0^{2\pi} - \frac{\sin 2\omega t}{4}\Big|_0^{2\pi}\right\}\right]^{1/2}$$

$$= \left[\frac{n}{m+n}\frac{V_m^2}{2\pi}\left\{\frac{1}{2}*(2\pi - 0) - \left(\cancel{\frac{\sin 4\pi - \sin 0}{4}}\right)^{0}\right\}\right]^{1/2} = \left[\frac{n}{m+n}\frac{V_m^2}{\cancel{2\pi}} * \frac{\cancel{2\pi}}{2}\right]^{1/2}$$

$$V_{o rms} = \left[\frac{V_m^2}{2} * K\right]^{1/2} \Rightarrow \boxed{V_{o(rms)} = \frac{V_m}{\sqrt{2}} * \sqrt{K}}$$

$\left[\text{Where } \underset{\text{duty cycle}}{\underbrace{K}} = \frac{n}{m+n}\right]$

NOTE:- W.K.T $V_m = \sqrt{2}\, V_s \Rightarrow \frac{V_m}{\sqrt{2}} = V_{s(rms)}$ (The rms value of supply voltage)

$$\boxed{\therefore V_{o(rms)} = V_{s(rms)}\sqrt{K}}$$

→ Rms load current $I_{o(rms)} = \frac{V_{o(rms)}}{R} = \frac{V_{s(rms)} * \sqrt{K}}{R}$

$$\boxed{I_{o(rms)} = \frac{V_m\sqrt{K}}{\sqrt{2} * R}.}$$

Power Factor:- It is the ratio of active load power to that of total rms input power

$$PF = \frac{\text{Active load power}}{\text{Total rms i/p power}}$$

$$\text{Active load power} = V_{o(rms)} \cdot I_{o(rms)} = \frac{V_{o(rms)}^2}{R} = I_{d(rms)}^2 R.$$

$$\text{Total rms i/p power} = V_{s(rms)} \cdot I_{s(rms)}$$

$$PF = \frac{V_{o(rms)} * I_{d(rms)}}{V_{s(rms)} * I_{s(rms)}}$$

Supply current is same as output current $\boxed{\therefore I_{orms} = I_{srms}}$

$$\therefore PF = \frac{V_{o(rms)} * \cancel{I_{o(rms)}}}{V_{s(rms)} * \cancel{I_{d(rms)}}}$$

$$= \frac{\cancel{V_{s(rms)}} * \sqrt{K}}{\cancel{V_{s(rms)}}}$$

$$\boxed{\therefore PF = \sqrt{K}}$$

Q) The single phase ACVC delivers a power of 5KW to the resistive load of $\boxed{R=5\Omega}$. If ON-off Control strategy is used and the supply voltage is 230V, 50Hz Calculate

a) rms o/p voltage and current

b) Duty rule c) power factor d) Rms & average values of thyristor current.

Given:- $R=5\Omega$, $V_s=230V$, $f=50Hz$, $P=5KW$

$$P=\frac{V_{o(rms)}^2}{R}$$

(a) $V_{o(rms)}^2 = P \cdot R = 5K(5) = 25000$

$V_{o(rms)} = \sqrt{25000} = 158.1V$

$I_{o(rms)} = \frac{V_{o(rms)}}{R} = \frac{158.1}{5} = 31.62A$

power factor $=\sqrt{K}$

$\frac{V_{o(rms)}\, I_{o(rms)}}{V_{s(rms)}\, I_{s(rms)}} = \sqrt{K}$

$\because I_{o(rms)} = I_{s(rms)}$

$\sqrt{K} = \frac{V_{o(rms)}}{V_{s(rms)}}$

$= \frac{158.1}{230} = 0.687$

$K = (0.687)^2$

$\boxed{K=0.472}$

(or) $V_o = V_s\sqrt{K}$

$K = \left(\frac{V_o}{V_s}\right)^2 = \left(\frac{158.1}{230}\right)^2$

$\boxed{K=0.472}$

Rms and average values of thyristor current

$I_{T(rms)} = \frac{V_{orms}}{\sqrt{2}\,R} = \frac{158.1}{\sqrt{2}\cdot 5} = 22.35A$

$I_{Tavg} = \frac{\sqrt{2}\,V_s\,k}{\pi R} = 9.77A$

Power factor $\sqrt{K} = \sqrt{0.472} = 0.687$

(1) An ACVC is provided with a load of 10Ω, supplied with an AC Voltage of 120V, 50Hz with 25 cycles ON & 75 cycles off. Calculate the power dissipated in the resistance, rm current in each SCR's and average current in each of the SCR & input power factor.

A: Given $n=25$ $m=75$, $V_s=120$ $f=50Hz$

Duty cycle $k = \frac{n}{m+n} = \frac{25}{100} = 0.25$

$V_{orms} = V_{s(rms)}\sqrt{K}$ $\qquad I_{orms} = \frac{V_{orms}}{R} = \frac{60}{10} = 6A$

$= 120 * \sqrt{0.25}$

$\boxed{V_{orms} = 60V}$

power dissipation $P = V_{orms} * I_{orms}$

$= 60 * 6$

$\boxed{P = 360 \text{ watts.}}$

Thyristor Current (Rms) $I_{T(rms)} = \frac{V_{o(rms)}}{\sqrt{2} * R}$

$= \frac{60}{\sqrt{2} * 10}$

$\boxed{I_{Trms} = 4.24A}$

average current

$I_{Tavg} = \frac{\sqrt{2}\, V_s\, k}{\pi R}$

$= \frac{\sqrt{2} * 120 * 0.25}{\pi \cdot 10}$

$\boxed{I_{Tavg} = 1.35A}$

power factor $PF = \sqrt{K}$

$= \sqrt{0.25} = 0.5$

(2) $R = 20Ω$, $V_s = 230V$, $f = 50Hz$ $n = 50$ $m = 150$

$K = \frac{n}{m+n} = \frac{50}{200} = 0.25$ $\qquad \sqrt{K} = \sqrt{(0.5)^2} = 0.5$

(1) $V_{orms} = V_{Srms}\sqrt{K}$

$\boxed{V_{orms} = 0.5(230) = 115V}$

(2) $I_{Tavg} = \frac{\sqrt{2}\, V_s\, k}{\pi R} = \frac{\sqrt{2} * 230 * 0.25}{\pi * (20)}$

(3) $I_{TRms} = \frac{V_s \sqrt{K}}{\sqrt{2}\, R} = \frac{230\sqrt{0.25}}{\sqrt{2}\, 20} = 4.065A$

(3) The single phase ACVC delivers a power of 5kW to the resistive load (R = 15Ω, if the ON-off Control strategy is used & the supply voltage is 230V, 50Hz. Calculate
a) Rms Output voltage and current b) duty cycle
c) power factor d) Rms and average value of thyristor current.

ans. $R = 5\Omega$, $V_s = 230V$, $f = 50Hz$, $P = 5KW$

$$P = \frac{V_{oRms}^2}{R} \Rightarrow V_{oRms} = \sqrt{R*P} = \sqrt{5*5K} = \sqrt{25000} = \underline{158.1V}$$

$$I_{orms} = \frac{V_{oRms}}{R} = \frac{158.1}{5} = 31.62\,A$$

$$V_o = V_s\sqrt{K}$$

$$\sqrt{K} = \frac{V_o}{V_s} = \frac{158.1}{230} = 0.687 \Rightarrow K = \sqrt{0.687} = 0.472\,//.$$

$$I_{T(Rms)} = \frac{V_{o(Rms)}}{\sqrt{2}\,R} = \frac{158.1}{\sqrt{2}*5} = 22.35\,A$$

$$I_{T(avg)} = \frac{\sqrt{2}\,V_s\,K}{\pi R} = 9.77A$$

power factor $= \sqrt{K} = 0.687$.

(4) A single phase full wave ac vc working on ON-off Control technique has supply voltage of 230V Rms, 50Hz, load 50Ω. The controller is ON for 30 cycles and OFF for 40 cycles. Calculate i) ON-off time intervals ii) Rms output voltage iii) power factor iv) Average and Rms thyristors currents.

Given $V_s = 230V$, $f = 50Hz$, $R = 50\Omega$, $n = 30$, $m = 40$, $T = \frac{1}{50} = 20\,msec$

i) ON period $T_{on} = n \cdot T = 30*(20m) = 0.6\,sec$

Off time period $T_{off} = m \cdot T = 40 * 20m = 80m = 0.8se.$

ii) $V_{orms} = V_s\sqrt{K} = 230\sqrt{0.6546} = 150.55V\,//$

iii) $PF = \sqrt{K} = 0.6546$

iv) $I_{Tavg} = \frac{K \cdot I_m}{\pi} = \frac{K \cdot V_m}{\pi R} = \frac{K \cdot \sqrt{2}\,V_s}{\pi \cdot R} = \frac{0.4 \times \sqrt{2} \times (230)}{\pi\,(50)}$

$$\boxed{I_{Tavg} = 0.88A}$$

$$I_{TRms} = \frac{I_m}{2}\sqrt{K} = \frac{V_m}{2R}\sqrt{K} = \frac{\sqrt{2}\,V_s\sqrt{K}}{2R} = \frac{\sqrt{2} \cdot V_{oRms}}{2R} = \underline{2.128A}$$

Principle of phase Angle control

• Half wave Ac Voltage Controller (or) unidirectional Controller.

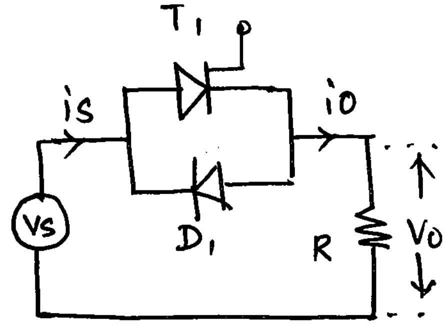
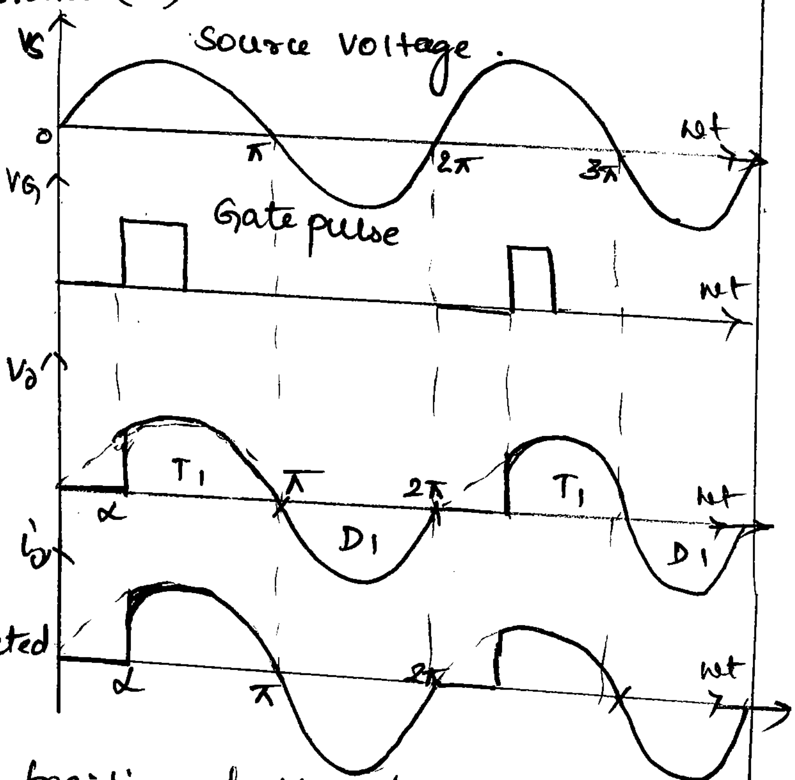


Fig1:- Circuit diagram.



→ Figure shows the circuit diagram of half wave ACVC.

→ The SCR T_1 and D_1 are connected in anti parallel direction.

Due to One SCR, only the positive half cycle will be controlled and entire negative half cycle of ac supply appears across the load without any changes as depicted in waveforms.

Working:-

→ During positive half cycle of an ac supply, SCR is turned ON at $\boxed{wt=\alpha}$ which makes load voltage positive & equal to the instantaneous ac supply voltage. The SCR will be turned off due to natural commutation at $\boxed{wt=\pi}$.

→ During negative half cycle of ac supply, D_1 diode is forward biased $\boxed{wt=\pi}$ and entire -ve cycle appears across the load and D_1 is turned off at $\boxed{wt=2\pi}$

→ Thus load voltage can be controlled by controlling the firing angle "α" of the SCR.

Advantages:-

1. Circuit seems to simple and since there's only SCR control circuit is simple thus circuit less expensive.

Disadvantages:-

- Output voltage is not controlled fully since negative half cycle is uncontrolled due to diode D_1.
- The supply current, output current and output voltage have DC component this is proplematic for inductive loads.

Applications:-

→ Limited for heating and lighting applications.

Rms output voltage

$$V_{orms} = \left[\frac{1}{2\pi}\int_0^T V_s^2\, d\omega t\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\int_\alpha^\pi V_s^2\, d\omega t + \int_\pi^{2\pi} V_s^2\, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_\alpha^\pi \sin^2\omega t\, d\omega t + \int_\pi^{2\pi} \sin^2\omega t\, d\omega t\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi}\int_\alpha^\pi \frac{1-\cos 2\omega t}{2} d\omega t + \int_\pi^{2\pi}\frac{1-\cos 2\omega t}{2} d\omega t\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi}\left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_\alpha^\pi + \left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_\pi^{2\pi}\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi}\left(\frac{\pi}{2} - \frac{\alpha}{2} - \frac{\cancelto{0}{\sin 2\pi}}{4} + \frac{2\pi}{2} - \frac{\pi}{2} - \frac{\cancelto{0}{\sin 4\pi}}{4} + \frac{\cancelto{0}{\sin 2\pi}}{4}\right)\right]^{1/2}$$

$$= V_m\left[\frac{1}{2\pi}\left[\pi - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4}\right]\right]^{1/2}$$

$$= \frac{V_m}{2}\sqrt{\frac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}} \quad \cdots (1)$$

//.

Interms of supply voltage V_s

W.K.T $V_m = \sqrt{2}V_s$

$$\therefore V_{orms} = \frac{\sqrt{2}V_s}{2}\left(\frac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}\right)^{1/2}$$

$$V_{orms} = \frac{V_s}{\sqrt{2}}\left(\frac{2\pi - \alpha + \sin 2\alpha/2}{\pi}\right)^{1/2}$$

The Rms current can be written as $\boxed{I_{orms} = \frac{V_{oRms}}{R}}$

NOTE:-

for $\alpha = 0$

(1) $V_{orms} = \frac{V_m}{2}\left(\frac{2\pi}{\pi}\right)^{1/2} = \frac{V_m}{\sqrt{2}}$

(2) for $\alpha = \pi$

$$V_{orms} = \frac{V_m}{2}\left(\frac{2\pi - \pi}{\pi}\right)^{1/2} = \frac{V_m}{2}$$

$$\boxed{V_{orms} = \frac{V_m}{2} = \frac{V_s}{\sqrt{2}}}$$

Average Value of output Voltage (V_{oavg} or V_{odc})

$$V_{o(avg)} = \frac{1}{T}\int_0^T V_s \, d\omega t$$

$$= \frac{1}{2\pi}\left[\int_\alpha^\pi V_m \sin\omega t \, d\omega t + \int_\pi^{2\pi} V_m \sin\omega t \, d\omega t\right]$$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\Big|_\alpha^\pi + -\cos\omega t\Big|_\pi^{2\pi}\right]$$

$$= \frac{V_m}{2\pi}\left[-\cancel{\cos}\pi + \cos\alpha - \cos 2\pi + \cancel{\cos}\pi\right]$$

$$\boxed{V_{avg} = \frac{V_m}{2\pi}\left[\cos\alpha - 1\right]}$$

NOTE: The average current is given by

$$\boxed{I_{o(avg)} = \frac{V_m}{2\pi R}(\cos\alpha - 1)}$$

NOTE: $V_{o(dc)} = V_{o(avg)} = \frac{V_s}{\sqrt{2}\pi}(\cos\alpha - 1)$

When $\alpha = 0$, $V_{o(avg)} = 0$

$\alpha = \pi$ $\quad V_{o(avg)} = \frac{V_s}{\sqrt{2}\pi}(\cos\pi - 1) = \frac{-2V_s}{\sqrt{2}\pi} = -\frac{\sqrt{2}V_s}{\pi}$

The dc component depends on the firing angle α.
If α is varied from 0 to π, V_{dc} varies from 0 to $-\sqrt{2}V_s/\pi$ and V_{orms} varies from V_s to $V_s/\sqrt{2}$.

1) A single phase half wave ac voltage controller has a resistance load $R = 5\Omega$ and input voltage $V_s = 120V$, 60Hz. The delay angle of SCR is $\alpha = \pi/3$, find i) Rms o/p voltage ii) power factor iii) Average input current.

Given:- $R = 5\Omega$, $V_s = 120V$, $f = 60Hz$ $\alpha = \pi/3$

$$V_{orms} = V_s\left(\frac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{2\pi}\right)^{1/2} = 120\left[\frac{2\pi - \pi/3 + \frac{\sin 2\pi/3}{2}}{2\pi}\right]^{1/2}$$

$$= 120(0.94) = 113.98 \simeq 114V$$

ii) power factor $PF = \dfrac{V_{o(rms)} I_{o(rms)}}{V_{s(rms)} I_{s(rms)}}$

$$PF = \frac{V_{o(rms)}}{V_{s(rms)}} = \frac{114}{120} = 0.95$$

iii) $V_{o(avg)} = \dfrac{V_m}{2\pi}(\cos\alpha - 1) = \dfrac{\sqrt{2}V_s}{2\pi}(\cos\pi/3 - 1)$

$$= \frac{\sqrt{2} * 120}{2\pi}\left(\frac{1}{2} - 1\right)$$

$\boxed{V_{o(avg)} = -13.5V}$ $\quad I_{o(avg)} = \dfrac{V_{o(avg)}}{R} = \dfrac{-13.5}{5} = -2.7A$

$\boxed{I_{o(avg)} = -2.7A}$

(2) A single phase ac vc has a resistive load of $R = 10\Omega$ & input voltage is $V_s = 120V$, 60Hz the delay angle of T_1 SCR is $\alpha = \pi/2$. find the foll. i) Rms output voltage ii) power factor iii) Average current.

Sol. Given $R = 10\Omega$, $\alpha = \pi/2$, $V_s = 120V$, $f = 60Hz$

i) $V_{oRms} = \dfrac{V_m}{2}\sqrt{\dfrac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}} = V_s\sqrt{\dfrac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{2\pi}} = 120\left(\dfrac{2\pi - \pi/2 + \frac{\sin\pi}{2}}{2\pi}\right)^{1/2}$

$= 120\sqrt{3/4}$ $\quad \therefore \boxed{V_{oRms} = 103.92V}$

ii) power factor $PF = \dfrac{V_{orms}}{V_{srms}} = \dfrac{103.92}{120} = \underline{\underline{0.866}}$

iii) Average output voltage $V_{oavg} = \dfrac{V_m}{2\pi}(\cos\alpha - 1) = \dfrac{\sqrt{2}V_s}{2\pi}(\cos\alpha - 1)$

$V_{o(avg)} = \dfrac{\sqrt{2} * 120}{2\pi}\left(\cos\dfrac{\pi}{2} - 1\right) = \dfrac{\sqrt{2} * 120}{2\pi}(-1)$

$\boxed{V_{o(avg)} = -27.009V}$ $\quad$ Now average current $I_{oavg} = \dfrac{V_{oavg}}{R} = \dfrac{-27.009}{10}$

$\boxed{I_{oavg} = -2.7A}$

NOTE:- -ve sign in I_{oavg} signifies that input current during +ve half cycle is less than that during the negative half cycle.

(3) A 1ϕ half wave ac vc has an input voltage of 120V, 60Hz & a load resistance of 10Ω. the firing angle of thyristor is 60°. find the i) Rms o/p voltage ii) Input power factor iii) Input average current

(July 2016)

Given :- $V_S = 120V$, $f = 60Hz$ $\alpha = 60° = \pi/3$

$$V_{o(rms)} = \frac{V_m}{2}\sqrt{\frac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}} = V_s\sqrt{\frac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{2\pi}}$$

$$= 120\left(\frac{2\pi - \pi/3 + \frac{\sin(2\pi/3)}{2}}{2\pi}\right)^{1/2} = 114V$$

ii) $PF = \frac{V_{o(Rm)}}{V_{s(Rm)}} = \frac{114}{120} = 0.95$

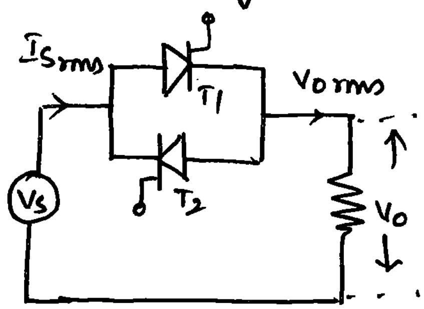
iii) $V_{o\,avg} = \frac{V_m}{2\pi}(\cos\alpha - 1) = \frac{V_m}{2\pi}(\cos 60° - 1)$

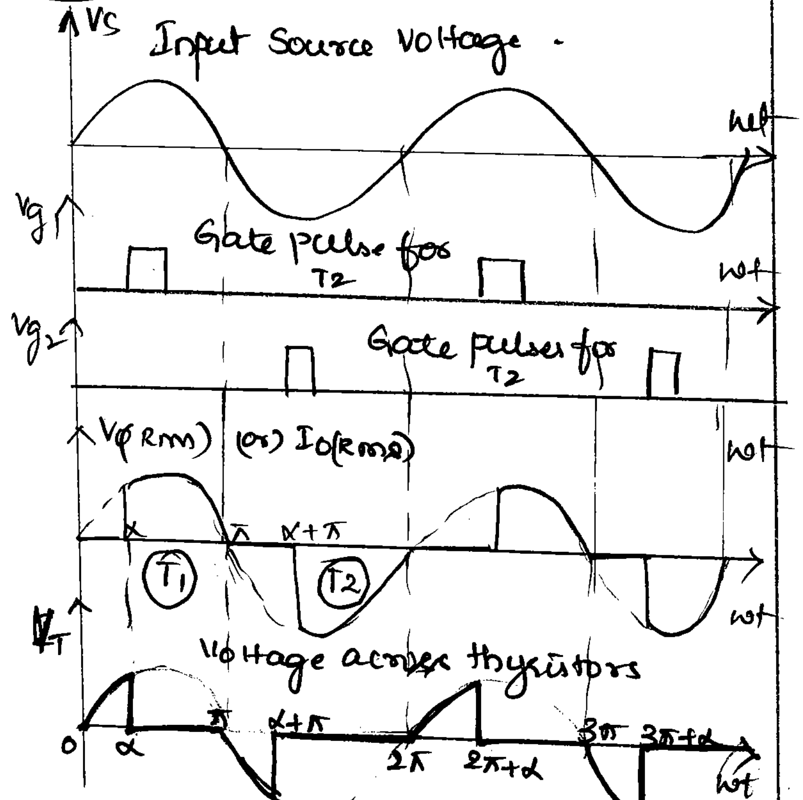
$= \frac{\sqrt{2}V_s}{2\pi}(-0.5) = \frac{\sqrt{2}\,120}{2\pi}(-0.5)$

$= -13.5V$

$$I_{o\,avg} = \frac{V_{o(avg)}}{R} = \frac{-13.5}{10} = -1.35A$$

Single phase full wave Ac voltage controller (or) Bidirectional Ac voltage controller with Resistive load.





→ The circuit diagram and relavant waveforms are shown above. The circuit has two SCR's namely T_1 and T_2 in antiparallel direction.

→ In the positive half cycle of the supply T_1 controls the power flow to the load and during negative half cycle T_2 controls the power flow to the load ∴ Thus the rms o/p voltage can be controlled by varying the firing angle α from 0 to 180°.

→ Since the power flow to the load is controlled in the both half cycle of ac supply the circuit is known as bidirectional controller.

→ The output voltage and current are in phase i.e, symmetric.

→ The SCR's turns off due to natural commutation at the end of corresponding half cycles of supply voltage.

Advantages:- ① power control to load is possible in both cycles.

② The average value of load current and supply current is zero ∴ there's no possibility of core saturation of Induction motor when used as a load.

Thyristor average and Rms current:-

$$I_{Tavg} = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t \, d\omega t$$

$$= \frac{I_m}{2\pi} \left(-\cos\omega t\right)_{\alpha}^{\pi}$$

$$= \frac{I_m}{2\pi}\left(-\cos\pi + \cos\alpha\right)$$

$$\boxed{I_{Tavg} = \frac{I_m}{2\pi}\left(1+\cos\alpha\right)}$$

Average diode current

$$I_{Davg} = \frac{1}{2\pi}\int_{\pi}^{2\pi} I_m \sin\omega t \, d\omega t$$

$$= \frac{I_m}{2\pi}\left(-\cos\omega t\right)_{\pi}^{2\pi}$$

$$= \frac{I_m}{2\pi}\left(-\cos 2\pi + \cos\pi\right)$$

$$= \frac{I_m}{2\pi}(-2)$$

$$\boxed{I_{Davg} = \frac{I_m}{\pi}}$$

Rms current

$$I_{TRms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$I_{TRms} = \left[\frac{I_m^2}{2\pi}\int_{\alpha}^{\pi} \frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_{\alpha}^{\pi}\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\pi}{2} - \frac{\alpha}{2} - \frac{\sin 2\pi}{4} + \frac{\sin 2\alpha}{4}\right)\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\pi - \alpha + \sin 2\alpha/2}{2}\right)\right]^{1/2}$$

$$= \left[\frac{I_m}{\sqrt{2}}\left(\frac{\pi - \alpha + \sin 2\alpha/2}{2\pi}\right)^{1/2}\right]$$

Diode Rms current

$$I_{DRms} = \left[\frac{I_m^2}{2\pi}\int_{\pi}^{2\pi} \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\int_{\pi}^{2\pi} \frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_{\pi}^{2\pi}\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{2\pi}{2} - \frac{\pi}{2} - \frac{\sin 4\pi}{4} + \frac{\sin 2\pi}{4}\right)\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\,\pi/2\right]^{1/2} = \left(\frac{I_m^2}{4}\right)^{1/2}$$

$$\boxed{I_{DRms} = \frac{I_m}{2}}$$

Disadvantages:-

① Due to the presence of two SCR's ckt becomes slightly complicated & expensive.

Applications

1. used in induction motors, pumps etc.
2. as an fan Regulator
3. Heater control 4. light dimmer.

Rms output Voltage:-

$$V_{o\,rms} = \left[\frac{1}{T}\int_0^T V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2} = \left[\frac{1}{2\pi}\int_\alpha^\pi V_m^2 \sin^2\omega t\, d\omega t + \int_{\pi+\alpha}^{2\pi} V_m^2 \sin^2\omega t\, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_\alpha^\pi \frac{1-\cos 2\omega t}{2} d\omega t + \int_{\pi+\alpha}^{2\pi} \frac{1-\cos 2\omega t}{2} d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_\alpha^\pi + \left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_{\pi+\alpha}^{2\pi}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\alpha}{2} - \cancel{\frac{\sin 2\pi}{4}}^{0} + \frac{\sin 2\alpha}{4}\right) + \frac{2\pi}{2} - \frac{(\pi+\alpha)}{2} - \cancel{\frac{\sin 4\pi}{4}}^{0} + \frac{\sin 2(\pi+\alpha)}{2}\right)^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left(\frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} + \pi - \frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin 2(\pi+\alpha)}{4}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left(\pi - \frac{\alpha}{2} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} + \frac{\sin 2\alpha}{4}\right]\right)^{1/2} \quad \because \sin 2(\pi+\alpha) = \sin(2\pi + 2\alpha) = \sin 2\alpha$$

$$= \left[\frac{V_m^2}{2\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$\boxed{V_{o(rms)} = V_m\sqrt{\frac{\pi - \alpha + \sin 2\alpha/2}{2\pi}}} = \sqrt{2}\,V_s\left(\frac{\pi - \alpha + \sin 2\alpha/2}{2\pi}\right)^{1/2}$$

Note:- When $\alpha = 0$, $V_{orms} = \frac{V_m}{\sqrt{2}} = V_s$

$\alpha = \pi$, $V_{orms} = 0$

Output can be Controlled from 0 to $V_{s(rms)}$ by varying firing angle α from π to zero. Since the output voltage and current as well as supply current is symmetric, their dc or average values are zero hence transformer saturation problem are absent.

Q. Show that average Voltage $V_{oavg} = 0$ for Bidirectional AC VC.

$$V_{oavg} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} V_m \sin\omega t + \int_{\pi+\alpha}^{2\pi} V_m \sin\omega t \, d\omega t\right]$$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\Big)_{\alpha}^{\pi} + -\cos\omega t\Big)_{\pi+\alpha}^{2\pi}\right]$$

$$= \frac{V_m}{2\pi}\left[-\cos\pi + \cos\alpha - \cos 2\pi + \cos(\pi+\alpha)\right] \quad \because \cos \pi+\alpha = -\cos\alpha.$$

$$= \frac{V_m}{2\pi}\left[\cancel{1} + \cancel{\cos\alpha} - \cancel{1} + (-\cancel{\cos\alpha})\right]$$

$$= \frac{V_m}{2\pi}(0)$$

$$\boxed{V_{oavg} = 0} \quad |||^{ly} \quad I_{oavg} = \frac{V_{oavg}}{R} = 0.$$

Average and Rms Current for thyristor.

$$I_{Tavg} = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t \, d\omega t$$

$$= \frac{I_m}{2\pi} -\cos\omega t\Big)_{\alpha}^{\pi}$$

$$I_{Tavg} = \frac{I_m}{2\pi}(1+\cos\alpha)$$

$$\boxed{I_{Tavg} = \frac{V_m}{2\pi R}(1+\cos\alpha)}$$

$$I_{Tavg} = \frac{V_s\sqrt{2}}{2\pi R}(1+\cos\alpha)$$

$$I_{TRms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\int_{\alpha}^{\pi} \frac{1-\cos 2\omega t}{2} \, d\omega t\right]^{1/2}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_{\alpha}^{\pi}\right]^{1/2}$$

$$= \left\{\frac{I_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\alpha}{2} - \frac{\cancel{\sin 2\pi}^{\,0}}{4} + \frac{\sin 2\alpha}{4}\right]\right\}^{1/2}$$

$$= \left(\frac{I_m^2}{2\pi}\left(\pi/2 - \alpha/2 + \frac{\sin 2\alpha}{2*2}\right)\right)^{1/2}$$

$$\boxed{I_{TRms} = \frac{I_m}{2}\sqrt{\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}}}$$

$$= \frac{V_m}{2R}\sqrt{\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}}$$

$$\boxed{I_{TRms} = \frac{V_{ORms}}{\sqrt{2}R}} \quad (or) \quad \boxed{I_{TRms} = \frac{I_{ORms}}{\sqrt{2}}}$$

A single phase full wave ac vc has a resistive load $R = 10\Omega$ & $V_s = 120V$ (Rms), 60Hz. The delay angles of thyristors T_1 and T_2 are $\alpha_1 = \alpha_2 = \alpha = \pi/2$. find the i) V_o Rms ii) input pF iii) Thyristor average current & Rms current

Sol, $R = 10\Omega$ $V_s = 120V$ $\alpha = \pi/2$, $V_m = \sqrt{2} * 120 = 169.7V$

$$V_o = V_s\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2} = V_s\left[\frac{1}{\pi}\left(\pi - \pi/2 + \frac{\sin 2 * \pi/2}{2}\right)\right]^{1/2}$$

$$V_o = V_s\left[\frac{1}{\pi} \pi/2\right]^{1/2}$$

$$= V_s/\sqrt{2} = \frac{120}{\sqrt{2}} = 84.85V$$

Rms load current $I_o = \frac{V_o}{R} = \frac{84.85}{10}$

$= 8.485$

$$P_o = I_o^2 R = 8.485^2 * 10$$

$$= 719.95W$$

$$P.F = \frac{V_o \text{ Rms } I_o \cancel{\text{Rms}}}{V_s \text{ Rms } I \cancel{\text{srms}}} = \frac{84.85}{120}$$

Thyristor average current

$$I_{Tavg} = \frac{I_m}{2\pi}(1 + \cos\alpha)$$

$$= \frac{V_m}{2\pi R}(1 + \cos\alpha)$$

$$= \frac{\sqrt{2} * V_s}{2\pi R}(1 + \cos\alpha)$$

$$= \sqrt{2}\frac{120}{2\pi \times 10} = \underline{\underline{2.7A}}$$

$$I_{TRms} = \frac{V_s}{\sqrt{2}R}\left(\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right)^{1/2}$$

$$= \frac{120}{2 \times 10} = \underline{\underline{6A}}$$

(or)

$$\boxed{I_{TRm} = \frac{V_o \text{ Rms}}{\sqrt{2}R} = \frac{84.85}{\sqrt{2} * 10} \approx \underline{6A}}$$

The thyristor ~~average~~ current

$$I_{TRms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= I_m\left[\frac{1}{2\pi}\int_{\alpha}^{\pi}\frac{1-\cos 2\omega t}{2}\, d\omega t\right]^{1/2}$$

$$= I_m\left[\frac{1}{2\pi}\left(\frac{\omega t}{2}\right)_{\alpha}^{\pi} - \frac{\sin 2\omega t}{2}\Big|_{\alpha}^{\pi}\right]^{1/2}$$

$$= I_m\left[\frac{1}{2\pi}\left\{\frac{\pi}{2} - \frac{\alpha}{2} - \cancel{\frac{\sin 2\pi}{2}} + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

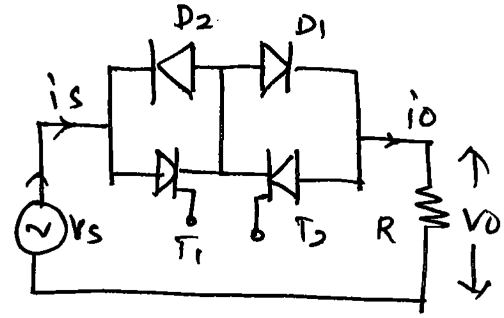
$$= I_m\left[\frac{1}{2\pi}\left(\frac{\pi - \alpha + \sin 2\alpha}{2}\right)\right]^{1/2}$$

$$I_{TRms} = \frac{I_m}{2}\left[\sqrt{\frac{\pi - \alpha + \sin 2\alpha}{\pi}}\right]$$

$$= \frac{V_m}{(\sqrt{2})^2 R}\left[\sqrt{\frac{\pi - \alpha + \sin 2\alpha}{\pi}}\right]$$

$$\boxed{I_{TRms} = \frac{V_{ORms}}{\sqrt{2}\, R}}$$

Single phase full-wave AC voltage controller with common cathode.



The circuit diagram of common cathode thyristors full-wave ACVC is as shown.

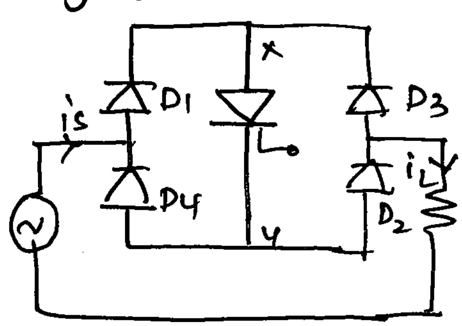
* The cathodes of two thyristors T_1 & T_2 are connected together.

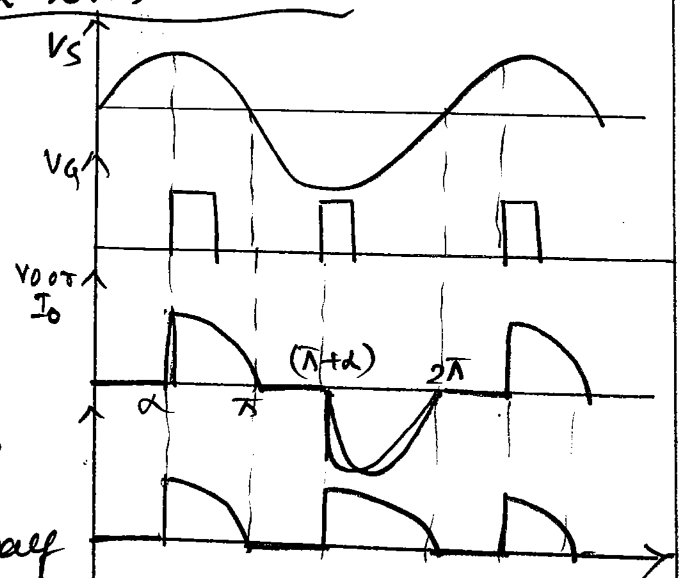
Working:-

* During positive half cycle of the supply SCR T_1 and diode D_1 conducts and in the negative half cycle SCR T_2 & diode D_2 conducts.
* In this circuit, the gate cathode drives need not to be isolated but isolation is normally provided between the control and power circuits.
* Four devices are required in this ckt, the efficency is slightly reduces due to increased power dissipation in the devices.

NOTE:- The input and output waveforms is same as that of full wave ac VC.

Single phase full wave controller with One SCR





* The circuit has 4 diodes in bridge connection with one SCR as shown above.
* The SCR is gated during every half cycle.

Operation:-

* During positive half cycle, SCR T_1 is triggered at $\boxed{\omega t = \alpha}$ the current will flow through $D_1 - T_1 - D_2$ & the load. At $\boxed{\omega t = \alpha}$ load voltage and current reduces to zero & SCR T_1 is turned off due to natural commutation.

* During -ve half cycle, SCR T_1 is triggered at $\pi + \alpha$, the current flows through $D_3 - T_1 - D_4$ & the load.

* During the time interval $\omega t = 0$ to α, & π to $(\pi + \alpha)$ the SCR is off, ∴ load voltage and load current will be zero during this time intervals.

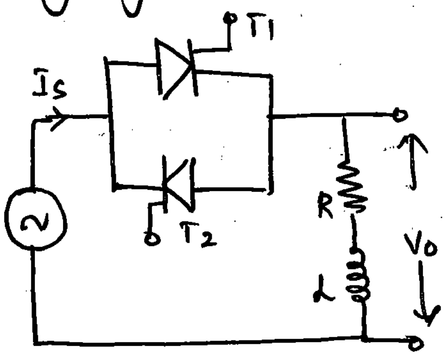
Advantages.

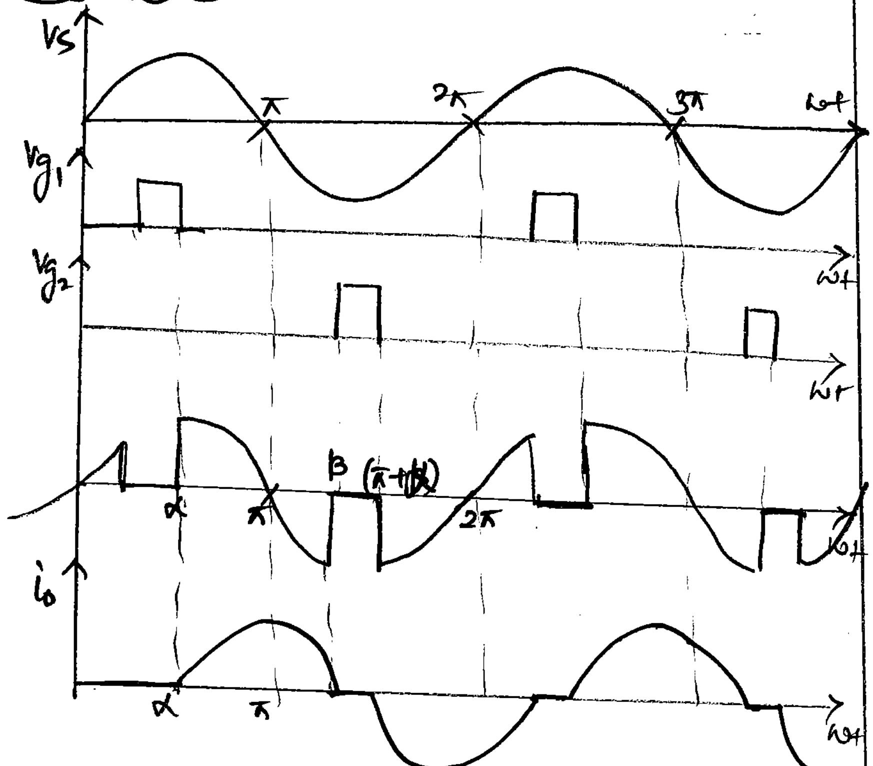
1. The input and output currents & voltage are symmetric hence there is no DC component.
2. Transformers and motors saturation problems are absent.

Applications

1. Full wave controllers are used extensively for induction motors, pumps, fans etc.

Single phase Controllers with Inductive loads





Working:- i) interval ωt = α to π

→ At $\boxed{\omega t=\alpha}$, SCR T_1 is turned ON by applying a triggering pulse. the load voltage is positive and equal to instantaneous supply voltage.

→ load current starts increasing gradually as the load is inductive. During this interval, load inductor will store energy.

ii) For ωt = π to β

At $\boxed{\omega t=\pi}$ the ac supply becomes negative, but due to stored energy, the load inductance will maintain SCR T_1 in ON state ie SCR T_1 conducts from π to β due to energy stored in the load inductance.

At ωt = β, the output current becomes zero hence T_1 turns by natural commutation.

iii) For ωt = β to π + α :-

During this interval, both the SCRs remain off ∴ output voltage and current is zero.

iv) for $\omega t = (\pi+\alpha)$ to 2π

* SCR T_2 is ON at $\omega t = (\pi+\alpha)$ the load voltage becomes negative and equal to the instantaneous supply voltage.
* load current increases gradually in the negative direction. as load voltage and current both are negative the load inductor will store energy. Due to this SCR T_2 continues to conduct even in next +ve half cycle as shown in figure
* This is due to the stored energy in the inductive load. The load current continues to be negative but load voltage becomes +ve. as the load voltage and current have opposite polarities, the load stored energy is returned to ac source.

Rms output voltage for Inductive load.

let supply voltage be $V_s = V_m \sin\omega t$

$$\therefore V_{o(rms)} = \left[\frac{1}{T}\int_0^T V_s^2 \, d\omega t\right]^{1/2} = \left[\frac{V_m^2}{\pi}\int_\alpha^\beta \sin^2\omega t \, d\omega t\right]^{1/2} = \left[\frac{V_m^2}{\pi}\int_\alpha^\beta \frac{1-\cos 2\omega t}{2}\, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left\{(\omega t)_\alpha^\beta - \left(\frac{\sin 2\omega t}{2}\right)_\alpha^\beta\right\}\right]^{1/2} = \left[\frac{V_m^2}{2\pi}\left(\beta - \alpha - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$\boxed{V_{ORms} = V_m\left[\frac{\beta - \alpha + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}}{2\pi}\right]^{1/2}}$$

NOTE:-

1. Average dc voltage and current values are zero as both are symmetric i.e. $\boxed{I_{dc} = V_{dc} = 0}$
2. The period T should be considered as $\boxed{T = \pi}$ because values of +ve and -ve half cycles are same for symmetric waveforms.

To determine the conduction angle β.

W.K.T

$$V_S = V_m \sin\omega t \quad \cdots (1)$$

When thyristor T_1 conducts, by applying KVL

$$V_S - i(t)R - L\frac{di(t)}{dt} = 0$$

$$V_S = i(t)R + L\frac{di(t)}{dt}$$

$$V_m \sin\omega t = i(t)R + L\frac{di(t)}{dt}$$

using laplace transform, above equation can be solved for $i(t)$ hence solution is

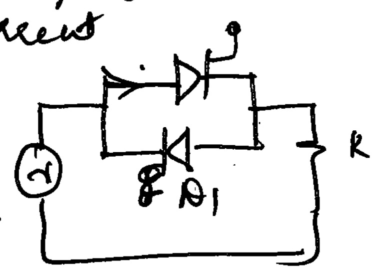
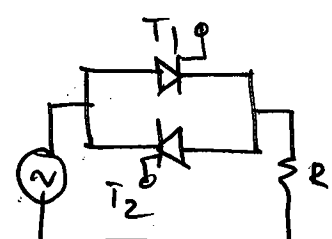
$$i(t) = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \left\{ \sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{\frac{R(\alpha - \omega t)}{\omega L}} \right\} \cdots (2)$$

where $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ from the output waveform $\omega t = \beta$ & $i(t) = 0$

$$\sin(\beta - \theta) - \sin(\alpha - \theta)\, e^{\frac{R(\alpha - \beta)}{\omega L}} = 0 \quad \cdots (3)$$

β can be solved, if α and θ values are given.

comparison of Half wave and full wave AC VC.

Half wave	Full wave.
1) only one half cycle of the supply is controlled	1) Both the half cycles of supply are controlled.
2. These controller generate Asymmetric voltage & current across load.	2) Generate Symmetric Voltage and current across load
3. Rms load Voltage $V_0 = \frac{V_m}{2}\left(\frac{(2\pi-\alpha)+\frac{\sin 2\alpha}{2}}{\pi}\right)^{1/2}$	(3) $V_{ORms} = \frac{V_m}{\sqrt{2}}\left(\frac{\pi-\alpha+\frac{\sin 2\alpha}{2}}{\pi}\right)^{1/2}$
4 $V_{dc} = \frac{V_m}{2\pi}(\cos\alpha - 1)$	(4) $V_{dc} = 0$.
5. $I_{dc} = \frac{V_{dc}}{R}$	(5) $I_{dc} = 0$
6. Core saturation is possible due to the presence of dc value for supply & load current	(6) Core saturation does not occurs as $I_{dc} = 0$.
(7) 	(7) 
(8) They are prefered for small loads	(8) Prefered for large loads.

ON - off Control	Phase Angle Control.
1. output voltage is controlled by controlling the no of ON cycles 'n' and no of off cycles 'm'.	1. output Voltage is controlled by controlling the phase angle or firing angle α of the thyristor.
2. Thyristors conducts 180° as it is triggered at $\alpha = 0°$	2. Thyristor is triggered at some angle depending upon o/p power.
3. No harmonics	3. Harmonics are present
4. load voltage is distorted	4. It's distorted
5. load average Voavg & Ioavg is zero	5. Vo(avg) + Ioavg is zero
6. Industrial heating, speed control of motor applications they are used.	6. Heat control, Light dimmer, fan regulator etc applications they are used.

What are the applications of AC VC.

1. fans & lighting controls
2. Speed control of AC motors
3. Saturable Core reactors
4. Industrial heating furances.

What is the Problem caused by sharp single pulse triggered in 1ϕ AC VC Controller when the load is Inductive? How can this be solved.

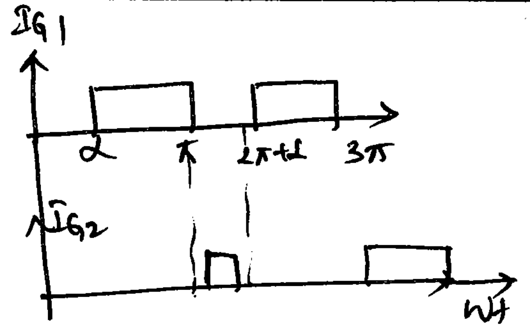
If the load is highly inductive, gating pulse is applied to T_2 at $(\pi+\alpha)$. But T_1 continues to conduct beyond $(\pi+\alpha)$ due to inductive load. hence T_2 doesnot turn on at $(\pi+\alpha)$ even if gate pulse is applied T_2 starts conducting when current of T_1 goes to zero.

Hence gate pulse of T_2 must be wide enough so that it is present at the time when T current current goes to zero. then Only T_2 will turn ON.

If gate pulse is of Short duration, then T_2 will not conduct since there is no gating pulse when it is forward biased

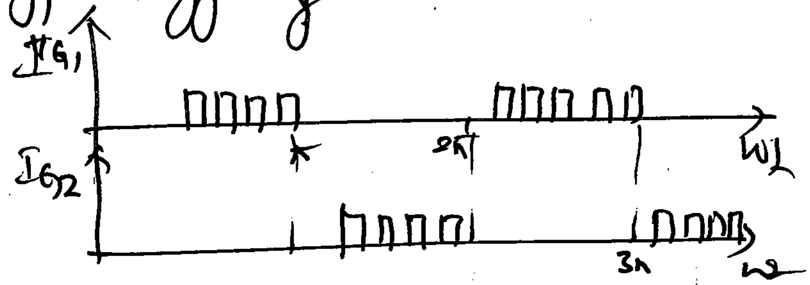
∴ short duration gating pulses are not suitable for inductive loads.

this problem can be solved by using a Continous gate pulse of duration $(\pi-\alpha)/w$ as shown in figure.



* as soon as T_1 current falls to zero at $\boxed{\omega t = \beta}$ T_2 will get forward biased & turned on since the gate pulse of T_2 is still available but this method of triggering causes higher gate power loss & switching loss as well.

So better to use a train of sharp high frequency pulses to trigger the thyristor as shown below. because this reduces the size of isolation pulse transformer required in the firing ckt, this reduces the switching & gate power losses in the switches. this type triggering is known as carrier frequency triggering



A single phase full wave ACVC, has a load Resistance of $R=10\Omega$ and input voltage of 120V, 60Hz, the delay angle for both thyristor is $\pi/2$ find 1) Rms o/p voltage 2) power factor 3) Average thyristor Current

July 2012

Given $R = 10\Omega$, $V_S = 120$ $f = 60Hz$ $\alpha_1 = \alpha_2 = \pi/2$

$V_m = \sqrt{2}\, V_S = \sqrt{2}\, 120 = 169.70$

$$V_{O\,rms} = V_m \sqrt{\frac{(\pi-\alpha) + \frac{\sin 2\alpha}{2}}{2\pi}} = 169.7 \sqrt{\frac{\pi - \pi/2 + \frac{\sin 2\times\pi/2}{2}}{2\pi}}$$

$$= 169.7 \sqrt{\frac{\pi/2}{2\pi}} = 169.7 \sqrt{\frac{\pi}{4\pi}} = 84.85V$$

$$I_{O\,rms} = I_{rms} = \frac{V_{o(rms)}}{R} = \frac{84.85}{10} = 8.485A$$

(2) $PF = \frac{V_o}{V_S} = \frac{84.85}{120} = 0.707$

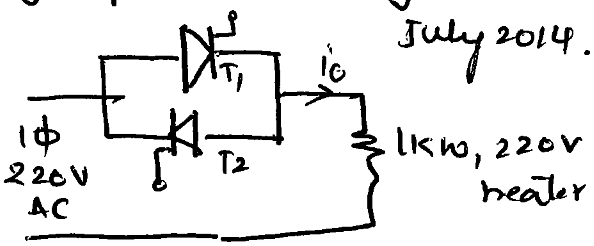
(3) $I_{Tavg} = \frac{I_m}{2\pi}(\cos\alpha + 1) = \frac{V_m}{2\pi R}(\cos\pi/2 + 1) = \frac{169.2}{2\pi(10)}$

$\boxed{I_{Tavg} = 2.7A}$

$I_{Trms} = \frac{I_{o(rms)}}{\sqrt{2}} = \frac{8.485}{\sqrt{2}} = 5.99A$

Find the Rms and average current flowing through the heater shown in fig. The delay angle of both the thyristors is 45°

July 2014.



$V_s = 220V$, $f = 50Hz$ $\alpha = \pi/4$

$P_{rated} = 1KW$, $V_{rated} = 220V$.

heater is 1KW, 220V it indicates it dissipates 1KW at 220V

$$P_{rated} = V^2_{rated}/R$$

$$R = \frac{V^2_{rated}}{P_{rated}} = \frac{(220)^2}{1K} = 48.4 \Rightarrow \boxed{R = 48.4\ \Omega}.$$

$$V_{orms} = V_s\left(\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}\right)^{1/2} = 220\left(\frac{\pi - \pi/4 + \frac{\sin \pi/2}{2}}{\pi}\right)^{1/2}$$

$$= 220\left(\frac{0.75*\pi + 0.5}{\pi}\right)^{1/2}$$

$$\boxed{V_{orms} = 209.76\ V}$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{209.76}{48.4} = 4.33\ A$$

Average current $I_{dc} = 0$.

iii) $P.F = \frac{V_{oRms}}{V_{s\,Rms}} = \frac{209.76}{220} = 0.95$

July 2018

(1) A single phase full wave AC VC has input of 230V + a load having 10 Ω, if the firing angle is 45°, Calculate the power absorbed by the load of 50 Ω.

Given:- $V_s = 230V_s$ $R = 10\,\Omega$ $\alpha = 45°$.

To find $P_o = V_{oRms} I_{oRms}$ (or) $\frac{V_{oRms}^2}{R}$.

$V_m = \sqrt{2} * 230$

$= 325.26V$

$V_{oRms} = V_s \sqrt{\frac{(\pi - \alpha) + \frac{\sin 2\alpha}{2}}{\pi}}$

$= \cancel{325.26}\ 230 \sqrt{\frac{(\pi - \pi/4) + \frac{\sin 2*\pi/4}{2}}{\pi}}$

$= 230 \sqrt{\frac{0.75*\pi + 0.5}{\pi}}$

$= 230 * 0.95 = 219.30V$

$P_o = \frac{V_{oRms}^2}{R}$

$= \frac{(219.30)^2}{10}$

$\boxed{P_o = 4.8\,KW}$