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# Module-4

## DC-DC Converters

1.Introduction,

2.principle of step-down operation and it's analysis with RL load,

3.principle of step-up operation,

4.Step-up converter with a resistive load,

5.Performance parameters,

6.Converter classification,

7.Switching mode regulators: Buck regulator, Boost regulator, Buck-Boost Regulators,

8. Chopper circuit design.

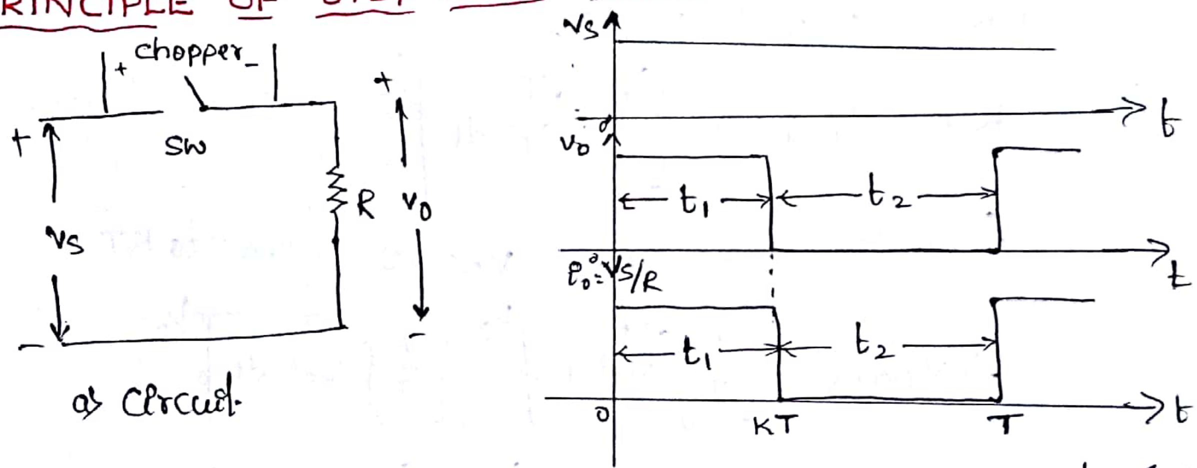
**Text Books:**

1. Mohammad H Rashid, Power Electronics, Circuits, Devices and Applications, 3rd/4th Edition, Pearson Education Inc, 2014.

## INTRODUCTION:

* DC to DC Converter is also called as DC chopper which Converts i/p DC voltage or fixed DC into variable DC.
* A DC converter can be considered as a dc equivalent to an AC Transformer with a continuously variable turns ratio.
* The o/p voltage $V_o$ can be greater or lesser than i/p hence chopper can be step up or step down type
* Choppers used in dc traction drives, trolley cars marine lifts etc.

## PRINCIPLE OF STEP DOWN CHOPPER:




a) circuit

* The above fig shows the circuit diagram of the basic step down chopper.
* Switch is known as chopper can be implemented by using a power transistor. SCR, GTO, power MOSFET, IGBT.
* when switch is closed for time $t_1$ the i/p voltage $V_S$ appears across the load ($V_o = V_S$)
* when switch is open for time $t_2$, the voltage across the load is zero.
* practically switches have a finite voltage drop ranging from 0.5 to 2V.

Average output voltage:

$$V_{o(avg)} = V_a = \frac{1}{T}\int_0^{t_1} V_o \cdot dt = \frac{1}{T}\int_0^{t_1} V_s \cdot dt \qquad \because V_o = V_s$$

$$V_a = \frac{V_s}{T}\left[t\right]_0^{t_1} = \frac{V_s}{T}(t_1 - 0)$$

$$V_a = \frac{V_s \cdot t_1}{T} = \underline{\underline{V_s \cdot K}}$$

where $K = \frac{t_1}{T}$ = duty cycle of chopper lies b/w 0 & 1

The avg load current is

$$I_{a(avg)} = \frac{V_{o(avg)}}{R} = \frac{V_s \cdot K}{R}$$

RMS output voltage:

$$V_{o(rms)} = \left[\frac{1}{T}\int_0^{T} V_o^2\, dt\right]^{1/2}$$

from the waveform, $V_o = V_s$ from 0 to KT

$$\therefore V_{o(rms)} = \left[\frac{1}{T}\int_0^{t_1} V_s^2\, dt\right]^{1/2} = \left[\frac{1}{T}\int_0^{KT} V_s^2 \cdot dt\right]^{1/2}$$

$$= \left[\frac{V_s^2}{T}\left[1\right]_0^{KT}\right]^{1/2} = \left[\frac{V_s^2}{\cancel{T}} \cdot K\cancel{T}\right]^{\frac{1}{2}}$$

$$V_{o(rms)} = \underline{\underline{V_s \cdot \sqrt{K}}}$$

RMS value of o/p current is $I_{o(rms)} = \frac{V_{o(rms)}}{R} = \frac{V_s \cdot \sqrt{K}}{R}$

Input & output power of the chopper.

Input power (For lossless chopper)

Assume a lossless chopper, the i/p power to chopper is same as o/p power & is given by

$$P_i = \frac{1}{T}\int_0^{t_1} V_s\, I_s\, dt \qquad I_s = \frac{V_s}{R}$$

$$= \frac{V_s \cdot V_s}{R \cdot T}\int_0^{KT} 1 \cdot dt \qquad \because t_1 = KT$$

$$= \frac{V_s^2}{RT}\left[t\right]_0^{KT}$$

$$= \frac{V_s^2}{RT}(KT-0)$$

$$= \frac{V_s^2}{R\cancel{T}} K \cdot \cancel{T}$$

$$\boxed{P_i = P_o = K \cdot \frac{V_s^2}{R}}$$

output power (For lossless chopper)

$$P_o = \frac{1}{T}\int_0^{t_1} V_o\, I_o \cdot dt = \frac{1}{T}\int_0^{KT} V_o \cdot \frac{V_o}{R} \cdot dt$$

$$= \frac{V_o^2}{TR}\int_0^{KT} 1\, dt = \frac{V_o^2}{R \cdot T}(KT-0)$$

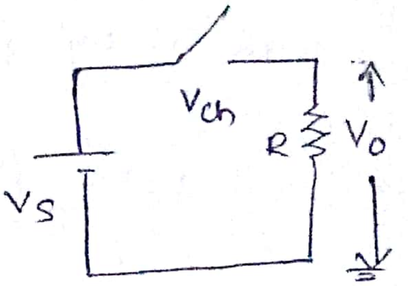
$$= \frac{V_o^2}{R\cancel{T}} \cdot K \cdot \cancel{T} \qquad \because V_o = V_s$$

$$P_o = \frac{V_o^2}{R} \cdot K$$

Input & o/p power for Lossy Chopper

If switching loss ($V_{ch}$) is considered then i/p power is given by

$$P_i = \frac{1}{T}\int_0^{t_1} V_s \cdot I_s \, dt$$




$$= \frac{1}{T}\int_0^{KT} V_s \left(\frac{V_s - V_{ch}}{R}\right) dt$$

$$V_o = V_s - V_{ch}$$

$$I = \frac{V_s - V_{ch}}{R}.$$

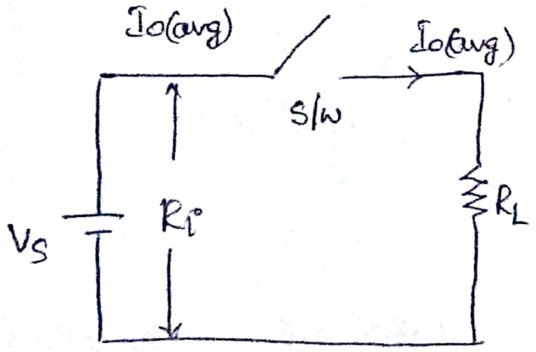
$$P_i = \frac{V_s \cdot (V_s - V_{ch}) \cdot K}{R}$$

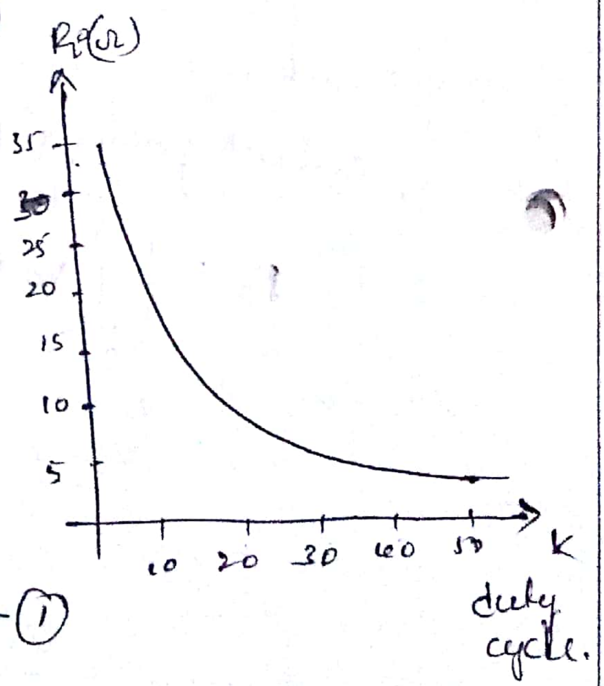
Why o/p power is given by

$$P_o = \frac{1}{T}\int_0^{t_1} V_o \cdot I_o \, dt = \frac{1}{T}\int_0^{t_1} V_o \cdot \frac{V_o}{R} \, dt = \frac{K \cdot V_o^2}{R} \qquad \because t_1 = KT$$

$$P_o = \frac{K \cdot (V_s - V_{ch})^2}{R}$$

Effective Input Resistance ($R_i$)




from the fig.

$$R_i = \frac{V_s}{I_o(avg)} = \frac{\cancel{V_s}}{\frac{K \cdot \cancel{V_s}}{R}} = \frac{R}{K} \quad \text{—} (1)$$

eqn (1) Indicates that the Converter makes the i/p resistance $R_i$ as variable resistance of $R/K$.

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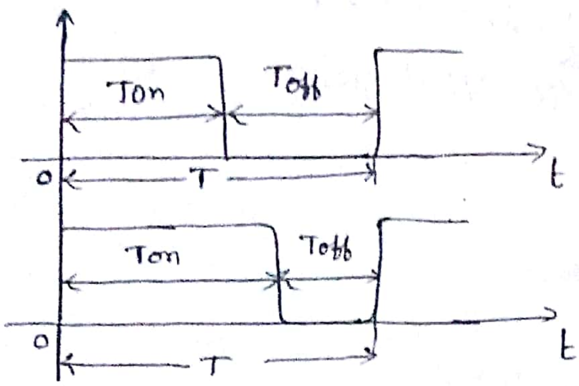
* the variation of normalized o/p resistance v/s duty cycle is shown in above fig.
* Duty cycle K can be varied from 0 to 1 by varying $t_1$
* o/p voltage $V_o$ varied from 0 to $V_s$ by controlling K & power flow can be controlled.

## Chopper Control Techniques (OR) Time Ratio Control Techniques.

1- Constant frequency operation:

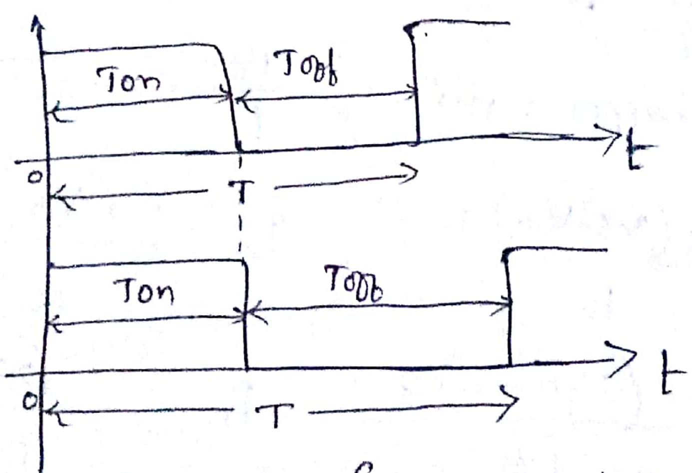



T = Constant

chopping frequency (f) is kept constant & on time is varied.

i.e the width of pulse is varied this type of control is known as pulse width modulation (PWM).

2. variable frequency operation.




T = varied.

chopping frequency f is varied by keeping either Ton or Toff constant this is called frequency modulation.

Problems.

(1) The DC chopper has a resistive load of $R = 10\Omega$, & the i/p voltage $V_S = 220V$ when chopper switch remain on its voltage drop is $V_{ch} = 2V$ chopping frequency is $f = 1KHz$. If the duty cycle is 50% determine

(a) the avg o/p voltage.

(b) rms o/p voltage

(c) chopper Efficiency.

(d) Effective i/p resistance $R_i$ of the chopper

(e) rms value of the fundamental component of o/p harmonic voltage.

Sol^n

Given data

$R_L = 10\Omega$

$V_S = 220V$

$V_{ch} = 2V$

$f = 1KHz$

$K = 50\% = 0.5$

$V_{avg} =$

$V_{orms} =$

$R_i = ?$

$\eta = ?$

(a) Avg o/p voltage.

$$V_a = K.(V_S - V_{ch})$$

$$V_a = 0.5(220-2) = 109V$$

(b) rms o/p voltage.

$$V_{orms} = \sqrt{K}.(V_S - V_{ch})$$

$$V_{o(rms)} = \sqrt{0.5}(220-2) = 154.15V$$

(c) $\eta$ = chopper Efficiency $= \frac{P_o}{P_i} \times 100\%$

$$P_i = \frac{V_S(V_S - V_{ch})K}{R}, \quad P_o = \frac{K(V_S - V_{ch})^2}{R}$$

$$P_i = \frac{220(220-2)0.5}{10}, \quad P_o = \frac{0.5(220-2)^2}{10}$$

$P_i = 2398W$ $\qquad P_o = 2376.2W$

$$\eta = \frac{P_o}{P_i} = \frac{2376.2}{2398} \times 100 = 99.09\%$$

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ⓓ $R_i = \frac{R}{K} = \frac{10}{0.5} = 20\Omega$

ⓔ o/p voltage $V_o$ can be expressed in Fourier Series

$$V_o(t) = K \cdot V_s + \frac{V_s}{n\pi} \sum_{n=1}^{\infty} \sin 2n\pi K \cos 2n\pi ft +$$

$$\frac{V_s}{n\pi} \sum_{n=1}^{\infty} (1 - \cos 2n\pi K) \sin 2n\pi ft$$

fundamental component (for $n=1$) of o/p vtg harmonic can be determined.

$$V_1(t) = \frac{V_s}{\pi}\left[\sin 2\pi K \cos 2\pi ft + (1 - \cancel{\cos 2\pi K}) \sin 2\pi ft\right]$$

$$= \frac{220 \times 2}{\pi}(\sin 2\pi \times 1000t)$$

$$= 140.06 \sin(6283.2t)$$ & its rms value.

$$V_1 = 140.06\sqrt{2} = 99.04V$$

② A DC chopper has a resistive load of $20\Omega$ & i/p voltage 220V when the chopper is on, its voltage drop is 1.5V & chopping freq is 10KHz. If the duty cycle is 80%. Determine the average o/p voltage, rms value of o/p voltage & chopper on time.

Soln

Given data

$V_s = 220V$

$V_{ch} = 1.5V$

$f = 10KHz$

$K = 80\% = 0.8$

ⓐ $V_o \text{ avg} = K(V_s - V_{ch})$

$= 0.8(220 - 1.5V) = 174.8V$

ⓑ $V_{o(rms)} = \sqrt{K}\left(V_s - V_{ch}\right) = \sqrt{0.8}(220 - 1.5)$

$= 195.43V$

ⓒ $K = \frac{T_{ON}}{T} = T_{on} \cdot f =$

$0.8 = T_{ON}\ 10 \times 10^3$ $\quad T_{ON} = 80\mu\text{sec}.$

# Step-Down Chopper with RL (RLE) Load.

* Choppers are used to drive the DC motors. moto, considered as RL (Inductive) Load.
* Since the load is inductive, free wheeling diode $D_m$ used in the circuit
* The operation of chopper can be divided into two modes. i.e mode 1 & mode 2.

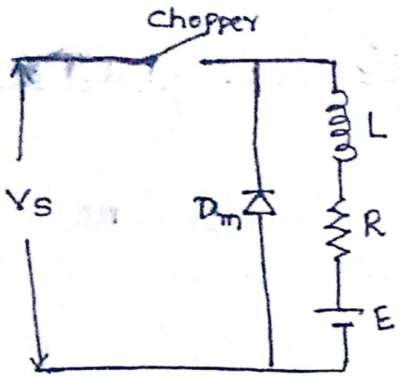
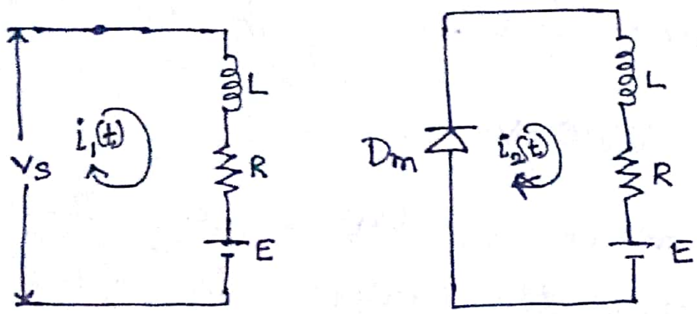
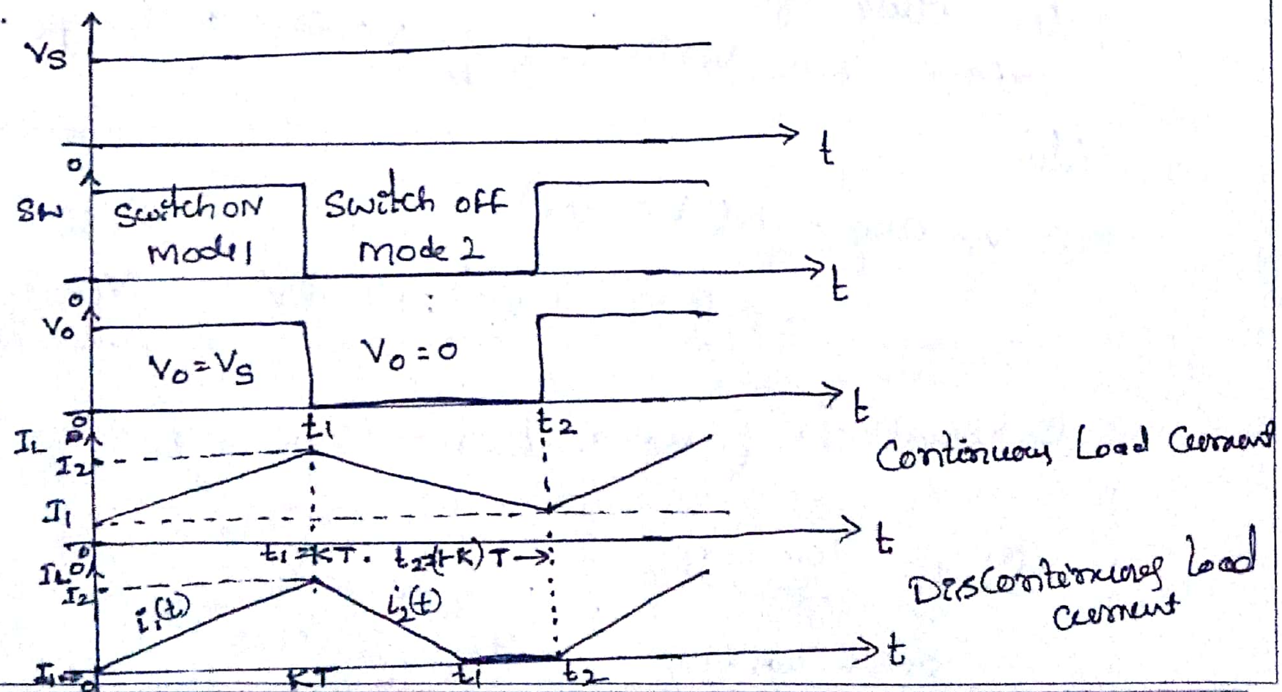



Fig 1: Circuit diagram.




Mode 1 Mode 2.

Fig 2: Equivalent Circuits.






$I_1$ = Imin
$I_2$ = Imax

## Continuous Load Current :

**Mode 1 :** The switch is turned ON for $t = t_1 = KT$ where K = duty cycle. the o/p voltage is equal to the supply voltage i.e $V_o = V_s$

The load current in mode 1 is $i_1(t)$ changes between $I_1 (= I_{min})$ to $I_2 (= I_{max})$

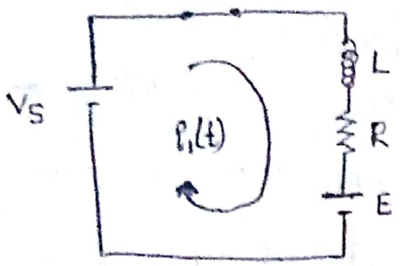



Fig 1

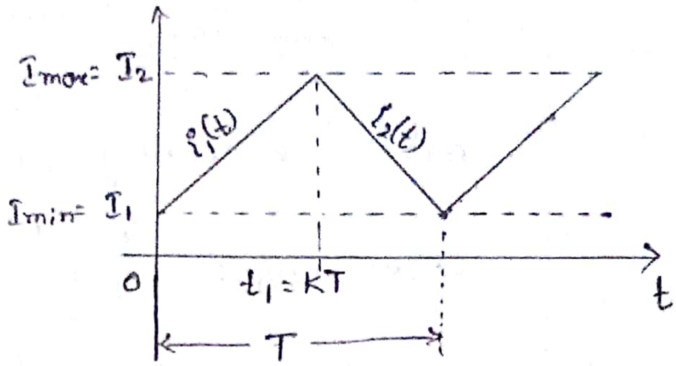



Fig 2.

Applying KVL to fig 1

$$V_s - L\frac{di_1(t)}{d(t)} - i_1(t)R - E = 0$$

$$V_s = L\frac{di_1(t)}{d(t)} + i_1(t)R + E \quad \text{—— (1)}$$

Taking Laplace transform & Inverse laplace transform of eqn (1)

$$i_1(t) = i_1(t=0)e^{-tR/L} + \frac{V_s - E}{R}\left[1 - e^{-tR/L}\right]$$

$i_1(t=0) = I_1$

$t = t_1 = KT$

at $t_1 = KT$

$$I_{max} = I_2 = I_1 e^{-KT\frac{R}{L}} + \frac{V_s - E}{R}\left[1 - e^{-KT\frac{R}{L}}\right]$$

**Note:** In mode 1 chopper is switched ON & current flows from supply to load.

**Mode 2 :**

Applying KVL to fig 2

$$0 = L\frac{di_2(t)}{dt} + i_2(t)R + E \quad \text{—— (2)}$$

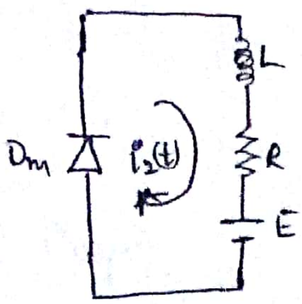



Fig (2)

Taking laplace transform & Inverse laplace transform of eqn (2)

$$i_2(t) = I_2 e^{-tR/L} - \frac{E}{R}\left[1-e^{-tR/L}\right]$$

at $t_2 = (1-K)T$

$$I_{min} = I_1 = I_2 e^{-(1-K)TR/L} - \frac{E}{R}\left[1-e^{-(1-K)TR/L}\right]$$

$T = t_1 + t_2$ *

$T = KT + t_2$

$T - KT = t_2$

$T(1-K) = t_2$

can be $t_2 = T(1-K)$

peak to peak load current (or ripple current) obtained by.

$$\Delta I = I_{O(p\text{-}p)} = I_{max} - I_{min}$$

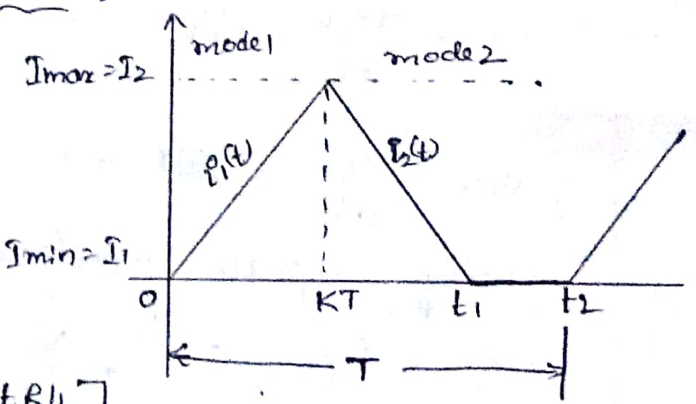
$$= I_2 - I_1$$

After substituting $I_{max}(=I_2)$ & $I_{min}(=I_1)$

$$\Delta I = I_{O(p\text{-}p)} = \frac{V_s}{R}\left[\frac{1-e^{-KTR/L}+e^{-TR/L}-e^{-(1-K)TR/L}}{1-e^{-TR/L}}\right]$$

Note: In mode 2 chopper is switched OFF & load current continuous to flow through freewheeling diode $D_m$.

Discontinuous Load Current:




Mode 1:

$$i_1(t) = I_1 e^{-tR/L} + \frac{V_s - E}{R}\left[1-e^{-tR/L}\right]$$

$$I_{max} = I_2 = \frac{V_s - E}{R}\left[1-e^{-t\cdot R/L}\right] \quad \text{---} \ (1)$$

Mode 2:

$$i_2(t) = I_2 e^{-tR/L} - \frac{E}{R}$$

$$\therefore 0 = I_2 e^{-t_1 R/L} - \frac{E}{R}\left[1-e^{-t_1 R/L}\right] \quad \text{---} \ (2)$$

Equn (1) & (2) represents the discontinuous load current

* To find the time $t_1$ at which load current becomes zero

Consider equation ② i.e.

$$0 = I_2 e^{-t_1 R/L} - \frac{E}{R}\left[1 - e^{-t_1 R/L}\right]$$

$$I_2 e^{-t_1 R/L} = \frac{E}{R} - \frac{E}{R} e^{-t_1 R/L}$$

$$e^{-t_1 R/L}\left[I_2 + \frac{E}{R}\right] = \frac{E}{R}$$

$$e^{-t_1 R/L} = \left[\frac{\frac{E}{R}}{I_{max} + \frac{E}{R}}\right] \qquad I_2 = I_{max}$$

Taking ln on both sides.

$$-t_1 \frac{R}{L} = \ln\left[\frac{E/R}{I_{max} + E/R}\right]$$

$$t_1 \frac{R}{L} = -\ln\left[\frac{E/R}{I_{max} + E/R}\right] \qquad \therefore -\ln(x) = \ln(1/x)$$

$$t_1 = \frac{L}{R} \ln\left[\frac{I_{max} + E/R}{E/R}\right]$$

$$\therefore t_1 = \frac{L}{R} \ln\left[\frac{I_{max} \cdot R}{E} + 1\right]$$

Problem.

① A chopper is feeding an RL load shown in fig. with $V_S = 220V$, $R = 5\Omega$, $L = 7.5mH$, $f = 1KHz$, $K = 0.5$ & $E = 0V$

Calculate.

ⓐ minimum instantaneous Load current $(I_1)$

ⓑ peak Instantaneous current $(I_2)$.

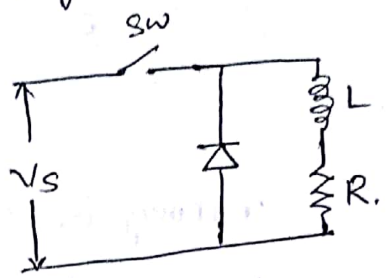
ⓒ maximum peak to peak load ripple current.

ⓓ Avg value of load current $I_a$

ⓔ rms Load current $I_o$.

ⓕ Effective i/p Resistance $R_i$ seen by source.

ⓖ rms Chopper Current $I_R$.




Soln:

Given data

$V_S = 220V$,

$R = 5\Omega$

$L = 7.5mH$

$f = 1KHz$.

$K = 0.5$

a) $I_{min} = I_{max}\, e^{-(1-K)TR/L} - \frac{E}{R}\left[1 - e^{-(1-K)TR/L}\right]$

$= I_{max}\, e^{-\frac{(1-0.5)5\Omega}{7.5mH \cdot 1KHz}}$ $\quad \therefore \frac{1}{f} = T$

$I_{min} = 0.7165\, I_{max}$ ——①

$I_{max} = I_{min}\, e^{-KTR/L} + \frac{V_S - \cancel{E}^{\,0}}{R}\left[1 - e^{-KTR/L}\right]$

$= I_{min}\, e^{-\left(\frac{0.5\times5}{1K\times7.5m}\right)} + \frac{220}{5}\left[1 - e^{-\left(\frac{0.5\times5}{7.5m\times1K}\right)}\right]$

$I_{max} = 0.7165\, I_{min} + 12.4726$ —②

Equn ① in ②

$I_{max} = 0.7165(0.7165\, I_{max}) + 12.4726$

$= 25.63A$

$I_{min} = 0.7165(25.63)$

$I_{min} = 18.36A$

(c) $\Delta I = I_{o(p-p)} = I_{max} - I_{min} = 7.27A$

(d) $I_{o(avg)} = \frac{I_{max} + I_{min}}{2} = \frac{25.63 + 18.36}{2} = 21.995A$.

(e) rms load Current

$$I_{o(rms)} = \left[ I_{min}^2 + \frac{I_{o(p-p)}^2}{3} + I_{o(p-p)} I_{min} \right]^{\frac{1}{2}}$$

$$= \left[ 18.36^2 + \frac{(7.27)^2}{3} + (18.36 \times 7.27) \right]^{1/2}$$

$I_{o(rms)} = 22.095A$

(f) $R_i = \frac{V_s}{I_{s(avg)}} = \frac{V_s}{K\, I_{o(avg)}} = \frac{220}{0.5(21.995)} = \frac{220}{11} = 20\Omega$

(g) $I_{T(rms)} = \sqrt{K} \cdot I_{o(rms)} = \sqrt{0.5}(22.095) = 15.62A$.

(2) A chopper circuit shown duty cycle: 0.5 & chopping frequency $f = 5KHz$. Determine

<i> min instantaneous load current.

<ii> peak

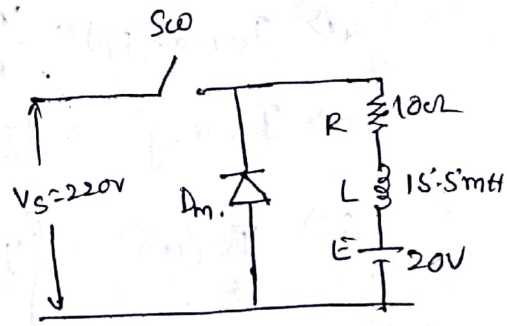
<iii> p-p current on load.

iv> max (p-p) ripple current

v> Avg load current.

vi> rms load current.

vii> Effective i/p resistance




Soln.

Given data

$R = 10\Omega$

$L = 15.5mH$

$E = 20V$

$V_S = 220V$

$K = 0.5$

$f = 5KHz$.

w.k.t.

$$I_{max} = I_{min}\, e^{-KTR/L} + \frac{V_S - E}{R}\left[1 - e^{-KTR/L}\right]$$

$$= I_{min}\, e^{-\left(\frac{0.5\times 10}{5k\times 15.5mH}\right)} + \left(\frac{220-20}{10}\right)\left[1 - e^{-\left(\frac{0.5\times 10}{5k\times 15.5mH}\right)}\right]$$

$$I_{max} = 0.9375 I_{min} + 1.2495 \quad \text{—— (1)}$$

$$I_{min} = I_{max}\, e^{-(1-K)TR/L} - \frac{E}{R}\left[1 - e^{-(1-K)T\frac{R}{L}}\right]$$

$$I_{min} = 0.9375 max - 0.12496 \quad \text{—— (2)}$$

Substitute eq (2) in (1)

(i) $I_{max} = 0.9375(0.9375 I_{max} - 0.12496) + 1.2495$

$I_{max} = 9.357A$

(ii) Using eqn (2)

$I_{min} = 0.9375(9.351) - 0.12496$

$I_{min} = 8.642A$

(iii) $I_{o(P.P)} = \Delta I = I_{max} - I_{min} = 0.709\ A$

(iv) $I_{omax(P.P)} = \frac{V_s}{R} \tanh\left(\frac{TR}{4L}\right) = \frac{220}{10} \tanh\left(\frac{10}{20k\times 15.5mH}\right) = 0.709A$

(v) $I_{o(avg)} = \frac{I_{max} + I_{min}}{2} = \frac{9.351 + 8.642}{2} = 8.99 \simeq 9A$

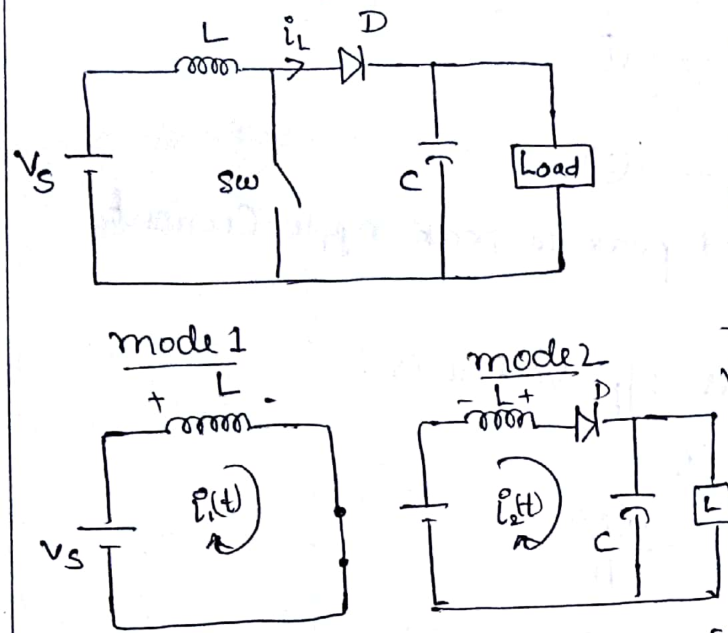
(vi) $I_{o(rms)} = \left\{I_{min}^2 + \frac{I_{o(P.P)}^2}{3} + I_{o(P.P)} \cdot I_{min}\right\}^{\frac{1}{2}}$

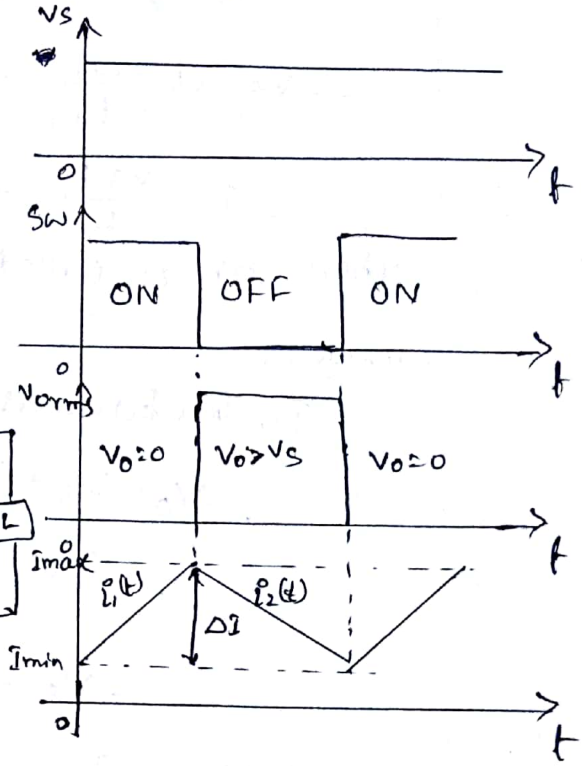
$= \left\{(8.642)^2 + \frac{(0.709)^2}{3} + (8.642 \times 0.709)\right\}^{\frac{1}{2}}$

$I_{o(rms)} = 8.99 \simeq 9A$

(vii) $R_i = \frac{V_S}{I_{S(avg)}} = \frac{220}{K\, I_{o(avg)}} = \frac{220}{0.5\times 9}$

## Principle of Step-up Chopper




* Here switch is connected across the inductance & supply.
* capacitor C is used a/c the load to make Vo smooth
* The diode D blocks the flow of current to the load when switch is turned ON.

**Mode 1:** when the switch (chopper) is closed for time $t_1$ the current flows through the inductance from the supply, the inductor current raises & energy is stored in the inductor 'L' ($V_L=V_S$) & o/p voltage is zero.

**Mode 2:** If the switch is opened at time $t_2$, then the energy stored in the inductor is transformed to load through diode D (forward biased) & o/p voltage $V_0>V_S$ (i.e $V_S+V_L$)

**output voltage:**

mode 1: when the chopper (SW) is ON voltage across L is

$$V_L = L\cdot\frac{di(t)}{dt}.$$

when $V_L = V_s$ , $di(t) = \Delta I$ , $dt = t_1$

$$\therefore V_s = L \cdot \frac{\Delta I}{t_1} \quad \text{--- (1)}$$

$$\Delta I = \frac{V_s \cdot t_1}{L} \quad \text{--- (2)}$$

where $\Delta I$ is called peak to peak ripple current.

mode 2:

Instantaneous o/p voltage is

$$V_0 = V_s + V_L$$

$$= V_s + L \frac{di_2(t)}{dt}$$

$$= V_s + L \cdot \frac{\Delta I}{t_2} \quad \text{--- (3)}$$

equ^n (2) in (3)

$$V_0 = V_s + \cancel{L} \cdot \frac{V_s t_1}{\cancel{L} t_2}$$

$$= V_s \left(1 + \frac{t_1}{t_2}\right)$$

$$= V_s \left(\frac{t_2 + t_1}{t_2}\right)$$

$$= V_s \left(\frac{1}{1-K}\right)$$

$$V_{0(avg)} = \frac{V_s}{1-K}$$

W.K.T $K = \frac{t_1}{t_1 + t_2}$

$$1 - K = 1 - \frac{t_1}{t_1 + t_2}$$

$$1 - K = \frac{\cancel{t_1} + t_2 - \cancel{t_1}}{t_1 + t_2}$$

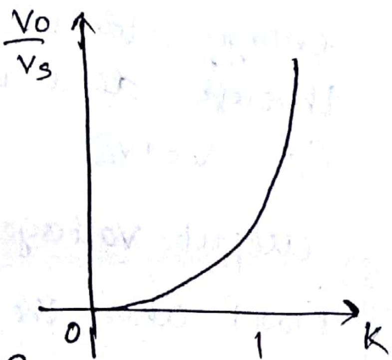
$$1 - K = \frac{t_2}{t_1 + t_2}$$

$$\frac{1}{1-K} = \frac{t_1 + t_2}{t_2}$$

Load voltage depends on duty cycle

for $K = 0$ , $V_0 = V_s$ $\frac{V_0}{V_s} = 1$

for $K = 1$ $\frac{V_0}{V_s} = \infty$




If a large capacitor C is connected a/c the load the o/p voltage will be continuous & $V_0$ would become equal to the average value.

Average output voltage:

Average voltage a/c inductor.

$$V_L = \frac{1}{T}\int_0^T V_L(t)\,dt = \frac{1}{T}\int_0^T L\cdot\frac{di_L(t)}{dt}\cdot \cancel{dt} = \frac{L}{T}\int_0^T d\,i_L(t).$$

above integration is w.r.t inductance current $di_L(t)$

∴ limit changes.

at $t=0$ $i_L(t) = I_{min}$

at $t=T$ $i_L(t) = I_{min}$

$$\therefore V_L = \frac{L}{T}\int_{I_{min}}^{I_{min}} di_L(t) = \frac{L}{T}\left[i_L(t)\right]_{I_{min}}^{I_{min}} = \frac{L}{T}\left[I_{min} - I_{min}\right]$$

$$\boxed{V_L = 0}$$

The avg voltage across the inductance is zero.

Note: The inductor stores the energy when the switch is ON & supplies the energy to the load when the switch is OFF

* voltage across switch.

$$V_{AB} = \frac{1}{T}\int_0^T V_{AB}(t)\cdot dt$$

from $KT$ to $T$, $V_{AB}(t) = V_0(avg)$ & rest of the period it's 0

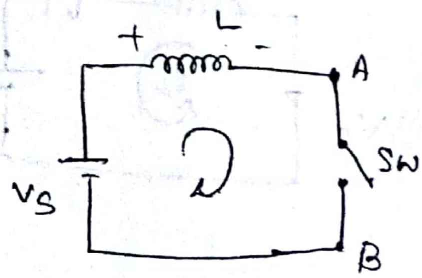
$$\therefore V_{AB} = \frac{1}{T}\int_{KT}^{T} V_0\,avg\; dt$$

$$V_0 = \frac{V_0(avg)}{T}\left[1\right]_{KT}^{T} = \frac{V_0\,avg}{T}\left[T - K\cdot T\right]$$

$$= \frac{V_0\,avg\cdot \cancel{T}\left[1-K\right]}{\cancel{T}}$$

$$V_{AB} = V_0(avg)(1-K)$$

Applying KVL to the fig.




$$V_S - V_L - V_{AB} = 0$$

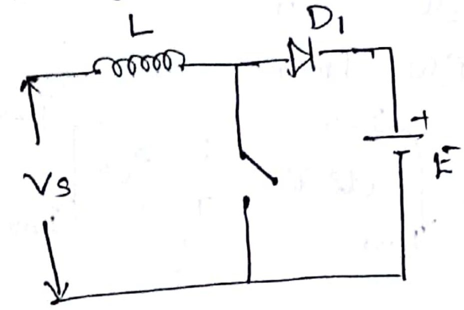
$$V_S = V_{AB} + V_L$$

$$V_S = V_{o(avg)}(1-K) + 0$$

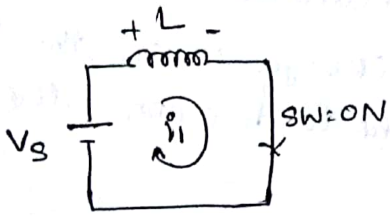
$$\therefore V_{o(avg)} = \frac{V_S}{1-K}$$

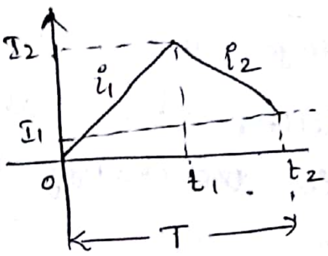
## Use of Step up operation for energy transfer:

Principle of step up chopper can be applied to transfer energy from one voltage source to another.




mode 1:




$$V_L = L\frac{di_1}{dt}$$

$$V_S \cdot dt = L \cdot di_1$$

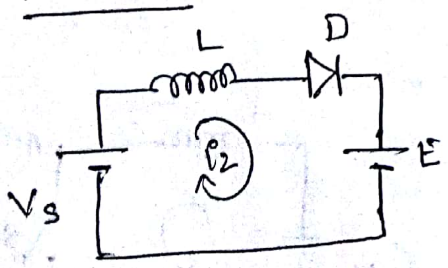
$$i_1 = \frac{V_S \cdot t}{L} + I_1$$

where $I_1$ = Constant & Initial Current through inductor.

During mode 1 current should rise & necessary Condition is

$$\frac{di_1}{dt} > 0 \quad \text{or} \quad V_S > 0.$$

mode 2:




Applying KVL

$$V_S = L \cdot \frac{di_2}{dt} + E$$

$$V_S - E = L\frac{di_2}{dt}$$

$$di_2 = \left(\frac{V_S - E}{L}\right) dt.$$

$I_2$ is the initial current for mode 2

For stable system the current must fall & the necessary condition is

$$\frac{di_2}{dt} < 0 \quad \text{or} \quad V_s < E$$

∴ The condition for controllable power transfer is

$$0 < V_s < E$$

$V_s < E$ to permit transfer of power from fixed or variable source to fixed dc voltage

when the chopper is turned ON energy is transfered from source $V_s$ to L. If chopper is turned off energy stored in the inductor is forced to battery E.

Problem.

① A step up chopper has i/p voltage of 220v & o/p voltage of 660v. If the non conducting time of thyristor chopper is 100µsec compute the pulse width of o/p voltage. In case pulse width is halved, for constant frequency operation. Find new o/p voltage

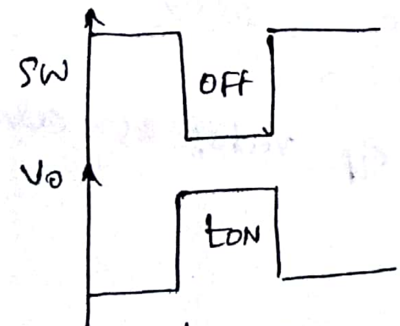
Soln

Given data

$V_o = 660V$

$V_s = 220V$

$t_{off} = 100µsec$




case 1) $t_{on} = ?$

case 2) $T = \text{constant}$, $t'_{on} = \frac{t_{on}}{2}$

$V_o(avg) = ?$

Case i) $V_{o(avg)} = \frac{V_S}{(1-K)}$

$$660 = \frac{220}{1-K}$$

$$1-K = \frac{220}{660}$$

$$K = 2/3 = 0.6$$

$$K = \frac{t_{on}}{T} = \frac{t_{on}}{t_{on}+t_{off}}$$

$$t_{on} = K t_{on} + K \cdot T_{off}$$

$$t_{on}(1-K) = K \cdot T_{off}$$

$$t_{on} = \frac{K \cdot T_{off}}{1-K} = \frac{0.6(100\mu)}{1-0.6}$$

$$\therefore t_{on} = 200 \mu sec.$$

Case ii) $T = Const = t_{on} + t_{off} = (100+200)\mu = 300 \mu sec$

$$t_{on}' = \frac{200\mu}{2} = 100 \mu sec$$

$$K = \frac{t_{on}}{T} = \frac{100\mu}{300\mu} = \frac{1}{3} = 0.3$$

$$V_{o(avg)} = \frac{V_S}{1-K} = \frac{220}{1-0.3} = 330V$$

as $t_{on}$ is halved, the o/p voltage is also reduced to half of its value.

Performance parameters:

The performance parameter of the Chopper are as follows.

1. Duty cycle (K):

The duty cycle of the chopper controls its o/p voltage the value of duty cycle lies b/w 0 & 1. Duty cycle can be controlled between a minm value (Kmin) & a maximum value (Kmax) thereby limitation of minimum & maximum value of o/p voltage.

2. Chopping frequency or Switching frequency (f)

Frequency of the chopper is $f = \frac{1}{T}$,

where T = period of o/p voltage waveform. the frequency should be as high as possible to reduce the load ripple current & minimize the size of series inductor in the load current.

3. operating speed of switch or devices.

operating speed of the device depends on turn ON & turn OFF times of the switch. Hence switching frequency of the Chopper depends on the speed of switching device.

ex: MOSFET has very small turn ON & turn OFF time hence it operates at very high speed.

## Converter classification or Chopper classification

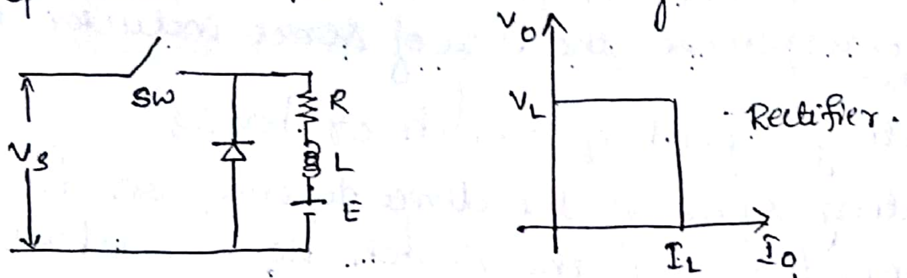
Based on the direction of current & voltage flow, Choppers can be classified as.

1. Class A Choppers [First quadrant chopper] [$V_0$ = +ve, $I_0$ = +ve]
2. Class B Choppers. [Second quadrant chopper] [$V_0$ = +ve, $I_0$ = −ve]
3. Class C Choppers. [First & Second quadrant] [$I_0$ = +ve/−ve, $V_0$ = +ve]
4. Class D Choppers [First & fourth quadrant] [$V_0$ = +ve/−ve, $I_0$ = +ve]
5. Class E Choppers. [Four quadrant choppers] [$V_0$ = +ve/−ve, $I_0$ = +ve/−ve]

## Class A Choppers:

In Class A both o/p voltage & load current are positive. this is called as I quadrant chopper. i.e. it is said to be operated as rectifier.

Step down chopper is basically class A chopper.

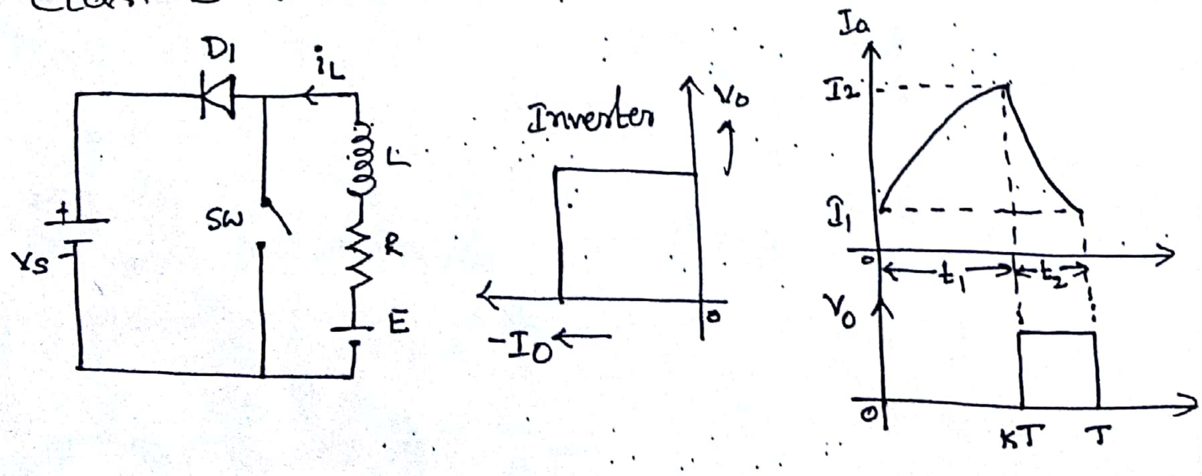



$I_0 = I_L$ = +ve it is flowing from source to load.

## Class B Chopper:

In class B load voltage is +ve & current is −ve it flows from load to source.

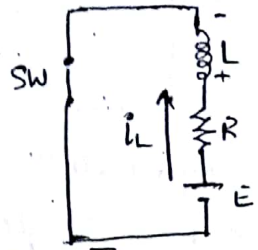
Class B said to be operated as an inverter.



In classB, load voltage is positive & current is -ve flows from load to source.
Class B is said to be operated as an inverter.

operation:

When Chopper is ON: the voltage E drives the current through inductor L & store energy in L & Load voltage $v_o = 0$




Load current is given by

$$0 = L\frac{di}{dt} + Ri_L + E \quad —① $$

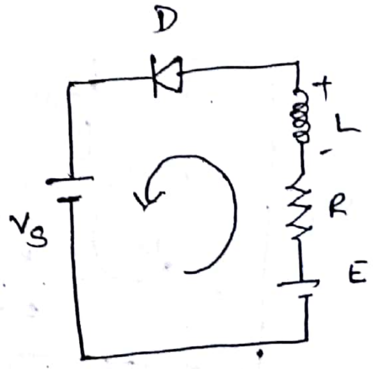
$$\text{i.e. } i_L(t) = I_1 e^{-tR/L} - \frac{E}{R}\left[1 - e^{-tR/L}\right] \quad —②$$

the above eqn is valid for $0 \le t \le t_1 [= KT]$

At time $t = t_1 = KT$ & load current $I_L$ becomes

$I_L = I_2$ at $t = t_1 = KT$

When Chopper is OFF




In this mode, load current falls & is given by

$$V_S = L\frac{di_L}{dt} + Ri_L + E$$

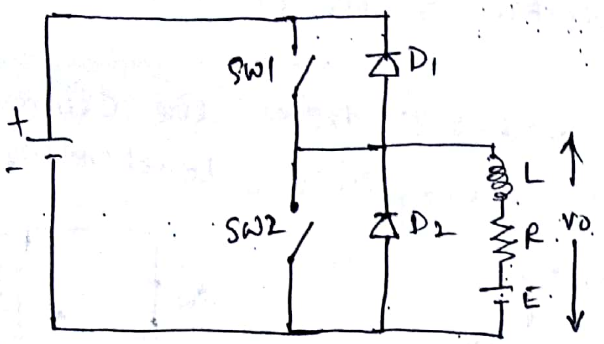
$$\therefore i_L(t) = I_2 e^{-tR/L} + \frac{V_S - E}{R}\left[1 - e^{-tR/L}\right]$$

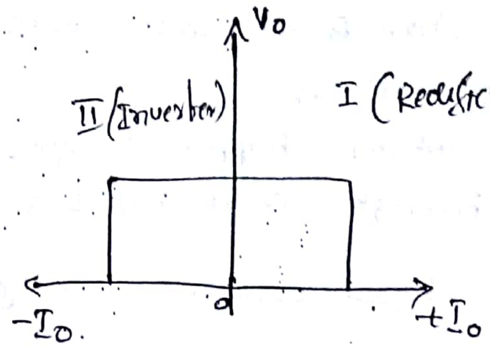
this is valid for $0 \le t \le t_2$ where $t_2 = (1-K)T$

## Class C Choppers:

In this chopper load current is either +ve or -ve & load voltage is always +ve, this is known as two quadrant chopper.

ClassA & ClassB are Combined to form class C Chopper




Operation

* For I quadrant operation $S_1$ & $D_1$ operates & $S_2$ & $D_2$ does not conduct.
* when SW1 is turned ON, dc supply gets connected to the load & $V_0 = +V_S$ = +ve. ∴ $I_0$ = +ve
* current flows from source to load

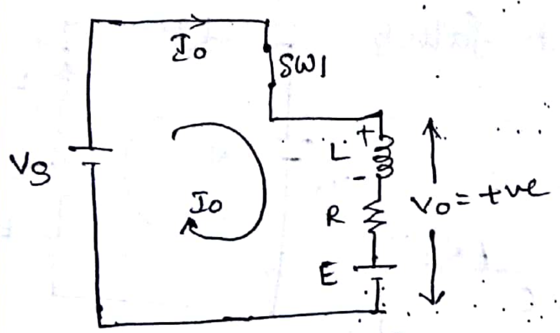



fig (a)

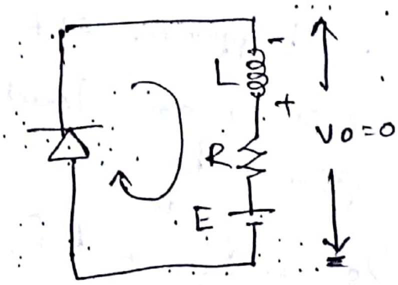



fig (b)

when switch is off free wheeling action takes place through diode $D_2$ as shown in fig (b). thus load voltage is zero & current is +ve.

* For II quadrant operation $S_2$ & $D_2$ conducts.

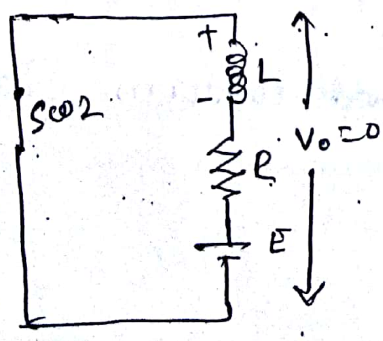



fig (c)

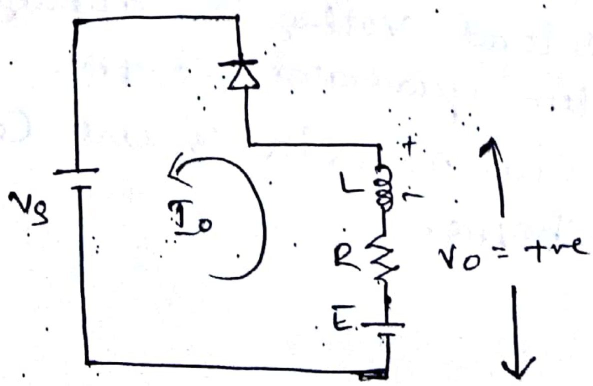



fig (d)

SCO2 Conducts, inductor stores energy from source E & free wheeling action takes place & $V_0 = 0$ & $I_0$ = -ve flows from load.

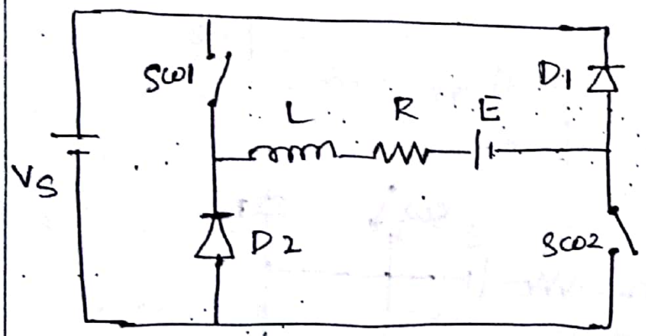
when SCO2 is turned off, induced emf in the inductor forces $D_1$ to conduct & stored energy is returned back. the load voltage is +ve & current is -ve as it is flowing from load to source.

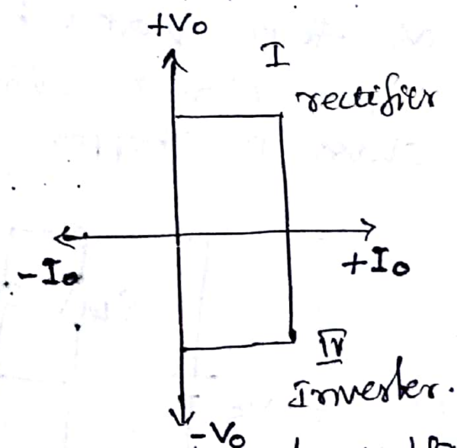
In II quadrant operated as Regenerative braking or inverter.

## Class D Choppers:

In Class D Chopper load voltage is +ve or -ve & Load current is always +ve.

It operated as rectifier or inverter.




when both switches are turned ON, the o/p voltage $V_0 = +V_s$ & load current is +ve as shown in fig(i) & inductor stores energy.

when both switches are turned off, induced emf of inductor forces $D_1$ & $D_2$ to conduct [L releases the energy stored]. So the diode provide path for a load current & it is maintained in the same direction

∴ $I_0$ = +ve & load voltage = -ve shown in fig(ii)

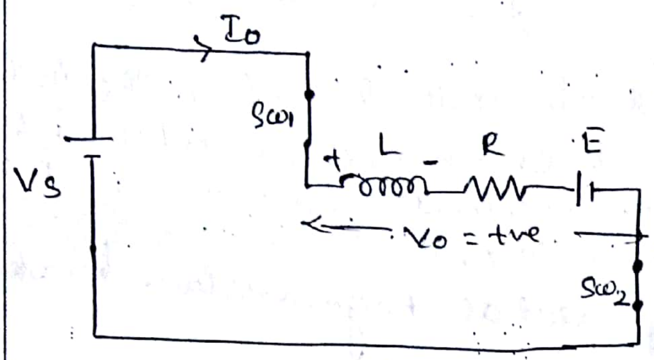



Fig (i)

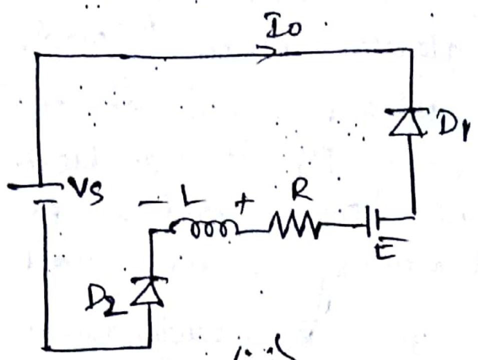
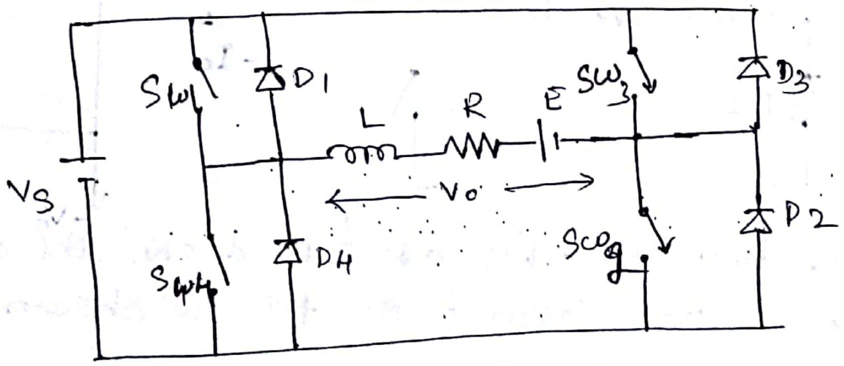


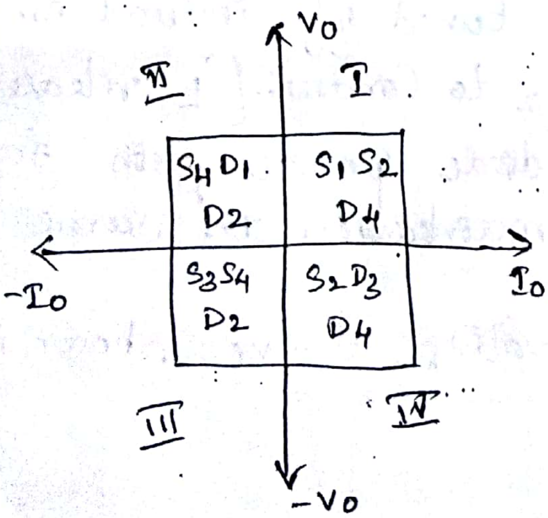

Fig (ii)

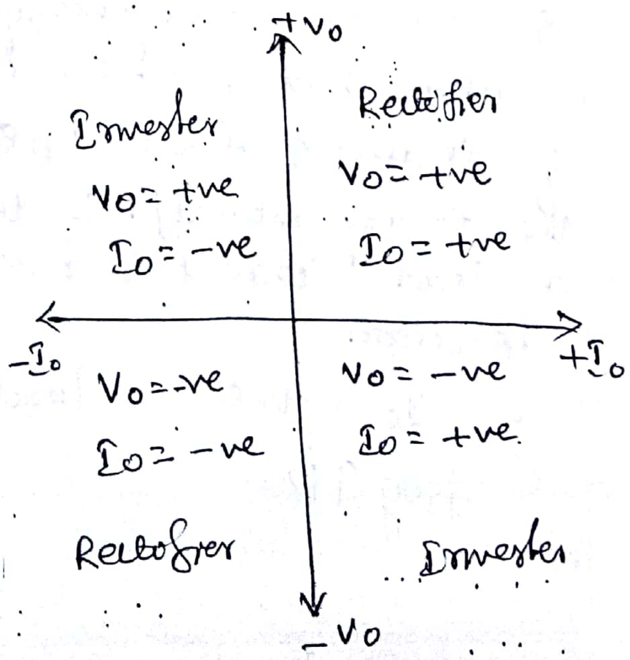
## Class E operation:

* In class E chopper, Load current is either +ve or -ve & Load voltage is also either +ve or -ve this is known as four quadrant chopper.

* Two class C choppers can be combined to form a class E chopper.



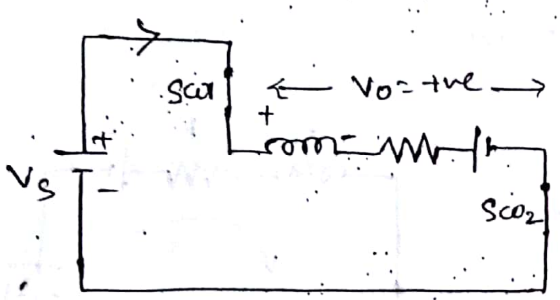



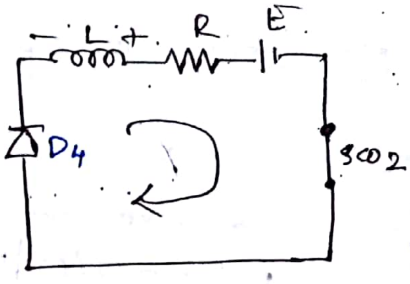



## I quadrant operation:

* To operate in 1st quadrant $S_{CH1}$ & $S_{CH2}$ is turned ON, $S_3$ & $S_4$ is off. Then current flows from source to load ∴ $I_O$ = +ve & $V_O$ = +ve ($V_S$ – $S_{CH1}$ – L – R – E – $S_2$)
* When $S_{CH1}$ is turned off, current flows in (L – R – E – $S_2$ – $D_4$), free wheeling action takes place.

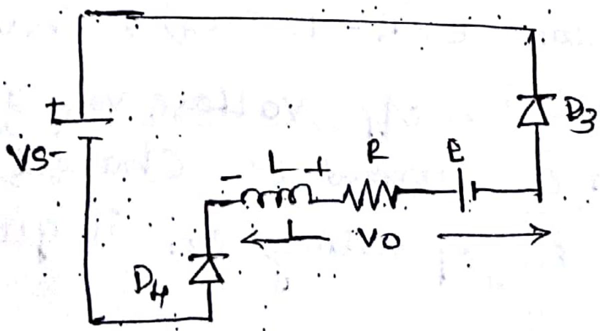
  $V_O = 0$ & $I_O$ = +ve.
* When both the switches are turned off stored energy in the inductor (– ⌇ +) forces current through $D_3$ & $D_4$ (L – R – E – $D_3$ – $V_S$ – $D_4$) since current maintained in same direction.




## IV quadrant operation:

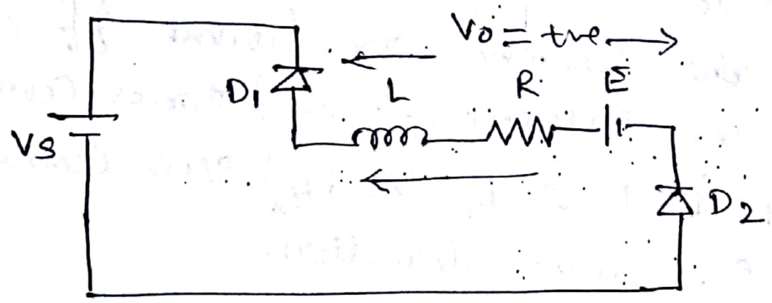
$I_O$ is +ve & $V_O$ is –ve ∴ chopper is operating in IV quadrant.



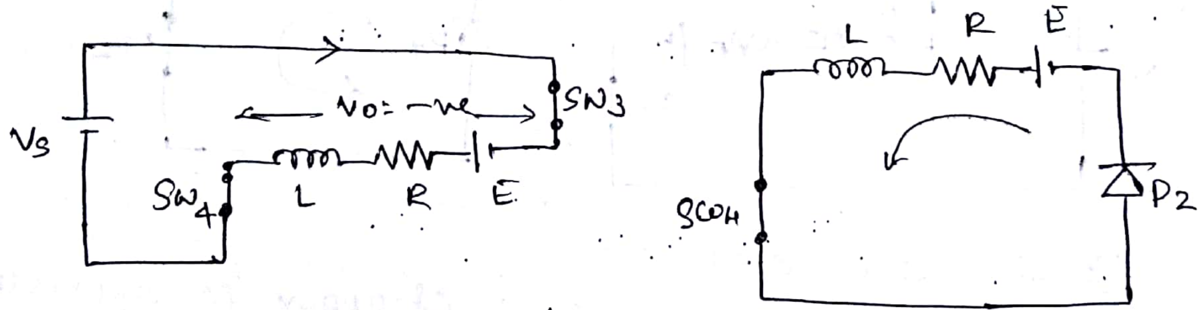

## II Quadrant operation.

When $S_{CH3}$ is turned off, current flows in the path (L – $T_4$ – $D_2$ – E – R) & freewheeling action (– ⌇ –) takes place & $V_O = 0$ & $I_O$ = –ve

* Now when $S_4$ is also turned off energy stored in inductor (+ooo−) flows current through $D_1$ & $D_2$) (i.e L − $D_1$ − $V_S$ − $D_2$ − E − R)
current flows through supply, thus current is
$I_o$ = −ve & $V_o$ = +ve ∴ Chopper operating is II quadrant.




III quadrant operation.




To reverse the direction of rotation of motor (E = −ve) when $SW_3$ & $SW_4$ is turned ON, current flows ($V_S$ − $SW_3$ − E − R − L − $SW_4$) & inductor stores energy (−ooo+). o/p voltage $V_o$ & $I_o$ is −ve (∵ direction of current is changed)
∴ Chopper is operating in III quadrant.

Note:
4 quadrant chopper has the capability to operate in all the four quadrants.

July 2014

## Switching Mode Regulators:

DC choppers can be used as switching mode regulators to convert a dc voltage i.e unregulated to a regulated dc output voltage.

Regulation is achieved by pulse width modulation at a fixed frequency and the switching device is normary power BJT, MOSFET or IGBT.

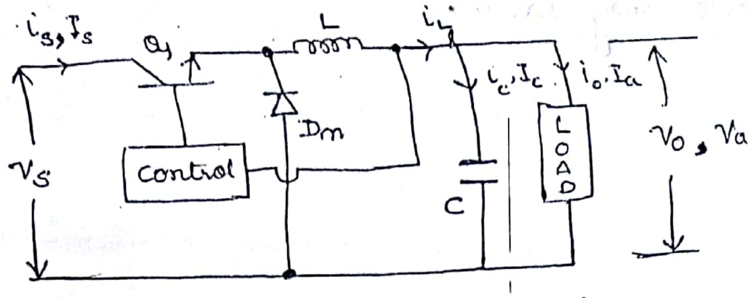
Different types of dc-dc converters are

1. Buck (step-down) converter.
2. Boost (step-up) converter.
3. Buck-Boost (stepdown-up) converter.
4. Cuk converter.

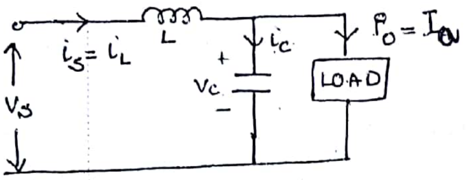
* Buck-Boost and Cuk converters are the combination of step-down and step-up converter.

### 1. Buck Converters or Regulators.

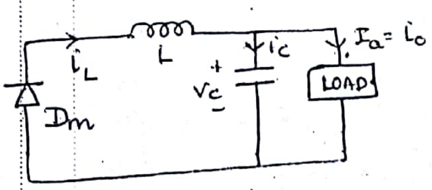
* In buck regulator avg output voltage is less than the input voltage $V_s$. Hence the name buck.
* Circuit diagram is as shown in the figure.
* By varying the duty cycle $K$ of the switch, the average o/p voltage can be controlled.
* very popular regulator

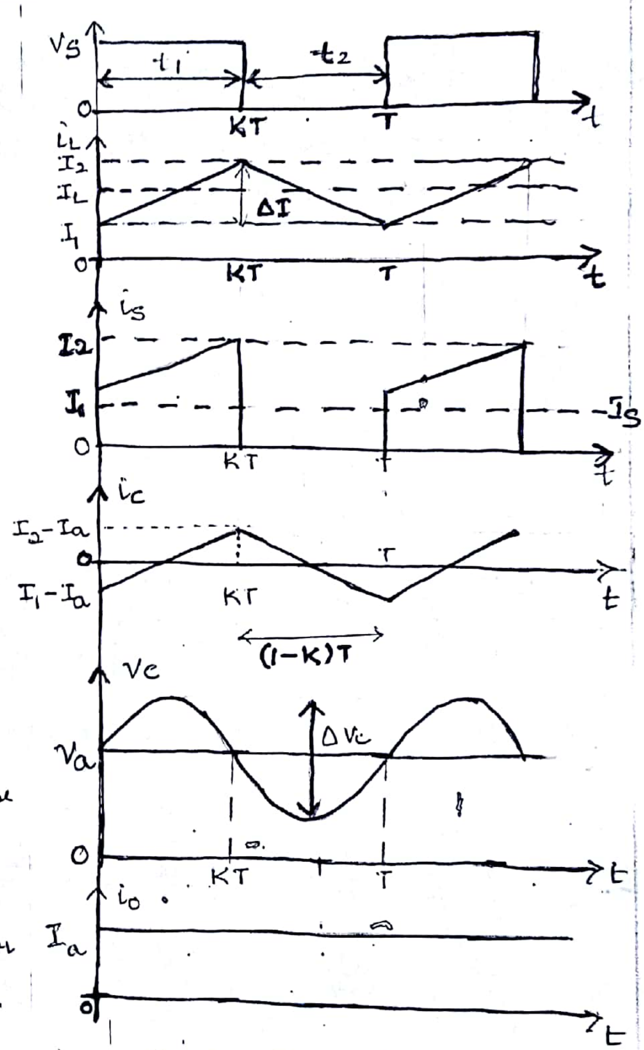



mode 1




mode 2




The circuit operation can be divided into two modes

mode 1 begins when transistor Q is switched ON at $t = 0$.

The input current which increases flows through L, C and load resistor R.

mode 2 begins when transistor Q is switched off at $t = t_1$. The free wheeling diode $D_m$ conducts due to energy stored in the inductor and current continues to flow through L, C and $D_m$.

Inductor current falls until $Q_1$ is switched on again in the next cycle

Voltage across the inductor L is

$$V_L = L \cdot \frac{di}{dt}$$

when current rises linearly from $I_1$ to $I_2$ in time $t_1$,

$$V_s - V_a = L\frac{(I_2 - I_1)}{t_1} = L \cdot \frac{\Delta I}{t_1}$$

$$\Rightarrow \boxed{t_1 = \frac{\Delta I \cdot L}{V_s - V_a}} \quad \& \quad \boxed{\Delta I = \frac{(V_s - V_a)t_1}{L}} \rightarrow (1)$$

at time $t_2$, the inductor current falls linearly from $I_2$ to $I_1$.

$$-V_a = -L\frac{\Delta I}{t_2}$$

$$\Rightarrow \boxed{t_2 = \frac{\Delta I \cdot L}{V_a}} \quad \& \quad \boxed{\Delta I = \frac{V_a t_2}{L}} \rightarrow (2)$$

Equating eq$^n$ (1) and eq$^n$ (2)

$$\Delta I = \frac{(V_s - V_a) t_1}{\cancel{L}} = \frac{V_a \cdot t_2}{\cancel{L}}$$

$$V_s t_1 - V_a t_1 = V_a \cdot t_2$$

$$V_a (t_1 + t_2) = V_s (t_1)$$

$$V_a = V_s \left(\frac{t_1}{t_1 + t_2}\right)$$

$$\boxed{V_a = V_s \cdot K} \rightarrow (3) \quad \text{Where } K = \frac{t_1}{t_1 + t_2} = \text{duty Cycle}$$

Assuming a lossless circuit,

$$V_s I_s = V_a I_a = K \cdot V_s \cdot I_a$$

and average input current $I_s = K \cdot I_a$

* Switching period T can be written as

$$T = t_1 + t_2 = \frac{\Delta I\ L}{V_s - V_a} + \frac{\Delta I\ L}{V_a} = \frac{\Delta I \cdot L \cdot V_s}{V_a (V_s - V_a)} \rightarrow (4)$$

$$\frac{1}{f} = \frac{\Delta I \cdot L \cdot V_s}{V_a(V_s - V_a)} \rightarrow (5)$$

$\therefore$ peak to peak ripple current of Inductor

$$\Delta I = \frac{V_a(V_s - V_a)}{f \cdot L \cdot V_s} = \frac{V_a\left(\frac{V_s}{V_s} - \frac{V_a}{V_s}\right)}{f \cdot L}$$

$$\Delta I = \frac{V_a(1-K)}{f \cdot L} = \frac{K \cdot V_s(1-K)}{f \cdot L} \rightarrow (6)$$

Using KCL, inductor current can be written as

$$i_L = i_c + i_o \rightarrow (7)$$

if we assume that load ripple current $\Delta i_o$ is very small and negligible $\Delta i_L = \Delta i_c \rightarrow (8)$

* average capacitor current which flows into for $\frac{t_1}{2} + \frac{t_2}{2} = \frac{T}{2}$

$$I_c = \frac{\Delta I}{4} \rightarrow (9)$$

* The capacitor voltage is expressed as

$$V_c = \frac{1}{C}\int i_c \, dt + v_c(t=0) \rightarrow (10)$$

P-P voltage of capacitor is

$$\Delta V_c = V_c - V_c(t=0) = \frac{1}{C}\int i_c \cdot dt + \cancel{V_c(t=0)} - \cancel{V_c(t=0)}$$

$$\Delta V_c = \frac{1}{C}\int_0^{T/2} \frac{\Delta I}{4} \cdot dt = \frac{\Delta I \, T}{8C} = \frac{\Delta I}{8fC} \rightarrow (11)$$

Substituting eq (6) in (11)

$$\Delta V_c = \frac{V_a(V_s - V_a)}{8LCf^2 V_s}$$

or

$$\boxed{\Delta V_c = \frac{V_s \cdot K(1-K)}{8 \cdot f^2 \cdot LC}}$$

* Condition for Continuous inductor current. If $I_L$ is avg inductor current then $\Delta I = 2I_L = 2I_a$

$$\frac{V_s(1-K) \cdot K}{f \cdot L} = \frac{2 \cdot K \cdot V_s}{R}$$

$$\boxed{L_c = L = \frac{R(1-K)}{2f}}$$ ($L_c$ = critical value)

$\|^{ly}$ Avg capacitor voltage $\Delta V_c = 2V_a$

$$\frac{V_s K(1-K)}{8 L \cdot C f^2} = 2 \cdot K \cdot V_s$$

$$\boxed{C_c = C = \frac{(1-K)}{16 f^2 L}}$$

* Since the buck regulator requires only one transistor it is simple and has high efficiency greater than 90%.
* The inductor L limits the $di/dt$ of the load current
* The i/p current is discontinuous and a smoothing i/p filter is required.
* It provides one polarity of output voltage and unidirectional output current. It requires a protection circuit in case of possible short circuit across the diode path.

## Problem

The buck regulator has an input voltage of $V_s = 12V$. The required output voltage is $V_a = 5V$, at $R = 500\Omega$ and peak to peak output ripple voltage is 20 mV. The switching frequency is 25 KHz. If the peak to peak ripple current of inductor is limited to 0.8 A, determine
a) duty cycle K
(b) filter inductance L
(c) filter capacitor C
d) critical value of L & C

Given data
$V_s = 12V$
$V_a = 5V$
$\Delta V_c = 20mV$
$f = 25kHz$
$\Delta I = 0.8A$
$K = ?$
$L = ?$
$C = ?$
$R = 500\Omega$

Soln Given $V_s = 12V$, $V_a = 5V$, $\Delta V_c = 20mV$ $f = 25KHz$. $\Delta I = 0.8A$

a) $V_a = K \cdot V_s \rightarrow K = \frac{V_a}{V_s} = \frac{5}{12} = 0.4167 = 41.67\%$

b) $\Delta I = \frac{V_s K(1-K)}{fL}$

$$L = \frac{V_s K(1-K)}{f \cdot \Delta I} = \frac{12 \times 0.41 \times (1-0.41)}{25K\,(0.8A)} = 145.83\,\mu H$$

c) $\Delta V_c = \frac{\Delta I}{8fC}$

$$C = \frac{\Delta I}{8 f \Delta V_c} = \frac{0.8}{8(25K)\,20mV} = 200\,\mu F$$

d) $L_c = L = \frac{(1-K)R}{2f} = \frac{(1-0.4167)\,500}{2 \times 25 \times 10^3} = 5.83mH$, $C_c = \frac{1-K}{16Lf^2} = 0.4\mu F$

## Boost Regulators

* In a boost regulator, the output voltage is greater than the input voltage – hence the name 'boost'.

* A boost regulator using a power MOSFET is shown in fig and is similar to a step-up chopper.

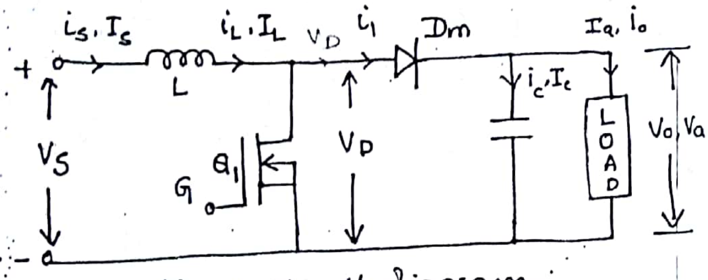


fig: circuit diagram

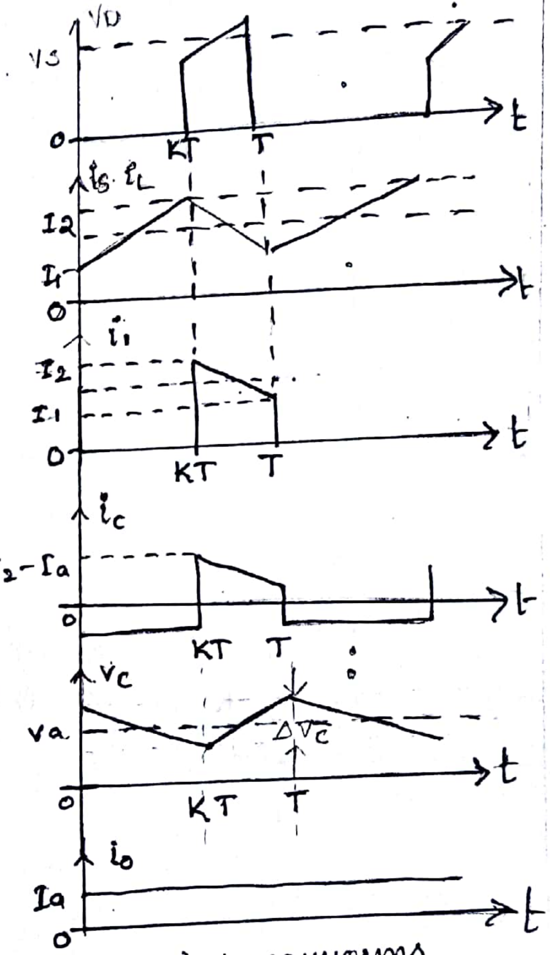


fig: waveforms

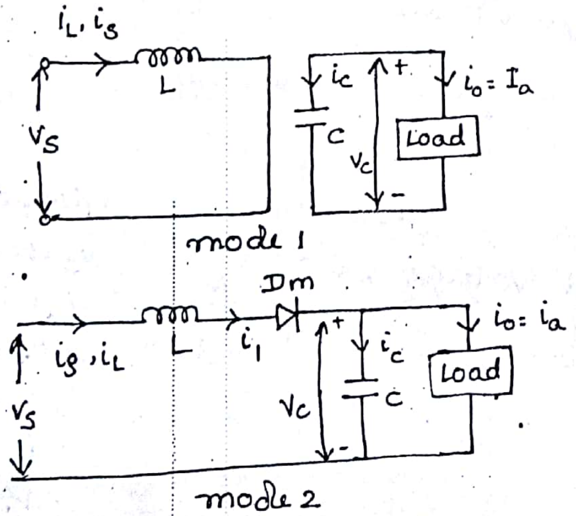



fig: Equivalent Circuits

* The circuit operation can be divided into two modes.

⊛ mode 1 begins when transistor $Q_1$ is switched on at $t=0$. The input current, which rises, flows through inductor L and transistor $Q_1$.

⊛ mode 2 begins when transistor $Q_1$ is switched off at $t=t_1$. The current which was flowing through the transistor would now flow through L, C, load and diode Dm. The inductor current falls until transistor $Q_1$ is turned on again in the next cycle.

* The energy stored in inductor L is transferred to the load.
* The equivalent circuits for the modes of operation are shown in figure. The waveforms for voltages and currents are shown for continuous load current.

Assuming that the inductor current rises linearly from $I_1$ to $I_2$ in time $t_1$,

$$V_S = L\frac{(I_2 - I_1)}{t_1} = L\frac{\Delta I}{t_1} \Rightarrow t_1 = \frac{\Delta I L}{V_S}$$

and the inductor current falls linearly from $I_2$ to $I_1$ in time $t_2$

$$V_S - V_a = -L\frac{\Delta I}{t_2} \Rightarrow t_2 = \frac{\Delta I L}{(V_a - V_S)}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L.

$$\therefore \Delta I = \frac{V_S t_1}{L} = \frac{(V_a - V_S) t_2}{L}$$

Substituting $t_1 = KT$ and $t_2 = (1-K)T$ yields avg o/p voltage

$$V_a = V_S \frac{T}{t_2} = \frac{V_S}{1-K} \qquad \therefore \frac{t_2}{T} = (1-K) \Rightarrow T/t_2 = \frac{1}{1-K}$$

$$\therefore (1-K) = \frac{V_S}{V_a}$$

$$\therefore t_1 = \frac{K}{f}$$

Substituting $K = t_1/T = f t_1$, $\quad t_1 = \frac{(V_a - V_S)}{V_a f} = \frac{K}{f}$

* Assuming a lossless circuit, $V_S I_S = V_a I_a = \frac{V_S I_a}{(1-K)}$ and the average input current is,

$$I_S = \frac{I_a}{1-K}$$

* The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_S} + \frac{\Delta I L}{(V_a - V_S)} = \frac{\Delta I L V_a}{V_S(V_a - V_S)}$$

$\therefore$ peak to peak ripple current $\Delta I = \frac{V_S(V_a - V_S)}{f L V_a} = \frac{V_S K}{f L}$

When the transistor is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time $t_1$ is $I_c = I_a$ and the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = V_c - V_c(t=0) = \frac{1}{C}\int_0^{t_1} I_c \cdot dt = \frac{1}{C}\int_0^{t_1} I_a = \frac{I_a t_1}{C}$$

Substituting $t_1 = \frac{(V_a - V_s)}{V_a f}$, $\Delta V_c = \frac{I_a (V_a - V_s)}{V_a f C} = \frac{I_a K}{f C}$

**Condition for continuous inductor current and capacitor voltage.**

If $I_L$ is the average inductor current, the inductor ripple current $\Delta I = 2 I_L$

$$\frac{K V_s}{f L} = 2 I_L = 2 I_a = \frac{2 V_s}{(1-K) R}$$

which gives the critical value of the inductor $L_c$ as

$$L_c = L = \frac{K(1-K) R}{2f}$$

If $V_c$ is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2 V_a$

W.K.T $\Delta V_c = \frac{I_a K}{f C} = 2 V_a = 2 I_a R$

which gives the critical value of the capacitor $C_c$ as

$$C_c = C = \frac{K}{2 f R}$$

A boost regulator has an input voltage of $V_s = 5V$. The average output voltage $V_a = 15V$ and the average load current $I_a = 0.5A$. The switching frequency is 25KHz. If $L = 150\mu H$ and $C = 220\mu F$, determine (a) duty cycle K (b) the ripple current of inductor $\Delta I_1$ (c) the peak current of inductor $I_2$ (d) the ripple voltage of filter capacitor $\Delta V_c$. (e) critical values of L and C.

Soln : Given $V_s = 5V$, $V_a = 15V$, $f = 25KHz$ $L = 150\mu H$ and $C = 220\mu F$

(a) $$V_a = \frac{V_s}{1-K}$$

$$15 = \frac{5}{1-K} \Rightarrow K = \frac{2}{3} = 0.6667 = 66.67\%$$

(b) $$\Delta I = \frac{V_s(V_a - V_s)}{f L V_a} = \frac{5(15-5)}{(25K)(150\mu)(15)} = 0.89 A$$

(c) $$I_s = \frac{I_a}{1-K} = \frac{0.5}{1-0.6667} = 1.5A$$

$$I_2 = I_s + \frac{\Delta I}{2} = 1.5 + \frac{0.89}{2} = 1.945 A.$$

(d) $$\Delta V_c = \frac{I_a K}{f c} = \frac{0.5(0.6667)}{(25K)(220\mu)} = 60.61 mV$$

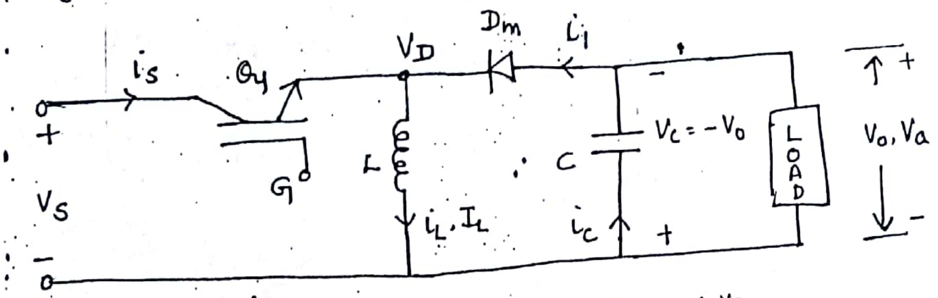
(e) $$R = \frac{V_a}{I_a} = \frac{15}{0.5} = 30\Omega$$

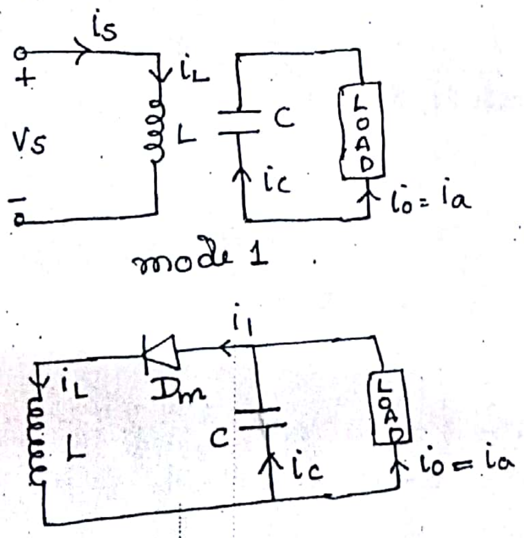
$$L_c = \frac{(1-K)KR}{2f} = \frac{(1-0.6667)\,0.6667 \times 30}{2 \times 25 \times 10^3} = 133\mu H$$

$$C_c = \frac{K}{2fR} = \frac{0.6667}{2 \times 25K \times 30} = 0.44\mu F$$

# Buck - Boost Regulators

* A buck boost regulator provides an output voltage that may be less than or greater than the input voltage hence the name "buck-boost". The output voltage is polarity is opposite to that of the input voltage. This regulator is also known as an inverting regulator.



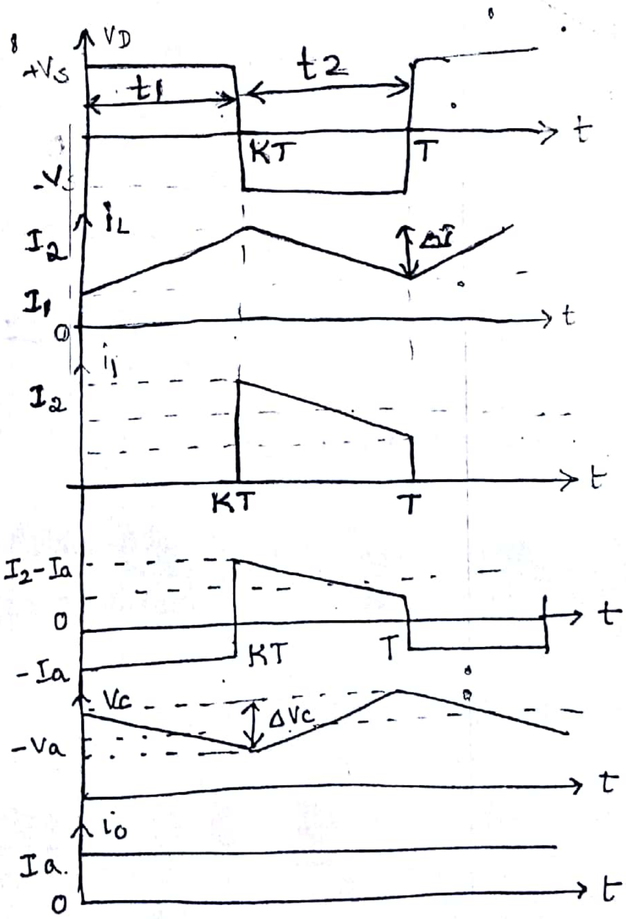




fig : Equivalent Circuits

* Circuit operation can be divided into two modes.

* During mode 1 transistor $Q_1$ is turned on and diode $D_m$ is reverse biased. The input current which rises flows

through inductor L and transistor $Q_1$.

* During mode 2, transistor $Q_1$ is switched off and the current; which was flowing through inductor L, would flow through L, C, $D_m$, and the load.

The energy stored in inductor L would be transferred to the load and the inductor current would fall until transistor $Q_1$ is switched on again in the next cycle.

* The equivalent circuits for the modes are shown in fig and waveforms for steady state voltages and currents of the buck boost regulator are shown for a continuous load current.

Assuming that the inductor current rises linearly from $I_1$ to $I_2$ in time $t_1$,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \Rightarrow t_1 = \frac{\Delta I L}{V_s}$$

and the inductor current falls linearly from $I_2$ to $I_1$ in $t_2$

i.e $$V_a = -L\frac{\Delta I}{t_2} \Rightarrow t_2 = \frac{-\Delta I L}{V_a}$$

$$\therefore \Delta I = \frac{V_s t_1}{L} = \frac{-V_a t_2}{L}$$

Substituting $t_1 = KT$ and $t_2 = (1-K)T$,

$$V_a = -\frac{V_s K}{(1-K)}$$

$$V_a = -\frac{V_s \cdot I_s}{I_a} = \frac{-V_s \cdot \cancel{I_a} \cdot K}{\cancel{I_a}(1-K)}$$

Assuming a lossless circuit, $V_s I_s = -V_a I_a = V_s I_a K/(1-K)$

and the average input current $I_s$ is related to the average output current $I_a$ by

$$I_s = \frac{I_a K}{1-K}$$

The Switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_s} - \frac{\Delta I L}{V_a} = \frac{\Delta I L (V_a - V_s)}{V_s \cdot V_a}$$

and this gives the peak to peak ripple current,

$$\Delta I = \frac{V_s V_a}{f L (V_a - V_s)} \quad \text{or} \quad \Delta I = \frac{V_s K}{f L} :$$

When transistor $Q_1$ is on, the filter capacitor supplies the load current for $t = t_1$. The average discharging current of the capacitor is $I_c = I_a$ and the peak to peak ripple voltage of the capacitor is

$$\Delta V_c = \frac{1}{C}\int_0^{t_1} I_c \cdot dt = \frac{1}{c}\int_0^{t_1} I_a \cdot dt = \frac{I_a t_1}{C}$$

when $t_1 = \frac{V_a}{(V_a - V_s) f}$, $\Delta V_c = \frac{I_a V_a}{(V_a - V_s) f C}$ or $\Delta V_c = \frac{I_a K}{f C}$

A buck-boost regulator provides output voltage polarity reversal without a transformer. It has high efficiency. Under a fault condition of the transistor, the di/dt of the fault current is limited by the inductor L and will be $V_s/L$. Output short-circuit protection would be easy to implement.

Condition for continuous inductor current and capacitor voltage.

If $I_L$ is the avg inductor current, the inductor ripple current $\Delta I = 2 I_L$.

$$I_a = \frac{V_a}{R} = \frac{V_s \cdot K}{(1-K)R}$$

$$\frac{K V_s}{f L} = 2 I_L = 2 I_a = \frac{2 K V_s}{(1-K) R}$$

which gives the critical value of the inductor $L_c$ as

$$L_c = L = \frac{(1-K) R}{2f}$$

If $V_c$ is the avg capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2 V_a$

$$\therefore \frac{I_a K}{C \cdot f} = 2 V_a = 2 I_a R$$

$$\frac{K}{2 f \cdot R} = C_c$$

1

which gives the critical value of the capacitor $C_c$ as

$$C_c = C = \frac{K}{2fR}$$

Problem

The buck boost regulator in figure has an input voltage of $V_s = 12V$. The duty cycle $K = 0.25$ and the switching frequency is 25 KHz. The inductance $L = 150\mu H$ and filter capacitance $C = 220\mu F$. The average load current $I_a = 1.25A$. Determine

(a) the average output voltage $V_a$

(b) peak to peak output voltage ripple $\Delta V_c$.

(c) peak to peak ripple current of inductor $\Delta I$

(d) peak current of the transistor $I_p$

(e) the critical values of L and C.

Soln Given $V_s = 12V$, $K = 0.25$, $I_a = 1.25A$, $f = 25KHz$ $L = 150\mu H$ and $C = 220\mu F$.

(a) $$V_a = \frac{-V_s(K)}{(1-K)} = \frac{-12(0.25)}{(1-0.25)} = -4V$$

(b) $$\Delta V_c = \frac{I_a K}{fC} = \frac{1.25 \times 0.25}{25K(220\mu)} = 56.8\,mV$$

(c) $$\Delta I = \frac{V_s K}{fL} = \frac{12(0.25)}{25K(150\mu)} = 0.8A$$

(d) $I_s = \frac{I_a K}{1-K} = 0.4167\,A$. Because $I_s$ is the average of duration $KT$, the peak-to-peak current of the transistor

$$I_p = \frac{I_s}{K} + \frac{\Delta I}{2} = \frac{0.4167}{0.25} + \frac{0.8}{2} = 2.067\,A$$

(e) $$R = \frac{-V_a}{I_a} = \frac{4}{1.25} = 3.2\Omega$$

$$L_c = \frac{(1-K)R}{2f} = \frac{(1-0.25)\,3.2}{2 \times 25 \times 10^3} = 450\mu H$$

$$C_c = \frac{K}{2fR} = \frac{0.25}{2 \times 25 \times 10^3 \times 3.2} = 1.56\mu F.$$