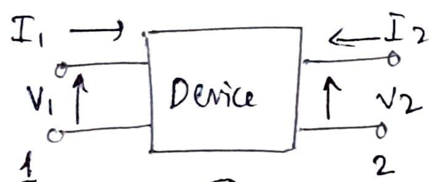
module - 2

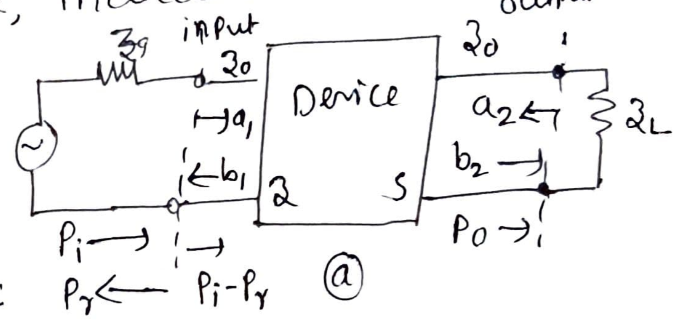
Microwave Network Theory:

A microwave network is formed when several microwave devices and components are coupled together by transmission lines for the desired transmission of a microwave signal. The point of inter connection of two or more devices is called a junction.

At low frequencies the physical length of the network is much smaller than the wavelength of the signal transmitted. The measurable input and output variables are voltage + current which can be related in terms of z, y, h or ABCD parameters.

consider a two port network, indicated below;





The parameters for this n/w:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad -(1)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad -(2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad -(3)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad -(4)$$

At microwave frequencies the physical length of the component or line is comparable to or much larger than the wavelength. measurement of z, y, h and ABCD parameters is difficult at microwave frequencies due to following reasons:

i) Non availability of terminal voltage and current measuring equipment.

ii) Short circuit and especially open circuit are not easily achieved for a wide range of frequencies.

iii) Presence of active devices makes the circuit unstable for short or open circuit.

Therefore microwave circuits are analysed using scattering or S-parameters which linearly relate the reflected waves' amplitude with those of incident waves.

Symmetrical Z and Y matrices for Reciprocal Network:

In a reciprocal network, the impedance and admittance matrices are symmetrical and the junction media are characterised by scalar electrical parameters μ and ϵ.

For a multiport network, let the incident wave amplitudes V_n^+ be so chosen that the total voltage $V_n = V_n^+ + V_n^- = 0$ at all ports $n = 1, 2, \ldots N$, except the i^{th} port where the fields are E_i, H_i. Similarly let $V_n = 0$ at all ports except j^{th} one where the fields are E_j, H_j. Then from the Lorentz reciprocity theorem,

$$\int_S (E_i \times H_j - E_j \times H_i) \cdot ds = 0 \quad -(5)$$

where S is the closed surface area of the conducting walls enclosing the junction and N ports in the absence of any source. Here non-zero integrals are considered over the reference planes so that

$$\sum_{n=1}^{N} \int_{tn} (E_i \times H_j - E_j \times H_i) \cdot ds = 0 \quad -(6)$$

Since all V_n except V_i and V_j are zero, $E_{ti} = n \times E_i$ and $E_{tj} = n \times E_j$ are zero on all reference planes at the corresponding ports except t_i and t_j respectively.

$$\therefore \int_{t_i} (E_i \times H_j) \cdot ds = \int_{t_j} (E_j \times H_i) \cdot ds \quad - (7)$$

$$\therefore P_{ij} = P_{ji} \quad - (8)$$

where P_{ij} represents the power at reference plane i due to an input voltage at plane j.

From the admittance matrix representation $[I] = [Y][V]$ and power relation $P = VI$ the above eqn reduces to

$$V_i V_j Y_{ij} = V_j V_i Y_{ji}$$

$$Y_{ij} = Y_{ji} \quad - (9)$$

$$Z_{ij} = Z_{ji} \quad - (10)$$

This proves that impedance and admittance matrices are symmetrical for a reciprocal junction.

Scattering or S-matrix Representation of multiport Network

Input power at the n^{th} port, $P_{in} = \frac{1}{2} |a_n|^2 \quad - (11)$

Reflected power at n^{th} port, $P_{rn} = \frac{1}{2} |b_n|^2 \quad - (12)$

where a_n and b_n represent the normalized incident wave amplitude and normalized reflected wave amplitude at the n^{th} port.

In two port network, normalized waves are represented by,

$$a_1 = \frac{V_{i1}}{\sqrt{Z_0}} = \frac{V_1 - V_{r1}}{\sqrt{Z_0}}, \quad a_2 = \frac{V_{i2}}{\sqrt{Z_0}} = \frac{V_2 - V_{r2}}{\sqrt{Z_0}} \quad - (13)$$

$$b_1 = \frac{V_{r1}}{\sqrt{Z_0}} = \frac{V_1 - V_{i1}}{\sqrt{Z_0}}, \quad b_2 = \frac{V_{r2}}{\sqrt{Z_0}} = \frac{V_2 - V_{i2}}{\sqrt{Z_0}} \quad - (14)$$

$V_1 = V_{i1} + V_{r1}$ - (15)

$V_2 = V_{i2} + V_{r2}$ - (16)

The total power flow into any port is given by,

$P = P_i - P_r = \frac{1}{2}(|a|^2 - |b|^2)$ - (17)

For a two port network, the relation between incident and reflected waves are expressed in terms of scattering parameters S_{ij}'s.

$b_1 = S_{11}a_1 + S_{12}a_2$ - (18)

$b_2 = S_{21}a_1 + S_{22}a_2$ - (19)

The physical significance of S-parameters can be described as follows:

$S_{11} = b_1/a_1 \big|_{a_2=0}$ = reflection coefficient Γ_1 at port 1 when port 2 is terminated with a matched load ($a_2=0$).

$S_{22} = b_2/a_2 \big|_{a_1=0}$ = Reflection Coefficient Γ_2 at port 2 when port 1 is terminated with a matched load ($a_1=0$)

$S_{12} = b_1/a_2 \big|_{a_1=0}$ = Attenuation of wave travelling from port 2 to port 1.

$S_{21} = b_2/a_1 \big|_{a_2=0}$ = Attenuation of wave travelling from port 1 to port 2.

For a multiport (N) networks, the s-parameters equations are expressed by,

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \quad - (20)$$

In a two port network if power fed at port 1 is P_i, power reflected at the same port is P_r, and output power at port 2 is P_o, then following losses are defined in terms of S-parameters:

$$\text{Insertion loss (dB)} = 10 \log \frac{P_i}{P_o} = 10 \log \frac{|a_1|^2}{|b_2|^2} = 20 \log \frac{1}{|S_{21}|}$$

$P_i = a_1^2$
$P_o = b_2^2$
$P_r = b_1^2$

$$= 20 \log \frac{1}{|S_{12}|} \quad - (21)$$

$$\text{Transmission loss (dB)} = 10 \log \frac{P_i - P_r}{P_o} = 10 \log \frac{1 - |S_{11}|^2}{|S_{21}|^2} \quad - (22)$$

$$= 10 \log \left(\frac{1 - P_r/P_i}{P_o/P_i} \right)$$

$$\text{Reflection loss (dB)} = 10 \log \frac{P_i}{P_i - P_r} = 10 \log \frac{1}{1 - |S_{11}|^2} \quad - (23)$$

$$\text{Return loss (dB)} = 10 \log P_i/P_r = 20 \log \frac{1}{|\Gamma|} = 20 \log \frac{1}{|S_{11}|} \quad - (24)$$

Properties of S-parameters:

a) Zero diagonal elements for perfect matched network:

For an ideal N-port network with matched termination, $S_{ii} = 0$, since there is no reflection from any port. ∴ Under perfect matched conditions, the diagonal elements of [S] are zero.

b) Symmetry of [S] for a reciprocal network:

A reciprocal device has the same transmission characteristics in either direction of a pair of ports and is characterised by a symmetric scattering matrix,

$$S_{ij} = S_{ji} \; (i \neq j) \quad - (25)$$

which results in $[S]^t = [S] \quad - (26)$

Proof:

For a reciprocal network, the impedance matrix equation is:

$$[V] = [Z][I] = [Z]([a] - [b]) = [a] + [b]$$

$$([Z] + [U])[b] = ([Z] - [U])[a]$$

$$[b] = ([Z] + [U])^{-1}([Z] - [U])[a] \quad - (27)$$

where $[U]$ is the unit matrix. The s-matrix equation for the network is:

$$[b] = [s][a] \quad - (28)$$

from (27) and (28)

$$[S] = ([Z]+[U])^{-1}([Z]-[U]) \quad - (29)$$

$$[R] = [Z]-[U], \quad [Q] = [Z]+[U] \quad - (30)$$

For a reciprocal network, the Z-matrix is symmetric.

$$\therefore [R][Q] = [Q][R]$$

$$[Q]^{-1}[R][Q][Q]^{-1} = [Q]^{-1}[Q][R][Q]^{-1}$$

$$[Q]^{-1}[R] = S = [R][Q]^{-1} \quad - (31)$$

Now the transpose of $[S]$ is:

$$[S]_t = ([Z]-[U])_t\,(([Z]+[U])^{-1})_t \quad - (32)$$

since Z-matrix is symmetrical,

$$([Z]-[U])_t = [Z]-[U] \quad - (33)$$

$$([Z]+[U])_t = [Z]+[U] \quad - (34)$$

$$\therefore \quad [S]_t = ([Z]-[U])([Z]+[U])^{-1}$$

$$= [R][Q]^{-1} = [S] \quad - (35)$$

$\therefore$ it is proved that $[S]_t = [S]$ for a symmetrical junction

(c) Unitary property for a lossless junction:

For any lossless network the sum of the products of each term of any one row or of any column of the s-matrix multiplied by its complex conjugate is unity.

For a lossless n-port device, the total power leaving N-ports must be equal to the total power input to these ports, so that

$$\sum_{n=1}^{N} |b_n|^2 = \sum_{n=1}^{N} |a_n|^2 \quad \text{or} \quad \sum_{n=1}^{N} \left| \sum_{i=1}^{n} S_{ni}\, a_i \right|^2 = \sum_{n=1}^{N} |a_n|^2$$

If only i^{th} port is excited and all other ports are matched terminated, all $a_n = 0$, except a_i so that,

$$\sum_{n=1}^{N} |S_{ni}\, a_i|^2 = \sum_{n=1}^{N} |a_i|^2 \quad - (37)$$

$$\sum_{n=1}^{N} |S_{ni}|^2 = 1 = \sum_{n=1}^{N} S_{ni}\, S_{ni}^* \quad - (38)$$

∴ for a lossless junction

$$\sum_{n=1}^{N} S_{ni} \cdot S_{ni}^* = 1 \quad - (39)$$

If all $a_n = 0$, except a_i and a_k,

$$\sum_{n=1}^{N} S_{ni} \cdot S_{nk}^* = 0 ;\ i \neq k \quad (40)$$

In matrix notation, these relations can be expressed as

$$[S^*][S]_t = [U]$$

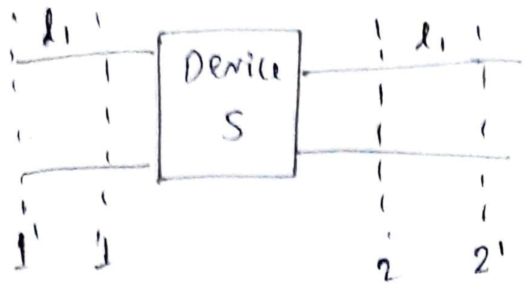
or $$[S^*] = [S]_t^{-1} \quad - (41)$$

A matrix [S] for lossless network which satisfies the above three conditions (39) to (41) is called a unitary matrix.

d) <u>phase shift property:</u>

Complex S-parameters of a network are defined with respect to the positions of the port. For a two-port network with unprimed reference planes 1 and 2 as shown below, S-parameters have definite complex values

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad - (42)$$



If this reference planes 1 and 2 are shifted outward to 1' and 2' by electrical phase shifts $\phi_1 = \beta_1 l_1$ and $\phi_2 = \beta_2 l_2$, then the new wave variables are $a_1 e^{j\phi_1}$, $b_1 e^{-j\phi_1}$, $a_2 e^{j\phi_2}$, $b_2 e^{-j\phi_2}$.

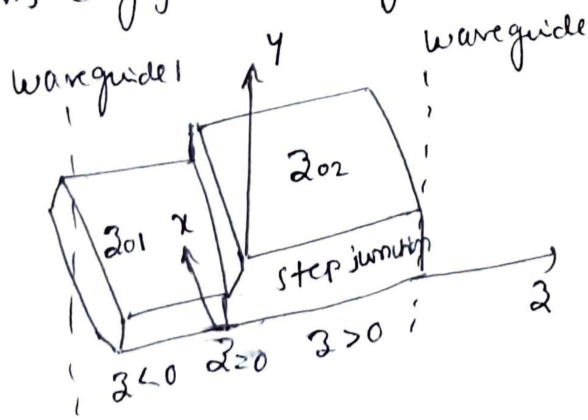
The new S-matrix S' is then given by,

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} \quad - (43)$$

This property is valid for any number of ports and is called the phase shift property applicable to a shift of reference planes

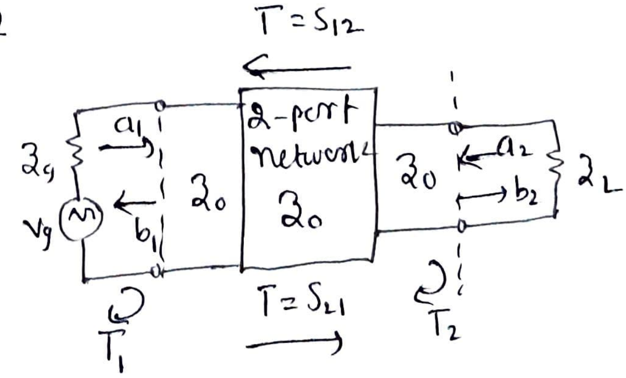
S-parameters of a Two-port Network with mismatched load:

A two port network is formed when there is a discontinuity between the input and output ports of a transmission line. Many configurations of such junctions practically exist,



(a)

waveguide step junction



(b)

A two port network

consider a two port network terminated by normalized load and generator impedance $\frac{Z_L}{Z_0}$ and $\frac{Z_g}{Z_0} = 1$. Then the load reflection coefficient

$$\Gamma_2 = \frac{a_2}{b_2} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} \quad - (44)$$

$$b_1 = S_{11} a_1 + S_{12} a_2 = S_{11} a_1 + S_{12} b_2 \Gamma_2 \quad - (45)$$

$$b_2 = S_{21} a_1 + S_{22} a_2 = S_{21} a_1 + S_{22} b_2 \Gamma_2 \quad - (46)$$

Solving for the input reflection coefficient

$$\Gamma_1 = b_1/a_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_2}{1 - S_{22}\Gamma_2} \quad - (47)$$

Therefore, for a mismatch load, input reflection coefficient $\Gamma_1 \neq S_{11}$. For a reciprocal network, $S_{12} = S_{21}$ so that

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2} \quad - (48)$$

If the junction is lossless,

$$S_{11} S_{11}^* + S_{12} S_{12}^* = 1 \quad - (49)$$

$$S_{22} S_{22}^* + S_{12} S_{12}^* = 1 \quad - (50)$$

$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0 \quad - (51)$$

For a lossless, reciprocal two port network, terminated by mismatch load,

$$|S_{11}| = |S_{22}| \quad - (52)$$

$$|S_{12}| = \sqrt{1 - |S_{11}|^2} \quad - (53)$$

and input reflection coefficient $\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2} \quad - (54)$

Comparision between [S], [Z] and [Y] Matrices

The [S] can be expressed in terms of [Z] and [Y] as given below:

$$[S] = \left([Z]/Z_0 - [U]\right)\left([Z]/[Z_0] + [U]\right)^{-1}$$
$$= \left([U] - [Y]/Y_0\right)\left([U] + [Y]/Y_0\right)^{-1} \quad \text{— (55)}$$

The following properties are common for [S], [Z] and [Y]

i) Number of elements are equal.

ii) For reciprocal devices both [Z] and [S] satisfy reciprocity properties, $Z_{ij} = Z_{ji}$, $S_{ij} = S_{ji}$

iii) If [Z] is symmetrical, [S] is also symmetrical.

iv) The following are the advantages of [S] over [Z] or [Y].

a) In microwave techniques the source remains ideally constant in power. The measurable parameters are VSWR, f, power, phase. These are essentially measurements of b/a, $|a|^2$ and $|b|^2$. such a direct correspondence is not possible with [Z] or [Y] representations.

b) The unitary property of [S] helps a quick check of power balance of lossless structures. No such immediate check is possible with [Z] or [Y].

c) [S] is defined for a given set of reference planes only. If the reference planes are changed, the S-coefficients vary only in phase. Since voltage & current are functions of complex impedance, both magnitude and phase change in [Z] and [Y].

Ex: Two transmission lines of characteristic impedance Z_1 and Z_2 are joined at plane PP'. Express S-parameters in terms of impedances.

i) The incident and scattered wave amplitude are related by
$$[b] = [S][a]$$
Assuming that the output lines to be matched, $a_2 = 0$, the input impedance Z_{in} at the junction = Z_2 = load of line Z_1.

$$\therefore S_{11} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \text{reflection coefficient on the input side.}$$

ii) For symmetry, assuming that input side is matched $a_1 = 0$

$$S_{22} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -S_{11}$$

iii) In general,
$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

with output line matched, for lossless line Z_{in} at the junction $= Z_2$. A pure shunt element.

$$\therefore b_2 = a_1 + b_1 = a_1 + S_{11} a_1 = a_1 (1 + S_{11})$$
$$b_2 / a_1 = 1 + S_{11}$$
$$S_{21} = 1 + S_{11} = 1 + \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{2Z_2}{Z_2 + Z_1}$$

iv) with input lines matched $(a_1 = 0)$
$$b_1 = a_2 + b_2 = a_2 + S_{22} a_2 = a_2 (1 + S_{22})$$
$$b_1 / a_2 = 1 + S_{22}$$
$$S_{12} = 1 + S_{22} = 1 + \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{2Z_1}{Z_1 + Z_2}$$

$$\therefore [S] = \begin{bmatrix} Z_2 - Z_1 / Z_2 + Z_1 & 2Z_1 / Z_1 + Z_2 \\ \dfrac{2Z_2}{Z_1 + Z_2} & \dfrac{Z_1 - Z_2}{Z_1 + Z_2} \end{bmatrix}$$

Relations of Z, Y and ABCD Parameters with S-Parameters

Consider

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\left.\begin{aligned} \therefore V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\} \quad (1)$$

and $$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} \quad (2)$$

In set 1, if $I_2 = 0$, $\frac{V_1}{I_1} = Z_{11}$ and $\frac{V_2}{I_1} = Z_{21}$

$\therefore \frac{V_1}{V_2} = \frac{Z_{11}}{Z_{21}}$ and $\frac{V_2}{I_1} = Z_{21}$

in set (2) if $I_2 = 0$, $V_1 = AV_2$ and $I_1 = CV_2$

$\therefore \frac{V_1}{V_2} = A$ and $\frac{I_1}{V_2} = C$

$\therefore A = \frac{Z_{11}}{Z_{21}}$, $C = \frac{1}{Z_{21}}$, $B = -\frac{(Z_{11}Z_{22} - Z_{12}Z_{21})}{Z_{21}}$

and $D = -\frac{Z_{22}}{Z_{21}}$

Similarly and y and ABCD parameters,

$Y_{11} = D/B$, $Y_{22} = -A/B$, $Y_{12} = 1/B$, $Y_{21} = C + \frac{AD}{2}$

$S_{11} = \frac{A - B - C + D}{A - B + C - D}$, $S_{22} = \frac{-A - B - C - D}{A - B + C - D}$, $S_{12} = \frac{-2(AD + BC)}{A - B + C - D}$

and $S_{21} = \frac{2}{A - B + C - D}$

Microwave passive Devices

Coaxial connectors and adapters :

Coaxial Cables are terminated to other cables and components by means of shielded standard connectors. The outer shield makes a 360° extremely low impedance joint to maintain shielding integrity. These connectors are of various types depending on the frequency range and the cable diameters. Commonly used microwave connecters are type N, BNC, TNC, APC etc. Adapters, having different connecters at the two ends, are also made for interconnection between two different ports in a microwave system. The type N connecter is 50 and 75 Ω connecter. It is suitable for flexible or rigid cables in a frequency range of 1-18 GHz. The BNC is suitable for 0.25 inch 50Ω or 75Ω flexible cables used upto 1GHz. The TNC (Threaded Navy Connecter) is like BNC, except that, the outer conducter has thread to make firm contact in the mating surface to minimise radiation leakage at higher frequencies.

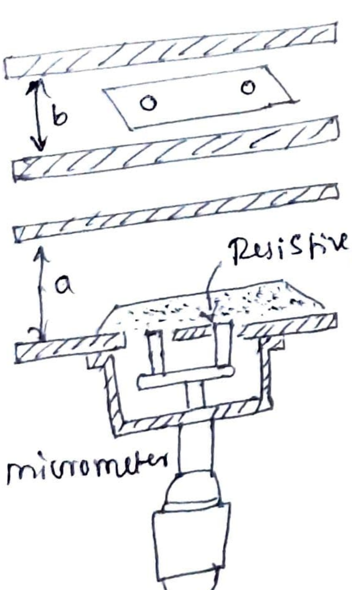
The SMA (Sub miniature A) connecters are used for thin flexible or semi rigid cables. The higher frequency is limited to 24GHz because of generation of higher order modes beyond this limit The APC-7 is a very accurate 50 ohm, low low VSWR connecter which can operate upto 18 GHz. Another APC-3.5 connecter is a high precision 50Ω, low VSWR connecter which can be either the male or female and can operate upto 34GHz.

Type	sex	Dielectric in mating place	Impedance
N	M/F	Air	50/75
BNC	M/F	solid	50/75
TNC	M/F	Solid	50/75
SMA	M/F	Solid	50
APC-7	Sexless	Air	50
APC-3.5	sexed	Air	50

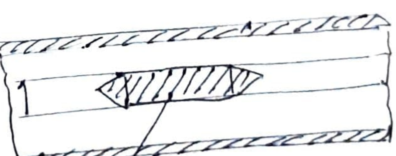
Attenuators: Attenuators are passive devices used to control power levels in a microwave system by partially absorbing the transmitted signal wave. Both fixed and variable attenuators are designed using resistive films.

A coaxial fixed attenuator uses a film with losses on the center conductor to absorb some of the power. The fixed waveguide type consists of a thin dielectric strip coated with resistive film and placed at the center of the waveguide parallel ~~strip coated with resistive film and placed at the center of the waveguide parallel~~ to the maximum E-field. Induced current on the resistive film due to the incident wave results in power dissipation, leading to attenuation of microwave energy.

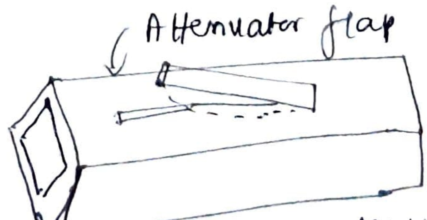
- The dielectric strip is tapered at both ends up to a length of $> \lambda/2$ to reduce reflections.



(a) coaxial line fixed attenuator



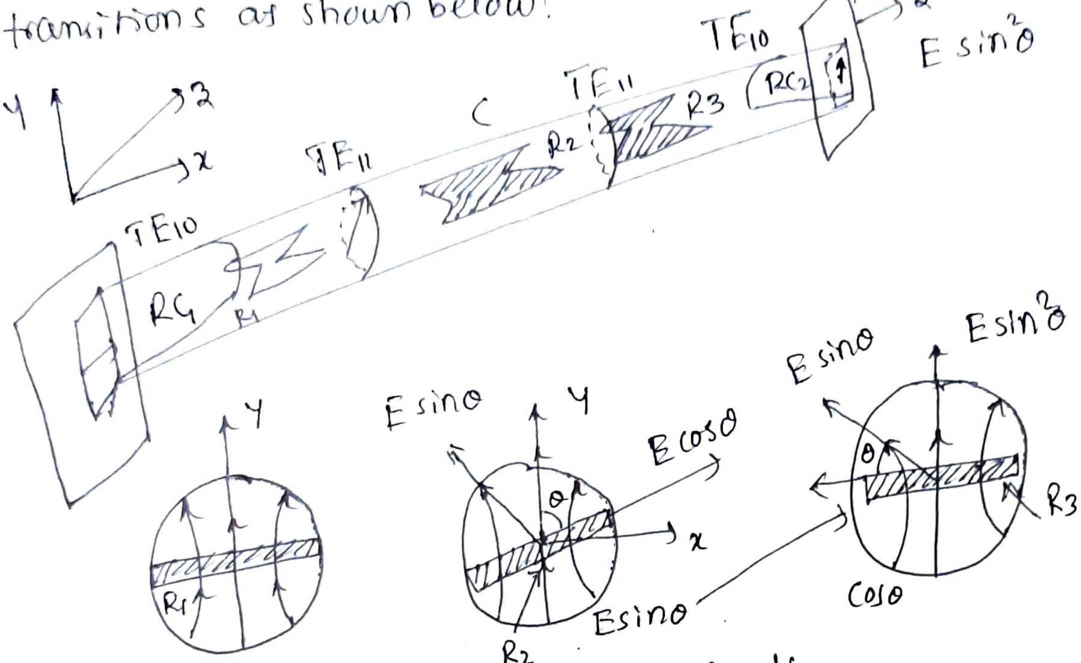
(b) waveguide attenuator.



(c) wave guide attenuator

A variable type attenuator can be constructed by moving the resistive vane by means of micrometer screw from one side of the narrow wall to the centre where the E-field is maximum. A maximum of 90dB attenuation is possible with VSWR of 1.05. The resistance card can be shaped to give a linear variation of attenuation with the depth of insertion.

A precision type variable type attenuator makes use of a circular waveguide section containing a very thin tapered resistive card R_2, to both sides of which are connected axi symmetric sections of circular to rectangular waveguide tapered transitions as shown below:



R1, R2, R3 - Tapered resistive cards

RC1 & RC2 - Rectangular to circular waveguide transitions

C - Circular waveguide section

The centre circular sections with the resistive card can be precisely rotated by 360° w.r.t. the two fixed sections of circular to rectangular waveguide transitions. The induced current on the resistive card R_2 due to the incident signal is dissipated as heat producing attenuation of the transmitted signal.

The incident TE_{10} dominant mode wave in the rectangular waveguide is converted into a dominant TE_{11} mode in the circular waveguide. A thin tapered resistive card placed ⊥r to E field at the circular end of each transition absorbs the parallel component of field. ∴ only pure TE_{11} mode is excited in the middle section.

If the resistive card in the centre section is kept at an angle θ relative to the E-field direction of the TE_{11} mode, the component E cosθ parallel to the card get absorbed, while the component E sinθ is transmitted without attenuation. The attenuation of the incident wave is:

$$\alpha = \frac{E}{E\sin^2\theta} = \frac{1}{\sin^2\theta} = \frac{1}{|S_{21}|}$$

$$\alpha(dB) = -40\log(\sin\theta) = -20\log|S_{21}|$$

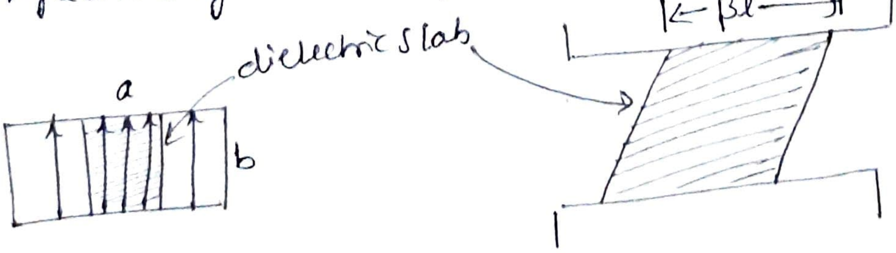
∴ Precision rotary attenuator produces attenuation which depends only on the angle of rotation θ of the resistive card with respect to the incident wave polarization. Attenuation are normally matched reciprocal devices, so that $|S_{21}| = |S_{12}|$

and $|S_{11}|$ or $|S_{22}| = \frac{VSWR-1}{VSWR+1} << 0.1$

S matrix of the ideal precision rotary attenuater is:

$$[S] = \begin{bmatrix} 0 & \sin^2\theta \\ \sin^2\theta & 0 \end{bmatrix}$$

Phase shifters: A phase shifter is a two-port passive device that produces a variable change in phase of the wave transmitted thru it. A phase shifter can be realised by placing a lossless dielectric slab within a waveguide parallel to and at the position of maximum E-field. A differential phase change is produced due to the change of wave velocity through the dielectric slab compared to that through an empty waveguide. Two ports are matched by reducing the reflections of the wave from the dielectric slab tapered at both ends



phase shifters

The propagation constant through a length l of a dielectric slab and of empty guide are,

$$\beta_1 = \frac{2\pi}{\lambda g_1} = \frac{2\pi\sqrt{\left[1-\left(\lambda_0/(2a\sqrt{\epsilon_r})\right)^2\right]}}{\lambda_0/\epsilon_r}$$

$$\beta_2 = \frac{2\pi}{\lambda g_2} = \frac{2\pi\sqrt{\left[1-\left(\frac{\lambda_0}{2a}\right)^2\right]}}{\lambda_0/\sqrt{\epsilon_r}}$$

The differential phase shift produced by the phase shifter is:

$\Delta\phi = (\beta_1 - \beta_2)\, l.$

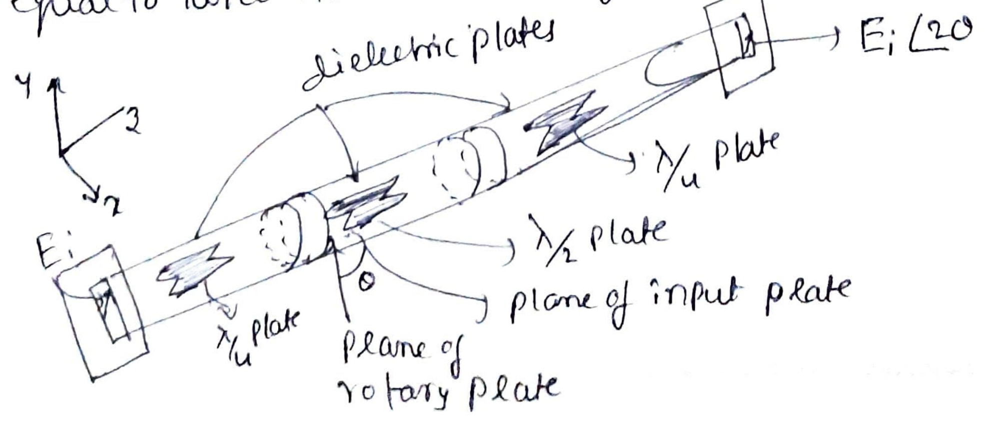
By adjusting the length l, different phase shifts can be produced. The S-matrix of an ideal phase shifter can be expressed by,

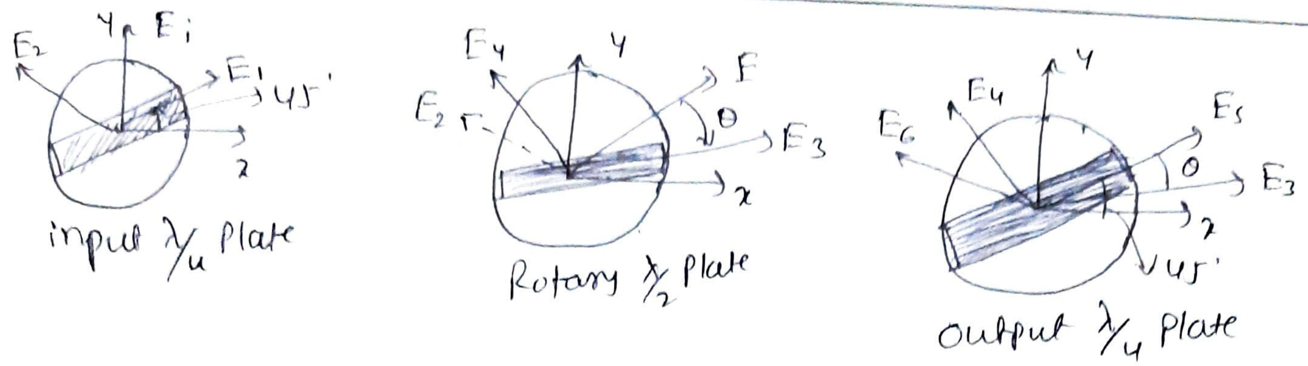
$$[S] = \begin{bmatrix} 0 & e^{-j\Delta\phi} \\ e^{-j\Delta\phi} & 0 \end{bmatrix}$$

Precision phase shifter:

A precision phase shifter uses a section of circular waveguide containing a lossless dielectric plate of length 2l called halfwave (180°) section. This section can be rotated over 360° precisely between two sections of circular to rectangular waveguide transitions each containing lossless dielectric plates of length l called quarter wave (90°) sections oriented at an angle of 45° w.r.t. broad wall of the rectangular waveguide ports at the input and output.

- The incident TE_{10} wave in the rectangular guide becomes a TE_{11} wave in the circular guide. The halfwave section produces a phase shift equal to twice its rotation angle θ w.r.t. quarterwave section.





Principle of operation:

The TE_{11} mode incident field E_i in the input quarterwave section can be decomposed into two transverse components, one E_1, polarised parallel and other, E_2 $\perp^r$ to quarterwave plate. After propagation through the quarter wave plate these components are:

$$E_1 = E_i \cos 45° \, e^{-j\beta_1 l} = E_0 e^{-j\beta_1 l}$$

$$E_2 = E_i \sin 45° \, e^{-j\beta_2 l} = E_0 e^{-j\beta_2 l}$$

where $E_0 = E_i/\sqrt{2}$.

The length l is adjusted such that these two components will have equal magnitude but a differential phase change of 90°. After propagation through the quarterwave plate, these field components become

$$E_1 = E_0 e^{-j\beta_1 l}$$

$$E_2 = j E_0 e^{-j\beta_1 l} = j E_1 = E_1 e^{j\pi/2}$$

∴ The quarterwave sections convert a linearly polarized TE_{11} wave to a circularly polarised wave and vice versa.

After emergence from the halfwave section, the field components parallel and $\perp^r$ to halfwave plate can be represented as,

$$E_3 = (E_1 \cos\theta - E_2 \sin\theta) e^{-j2\beta_1 l} = E_0 e^{-j\theta} e^{-j3\beta_1 l}$$

$$E_4 = (E_2 \cos\theta + E_1 \sin\theta) e^{-j2\beta_2 l} = E_0 e^{-j\theta} e^{-j3\beta_1 l} e^{-j\pi/2}$$

since $2(\beta_1 - \beta_2) l = \pi$,

After emergence from the halfwave section the field components E_3 and E_4 may again be decomposed into Two TE_{11} modes, polarized Parallel and $\perp^r$ to the output quarterwave plate. At the o/p end of this quarterwave plate these components can be written as,

$$E_5 = (E_3 \cos\theta + E_4 \sin\theta)\, e^{-j\beta_1 l} = E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$

$$E_6 = (E_4 \cos\theta - E_3 \sin\theta)\, e^{-j\beta_2 l} = E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$

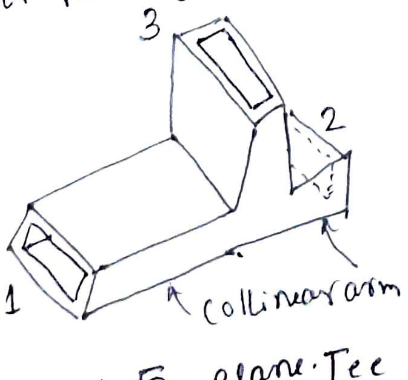
$\therefore$ parallel component E_5 and $\perp^r$ component E_6 at the output end of the quarterwave plate are equal in magnitude and in phase to produce a resultant field which is a linearly polarized TE_{11} wave.

$$E_{out} = \sqrt{2}\, E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$

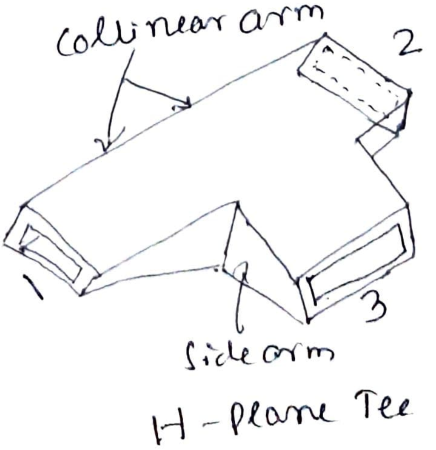
$$= E_i\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$

having the same direction of polarization as the incident field E_i with a phase change of $2\theta + 4\beta_1 l$. since θ can be varied and a phase shift of 2θ can be obtained by rotating the half wave plate precisely through an angle θ, w.r.t. quarterwave plates.

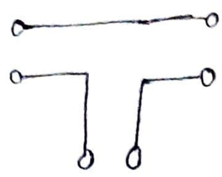
<u>waveguide Tees</u>: These are ~~two~~ three port components. They are used to connect a branch or section of the waveguide in series or parallel with the main waveguide transmission line for providing means of splitting and also of combining power in a waveguide system. The two basic types, E-plane (series) T and H-plane (shunt) T, are constructed as shown below:



E - plane. Tee

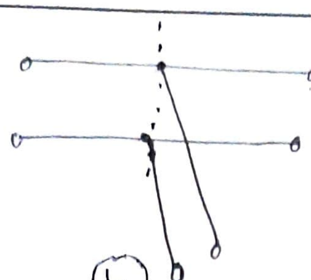


H - Plane Tee



(a)

E - Tee or Series - T



(b)

H - Tee or Shunt - T

waveguide tees are poorly matched devices because of the junction. Because of symmetry and absence of non-linear elements in the junction, the S-matrix is symmetric: $S_{ij} = S_{ji}$; $i = 1, 2$; $j = 1, 2$.

The general S-matrix for a tee junction is:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

E - Plane Tee:

From considerations of symmetry and the phase relationship of the waves in each of the arms, it can be seen that a wave incident at port 3 will result in waves at ports 1 and 2, which are equal in magnitude but opposite in phase, i.e. $S_{31} = S_{13} = -S_{23} = -S_{32}$, $S_{12} = S_{21}$.

- If two input waves are fed into ports 1 and 2 of the collinear arm, o/p wave at port 3 will be opposite in phase and substractive. This port is called difference arm. The tee junction cannot be matched to all the three arms simultaneously. Assuming that port 3 is matched, S-matrix of a E-plane T can be derived as follows:

Denoting the incident and outgoing signal variables at the ith port by a_i and b_i respectively, for an input power at port 3, the net input power to port 3 is $|a_3|^2 - |b_3|^2 = |a_3|^2(1 - |S_{33}|^2)$, and o/p power is $|b_1|^2 + |b_2|^2 = 2|a_3|^2|S_{13}|^2$ since $|S_{31}| = |S_{32}|$ by symmetry.

since the junction is assumed lossless, the input power must be equal to the output power, i.e.

$$(1-|S_{33}|^2) = 2|S_{13}|^2$$

By suitable matching element we can make $S_{33}=0$, so that $|S_{13}| = \frac{1}{\sqrt{2}}$. From the symmetry characterics described above,

$$S_{13} = S_{31} = \frac{1}{\sqrt{2}}, \quad S_{23} = S_{32} = -\frac{1}{\sqrt{2}}$$

After matching the port 3, if one attempts to match either port 1 or 2 by similar method, the matching elements, will interact with each other and matching at port 3 would be disturbed. Based on power consideration it can also be shown that $S_{11} = S_{22} = \frac{1}{2}$ and $S_{12} = S_{21} = \frac{1}{2}$ for $S_{33}=0$.

∴ with matching at port 3, the S-matrix of a E-plane T can be expressed by real values with proper choices of reference plane:

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

H-plane tee:

In a H-plane tee if two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive. Because of this, the third port is called the sum arm. Reversely an input wave at port 3 will be equally divided into ports 1 and 2 in phase. For a symmetrical and lossless junction, in absence of non linear elements at the H-plane junction, the S-parameters are written as, when matched at port 3.

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Because of mismatch at any two ports, the VSWR at the mismatch port of either E or H-plane tee junction is very high.

Ex: A 20mW signal is fed into one of collinear port 1 of a lossless H-plane T-junction. Calculate the power delivered through each port when other ports are terminated in matched load.

Since ports 2 and 3 are matched terminated, $a_2 = a_3 = 0$, $S_{11} = \frac{1}{2}$.

The total effective power input to port 1 is:

$$P_1 = |a_1|^2 (1 - |S_{11}|^2) = 20(1 - 0.5^2) = 15\text{mW}$$

The power transmitted to port 3 is:

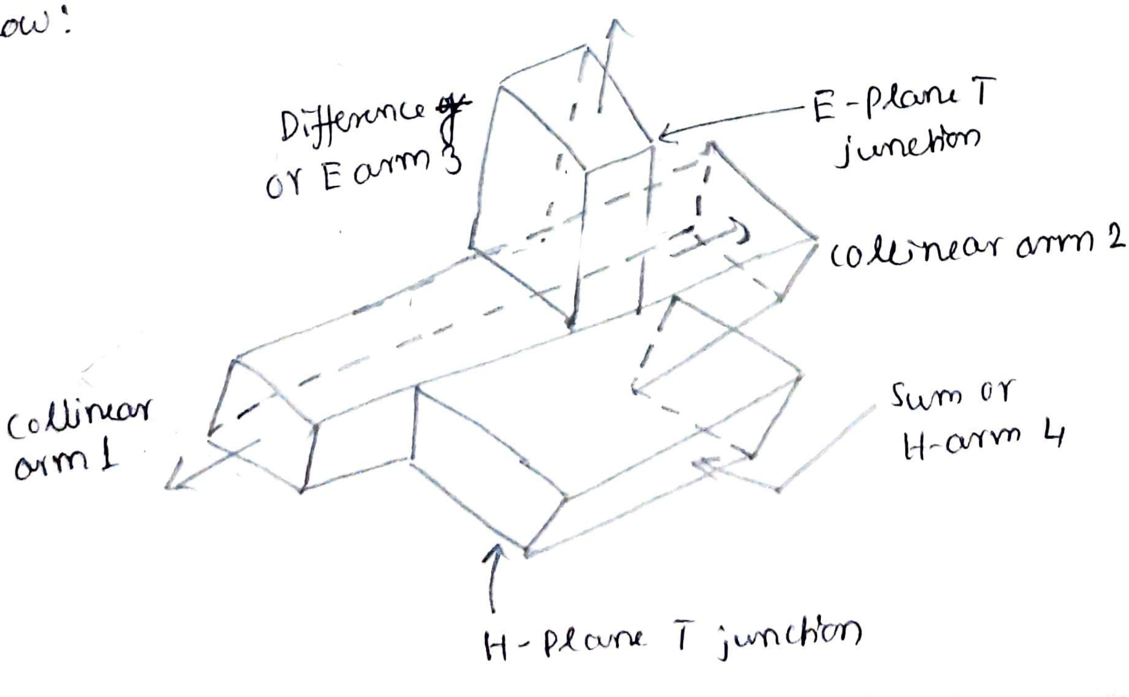
$$P_3 = |a_1|^2 |S_{31}|^2 = 20\left(\frac{1}{\sqrt{2}}\right)^2 = 10\text{mW}$$

The power transmitted to port 2 is:

$$P_2 = |a_1|^2 |S_{21}|^2 = 20 * \left(\frac{1}{2}\right)^2 = 5\text{mW}$$

$$\therefore \quad P_1 = P_3 + P_2$$

Hybrid or magic T: A combination of the E-plane and H-plane tee forms a hybrid tee, called a magic-T having 4 ports as shown below:



The magic T has the following characteristics when all the ports are terminated with matched load:

→ If two inphase waves of equal magnitude are fed into ports 1 and 2 the output at port 3 is subtractive and hence zero. Total output will appear additively at port 4. Hence port 3 is called the difference or E-arm and 4 the sum or H-arm.

→ A wave incident at port 3 (E-arm) divides equally between ports 1 and 2 but opposite in phase with no coupling to port 4.

$\therefore S_{13} = S_{31} = 1/\sqrt{2} = S_{24} = S_{42}$ and $S_{34} = 0$

→ A wave incident at port 4 (H-arm) divides between ports 1 and 2 inphase with no coupling to port 3.

$\therefore S_{14} = S_{41} = 1/\sqrt{2} = S_{24} = S_{42}$ and $S_{34} = 0$

→ A wave fed into one collinear port 1 or 2, will not appear in the other collinear ports 2 or 1. Hence two collinear ports 1 and 2 are isolated from each other, making

$S_{12} = S_{21} = 0$.

For ideal lossless magic-T matched at ports 3 and 4, $S_{33} = S_{44} = 0$.

The procedure of derivation of the S-matrix considers the symmetry property at the junction for which $S_{14} = S_{41} = S_{24} = S_{42}$, $S_{31} = S_{13} = -S_{23} = -S_{32}$, $S_{34} = S_{43} = 0$, $S_{12} = S_{21} = 0$.

$\therefore$ S-matrix for a magic-T matched at ports 3 and 4 is given by,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix}$$

From unitary property applied to rows 1 and 2, we get

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad -(1)$

$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad -(2)$

Subtracting above equations, $|S_{11}|^2 - |S_{22}|^2 = 0 \quad \therefore |S_{11}| = |S_{22}| \quad -(3)$

Unitary property applied to rows 3 and 4

$2|S_{13}|^2 = 1, \quad |S_{13}| = 1/\sqrt{2} \quad -(4)$

$2|S_{14}|^2 = 1, \quad |S_{14}| = 1/\sqrt{2} \quad -(5)$

Substituting these values in (1) we get

$|S_{11}|^2 + |S_{12}|^2 + 1/2 + 1/2 = 1 \quad -(6)$

$\therefore |S_{11}|^2 + |S_{12}|^2 = 0$

which is valid if $|S_{11}| = |S_{12}| = 0$

Since $|S_{11}| = |S_{22}| \quad \therefore S_{22} = 0$

$$\therefore [S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{13} \\ 0 & 0 & -S_{13} & S_{13} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{13} & S_{13} & 0 & 0 \end{bmatrix} \quad -(7)$$

where $|S_{13}| = 1/\sqrt{2} = |S_{14}|$

By proper choice of reference planes in arms 3 and 4, it is possible to make both S_{13} and S_{14} real, resulting in the final form of S-matrix of magic-T

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Q: A magic-T is terminated at colinear ports 1, and 2 and difference port 4 by impedances of reflection coefficients $\Gamma_1 = 0.5$, $\Gamma_2 = 0.6$ and $\Gamma_4 = 0.8$ resp. If 1W power is fed at sum port 3, calculate the power reflected at port 3 and power transmitted to other three ports.

S matrix for a matched magic-T with collinear ports 1 and 2 and sum and difference ports 3 and 4, respectively, is given by

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

If a_1, a_2, a_3 and a_4 be the normalised input voltages and b_1, b_2, b_3 and b_4 are the corresponding output voltage at ports 1, 2, 3 and 4 respectively, then

$a_1 = \Gamma_1 b_1$, $a_2 = \Gamma_2 b_2$, a_3 = input applied voltage and

$a_4 = \Gamma_4 b_4$.

Now, $P_i = |a_3|^2 = 1\,W$, or $a_3 = 1\,V$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0.5 b_1 \\ 0.6 b_2 \\ 1.0 \\ 0.8 b_4 \end{bmatrix}$$

$\therefore b_1 - 0.8 b_4/\sqrt{2} = 1/\sqrt{2}$

$b_2 + 0.8 b_4/\sqrt{2} = 1/\sqrt{2}$

$-0.5 b_1/\sqrt{2} - 0.6 b_2/\sqrt{2} + b_3 = 0$

$-0.5 b_1/\sqrt{2} + \frac{0.6 b_2}{\sqrt{2}} + b_4 = 0$

$\therefore b_1 = 0.6566\,V$ and $b_2 = 0.7576\,V$

$b_3 = 0.5536\,V$, $b_4 = -0.0893\,V$

$\therefore$ power transmitted at port 1 $= |b_1|^2 = 0.4309\,W$

" " " port 2 $= |b_2|^2 = 0.5738\,W$

" " " port 4 $= |b_4|^2 = 0.00797\,W$

" " reflected " port 3 $= |b_3|^2 = 0.3065\,W$