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Module - 4

Deepthi HS

Part1: Color Image Processing

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→ Color models

→ Color Convertions ⇒ RGB ↔ CMY, RGB ↔ HSI

→ Pseudocolor image processing

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→ Basics

→ Image Pyramid, Subband coding

→ Multiresolution Expansions.

Part3: Morphological Image Processing

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↳ Hole filling

↳ Extraction of Connected Components

↳ Convex hull

↳ Thinning

↳ Thickening

↳ Skeletons

↳ pruning.

Module-4

Part I: Color Image Processing

Why Color image Processing?

* Color is a powerful descriptor that often simplifies object identification.
* Humans can discern thousands of color shades & intensities.

Color fundamentals:

* Sir Isaac Newton showed that when a beam of Sunlight passes through a glass prism, the emerging beam of light consist of continuous spectrum of colors ranging from violet at one end to red at the other end which span the electromagnetic spectrum at different wavelengths is as shown in fig. below. i.e. (400 to 700nm)

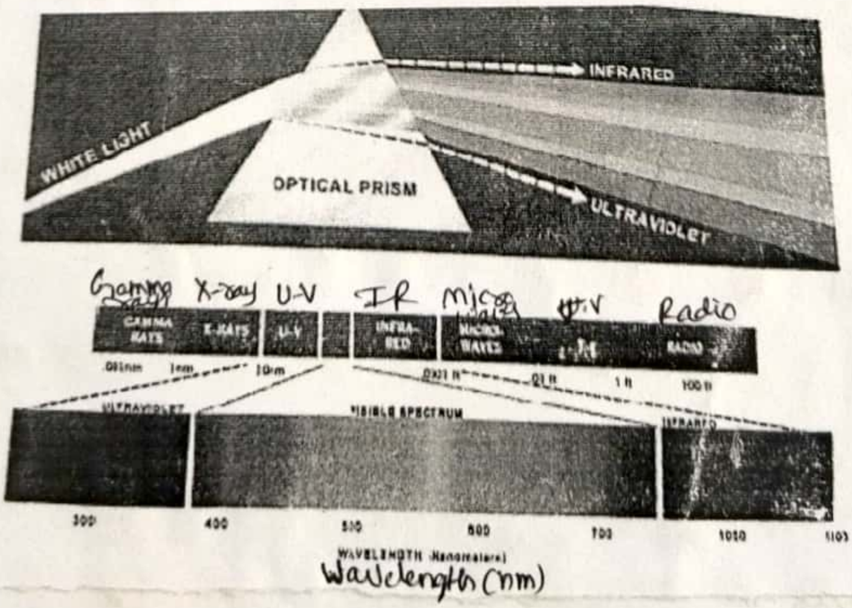
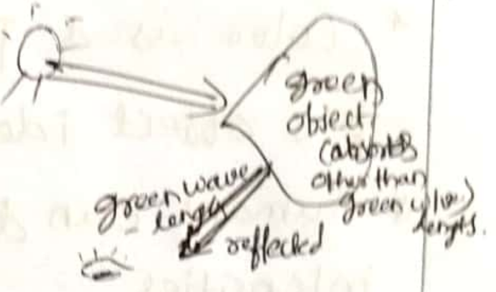


Figure: Wavelengths comprising the visible range of EM spectrum.

* The color of an object is determined by the nature of the light reflected from the object. For Ex: Green object reflect light with wavelengths primarily in the 500 to 570nm range while absorbing most of other wavelengths.



* Light can be < achromatic / Chromatic

* Achromatic light [No color], it only attributes its intensity.

* Chromatic light spans the EM spectrum from approximately 400 to 700nm.

* 3 - Quantities are used to describe the quantity of a chromatic light source

1) <u>Radiance</u>: Radiance is total amount of Energy that flows from the light Source & is measured in watts.

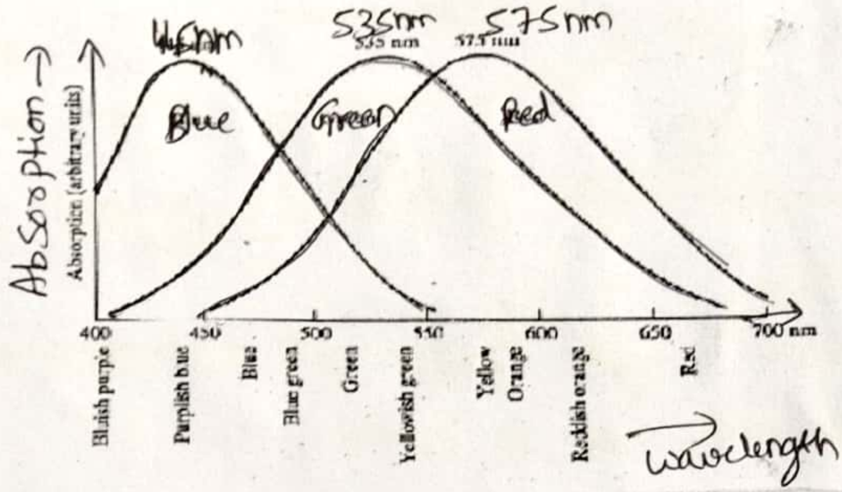
2) <u>Luminance</u>: It is the measure of amount of Energy an observer perceives from light Source.

3) <u>Brightness</u>: Brightness is a subjective descriptor that is practically impossible to measure.

* Cones are sensors in Eye responsible for color Vision. Rods in an Eye detect average intensity across spectrum.

Scanned with CamScanner

* There are total of 6 to 7 millions of Cones in human -Eye. out of which 65% of the Cones are Sensitive to red color, 33% of Cones are sensitive to green Color & only 2% are Sensitive to blue Color.



* Above fig. Shows an average Experimental Curves detailing the absorption of light by the red, green & blue Cones in the Eye.

* Primary & Secondary Colors :-

* Due to the absorption characteristics of human Eye, colors are seen as a Combination of Primary colors i.e. Red (R), Green (G) & Blue (B).

* Primary Colors Can be added to produce the Secondary colors of light i.e. Cyan (C), Magenta (M) & yellow (Y).

i.e. C = Green (G) + Blue (B)

M = Red (R) + Blue (B)

Y = Red (R) + Green (G)

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = 1 - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

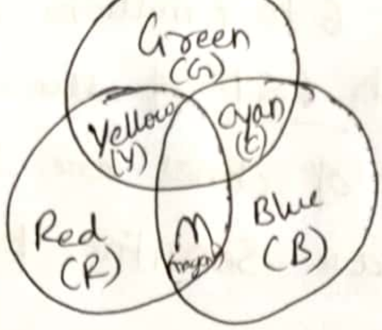


fig. Primary & Secondary Colors of light.

<u>Primary Colors</u>: R, G, B

* a proper Combination of R, G, B will produce white color.

i.e. R + G + B ⇒ White

* <u>Secondary Colors</u>: C, M, Y

<u>C + M + Y ⇒ Black</u>

* The Characteristics generally used to distinguish one Color from another are

i) <u>Hue</u>: Hue is a color attribute that represents dominant Color as perceived by an observer.

Ex: pure red

ii) <u>Saturation</u>: Saturation refers to amount of white light mixed with hue.

iii) <u>Intensity</u>: Intensity refers to brightness of an image, which is a subjective descriptor.

Hue + Saturation is called <u>Chromaticity</u>.

Chromaticity + Brightness is called <u>Color</u>.

* <u>Tristimulus values</u>: The amount of R, G & B needed for any color is called tristimulus Values.

Color Models :

* Color models are used to specify the colors in some Standard way. 3-Color models → RGB Color model / CMY/CMYK color model / HSI color model.
* These are based on Cartesian Co-ordinate System i.e. 3-D Co-ordinate System. In this 3-D Subspace each Color is represented by a single point.
* Most color models are oriented towards hardware i.e. (Color monitor, printer, Scanner Etc) Ex: RGB color model, CMY/CMYK color model.

(or) Software oriented Ex: HSI color model.

(1) RGB Color model :

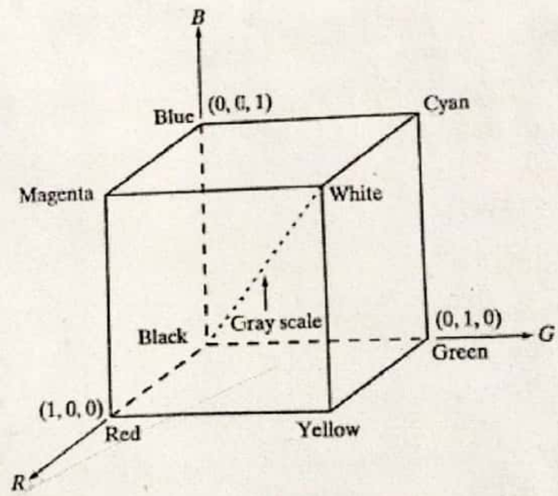
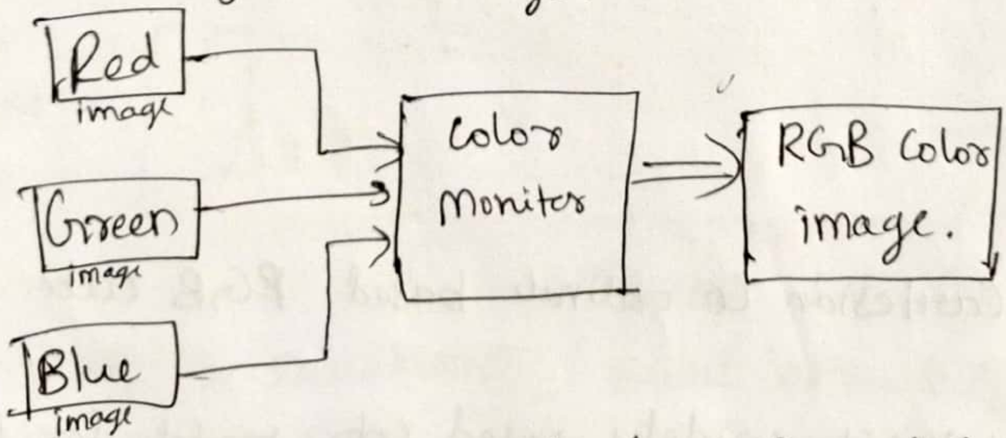


Figure: Cartesian Co-ordinate based RGB cube.

* RGB is most widely used color model for H/w oriented applications such as monitors, Cameras Etc.
* In this model, each color is represented by 3 values i.e. R, G & B indicating the amount of red, green & blue — which makes the color.

* R,G,B color model is based on Cartesian co-ordinate system.
* RGB color model is represented by a cube of unit length as shown in above fig.
* Black (0,0,0) is at origin & white (1,1,1) is at opposite end of black.
* All the values are assumed to be normalized to the range [0,1].
* Primary colors R(1,0,0), G(0,1,0) & B(0,0,1) are at the 3 corners.
* Secondary colors Cyan(0,1,1), Magenta(1,0,1) & yellow(1,1,0) are at the other 3 corners.
* Line joining black & white [Diagonal line through cube] represent all shades of gray.
* Generating RGB image:



* The number of bits used to represent each pixel in RGB is called pixel depth. If each Red, Green & Blue image is an 8-bit image then the depth of RGB color image pixel is $8+8+8 = 24$ bits, i.e. RGB color

image is supporting $(2^8)^3 = 2^{24} = 16,777,216$ colors.

* Many Systems used in today are limited to ⇒ 256 colors.
* Out of 256 colors only 216 colors have become standard & they are called safe colors, remaining 256-216 = 40 colors are processed differently by various operating system.
* From 216 safe colors, the following 6 ($6^3 = 216$) colors combination (difference of 51) is used to generate different shades of color

i.e.

Number System	Color Equivalents					
Decimal	0	51	102	153	204	255
Hex	00	33	66	99	CC	FF

R G B
00 00 00 ⇒ Black
FF FF FF ⇒ white
FF 00 00 ⇒ Pure Red
00 FF 00 ⇒ Pure Green Etc.

CMY & CMYK Color Model:

* Cyan(C) Magenta(M) & yellow (Y) are secondary colors of light

i.e. $\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$ ⇒ ① ⇒ * i.e. from Cyan surface Red will be absorbed, G+B reflected.

* From Magenta surface, Green will be absorbed, Red & Blue reflected.

* According to Equation ①, Equal amount of cyan, magenta & yellow should produce black. In practice combining these colors for printing produces a muddy looking black. So, inorder to produce true black, a fourth color black(K) is added giving rise to CMYK (cyan, magenta, yellow, K = Black) color model.

3) * The HSI Color Model :

* In the previous Sessions, we discussed with RGB & CMY models & changing from one model to other is a straightforward process.

* These (RGB & CMY) Color systems are ideally - Suitable for hardware implementations.

* Unfortunately, the RGB, CMY & other similar color models are not well suited for describing colors that are practical for human interpretation.

* when humans view a color object, we describe it by its Hue (H), Saturation (S) & Intensity (I).

→ Hue (H): hue is a color attribute that describes a pure color (Ex: pure red, pure yellow etc)

→ Saturation: It is the measure of the degree to which a pure color is diluted by white light.

→ Intensity (Brightness): is a subjective descriptor that is practically impossible to measure.

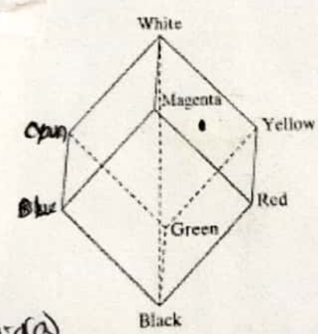
* The model that we are about to present, called the HSI (hue, Saturation, intensity) Color model, decouples the Intensity Component from the - Color-carrying information (hue & Saturation) in a Color - image.

* As a result, the HSI model is an ideal tool for developing image processing algorithms based on Color descriptions.

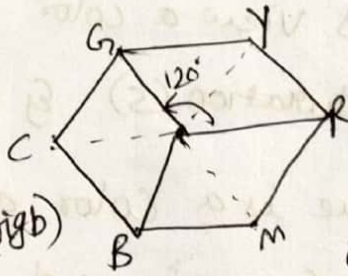
Note: we can summarize that RGB is ideal for image Color generation, but its use for color description is much more limited. So, we go for HSI model.

* How to obtain HSI Components?

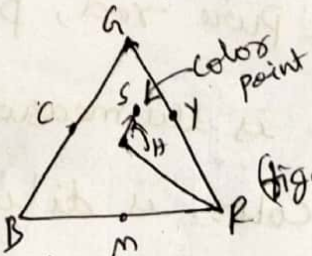
⇒ HSI Components can be obtained from RGB model. To show this, we will refer following fig's.

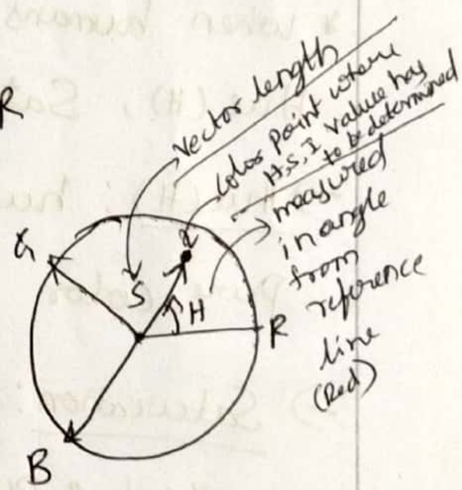


fig(a)
*cube in vertical direction.



(figb)



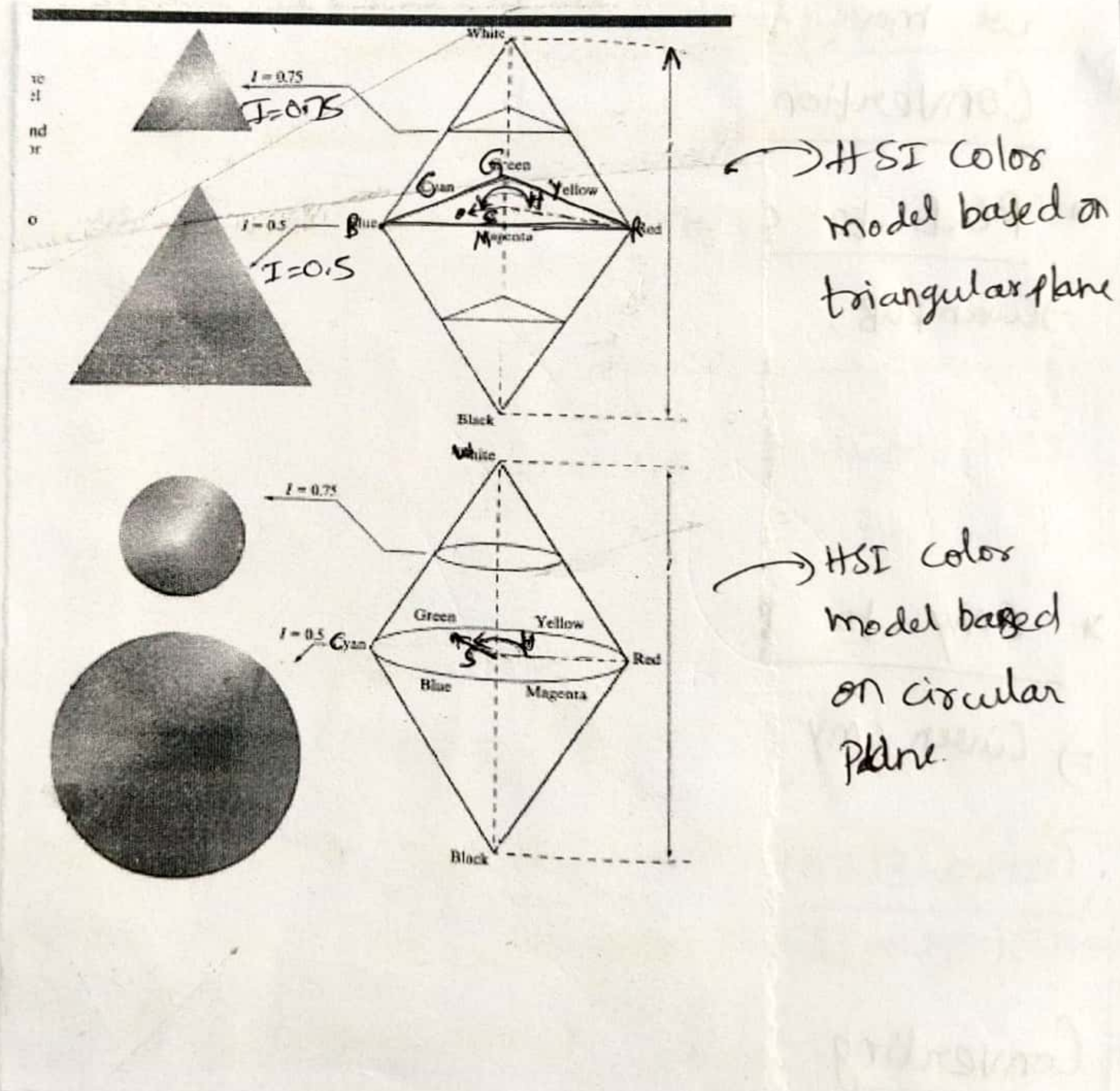


(figc)

* fig(a) Shows, RGB cube placed in vertical direction. The line joining Black (0, 0, 0) & white (1,1,1) vertices gives intensity value which is a vertical line.

* If we want to determine the intensity Component of any color point in fig(a), we would Simply Pass a plane perpendicular to the intensity axis & Containing the color point.

* The Point of intersection gives intensity value in the range [0,1].
* As shown in above fig (b) & (C) the primary colors (R,G,B) are seperated by 120° & secondary colors also by 120°. The distance between primary & secondary color is 60°.
* The HSI values can be obtained from RGB color cube, along with shape Hexagonal, circle (or) triangle shape can be used as shown in fig. below.



* Intensity (I) at any point is determined by passing a plane perpenticular to intensity axis (line joining Black & white) containing that point.

* Hue (H) ⇒ The hue of the point is an angle from some refrence axis (Ex: red axis as reference).

* Saturation (S):- is the length of the vector from the origin to color point.

⇒ As shown in above fig. the intensity (I) increases as we move from Black towards white.

Convertions between Color models:-

RGB ⇄ CMY

RGB ⇄ HSI

* RGB to CMY:-

⇒ Given RGB,

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

* CMY to RGB!

⇒ Given CMY

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

Converting Colors from RGB to HSI:-

Given an image in RGB Color format, the H, S & I Components obtained by using following Equations ⇒

* wkt 'H' is nothing but angle from reference axis. So,

$$\boxed{H = \begin{cases} \theta & \text{if } B \le G \\ 360^\circ - \theta & \text{if } B > G \end{cases}} \quad \text{—} (1)$$

where $\theta = \cos^{-1} \left\{ \dfrac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2 + (R-B)(G-B)]^{1/2}} \right\}$

* The Saturation Component is given by

$$\boxed{S = 1 - \frac{3}{(R+G+B)} [\min(R,G,B)]} \quad \text{—} (2)$$

* The intensity component is given by

$$\boxed{I = \frac{1}{3}(R+G+B)} \quad \text{—} (3)$$

Problem!

(1) Given RGB = (0.683, 0.1608, 0.1922) convert this to HSI.

Soln:

$H = 360^\circ - \theta \quad (\because B > G)$

where $\theta = \cos^{-1} \left[\dfrac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2 + (R-B)(G-B)]^{1/2}} \right]$

$= \cos^{-1} \left[\dfrac{\frac{1}{2}[(0.683-0.1608)+(0.683-0.1922)]}{(0.683-0.1608)^2 + (0.683-0.1922)(0.1608-0.1922)} \right]$

$\theta = 3.010$

$\Rightarrow H = 360 - \theta = 360 - 3.010 = 356.99^\circ$

$S = 1 - \dfrac{3}{R+G+B}\min(R,G,B)$

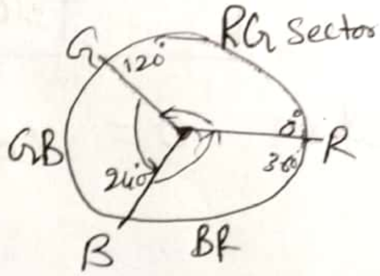
$= 1 - \dfrac{3 \times (0.1608)}{R+G+B} = 0.534$

$I = \dfrac{1}{3}(R+G+B)$

$I = 0.3453$

* Converting colors from HSI to RGB:-

There are 3-sectors of interest, corresponding to 120° intervals of Hue i.e.
- i) RG Sector
- ii) GB Sector
- iii) BR Sector



i)

RG Sector

$0 \leq H < 120°$

$$B = I(1-S)$$
$$R = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$$
$$G = 1 - (R+B)$$

GB Sector

$120° \leq H < 240°$

where $\boxed{H = H - 120°}$

$$R = I(1-S)$$
$$G = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$$
$$B = 1 - (R+G)$$

BR Sector

$240° \leq H < 360°$

where $\boxed{H = H - 240°}$

$$G = I(1-S)$$
$$B = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$$
$$R = 1 - (G+B)$$

(Pb) Given HSI = (356.99, 0.534, 0.3453) Convert to RGB model.

Solun: $H = H - 240°$ ($\because$ H = 356.99 belongs to BR Sector)

$H = 356.99 - 240$

$\boxed{H = 116.99}$

* $G = I(1-S)$

$= 0.3453(1-0.534)$

$\boxed{G = 0.1608}$

* $B = I\left[1 + \frac{S \cos H}{\cos(60° - H)}\right]$

$= 0.3453\left[1 + \frac{0.534 \cos(116.99)}{\cos(60° - 116.99)}\right]$

$\boxed{B = 0.1922}$

* $R = 1 - (G + B)$

$= 1 - (0.1608 + 0.922)$

$\boxed{R = 0.683}$

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* Pseudo Color image Processing:-

* Pseudo means false. Pseudo color image processing is assigning false colors to gray values based on specified criteria which is application dependent.
* The principal use of pseudo color processing is for human visualization & interpretation of gray-scale events in an image.
* Different methods of Pseudo color image processing are
  1) Intensity slicing
  2) Gray level to color transformation (or) Intensity to Color transformation.

1) Intensity Slicing:-

* This is the simplest method of assigning any colors to certain gray level based on some criteria.

*

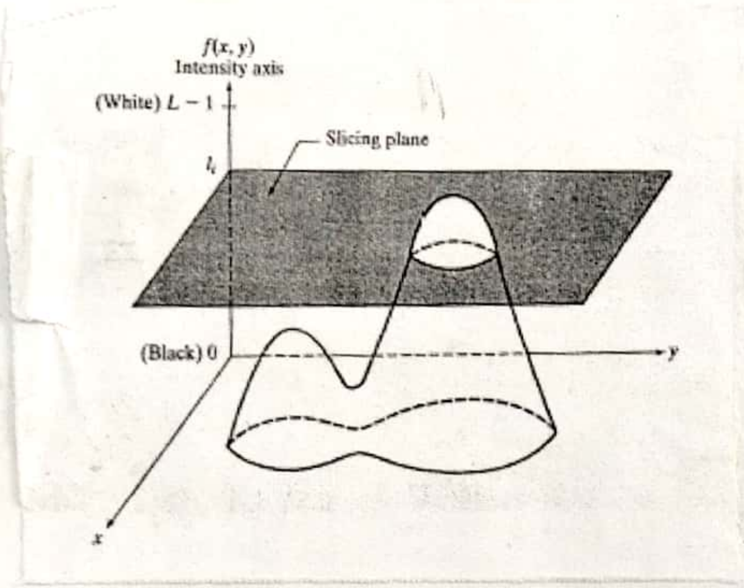


Figure: Geometric interpretation of the intensity slicing technique.

* As shown in above fig, if the gray levels are above the plane, then one particular color is assigned. If the gray levels are below the plane then second color is assigned.

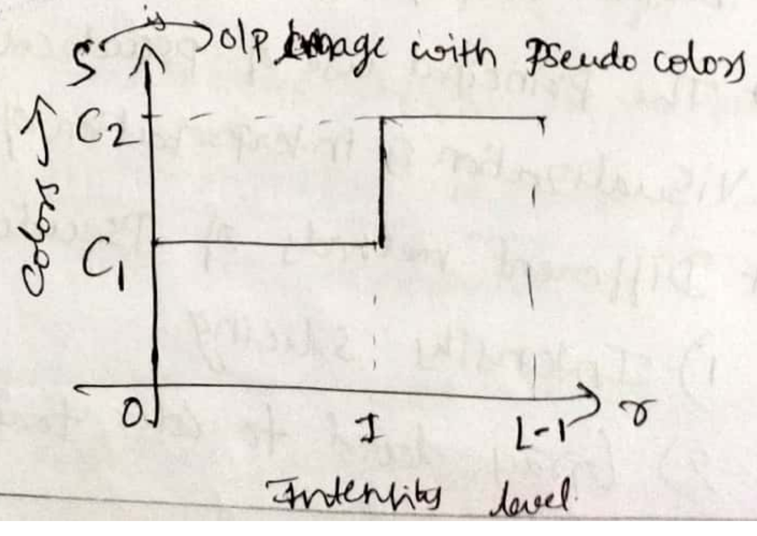
* If the gray levels are at the plane itself, then arbitrary color (out of these 2 colors) will be assigned.

* For one slicing (1 plane), we will get 2 different colors, for 2 slicing 4 different colors and so on.

Case 1: $S = T(r)$

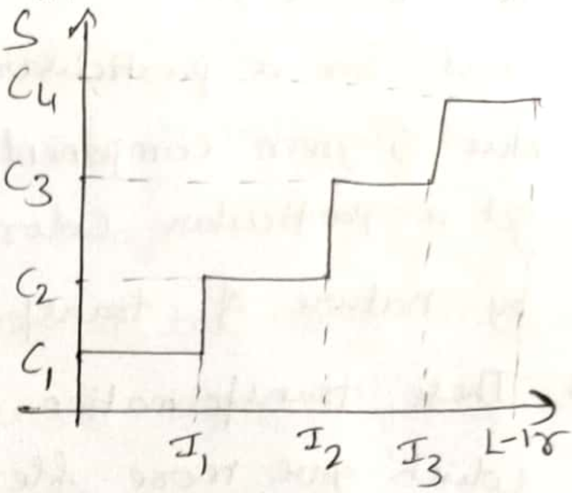
$$S = \begin{cases} C_1 & \text{if } r \leq I \\ C_2 & \text{if } r > I \end{cases}$$

i.e. when I/p 'r' is upto intensity level I, the color assigned is C_1.
If r is $> I$, C_2 color assigned.



Case 2: Color assignment is done at multiple interval as defined below:

$$S = \begin{cases} C_1 & \text{if } r \le I_1 \\ C_2 & \text{if } I_1 < r \le I_2 \\ C_3 & \text{if } I_2 < r \le I_3 \\ C_4 & \text{if } r > I_3 \end{cases}$$



* As shown here, 4 different colors are assigned for different intensity levels.

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② Gray level to color transformation :-
(Intensity level)

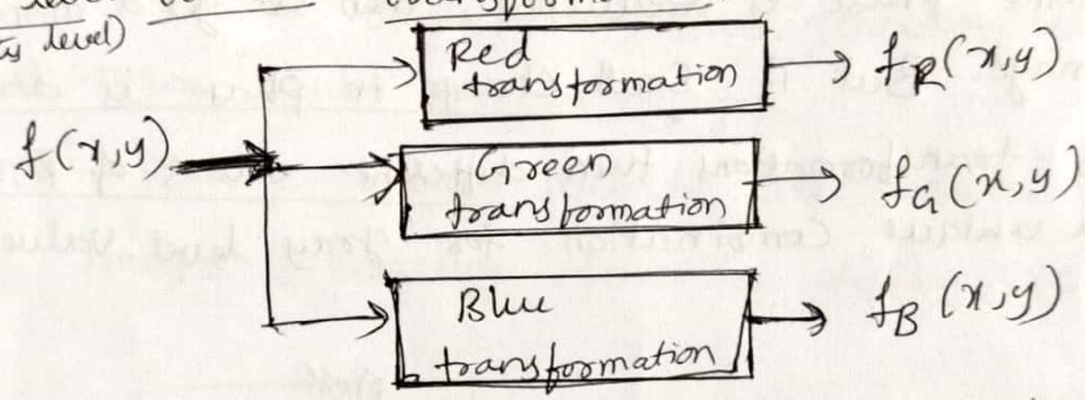


fig: Block diagram of gray level to color transformation.

* In this transformation 3-independent transformations i.e. Red, Green & Blue are applied on the intensity of I/P pixels. The results are fed separately into red, green & Blue channels of a color TV monitor.

* This transformation is particularly useful in certain cases where the objective is to highlight a portion of a image which is hidden under the background.

* Thus different transformation are used as shown in below fig, inorder to highlight a portion of the image in the best way.

* Red, Blue & green transformation can vary in phase so that for a particular intensity value, we add red, blue & green component in different proportions and get a particular color. Thus color content is modulated by nature of transformation function.

* These transformation are smooth non linear functio which give more flexibility than intensity slicing method.

* As shown in below fig. if all the 3 transformations have same phase & frequency, then we get a monochrome-image. But if small change in phase is done between 3-transformations then different colors of R, G, B generate a unique combination for gray level values.

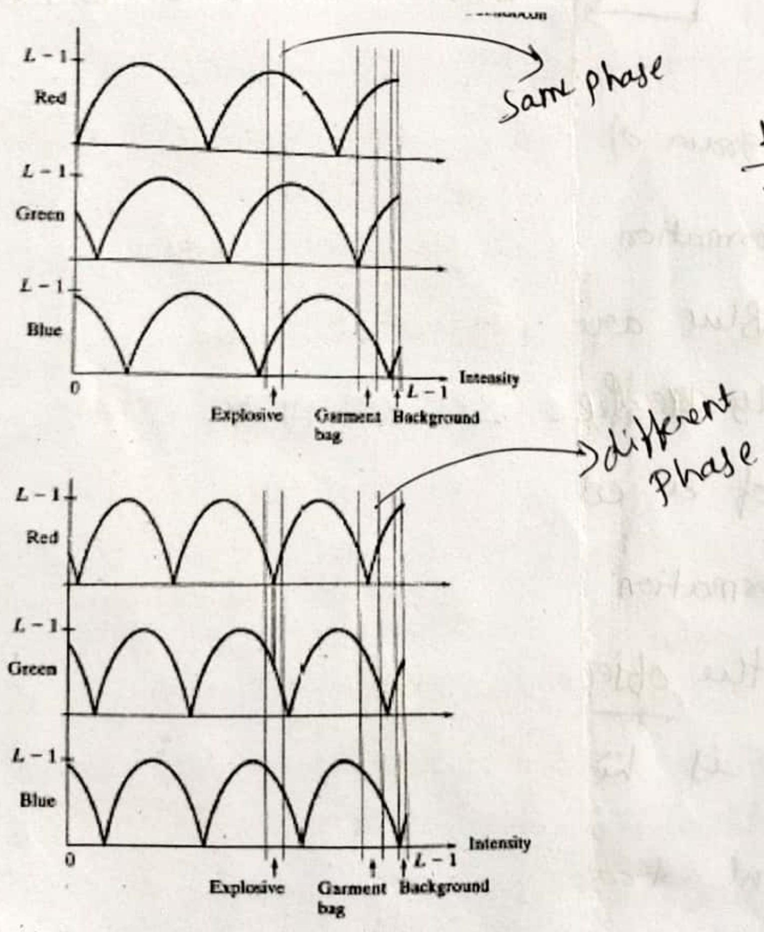


figure:
* Transformations used to assign different colors to the images based on R, G, B phases.

* Another approach in gray level to color transformation is to combine several monochrome images into a single color image. Generally it is used in multi spectral image processing (or) multisensor image processing.
* As shown in below fig $f_1(x,y)$ is the image in first spectral band (or) from first sensor. $f_2(x,y)$ is the image in other spectral band & so on.
* The transformation function for each spectral band (or) each sensor is a single color, thus generating color image for multisensor | multi spectral images.
* Additional processing may include color balancing, combining images & selecting few for display etc.

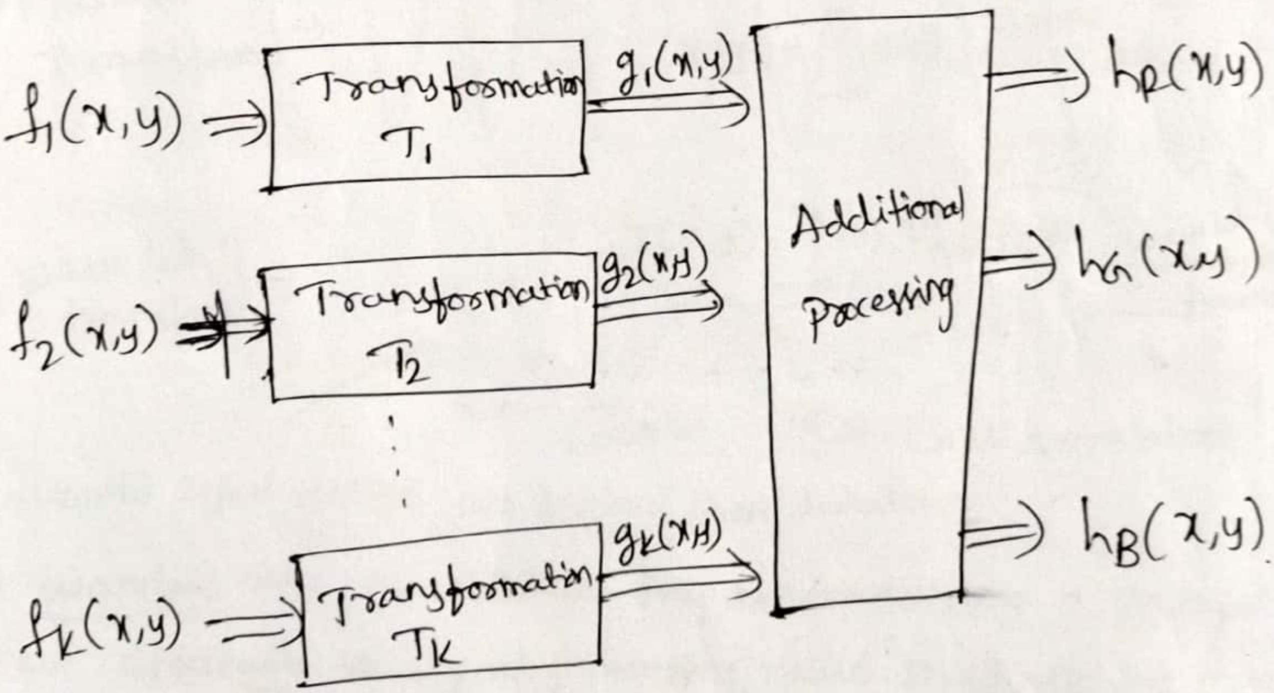


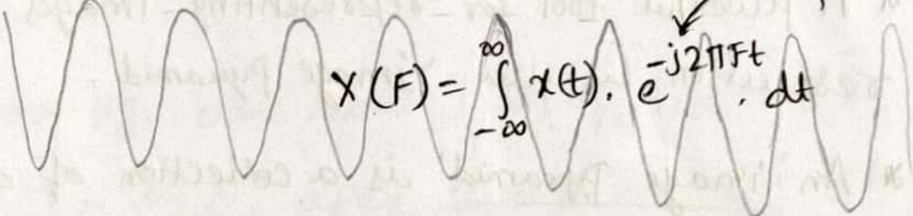
fig: Pseudo color coding approach used when several monochrome images are available.

Module-4
(Part-II)

Wavelets & Multiresolution Processing

* Since the late 1950, Fourier transform has been the mainstay of transform-based image processing.
* A more recent transformation, called the 'Wavelet transform', is now making it even easier to compress, transmit and analyze many images.
* Unlike the **Fourier** transform, whose basis functions are Sinusoids, wavelet transforms are based on Small waves, called wavelets, of varying frequency and limited duration.

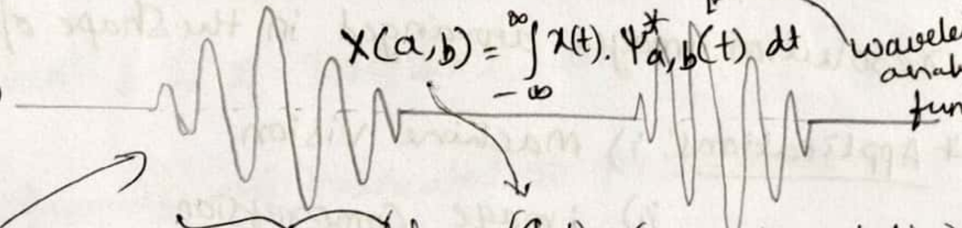
* Fourier Transform } ⇒



$$X(F) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi Ft} \cdot dt$$

Analysing function

* wavelet transform } ⇒



$$X(a,b) = \int_{-\infty}^{\infty} x(t) \cdot \psi^{*}_{a,b}(t)\, dt$$

wavelet analysing function

wavelet

$(a,b) = $ (Scale, translation)

⇒ Wavelet Signal does not last forever (Short duration).

* Wavelets are foundation for understanding a powerful new approach to signal processing called 'multiresolution' theory.
* Multiresolution Process means representing & analyzing signals at more than one resolution.

* Multiresolution theory incorporates techniques from a variety of disciplines, including 'Subband Coding' from Signal Processing, quadrature 'mirror filtering' from digital speech recognition, and 'pyramidal' image processing.

* Multiresolution theory is concerned with the representation & analysis of signals (or images) at more than one resolution. The objective of approach is - features that might go undetected at one resolution may be easy to detect at another.

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* Image Pyramids :

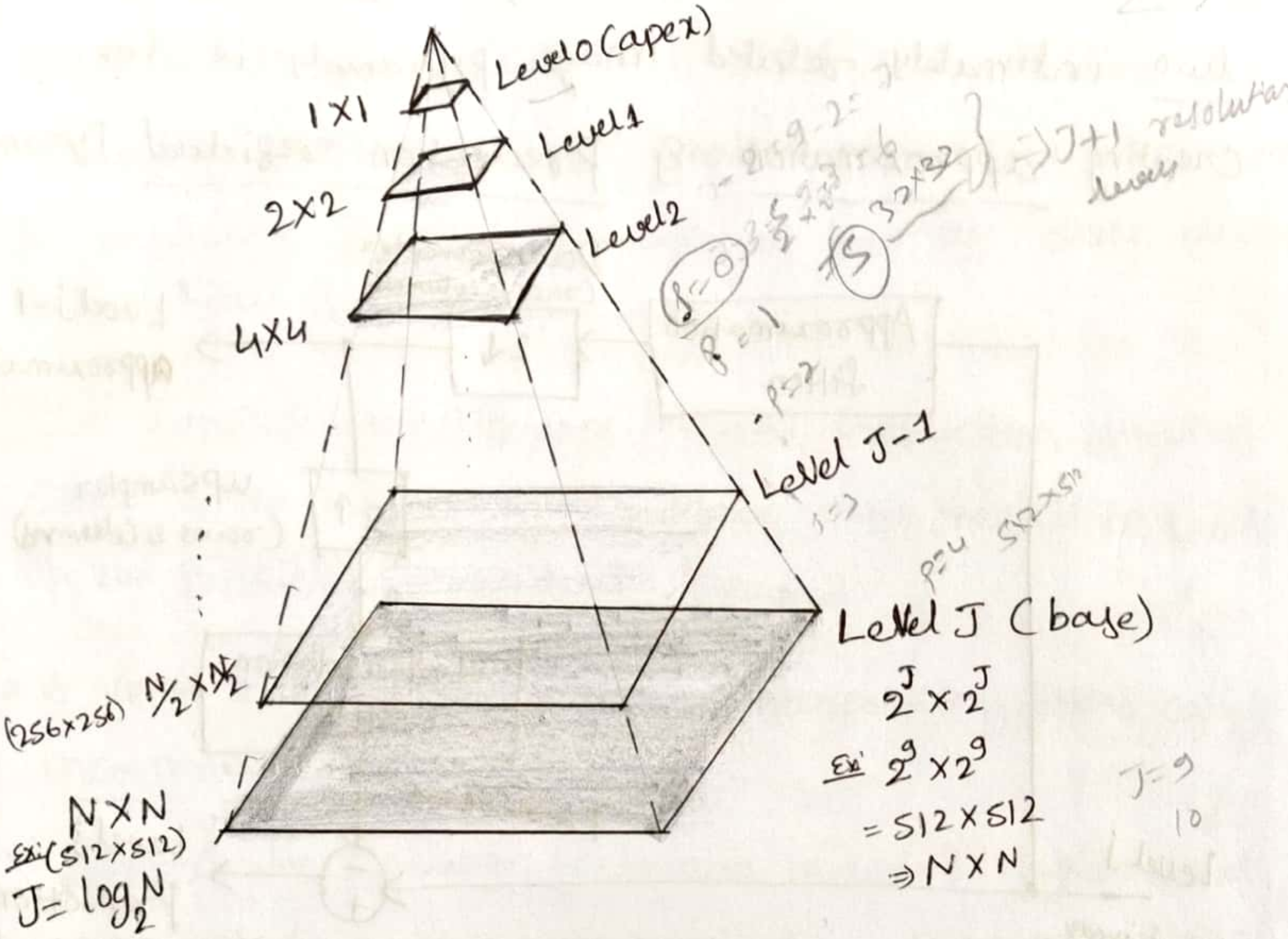
* A powerful tool for representing images at more than one resolution is the 'image pyramid'.

* An 'image pyramid' is a collection of decreasing - resolution images arranged in the shape of a pyramid.

* Applications: i) machine vision
ii) image compression.

* The base of the pyramid contains a high-resolution representation of the image being processed - whereas the apex contains a low - resolution approximation.

* As we move up the pyramid, both the size & resolution decrease as shown in fig below.



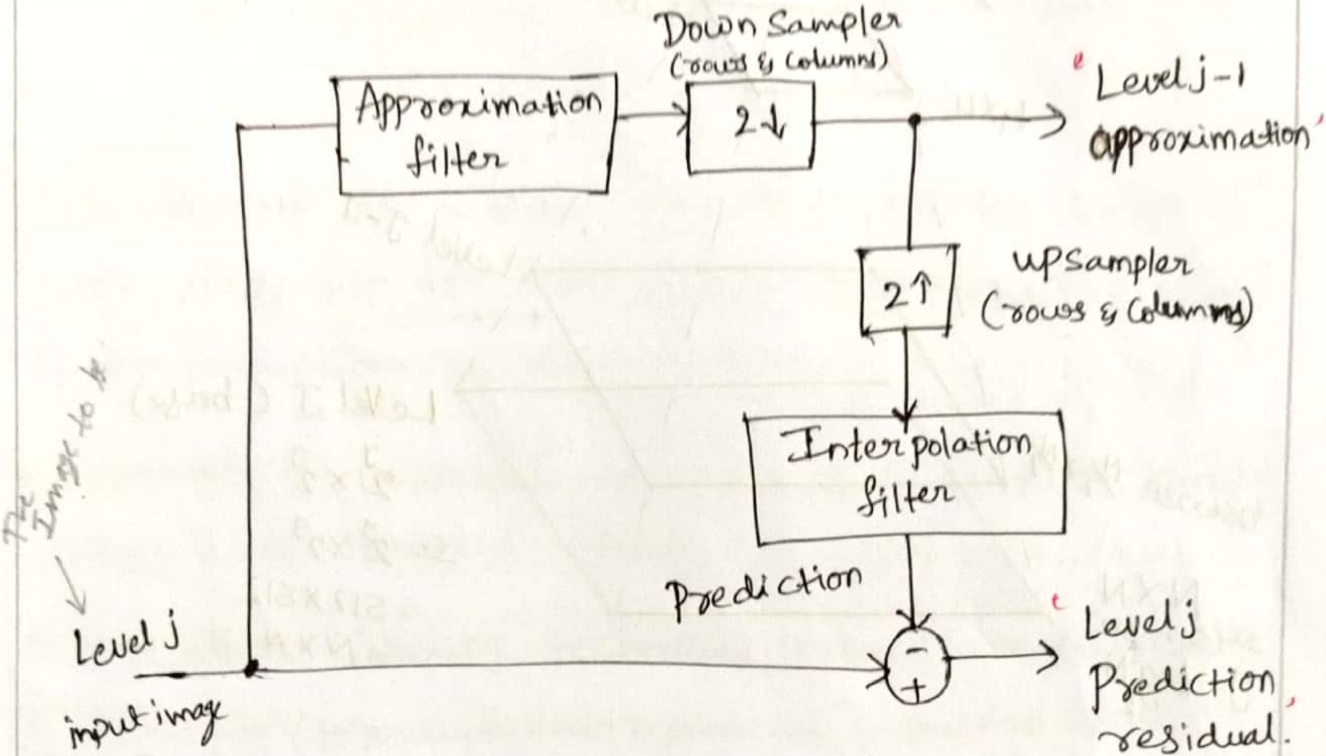
* As shown in above fig. base level J is of size $2^J \times 2^J$ (or) $N \times N$, where $J = \log_2 N$, apex level 0 is of size 1×1.
* And general level j is of size $2^j \times 2^j$, where $0 \le j \le J$.
* Most image pyramids are truncated to $P+1$ levels where $1 \le P \le J$ i.e. $j = J-P, \ldots, J-2, J-1, J$. i.e. we normally limit ourselves to P reduced resolution approximations of the original image.
* The total no. of pixels in a $P+1$ level pyramid for $P > 0$ is

$$N^2\left(1 + \frac{1}{(4)^1} + \frac{1}{(4)^2} + \cdots + \frac{1}{(4)^P}\right) \le \frac{4}{3}N^2$$

$$(512)^2\left(1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4}\right) = \frac{4}{3}N^2$$

Scanned with CamScanner

* Now, let us consider a simple system for creating two intimately related image pyramids i.e for creating 'approximation' & 'prediction residual' pyramids.



* From the above fig., the 'Level j-1 approximation' o/p provides the images needed to build an approximation pyramid, while the 'Level j prediction residual' o/p is used to build a complementary prediction residual pyramid.

* As above fig suggests, both approximation & prediction residual pyramids are computed in an iterative fashion.

Step1: Compute a reduced-resolution approximation of the Level j I/p image. This is done by filtering & downsampling the filtered o/p by a factor of 2. Place the resulting approximation at level j-1 of the approximation pyramid.

Step2: Create an Estimate of the Level j I/p image by taking the image generated in Step1. This is done by upsampling & filtering the generated approximation image in Step1. The resulting prediction image will have the same dimensions as the Level j I/p image.

Step3: Compute the difference b/w the prediction image of Step2 & the I/p to Step1. Place this result in Level j of the prediction residual pyramid.

* A variety of approximation & interpolation filters can be incorporated into the system.

=) Approximation filtering techniques include: neighborhood averaging (to produce mean pyramids), Lowpass Gaussian filtering (to produce Gaussian Pyramids), no filtering (Subsampling Pyramids).

=) Interpolation methods include: nearest neighbor, bilinear & bicubic.

* Upsampling is used to double & downsampling is used to halve the spatial dimensions.

* For a integer variable n and 1-D sequence of samples f(n), upsampled sequence $f_{2\uparrow}(n)$ is defined as:

$$f_{2\uparrow}(n) = \begin{cases} f(n/2) & \text{if } n \text{ is even} \\ 0 & \text{otherwise.} \end{cases}$$

* Signal divided by integer results in expansion

* Downsampling by a factor of 2 is defined as: $f_{2\downarrow}(n) = f(2n)$

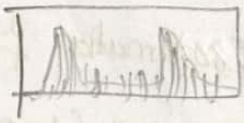
* Signal multiplied results in compression.

* upsampling can be thought of as inserting a '0'(zero) after every sample in a seauence; downsampling can be viewed as discarding every other sample.

* <u>Conclution</u>: i) the lower resolution levels of a Pyramid can be used for the analysis of large structures (or) overall image content.

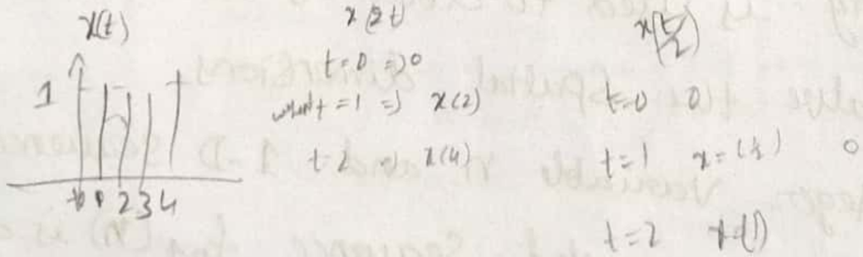
ii) the high resolution images are appropriate for analyzing individual object characteristics.

iii) the prediction residual Pyramids histogram is highly peaked around zero; the approximation histogram not.



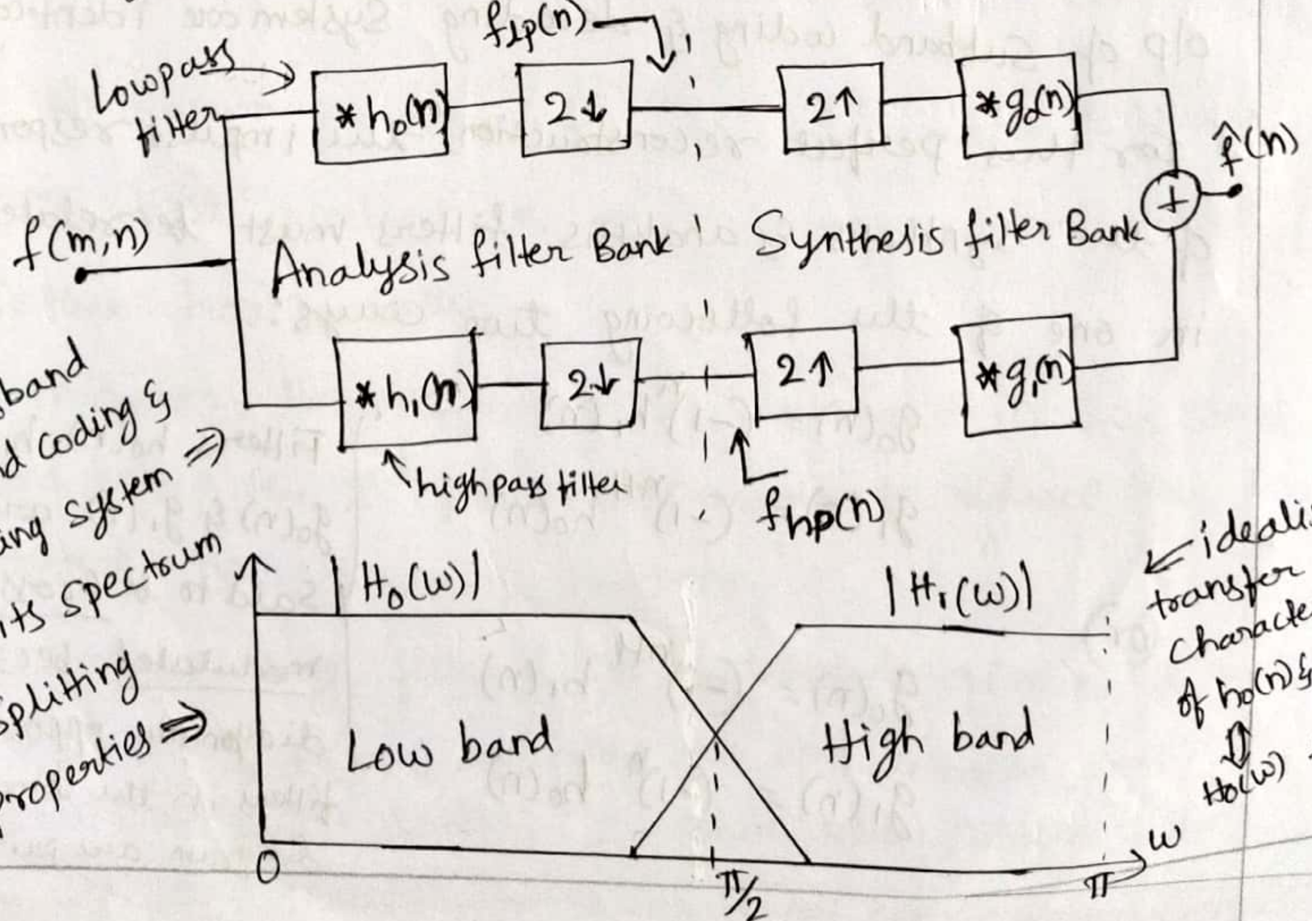
iv) unlike approximation images, prediction residual images can be highly compressed by assigning fewer bits.

* <u>Subband</u> <u>Coding</u> :



* Subband Coding:

* Another important imaging technique which ties to multiresolution analysis is Subband Coding.
* In Subband Coding, an image is decomposed into a Set of bandlimited Components, called Subbands.
* The decomposition is performed So that the Subbands Can be reassembled to reconstruct the original image without Error.
* This decomposition & reconstruction are performed by means of digital filters.
* Let us consider a two-band Subband coding & decoding system as shown in fig. below



* A twoband subband coding & decoding system ⇒ & its spectrum

* Splitting properties ⇒

← idealised transfer characteristic of $h_0(n)$ & $h_1(n)$ ⇕ $H_0(\omega)$ $H_1(\omega)$

* As shown in above fig., the system is composed of two filter banks i.e. Analysis filter bank & Synthesis filter bank, Each containing two FIR filters (Digital filter type).

* The analysis filter bank, which includes filters $h_0(n)$ & $h_1(n)$, is used to break input sequence $f(n)$ into two half length sequences $f_{lp}(n)$ & $f_{hp}(n)$ subbands.

* Synthesis bank filters $g_0(n)$ & $g_1(n)$ combine $f_{lp}(n)$ & $f_{hp}(n)$ to produce $\hat{f}(n)$.

* The goal in subband coding is to select $h_0(n)$, $h_1(n)$, $g_0(n)$ & $g_1(n)$ so that $\hat{f}(n) = f(n)$ i.e. I/P & o/p of subband coding & decoding system are identical.

* For this perfect reconstruction, the impulse responses of the synthesis & analysis filters must be related in one of the following two ways:

$$g_0(n) = (-1)^n h_1(n)$$
$$g_1(n) = (-1)^{n+1} h_0(n)$$

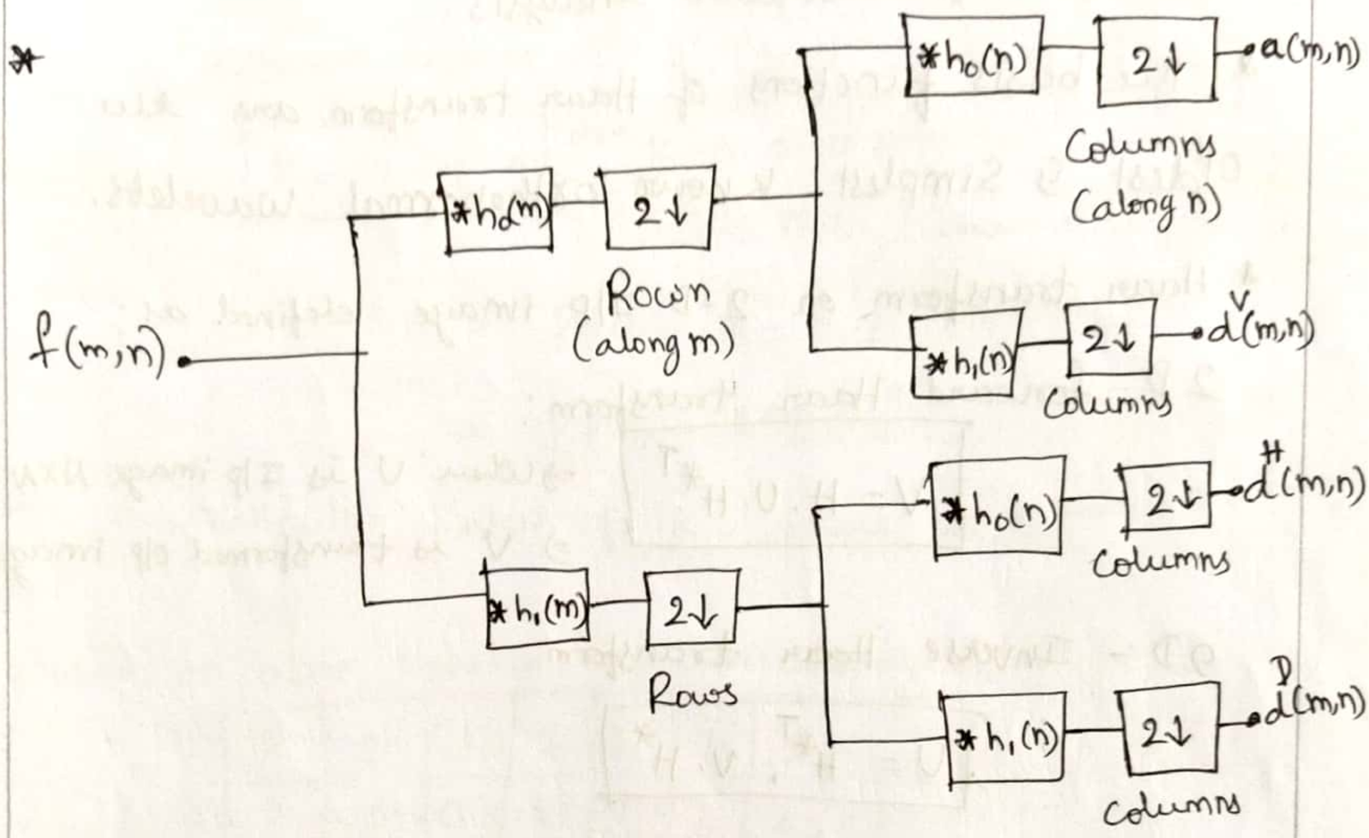
(or)

$$g_0(n) = (-1)^{n+1} h_1(n)$$
$$g_1(n) = (-1)^n h_0(n)$$

Filters $h_0(n)$, $h_1(n)$, $g_0(n)$ & $g_1(n)$ are said to be **cross-modulated** becz diagonally opposed filters in the block diagram are paired.

* One more approach for Subband Coding is use of 1-D orthonormal & biorthogonal filters can be used as 2-D Separable filters for the processing of images.

*



* As Shown in above fig. the separable filters are first applied in one-dimension (ex: vertically) & then horizontally.

* moreover, downsampling is performed in two stages, one before the 2nd filtering operation to reduce the overall no. of computations.

* The resulting filtered o/p's, denoted $a(m,n)$, $d^V(m,n)$, $d^H(m,n)$ & $d^D(m,n)$ are called the approximation, vertical detail, horizontal detail & diagonal detail subbands of the - I/p image, respectively. These subbands can undergo split again & so on.

* The Haar Transform :

↳ This is one more imaging-related operation which ties to multiresolution analysis.

* The basis functions of Haar transform are the Oldest & Simplest known orthonormal wavelets.

* Haar transform on 2-D I/p image defined as:

2D - forward Haar transform:

$$\boxed{V = H \cdot U \cdot H^{*T}}$$

⇒ when 'U' is I/p image N×N

⇒ 'V' is transformed o/p image.

2D - Inverse Haar transform:

$$\boxed{U = H^{*T} \cdot V \cdot H^{*}}$$

* H is a N×N, Haar transformation matrix, which Consists of Haar basis functions, $h_K(x)$. These functions are defined over Continuous interval $x \in [0, 1]$.

* To generate H, we define the integer 'K' such that $K = 2^p + q - 1$. where $0 \leq p \leq n-1$

* The steps to be followed to obtain Haar matrix H, are discussed below.

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Haar Transform:

* Basis functions of Haar Transform are non-sinusoidal functions.
* It is defined over continuous interval $x \in [0, 1]$.
* There are several steps to find Haar basis matrix. They are

Step 1: For different values of 'K', find values of P, q.

i.e. $\boxed{K = 2^P + q - 1}$ where, $K \Rightarrow 0$ to $N-1$

$0 \le P \le n-1$, where $n = \log_2 N$

* $P = 0$, $q = 0, 1$
* $P \neq 0$, $1 \le q \le 2^P$

* For a perticular value of 'K', there is a unique value of P & q.

* Based on above equation, we can construct table as,

K	0	1	2	3	4	5	6	7	...
P	0	0	1	1	2	2	2	2	...
q	0	1	1	2	1	2	3	4	...

* 'K' value gives no. of rows in Haar matrix

Step 2: Find interval $\boxed{x = \frac{m}{N}}$, for $x \in [0, 1]$

where $\boxed{0 \le m \le N-1}$

* $x = \frac{m}{N}$ gives no of columns in Haar matrix.

Step 3: i.e.

$$H = \begin{array}{c} K=0 \\ K=1 \\ K=2 \\ \vdots \\ K=N-1 \end{array} \begin{array}{c} \begin{array}{cccc} x=\frac{m}{N} & x=\frac{m}{N} & \cdots & x=N-1 \end{array} \\ \begin{bmatrix} h_K(x) & h_K(x) & h_K(x) & \cdots \\ h_1(x) & h_1(x) & \cdots & \\ h_2(x) & h_2(x) & \cdots & \\ - & - & - & \end{bmatrix} \end{array}$$

* To find contents $h_k(x)$ of Haar matrix.

$\Rightarrow$ If $\underline{\underline{K=0}}$, then o/p is $= \frac{1}{\sqrt{N}}$

ie. $h_k(x) = h_0(x) = \frac{1}{\sqrt{N}}$, $x \in [0,1]$

$\Rightarrow$ If $K \neq 0$

$$h_k(x) = h_{pq}(x) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2} ; & \frac{q-1}{2^p} \leq x < \frac{q-\frac{1}{2}}{2^p} \\ -2^{p/2} ; & \frac{q-\frac{1}{2}}{2^p} \leq x < \frac{q}{2^p} \\ 0 ; & \text{otherwise.} \end{cases}$$

(Pb) Find 2X2 Haar matrix H_2.

<u>Soluⁿ:</u> Here $\boxed{N=2}$, So, $n = \log_2 N$

$\boxed{n=1}$

<u>Step1:</u>

$K = 0$ to $N-1$

$K = 0$ to 1

K	0	1
p	0	0
q	0	1

* Haar matrix will have 2-rows

$$\begin{array}{c} K=0 \\ K=1 \end{array} \begin{bmatrix} & \\ & \end{bmatrix}$$

<u>Step2:</u> $x = \frac{m}{N}$

$0 \leq m \leq N-1$

$0 \leq m \leq 1$ ie. $m=0$; $m=1$

$x = \frac{0}{2} = 0$; $\cancel{m}\, x = \frac{1}{2}$

* Haar matrix will have 2 columns.

$$\begin{array}{c|cc} & x=0 & x=\frac{1}{2} \\ K=0 & & \\ K=1 & & \end{array}$$

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Step3:-

$$H_2 = \begin{array}{c|cc} & x=0 & x=\frac{1}{2} \\ k=0 & h_0(0) & h_0(\frac{1}{2}) \\ k=1 & h_1(0) & h_1(\frac{1}{2}) \end{array}$$

For $k=0$, $o/p = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2}}$

$\Rightarrow H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ h_1(0) & h_1(\frac{1}{2}) \end{bmatrix}$

i.e. $h_k(x) = h_0(x) = h_0(0) = h_0(\frac{1}{2}) = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2}}$

For $k \neq 0$, i.e. $k=1$, $p=0$, $q=1$ [$\because$ from Step1 table]

* Find 'x' interval

1^{st} interval $\frac{q-1}{2^p} \leq x < \frac{q-\frac{1}{2}}{2^p}$

$$\frac{1-1}{2^0} \leq x < \frac{1-\frac{1}{2}}{2^0}$$

$\boxed{0 \leq x < \frac{1}{2}} \Rightarrow$ For this o/p is $\frac{1}{\sqrt{N}} 2^{p/2} = \frac{1}{\sqrt{2}} 2^{0/2} = \frac{1}{\sqrt{2}}$

2^{nd} interval say: $\frac{q-\frac{1}{2}}{2^p} \leq x < \frac{q}{2^p}$

$$\frac{1-\frac{1}{2}}{2^0} \leq x < \frac{1}{2^0}$$

$\boxed{\frac{1}{2} \leq x < 1} \Rightarrow$ For this o/p is $-\frac{1}{\sqrt{N}} 2^{p/2} = -\frac{1}{\sqrt{2}}$

Now, $x=0$ i.e. $h_k(x) = h_1(0) \Rightarrow$ fall in Ist interval range, so o/p $= \frac{1}{\sqrt{2}}$

then $x = \frac{1}{2} \rightarrow$ fall in 2^{nd} interval, so, o/p $= -\frac{1}{\sqrt{2}}$

So, Haar matrix contents given by

$$H_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

(Q6) Find 4X4 Haar matrix H_4.

Soln:- Here $N=4$, $n=2$

Step 1: For different values of 'k', find values of P & q.

$k = 0$ to $N-1$

$k = 0$ to 3

K	0	1	2	3
P	0	0	1	1
q	0	1	1	2

*** Haar matrix Equation:-**

* If $k=0$, then o/p is $= \frac{1}{\sqrt{N}}$

i.e $h_k(x) = h_0(x) = \frac{1}{\sqrt{N}}$

* If $k \neq 0$

$$h_k(x) = h_{pq}(x) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2} ; & \frac{q-1}{2^p} \leq x < \frac{q-\frac{1}{2}}{2^p} \\ -2^{p/2} ; & \frac{q-\frac{1}{2}}{2^p} \leq x < \frac{q}{2^p} \\ 0 ; & \text{otherwise} \end{cases}$$

Step 2:- $x = \frac{m}{N}$; $0 \leq m \leq N-1$

$0 \leq m \leq 3$

So, $m = 0, 1, 2, 3$

$x = 0$, $x = \frac{1}{4}$, $x = \frac{2}{4}$, $x = \frac{3}{4}$

Step 3:- Haar matrix

$$H_4 = \begin{array}{c c} & \begin{array}{cccc} x=0 & x=\frac{1}{4} & x=\frac{2}{4} & x=\frac{3}{4} \end{array} \\ \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} & \begin{bmatrix} h_0(0) & h_0(\frac{1}{4}) & h_0(\frac{2}{4}) & h_0(\frac{3}{4}) \\ h_1(0) & h_1(\frac{1}{4}) & h_1(\frac{2}{4}) & h_1(\frac{3}{4}) \\ h_2(0) & h_2(\frac{1}{4}) & h_2(\frac{2}{4}) & h_2(\frac{3}{4}) \\ h_3(0) & h_3(\frac{1}{4}) & h_3(\frac{2}{4}) & h_3(\frac{3}{4}) \end{bmatrix} \end{array}$$

$h_k(x)$

Step4: Find the Elements of Haar matrix.

(a) For $k=0$, o/p is $\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

i.e. $h_k(x) = h_0(x) = h_0(0) = h_0(\frac{1}{4}) = h_0(\frac{2}{4}) = h_0(\frac{3}{4}) = \frac{1}{2}$

(b) for $k=1$, $p=0, q=1$

1^{st} interval $\frac{q-1}{2^p} \le x < \frac{q-\frac{1}{2}}{2^p}$

$\Rightarrow \boxed{0 \le x < \frac{1}{2}} \Rightarrow$ for 'x' in this range o/p is $= \frac{1}{\sqrt{N}} 2^{p/2} = \frac{1}{2} 2^{0} = \frac{1}{2}$

2^{nd} interval $\frac{q-\frac{1}{2}}{2^p} \le x < \frac{q}{2^p}$

$\boxed{\frac{1}{2} \le x < 1} \Rightarrow$ for 'x' in this range o/p is $= -\frac{1}{\sqrt{N}} 2^{p/2} = -\frac{1}{2}$

Now, $x=0$ i.e. $h_1(0) \Rightarrow$ falls in 1^{st} interval. So o/p $= \frac{1}{2}$

$x = \frac{1}{4}$ i.e. $h_1(\frac{1}{4}) \Rightarrow 1^{st}$ interval. So o/p $= \frac{1}{2}$

$x = \frac{2}{4}$ i.e. $h_1(\frac{2}{4}) \Rightarrow 2^{nd}$ interval, So, o/p $= -\frac{1}{2}$

$x = \frac{3}{4}$ i.e. $h_1(\frac{3}{4}) \Rightarrow 2^{nd}$ interval, So o/p $= -\frac{1}{2}$

(c) For $k=2$, $p=1$, $q=1$ ($\because$ from step1 table).

$k=2,\ P=1,\ q=1.$

$q=1$

1^{st} interval range $\Rightarrow \frac{1-1}{2^1} \le x < \frac{1-\frac{1}{2}}{2^1}$

$\boxed{0 \le x < \frac{1}{4}} \Rightarrow$ For this o/p is $\frac{1}{\sqrt{N}} 2^{\frac{p}{2}} = \frac{1}{\sqrt{4}} 2^{\frac{1}{2}}$

$= \frac{\sqrt{2}}{\sqrt{4}}$ ~~$\frac{1}{\sqrt{2}}$~~

$= \frac{1}{\sqrt{2}}$

2^{nd} interval range $\Rightarrow \frac{q-\frac{1}{2}}{2^p} \le x < \frac{q}{2^p}$

$\frac{1-\frac{1}{2}}{2^1} \le x < \frac{1}{2^1} \Rightarrow \boxed{\frac{1}{4} \le x < \frac{1}{2}}$

For this o/p is $-\frac{1}{\sqrt{N}} 2^{\frac{p}{2}} = -\frac{1}{\sqrt{2}}$

Now, for $x=0$ i.e. $h_k(0) = h_2(0) \Rightarrow 1^{st}$ interval. So o/p $= \boxed{\frac{1}{\sqrt{2}}}$

$x = \frac{1}{4} \Rightarrow h_2(\frac{1}{4}) \Rightarrow$ falls in 2^{nd} interval, o/p $= \boxed{-\frac{1}{\sqrt{2}}}$

$x = \frac{2}{4} \Rightarrow h_2(\frac{2}{4}) \Rightarrow$ i.e $x=\frac{1}{2} \Rightarrow$ out of range. so o/p is $\boxed{0}$

$x = \frac{3}{4} \Rightarrow h_2(\frac{3}{4})$ i.e. $x=\frac{3}{4} \Rightarrow$ out of range. Neither in 1^{st} nor in 2^{nd} interval. So, o/p $= \boxed{0}$

(d) For $K=3,\ p=1,\ q=2.$

1^{st} interval $\frac{q-1}{2^p} \le x < \frac{q-\frac{1}{2}}{2^p} \Rightarrow \frac{2-1}{2} \le x < \frac{2-\frac{1}{2}}{2}$

i.e. $\boxed{\frac{1}{2} \le x < \frac{3}{4}} \Rightarrow$ For this o/p is $\frac{1}{\sqrt{N}} 2^{\frac{p}{2}} = \frac{1}{\sqrt{4}} 2^{\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{4}}$

$\boxed{= \frac{1}{\sqrt{2}}}$

2^{nd} interval: $\frac{q-\frac{1}{2}}{2^p} \le x < \frac{q}{2^p} \Rightarrow \boxed{\frac{3}{4} \le x < 1} \Rightarrow$ o/p is $-\frac{1}{\sqrt{N}} 2^{\frac{p}{2}} = \boxed{-\frac{1}{\sqrt{2}}}$

Now, $x=0 \Rightarrow h_3(0) \Rightarrow$ out of range. o/p $\boxed{0}$

$x=\frac{1}{4} \Rightarrow$ out of range. So, o/p $\boxed{0}$

$x=\frac{2}{4} \Rightarrow 1^{st}$ interval. So, o/p $\boxed{\frac{1}{\sqrt{2}}}$

$x=\frac{3}{4} \Rightarrow 2^{nd}$ interval. So, o/p $\boxed{-\frac{1}{\sqrt{2}}}$

Thus, Haar matrix for $N=4$ is

$$H_4 = \begin{array}{c} \\ k=0 \\ k=1 \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} x=0 & x=\frac{1}{4} & x=\frac{2}{4} & x=\frac{3}{4} \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \end{array}$$

Assignment:

(Pb) Find 8X8 Haar transform matrix.

* Properties of Haar matrix:

* Let us consider 4x4 Haar matrix i.e.

$$H_4 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

(1) H is real.

As $H = H^* \Rightarrow H$ is real

i.e.

$$H_4 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \quad H_4^* = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = H$$

(2) H is not Symmetrical.

$$H^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} \neq H \qquad H^T \neq H$$

So, 'H' is not Symmetrical.

(3) H is unitary.

$$H . H^{*T} = I$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} . \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

4) It has poor Energy Compaction for images.

5) It is used in Edge detection applications.

6) Haar transform is a very fast algorithm. It can be implemented in $O(N)$ operations for N-dimensional vector.

* Note:

* Multiresolution Expansions :

* Multiresolution is concerned with the representation and analysis of Signals (or images) at more than one resolution.

basis fun
Expansion fun

* In Multi-Resolution Analysis (MRA), a Scaling function is used to create a Series of approximations of a function or image, Each differing by a factor of 2 in resolution from its nearest neighboring approximations.

* Wavelets are then used to Encode the difference in information between adjacent approximations.

⇒ Series Expansions:

* A Signal (or) function $f(x)$ can be analyzed as a linear Combination of Expansion functions

$$\boxed{f(x) = \sum_K \alpha_K \cdot \phi_K(x)}$$

where $K \rightarrow$ is an integer index

$\alpha_K \rightarrow$ are real valued expansion Co-Efficients

$\phi_K(x) \rightarrow$ are real valued expansion functions (or) basis functions.

* These basis functions form a function Space V i.e.

$$\boxed{V = \overline{\underset{K}{\text{Span}} \{\phi_K(x)\}}}$$

$\Rightarrow$ such that $f(x) \in V$.
i.e. $f(x)$ is in the closed - span of $\{\phi_K(x)\}$.

* For any $f(x) \in V$,
where V is a function space with corresponding Expansion set $\{\phi_K(x)\}$, there exist dual function set $\{\tilde{\phi}_K(x)\}$, which can be used to compute α_K Co-efficients as:

$$\boxed{\alpha_K = \underbrace{\langle \tilde{\phi}_K(x), f(x) \rangle}_{\text{inner product}} = \int \tilde{\phi}_K^*(x) \cdot f(x)\, dx}$$

inner product - between dual function $\tilde{\phi}_K(x)$ & $f(x)$.

'$*$' denotes the Complex conjugate - operation.

~~Basis fun~~ as

⇒ Scaling functions:

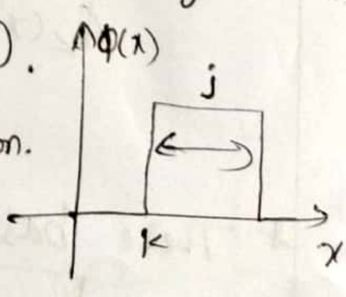
* Now, lets define a Expansion function $\phi(x)$, which is Composed of integer translations & binary Scalings is given by,

$$\boxed{\phi_{j,K}(x) = 2^{j/2} \cdot \phi(2^j \cdot x - K)}$$

for all $j, K \in Z \rightarrow$ (is the set of integers)

& $\phi(x) \in L^2(R) \rightarrow L^2(R)$ are Square integrable - functions.

* Here, K determines the position of $\phi_{j,K}(x)$ along x-axis, & j determines the width of $\phi_{j,K}(x)$.
* $2^{j/2}$ Controls the amplitude of the function.



* Since the shape of $\phi_{j,K}(x)$ changes - with j, So, $\phi(x)$ is called a Scaling function.

* In general, we will denote the Subspace Spanned over K for any j as

$$V_j = \overline{\underset{K}{\text{Span}} \{ \phi_{j,K}(x) \}}$$

& Corresponding $f(x) \in V_j$ as

$$f(x) = \sum_K \alpha_K \phi_{j,K}(x)$$

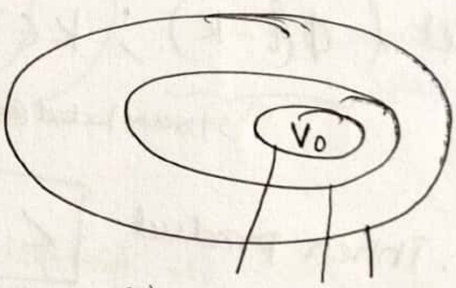
* **Properties of Scaling function:**

i) Nesting: The Subspaces Spanned by Scaling function are nested i.e One Subspace is contained in the Other Subspace.

i.e. $V_j \subset V_{j+1}$

$\Rightarrow V_j$ contained in V_{j+1}

i.e. $V_{-\infty} \subseteq \cdots \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \cdots \subset V_{\infty}$



Ex: $V_0 \subset V_1 \subset V_2$

ii) Closure: The closure of (or) union of all spaces Covers $L^2(R)$ (Square integrable functions).

i.e. $$\left(\bigcup_{j \in Z} V_j \right) = L^2(R)$$

implies $\Rightarrow$ Every function in $L^2(R)$ has a representation using elements in one of the nested subspaces.

iii) <u>Shrinking</u>: The intersection of all these spaces - Should be trivially zero.

$$\boxed{\bigcap_{j \in \mathbb{Z}} V_j = \{0\}}$$

→ zero Element.

Note: The only function that is common to all V_j is $f(x) = 0$.

iv) <u>Scaling</u>: If $f(x) \in V_j$,

then $f(2^{-j} \cdot x) \in V_0$

→ 2 ⇒ indicates Haar wavelet based diadic decomposition.

v) <u>Shift</u> <u>orthonormality</u>:- The function $\phi(t) \in V_0$ & set $\{\underbrace{\phi(t-k)}_{\text{translated (or) Shifted } \phi(t)} ; k \in \mathbb{Z}\}$ is an orthonormal basis for space V_0.

i.e. inner product $\boxed{\langle \phi(t), \phi(t-k) \rangle = 0}$ (for $k \neq 0$)

→ form an orthonormal basis for this space V_0.

Note: the choise of reference Subspace V_0 is arbitrary.

---

* So, if all these properties are satisfied, then it forms a multi resolution analysis.

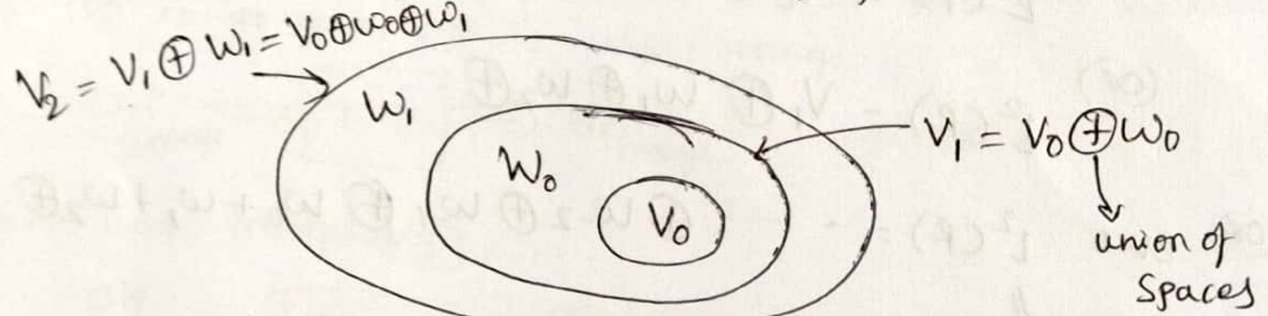
<u>Note</u>: Haar basis functions have finite / Compact Support unlike Sine & cosine functions which are having infinite dimension, infinite Expansion to represent finite signal. finite signal, $f(x)$.

* Wavelet Functions:

* Wavelets are used to encode the difference in information between adjacent approximations.

* Given a scaling function $\phi_{j,k}(x)$, that meets the MRA requirements (properties) discussed in previous section, we can define a wavelet function $\psi(x)$ that, together with its integer translates & binary scalings, spans the difference between any two adjacent scaling subspaces, V_j & V_{j+1}.

* The situation is illustrated graphically as:



* $\psi_{j,k}(x)$ of wavelets is defined as

$$\boxed{\psi_{j,k}(x) = 2^{j/2}\,\psi\left(2^j x - k\right)} \quad \text{for all } k \in \mathbb{Z}.$$

* Span of $\psi_{j,k}(x)$ results in W_j spaces i.e.

$$W_j = \overline{\operatorname{Span}_k \{\psi_{j,k}(x)\}}$$

* If $f(x) \in W_j$

$$\Rightarrow f(x) = \sum_k \alpha_k \psi_{j,k}(x)$$

* The Scaling & wavelet function subspaces as shown in graphical representation, related by

$$\boxed{V_{j+1} = V_j \oplus W_j}$$ where $\oplus$ denotes the union of spaces.

* All the members of V_j are orthogonal to the members of W_j. Thus

$$\langle \phi_{j,k}(x), \psi_{j,l}(k) \rangle = 0 \quad , \forall j,k,l \in Z$$

* We can now Express the space of all measurable, square-integrable functions as,

$$L^2(R) = V_0 \oplus W_0 \oplus W_1 \oplus \cdots$$

(or)

$$L^2(R) = V_1 \oplus W_1 \oplus W_2 \oplus \cdots$$

(or) Even $L^2(R) = \cdots \oplus W_{-2} \oplus W_{-1} \oplus W_0 + W_1 + W_2 \oplus \cdots$

$\Downarrow$

This Eqn. Eliminates the Scaling function, & represents a function in terms of <u>wavelets alone</u>.

Module-4 (Part-III)

Morphology

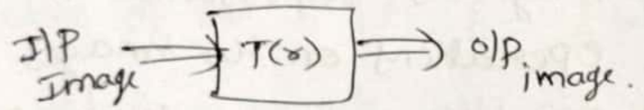
* Morphological Image Processing :-

* What is Morphology ? ⇒ The word morphology widely used in biology to define shape & structures of plants & animals

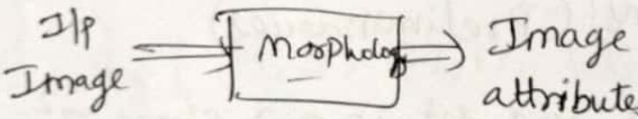
* Now, with respect of image processing morphology is a

Def: "mathematical tool for Extracting image components that are useful in the representation & description of region shape, such as boundaries, skeletons and the convex hull & etc.

* The enhancement methods discussed in previous modules where I/P is image & O/P is also image.



* Now, wrt morphology



attributes (Ex: Edges, connected points etc)

* Basic assumption :-

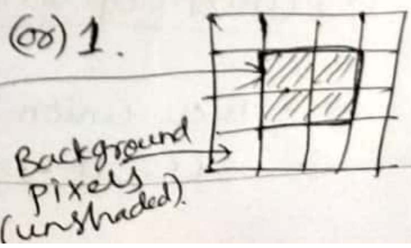
* Here we represent the image by a Point set. so, basics of set theory must be known.

* Here, we concentrate only on binary images in 2-D, Z^2 space, which is called binary Morphology.

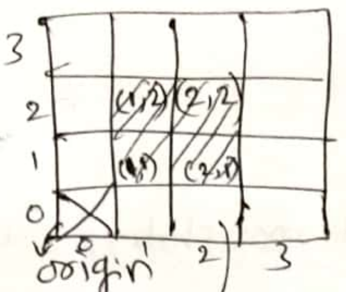
* If it is a gray scale image then graylevel morphology in Z^3 space

* Binary image means only 2-colors i.e. 0 (or) 1.

* Shaded region represents object region.



Ex: Binary image (A)



⇒ This binary image represented by set points (or) set pixels.

* The set will be consisting of only object region pixels.

i.e Set $\boxed{A = \{(1,1), (2,1), (1,2), (2,2)\}}$

⇓

This is binary image interms of set.

* This (2,0) is a background pixel. So it will not goto set A. It will be in A complement set i.e. A^c.

$A^c = \{ \quad \}$ ⇒ is a set of pixels which doesnot belong to A.

Conclusion; This implies an image can be represented in set point & once the image is represented in set, then all the morphological operations on the images are nothing but some set based operations on these point sets.

---

* Basics of set operations: (Preliminaries)

* w.r.t binary image, 'A' is a set in 2-D space Z^2. Each set point represented by (x,y) co-ordinate.

* Notations:

i) $a \in A$ ← belongs   ii) $a \notin A$ ← doesnot belongs to

iii) Subset operation:

$\boxed{A \subseteq B}$ ⇒ A is subset of B means every element in set A also belong to the set B, but reverse may not be true.

iv) Union operation: $\boxed{A \cup B}$ ⇒ elements of A & B together.

* This union operation is equivalent to logical 'or' operation.

i.e. A or B i.e. $\boxed{A + B \equiv A \cup B}$

V) <u>Intersection</u>: $\boxed{A \cap B}$ $\Rightarrow$ Common Elements b/w set A & B.
This operation is Equivalent to Logical '<u>and</u>' - operation

$\Rightarrow$ A and B i.e. $\boxed{A.B \equiv A \cap B}$

Vi) <u>Difference operation</u>: $\boxed{A-B}$

$A-B = \{w \mid w \in A \; \& \; w \notin B\}$ $\Rightarrow$ The Common elements from set ~~A~~ & B taken & those common elements removed from set A, which results in (A-B) set.

<u>Ex</u>: $A = \{(1,2), (2,1), \cancel{(2,2)}, (5,3)\}$

$B = \{(2,2), (4,5)\}$

<u>$A-B$</u> $= \{(1,2), (2,1), (5,3)\}$

Vii) <u>Set Reflection</u> : $(\hat{B})$

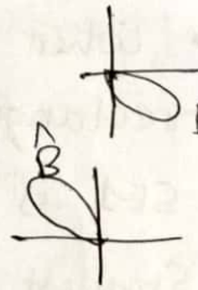
* Reflection of set B i.e. $\hat{B} = \{w \mid w = -b \text{ for } b \in B\}$.

i.e Take set B $\Rightarrow$ Negate all set elements, that results in $\hat{B}$.

i.e. (x, y) Co-ordinates replaced by $(-x, -y)$.

<u>Ex</u>: $B = \{(1,2), (-2,1), (3,2)\}$

$\hat{B} = \{(-1,-2), (2,-1), (-3,-2)\}$.

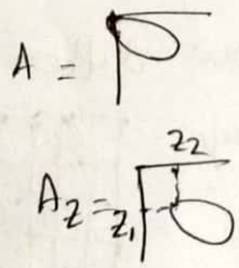


Viii) <u>Translation</u>: This translate the set A by a vector say Z.

i.e if $Z = (z_1, z_2)$ then in set A, $(x, y) \xrightarrow[\text{-ced by}]{\text{repla}} (x+z_1, y+z_2)$

i.e. $A_Z = \{c \mid c = a+z \text{ for } a \in A\}$

Set A, translated by 'Z' times



<u>Ex</u>: $A = \{(1,2), (2,1)\}$

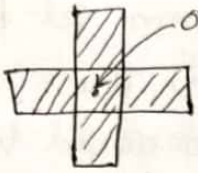
If $Z = (3,2)$ then

$A_Z = \{(1+3, 2+2), (2+3, 1+2)\} = \{(4,4), (5,3)\}$

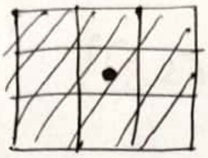
* Structuring Element :- (S.E)

* The SE play Vital role in morphology operation.
* This S.E is a Small sets (or) Subimages used on input image which is under study to Extract the properties of interest.

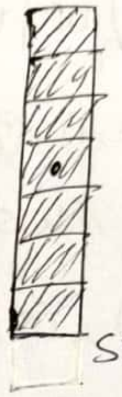
Ex:



S.E 1



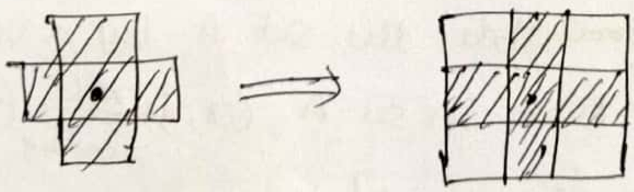
SE 2



SE3.

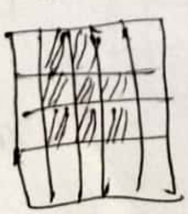
* Each **Shaded** Square represents member of Structuring Element.
* The origin of S.E also must be specified.
  When S.E is Symmetric & origin is not shown means by default it is at Center of Symmetry.
* When working with images, we require S.E's to be in rectangular array as show in fig as SE2 & SE3.
  SE1 is not in rectangular shape, So we add or append Smallest possible no. of back ground Elements (unshaded).

i.e.

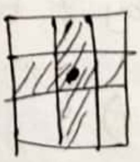


* How different Morphological operations performed?

Binary Image



⇒ represented in set A



⇒ Structuring element set B.

⇒ by applying m. operations a new set is created by running B over A. We make sure that origin of B visits every object-element of A.

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** Erosion & Dilation :

* These two operations are fundamental to morphological Processing. Many of the morphological algorithms - discussed in future are based on these two Primitive operations.

i) Erosion :

With A & B as sets in Z^2, the Erosion of A by B, denoted $A \ominus B$, is defined as

$$\boxed{A \ominus B = \{ z \mid (B)_z \subseteq A \}}$$

A ⇒ image, object region set

B ⇒ Structuring Element.

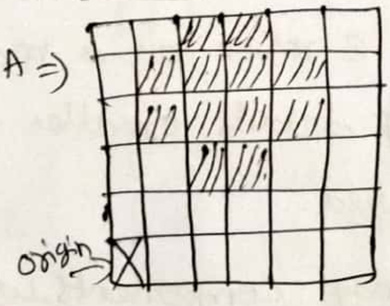
* The above Equation indicates that the Erosion of A by B is set of all points z such that B, translated by z i.e. $(B)_z$ is Contained in A. i.e. $(B)_z$ is not sharing any common Elements with the background. So, we can express Erosion in the following Equivalent form as

$$\boxed{A \ominus B = \{ z \mid (B)_z \cap A^c = \phi \}}$$

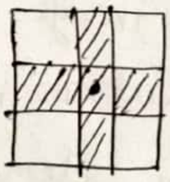
⇒ A^c is Complement of A.
ϕ is null set.

* Example for Erosion :

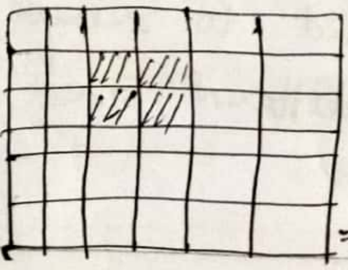
Set A ⇒



S.E
Set B



$A \ominus B$ =
Eroded set



→ This is Eroded Set. All Boundary pixels are eliminated.
⇒ Erosion shrinks (or) thins object region.

EX2:

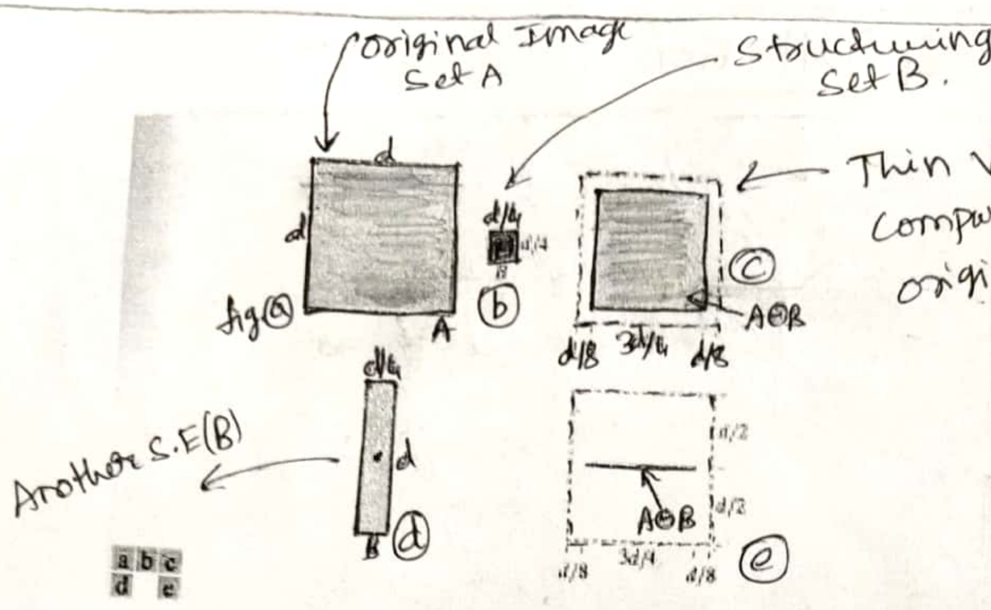


FIGURE 9.4 (a) Set *A*. (b) Square structuring element, *B*. (c) Erosion of *A* by *B*, shown shaded. (d) Elongated structuring element. (e) Erosion of *A* by *B* using this element. The dotted border in (c) and (e) is the boundary of set *A*, shown only for reference.

* $A \ominus B$ => all the elements of Set B, Should be covered by object region Pixel. That will go to Erosion set.

* AS shown in above EX-2, the Elements of A & B are shown Shaded.
* when S.E B is moved over Set A, the Eroded i.e. thinned Version of image is shown in fig c, with dotted line at the boundary indicates removed part of image. (dotted + unshaded)
* fig. d Shows an elongated Structuring Element & fig(e) Shows Erosion of A by this Element. Note that the original set was Eroded to a line.

Conclusion: we see that Erosion Shrinks (or) thins objects in a binary image. we can View Erosion as a morphological filtering operation in which image details smaller than the Structuring element are filtered.

* Erosion used to remove image components, using structuring elements of different sizes, Say 11X11, 15X15, 45X45 and soon

Dilation :-

With A & B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

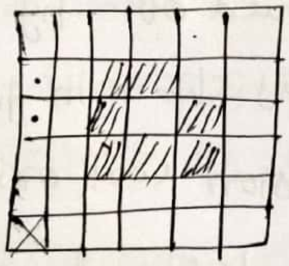
$$\boxed{A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \phi \}}$$

* The above Equation is based on reflecting B about its origin, and shifting this reflection by z.
* The dilation of A by B then is the set of all displacements z, such that $\hat{B}$ & A overlap by at least one element.
* Based on this interpretation, the above eqn can be written equivalently as:

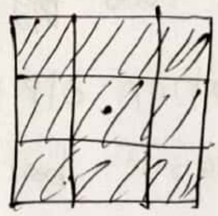
$$\boxed{A \oplus B = \{ z \mid [(\hat{B})_z \cap A] \subseteq A \}}$$

* unlike Erosion, which is a shrinking (or) thinning operation, dilation performs opposite operation i.e dilation → "grows" (or) "thickens" object boundary in a binary image.

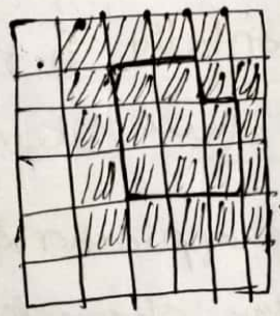
Ex1: Set A ⇒



S.E
Set B



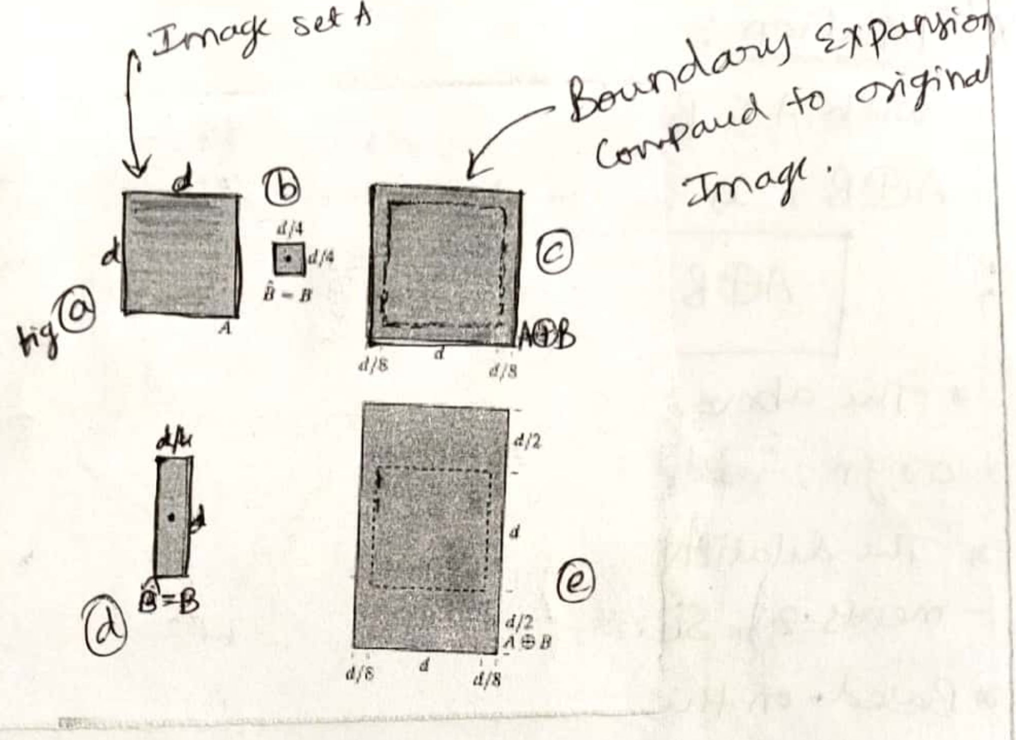
$A \oplus B =$



← Boundary Expansion done compared to original image A.

Note: If any one element of Set B covered by object region pixel in Set A then that goes to $A \oplus B$ set.

Ex2: Dilation:



* The Expansion of boundary (or) thickening is controlled by the shape of the structuring element used.
* The above fig a & b shows image & S.E. In this S.E $B = \hat{B}$ ($\because$ Symmetric about its origin).
* When this fig b, S.E is moved over fig. a image set A, then dilation o/p produced is as shown in fig. c where we can observe boundary expansion wrt original image.
* fig. d shows a another S.E designed to achieve max dilation vertically than horizontally. & fig(e) shows the dilation achieved with this element.

*Application: one of the simplest application of dilation is for bridging gaps i.e. gap of broken characters similar to lowpass filtering operation.

* Duality:-

* Erosion & dilation are duals of Each other wrt to set complementation & reflection i.e.

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$\& \; (A \oplus B)^c = A^c \ominus \hat{B}$$

Proof: LHS: $(A \ominus B)^c = \{ z \mid (B)_z \subseteq A \}^c$ By defn of Erosion $A \ominus B$

$= \{ z \mid (B)_z \cap A^c = \phi \}^c$ By 2nd defn of erosion

Applying complement

$= z \mid (B)_z \cap A^c \neq \phi$

$(A \ominus B)^c = A^c \oplus \hat{B}$ RHS Proved

---

* Opening & closing:-

* WKT dilation Expands the components of an image & Erosion shrinks them.
* If we use dilation & Erosion together one after other then that results in opening & closing operation.

i.e. Opening:- The opening of set A by structuring element B, denoted by $A \circ B$, is defined as

$$\boxed{A \circ B = (A \ominus B) \oplus B}$$

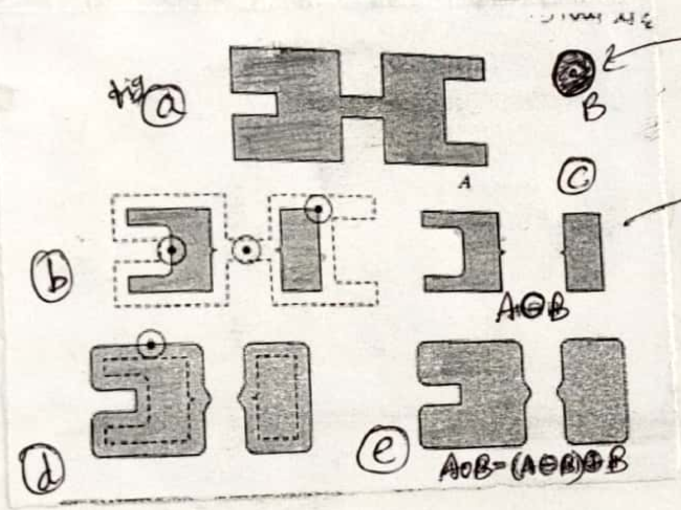
Thus, the opening A by B is the Erosion of A by B, followed by a dilation of the result by same S.E B.

*Similarly, Closing : the closing of set A by S.E B, denoted $A \bullet B$ is defined as,

$$\boxed{A \bullet B = (A \oplus B) \ominus B}$$

$\Rightarrow$ which says that the closing of A by B is simply the dilation of A by B, followed by the Erosion of the result by B.

Ex: for opening :-



* The fig a shows set A, & fig.b shows various positions of "rolling ball" shaped structuring element B, during Erosion process.
* o/p of Erosion is shown in fig c i.e. disjoint figure with reduced dimentions, then dilation applied fig d & o/p of complete opening is shown in fig e.
* Note that Elimination of the bridge b/w two main sections. Then, outward pointing corners were rounded, whereas inward pointing corners were not affected.

* The opening operation has a simple geometric interpretation as shown in fig. below

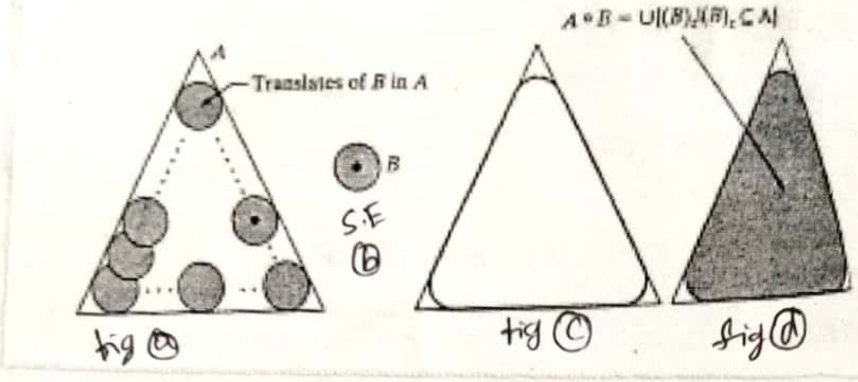


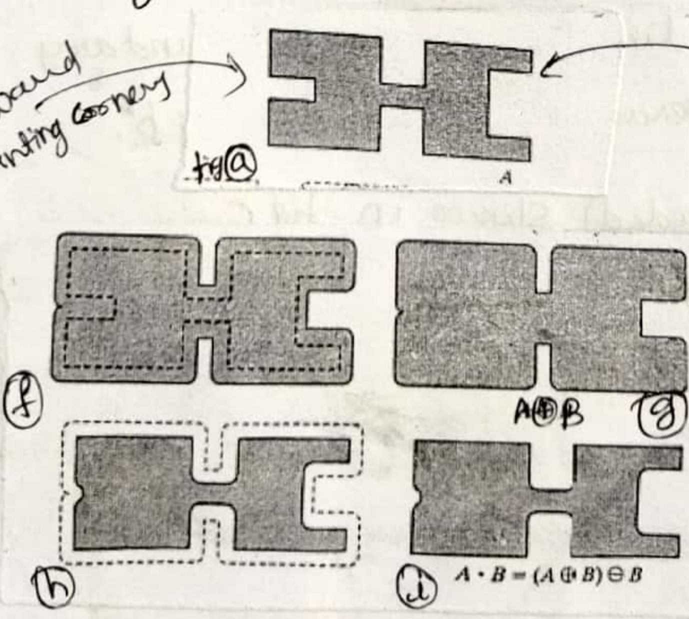
fig (a) fig (c) fig (d)

* Suppose that if we view the S.E B as a flat "rolling ball" then it is moved along the inner boundary of A.
* In fig c the heavy line is the outer boundary of the opening & fig d is complete opening (Shaded).
* we can observe that opening generally smoothes the Contour of an object, breaks narrow isthmuses & Eliminates thin protrusions.

<u>Ex: for</u> <u>closing</u> <u>operation</u>: $\boxed{A \bullet B = (A \oplus B) \ominus B}$

Inward pointing corners

outward pointing corners

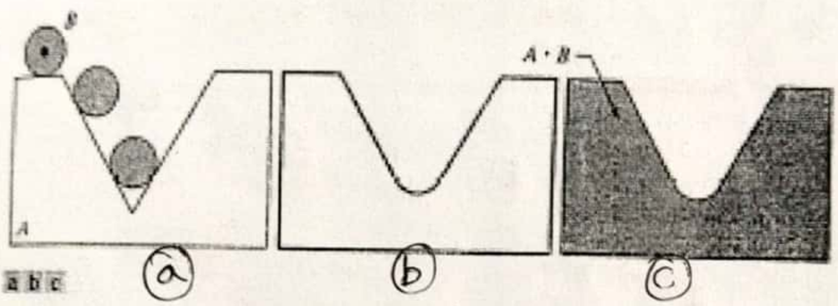


After dilation Boundary expansion.

* fig's (f) through (i) shows the results of closing A with the same 'rolling ball' structuring element. Here we observe that first dilation applied & then erosion as shown in fig g & fig h respectively.

* Here we observe that inward pointing corners are rounded, where as the outward pointing corners - remained unchanged.

* wrt geometric interpretation of closing operation:



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

* Here, rolling S.E 'B' is moved over outer boundary of set A. The heavy line in the outer boundary ~~of~~ the ~~clos~~ o/p of closing operation as shown in fig b.

* Complete closing (shaded) shown in fig c.

* Closing also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks & long thin gulfs, eliminates small holes & fills gaps in the contour.

* As in the case with dilation & Erosion, opening & closing are duals of each other with respect to set complementation & reflection. That is,

opening operator

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

and

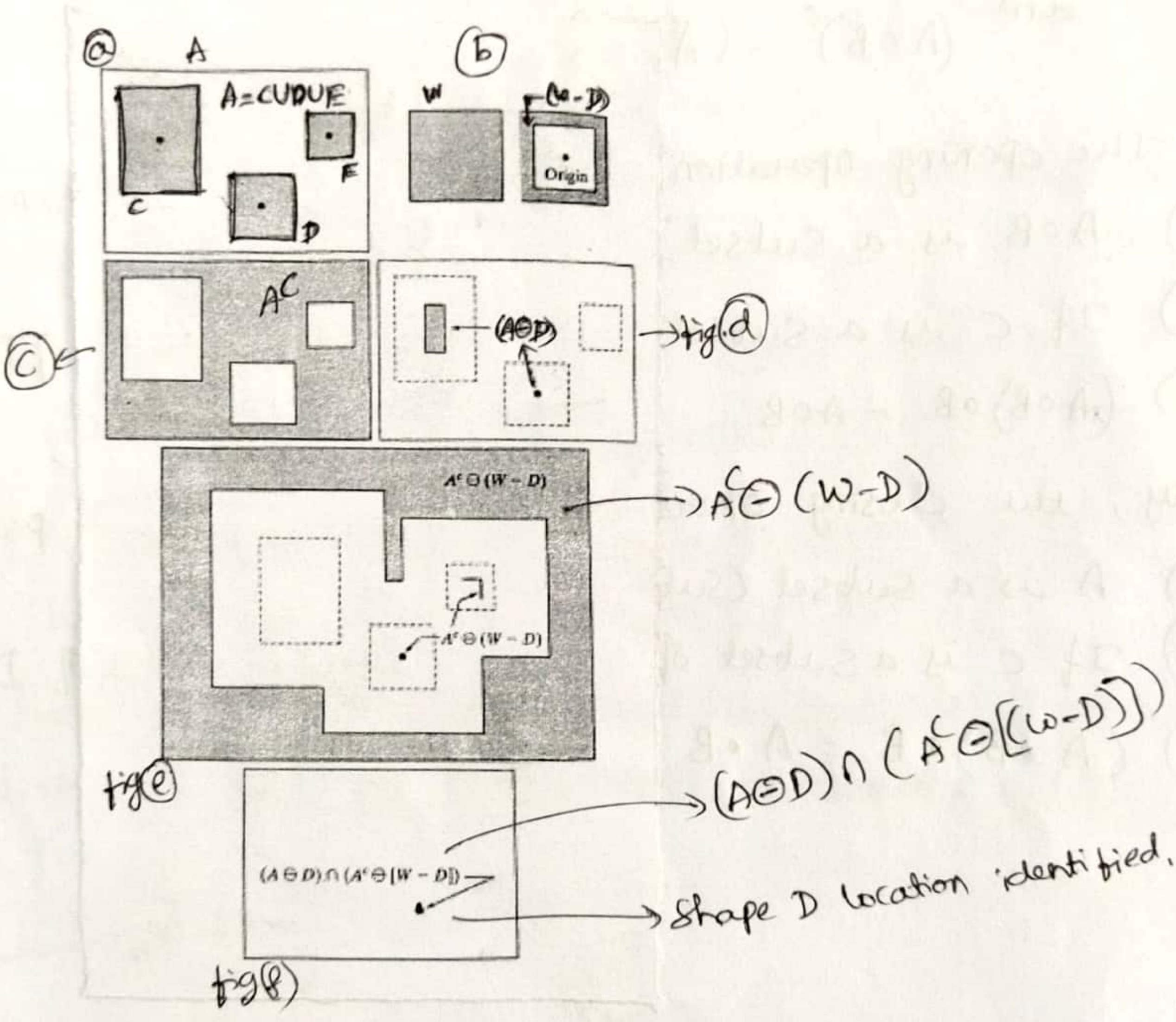
$$(A \circ B)^c = (A^c \bullet \hat{B})$$

* The opening operation ($\circ$) satisfies the following properties:
i) $A \circ B$ is a subset (subimage) of A.
ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
iii) $(A \circ B) \circ B = A \circ B$.

Illy, the closing operation satisfies the following properties:
i) A is a subset (subimage) of $A \bullet B$
ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
iii) $(A \bullet B) \bullet B = A \bullet B$

The Hit-or-Miss Transformation :

* The morphological hit or miss transform is a basic tool used for locating (or) detecting a position of particular shape.
* This can be explained with the help of below fig.



* fig a, shows a set A consisting of three shapes denoted by C, D & E.
* here we use hit-or-miss transform to find the location of one of the shapes, say D.

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* We take D as Structuring Element & Enclose it in a window W, as shown in fig.b.

* Then we implement the following Eqn. to perform hit-or-miss operation i.e to locate shape D.

$$\boxed{\underbrace{A \circledast B}_{\text{hit (or) miss operation}} = (A \ominus D) \cap [A^c \ominus (W-D)]} \quad \text{--- (1)}$$

* The set A is Eroded with Set D to perform $A \ominus D$, Shown in fig.d ⟹ which helps to locate shape D.

* To perform other part of above Eqn. (1), A complement i.e. A^c is considered as shown in fig.c

* Then A^c is Eroded with (W-D) to search for possibility of Shape-D in background as shown in fig.e

* Then intersection between fig.d & fig.e is performed which gives location of Shape-D as shown in fig. f.

Relation between opening and closing (24)

Opening and closing are dual to each other with respect to set complementation and reflection i.e

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

Hit or Miss Transformation

* It is a basic tool used for shape detection.
* We will use hit or mass transformation for finding a particular location of shape with the consideration of background also.
* Let us consider a set A that has 3 shapes denoted by C, D and E. Our objective is to find the location of shape D

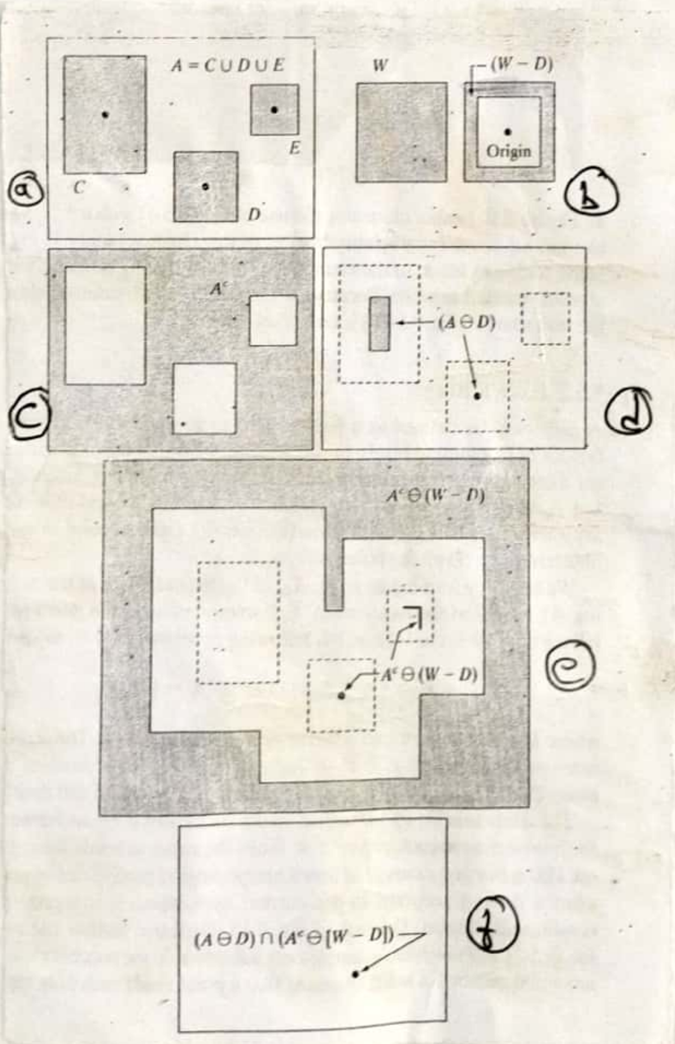


fig:-

a) set A

b) window W and W − D

c) complement of A

d) Erosion of A by D

e) Erosion of A^c by (W − D)

f) Intersection of d and e, showing the location of D

Note: The dot indicate the origin of C, D & E

To find the location of shape D, we apply ~~the~~ ~~consider~~ following expression

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

Where $\circledast$ represents hit or miss transformation.

* Here $(A \ominus B_1)$ represents erosion of A with object itself to find the location of D

so $B_1 = D$

$(A \ominus B_1) = (A \ominus D)$ without considering background

* If we want to consider the background then instead of A, we have to consider ~~the~~ A^c. Simultaneously we have to use a window that encloses the D for showing the local background of D itself.

Then local background of $D = (W - D) = B_2$ and the total background of A is given by $A^c \ominus B_2$

Basic Morphological Algorithm

Morphological algorithm are useful for extracting image components such as boundaries, connected components, the convex hull and skeleton of a region.

① Boundary extraction

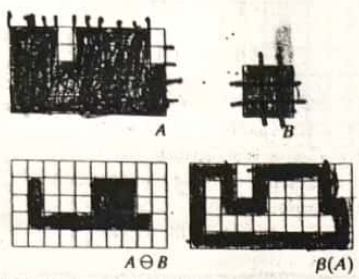
The boundary of A is denoted by $\beta(A)$. It can be obtained by first eroding A by B and then subtracting it by A. i.e

(24)

i.e $\beta(A) = A - (A \ominus B)$

Where B is suitable structuring element

Below fig shows the process of boundary extraction.



a b
c d

a) set A

b) structuring element

c) A eroded by B

d) Boundary

$A - (A \ominus B)$

fig: Process of boundary extraction

② Region Filling (Hole filling)

This algorithm is used for hole or region filling. Let image A denote a set, whose elements are 8 connected boundaries. The process can be represented by

$$X_K = (X_{K-1} \oplus B) \cap A^C, \quad K = 1, 2, 3 \text{ - - -}$$

* The objective is to fill all the holes with 1's (Black).

i.e (considering 1 pixel which we want to fill with Black(1)

* Here $X_0 = P$ i.e $X_0 = (A \oplus B) \cap A^C$

then $X_1 = (X_0 \oplus B) A^C$ and so on

Radhakrishna. M

GAT, B'lore-98

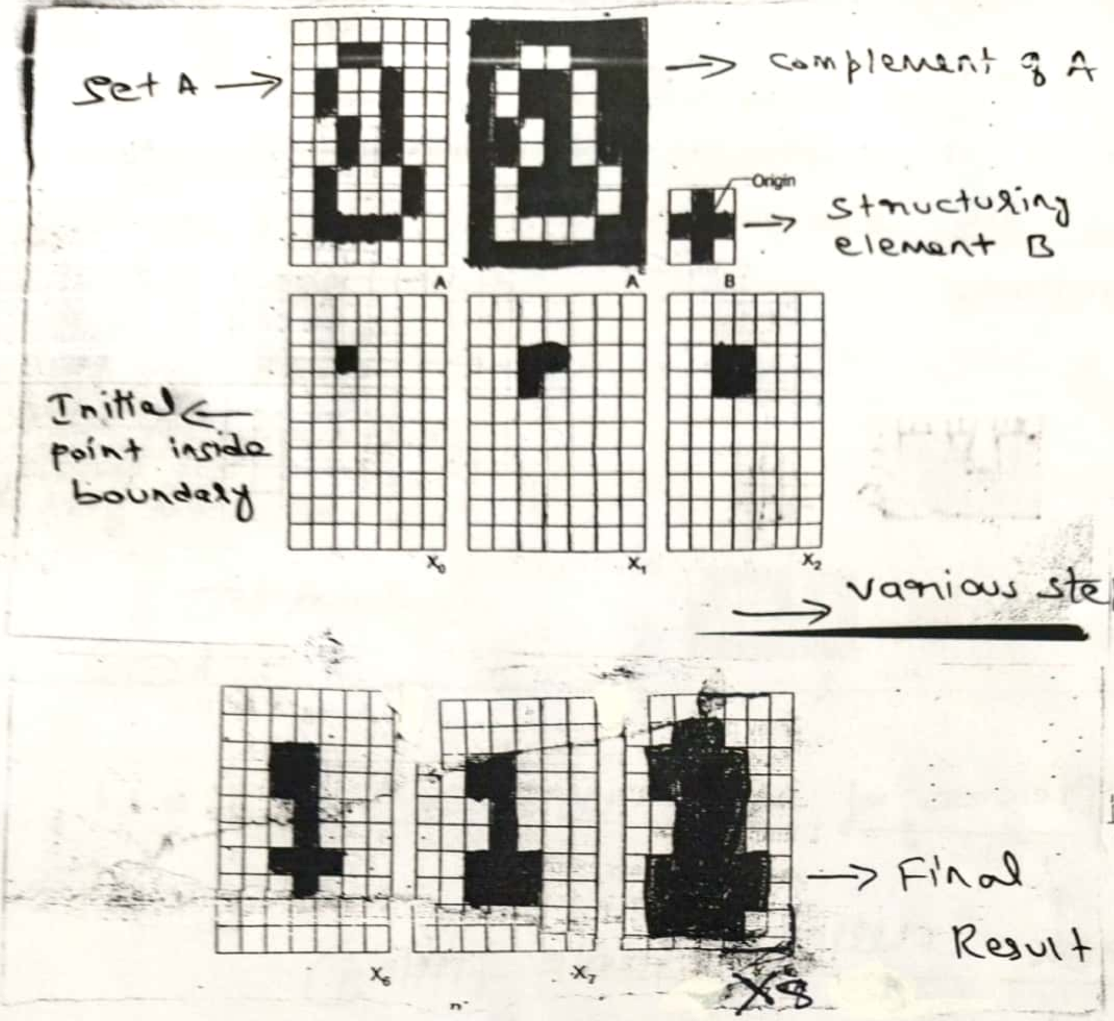


fig:- Hole filling process

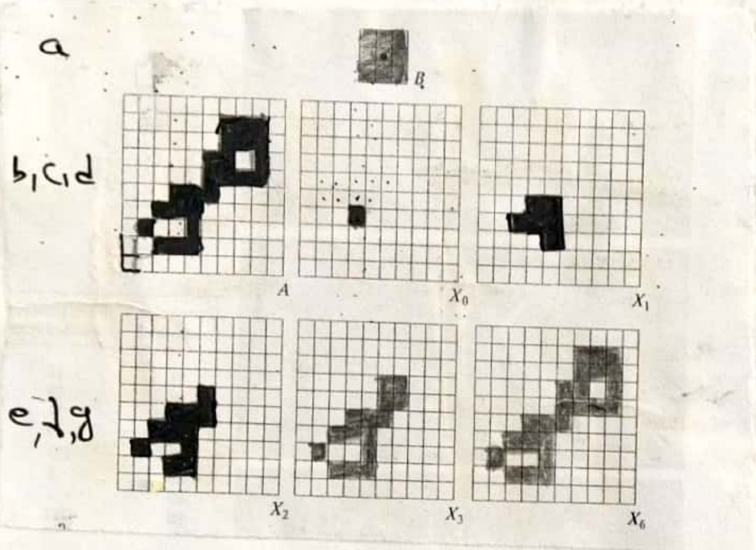
③ Extraction of connected components

* Let A be set containing one or more connected components
* Form an array X_0 of same size as A.
  Here, all elements of X_0 are 0 except for one ~~component~~ point in each connected component set to 1 (i.e X_0 in figure)
* The objective is to start with X_0 and find all the connected components, using the iterative procedure

$$X_K = (X_{K-1} \oplus B) \cap A \quad K = 1, 2, 3 \text{ ----}$$

Here B is suitable structuring element (26)

* The procedure terminates when $X_K = X_{K-1}$
* The procedure is same as hole filling but here here we use A instead of A^C, because here we are looking for foreground points, while the objective of hole filling is to find a background points.
* Below fig illustrates the mechanism of extraction of connected components, which is acheived for k=6



Extracting connected components

a) Structuring element
b) Array containing a set with one connected component
c) Initial array
(d to g) -> Various Iterations

fig: Extracting connected Components

(4) Convex hull

* A set A is said to be convex if the straight line segment joining any 2 points in A lies entirely within A.

* Let B^i, where $i = 1, 2, 3, 4$ represents 4 structuring element as shown in below fig.

* The procedure of convex hull consist of implementing the equation

$$X_K^i = (X_{K-1} * B^i) \cup A \quad \text{where } i = 1, 2, 3, 4$$
$$\text{and } K = 1, 2, 3, ---$$

$$\text{with } X_0^i = A$$

and

$$D^i = X_K^i$$

$$C(A) = \bigcup_{i=1}^{4} D^i$$

i.e in other words

(i) simply apply hit or miss transform to A with B^1 as

$$K = 1; \; X_1^i = (X_0 * B^i) \cup A$$

$$\text{for } i = 1; \; X_1^1 = (X_0 * B^1) \cup A$$

and then perform union with A and call result D^1

(ii) Again repeat (i) with B^2 and call the result D^2

(iii) By same process; calculate D^3 and D^4

(iv) After calculating D^1, D^2, D^3 and D^4, Calculate the union of D^1, D^2, D^3 and D^4 that will be the convex hull

* Below figure shows whole process of convex hull, here x denotes dont care and B^i is the clock wise rotation of B^{i-1} by 90°

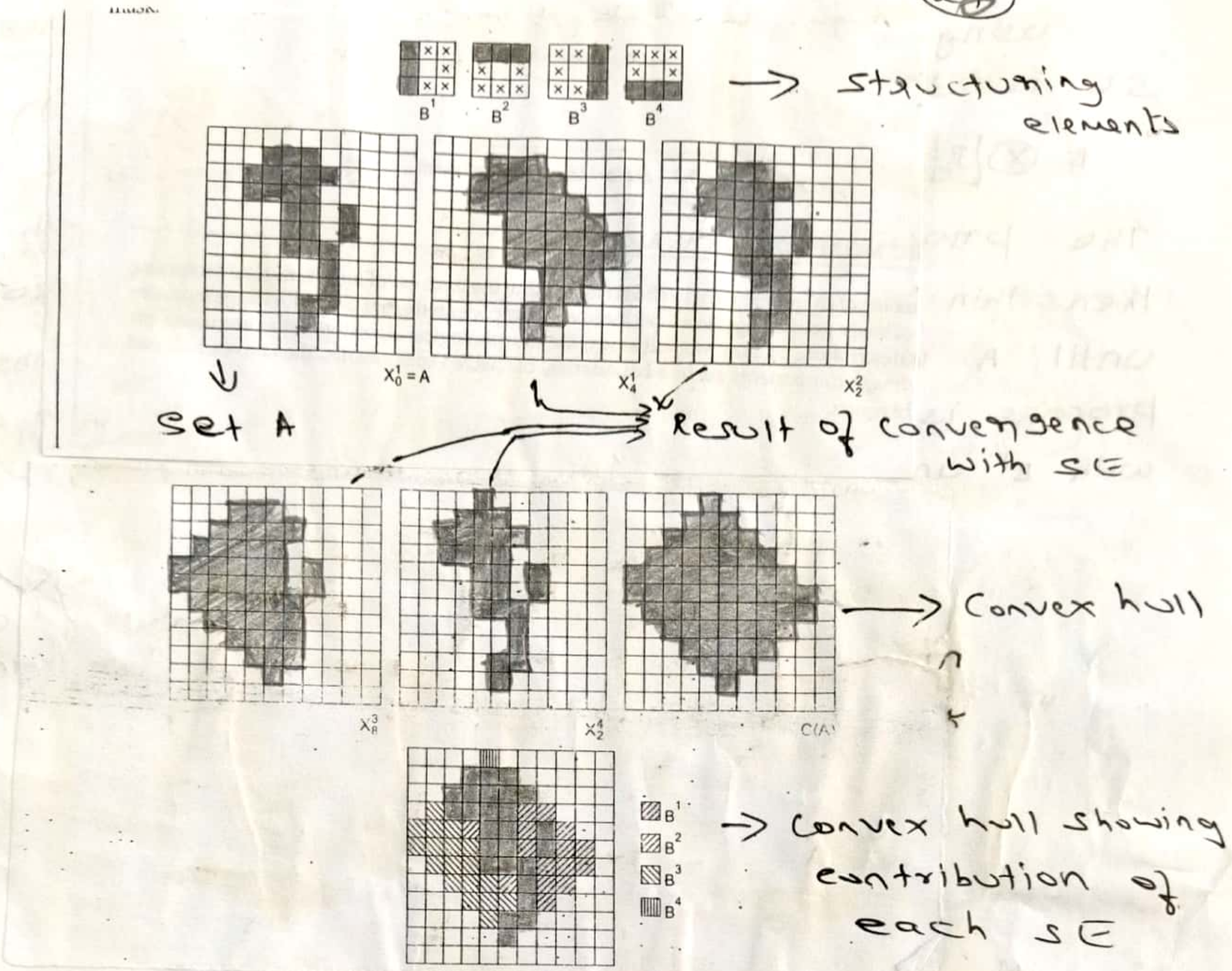


fig: convex hull Process

5) Thinning

The thinning of set A by a structuring element is denoted by $A \otimes B$, can be defined in terms of hit or miss transformation.

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

* A more useful expression for thinning A symmetrically is based on a sequence of structuring elements

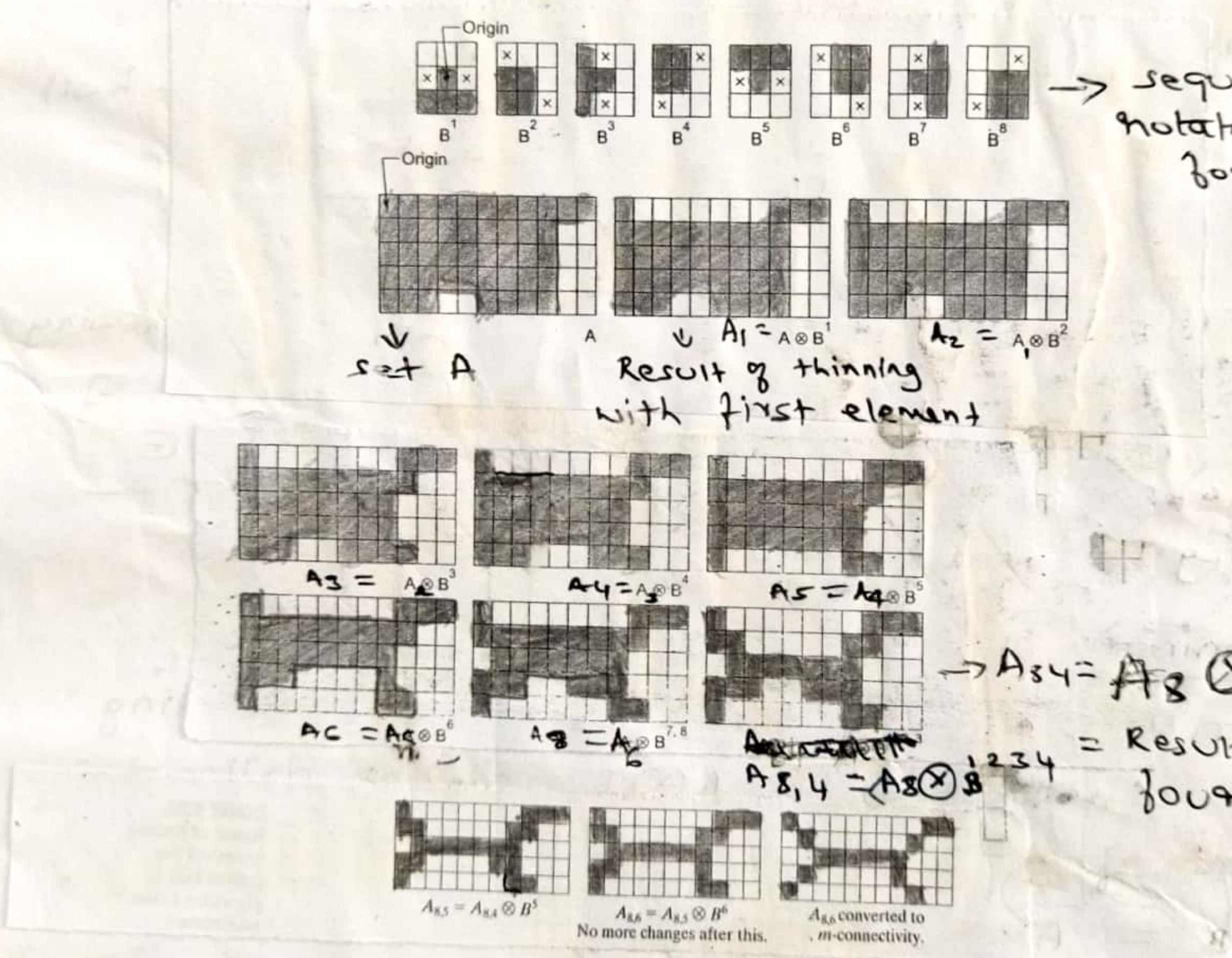
$$\{B\} = \{B^1, B^2, B^3, --- B^n\}$$

where B^i is the rotated version of B^{i-1}

Using this concept, thinning by a sequence of SE is defined as

$$A \otimes \{B\} = ((\cdots((A * B^1) * B^2)\cdots) * B^n)$$

The process is to thin A by one pass with B^1, then thin this result with one pass B^2 and so on until A is thinned with one pass B^n. The whole process is shown in below fig. Here thinning starts with B^1 and goes on till the thinning occurs at B^8



Result after Convergence

Conversion to m connectivity.

fig:- Thinning Process

6 Thickening

(2)

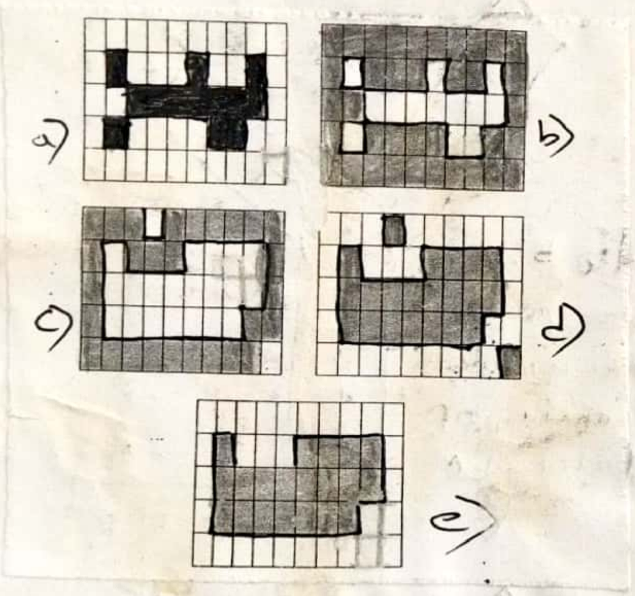
Thickening process is dual of thinning. It is defined by

$$A \odot B = A \cup (A \circledast B)$$

As in thinning, thickening can be defined as a sequential operation.

$$A \odot B = ((\cdots((A \odot B^1) \odot B^2 \cdots) \odot B^n)$$

* The structuring element is same as used for thinning process. But now '1' is replaced by 0 and 0 is replaced by '1'. The whole process is shown in below fig.



a) set A

b) Complement of A

c) Result of thinning the complement of A

d) Thickened set obtained by complementing c.

e) Final result with no disconnected points.

fig: Thickening process

7. Skeleton

Skeleton is defined by

$$S(A) = \bigcup_{K=0}^{K} S_K(A)$$

where $S_K(A) = (A \ominus KB) - (A \ominus KB) \circ B$

B is SE and $A \ominus KB$ indicates K successive erosions of A

$$(A \ominus KB) = ((\cdots((A \ominus B) \ominus B) \ominus \cdots) \ominus B$$

K is the last iterative step before A erodes to an empty set. i.e

$$K = \max\{k \mid (A \ominus kB) \neq \phi\}$$

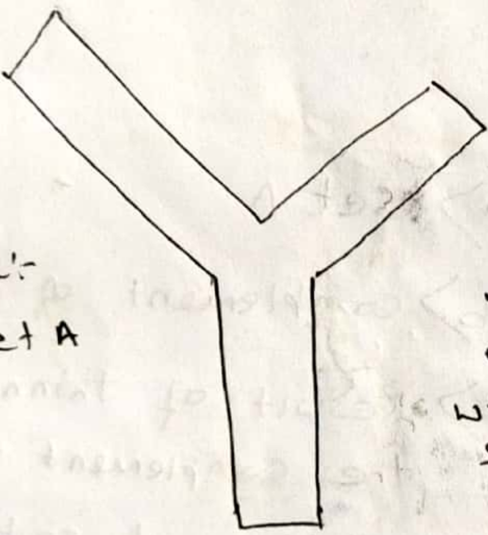


fig 'a':- set A

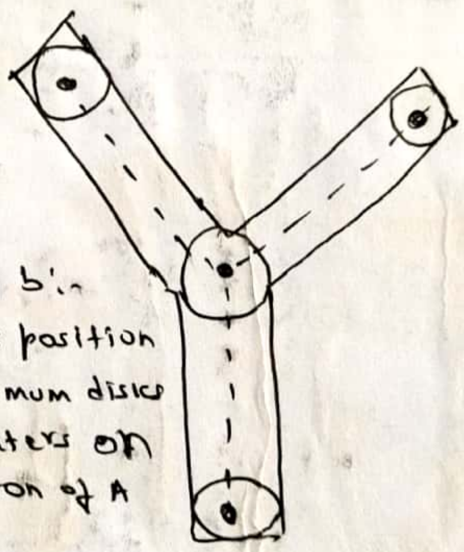


fig 'b':- Various position of maximum disks with centers on skeleton of A

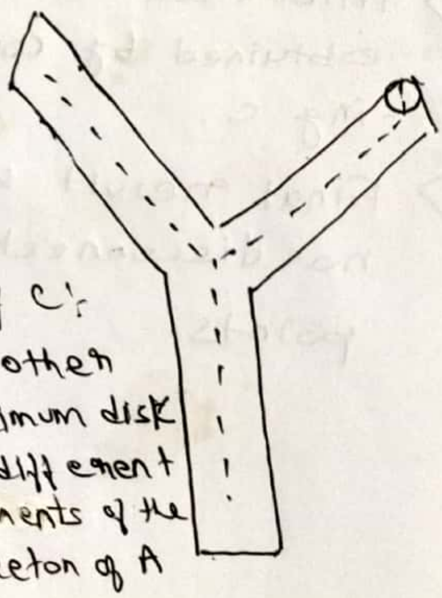


fig c':- An other maximum disk on different segments of the skeleton of A

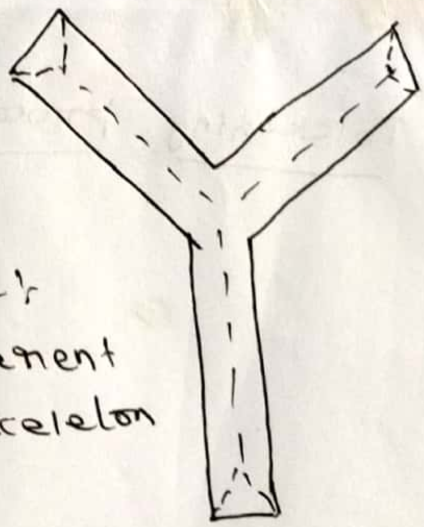
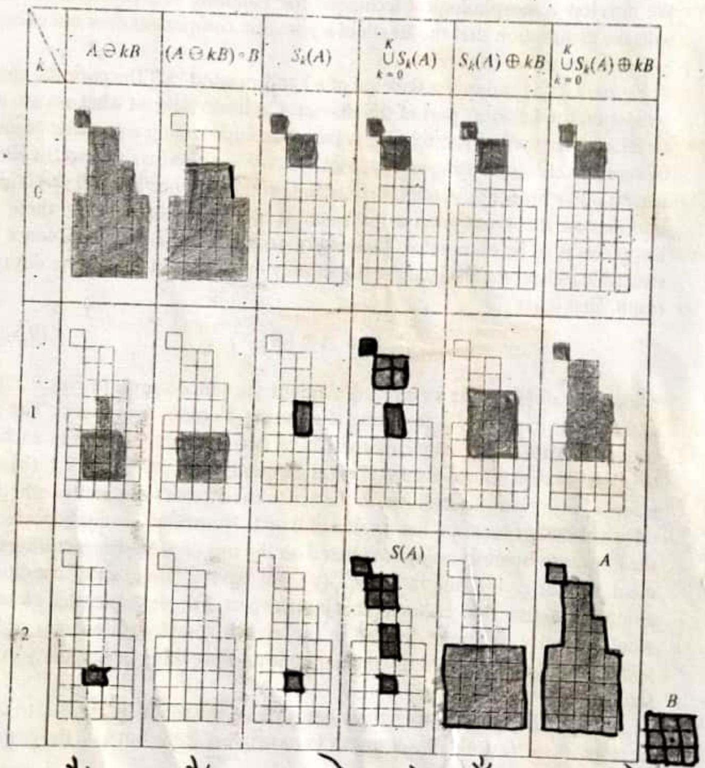


fig d':- Different skeleton

Below fig illustrates the concept of skeleton. The first column shows original set at top and two erosions by the structuring element B (one more erosion of A would yield in empty set so k=2 in this case). The second column shows the opening of the sets in the first column by B. The 4th column shows the skeleton result and 6th column shows the reconstructed image.

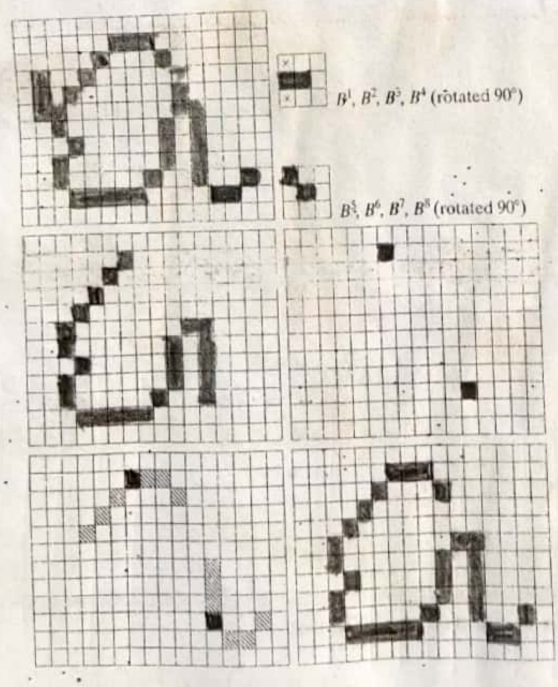


$A \ominus k.B$ $\quad (A \ominus kB) \circ B \quad S_K(A) \quad \bigcup_{k=0}^{K} S_K(A) \quad S_K(A) \oplus kB \quad \bigcup_{k=0}^{K} S_K(A) \oplus kB$

8 Pruning

* It is complement to thinning and skeletonizing algorithms to remove unwanted parasitic components.
* Ex:- Automated recognition of hand printed characters to analyse the shape of skeleton of each character
* Below fig shows the skeleton of a hand printed "a". The parasitic component on the leftmost part should be removed.
* Assumption is made that any branch ≤ 3 pixels will be removed
* Removal of parasitic branch is achieved with thinning input set A with a sequence of SE's designed to detect only end points.

i.e let $X_1 = A \circledast \{B\}$



a b
c
d e
f g

FIGURE (a) Original image. (b) and (c) Structuring elements used for deleting end points (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

* Form a set X_2 containing all end points in X_1 (fig e) 29

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

* Next step is dilation of end points three times, using set A as delimiter (fig f)

$$X_3 = (X_2 \oplus H) \cap A$$

where H = 3×3 SE of 1's and intersection with A is applied after each step.

* The final result comes from fig (g)

$$X_4 = X_1 \cup X_3$$

9. <u>Morphological Reconstruction</u>

* It works on two images and a SE. One image is called marker and contains straight points for transformation. Second image is called mask and contains the transformation or constraint. SE is used to define connectivity

i) Geodesic dilation and erosion

- Let F be the marker image and G be the mask image.
- F and G are binary image and $F \subseteq G$
- Geodesic dilation is defined as [for size 1 of F with respect to G]

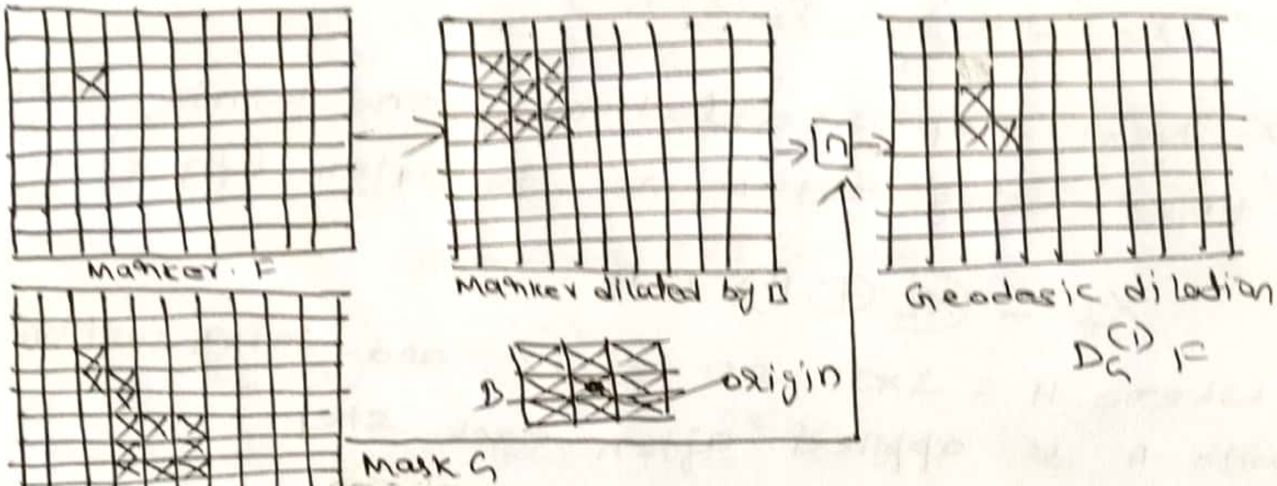
$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

- Geodesic dilation of size n w.r.t G is defined as

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right] \text{ with } D_G^{(0)}(F) = F$$

Scanned with CamScanner

* Intersection is performed at each step and mask G limits the growth of marker F



Geodesic erosion

Geodesic erosion of size 1 of F w.r.t G is defined as

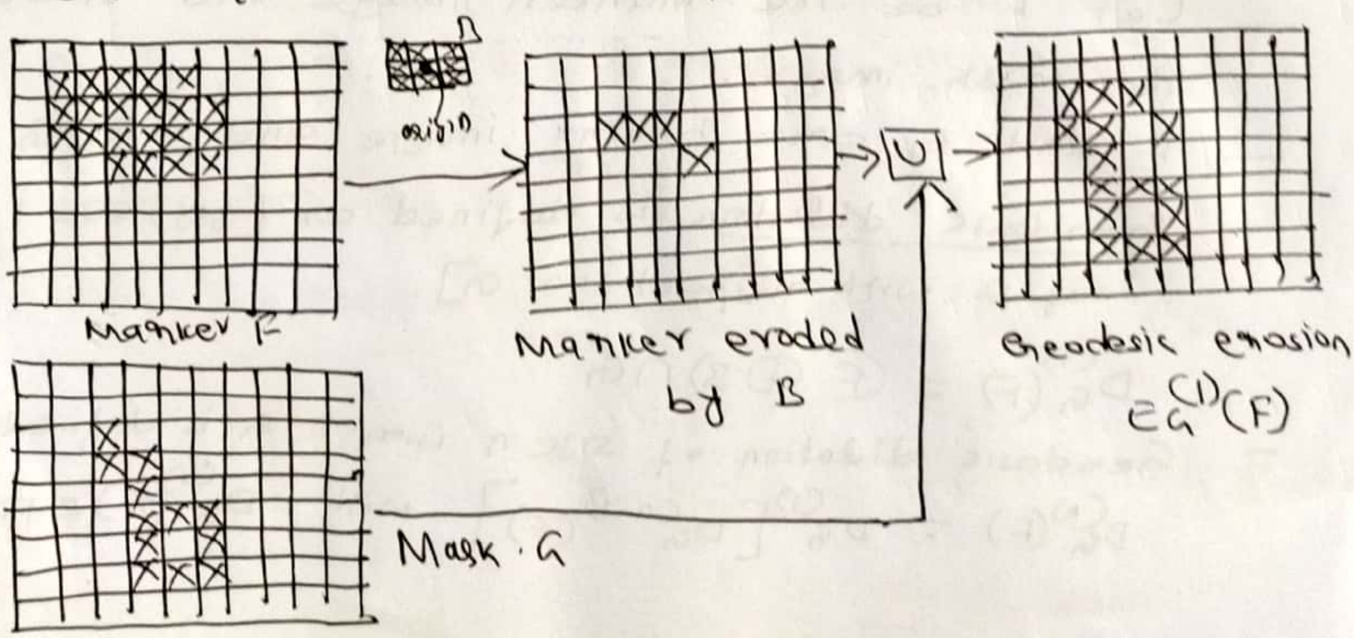
$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

Geodesic erosion of size n of F w.r.t G is defined as

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$$

with $E_G^{(0)}(F) = F$

* Set union is performed at each stage.
* In Godesic erosion image remain greater than or equal to its mask.



Scanned with CamScanner

(ii) Morphological reconstruction by dilation and by erosion

Morphological reconstruction by dilation ($R_G^D(F)$)

* Given mask image G and marker image F.
* It is defined as the geodesic dilation of F w.r.t G iterated till stability is achieved

$$R_G^D(F) = D_G^{(k)}(F)$$

with k such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$

Morphological reconstruction by erosion ($R_G^E(F)$)

* Given mask image G and marker image F
* It is defined as the geodesic erosion of F w.r.t G iterated till stability is achieved.

$$R_G^E(F) = E_G^{(k)}(F)$$

with k such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$

Scanned with CamScanner

(iii) Sample applications

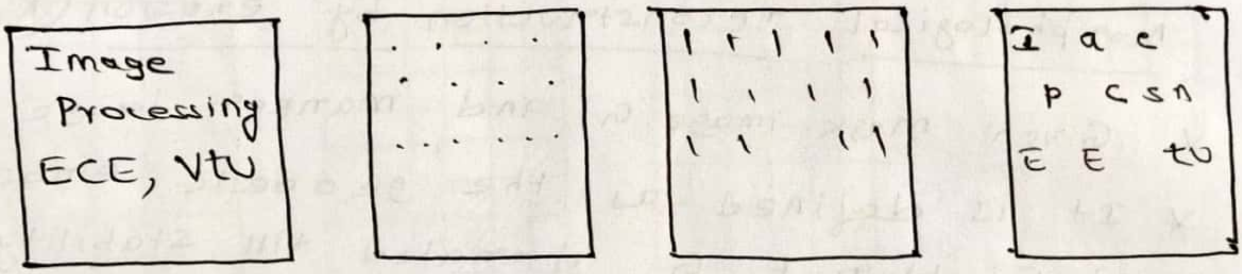
Morphological reconstruction has a broad practical applications.

(a) Opening by reconstruction

* In morphological opening, erosion removes small objects and dilation restores the shape of the object. Accuracy depends on shape of the object and SE
* Opening by reconstruction restores exactly the shape of object after erosion.
* The opening by reconstruction of size n of an image F is defined as the reconstruction by dilation of F from the erosion of size n of F

$$O_R^{(n)}(F) = R_F^D\left[(F \ominus nB)\right]$$

Where $F \ominus nB$ indicates n erosions of F by B



a) Image (A) b) Erosion of A c) Opening of (A) with same SE d) Result of opening by Reconstruction

fig: opening by Reconstruction

* In fig (d) characters containing long vertical strokes were restored and all other characters are removed.

(b) Filling holes

* Let $I(x,y)$ denote a binary image and suppose that we form a marker image F that is 0 (white) everywhere, except at the image border, where it is set to $1-I$, i.e

$$F(x,y) = \begin{cases} 1 - I(x,y), & \text{if } (x,y) \text{ is on the border of } I \\ 0, & \text{otherwise} \end{cases} \quad \text{— (1)}$$

then $H = \left[R^{D}_{I^c}(F)\right]^c$ is a binary image equal to I with all holes filled.

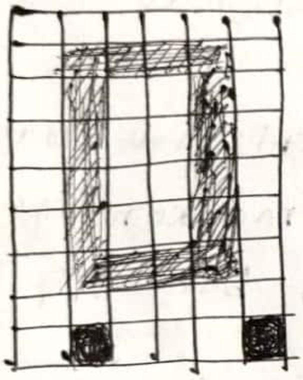


Fig a -> I

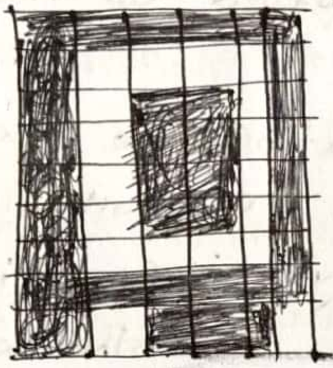


Fig b :- I^c

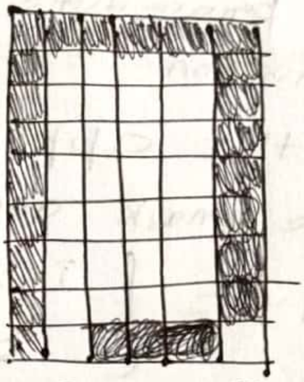


Fig c :- F

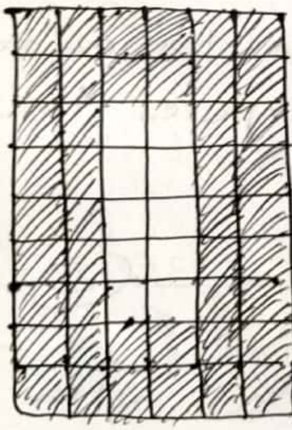


Fig d :- $F \oplus B$

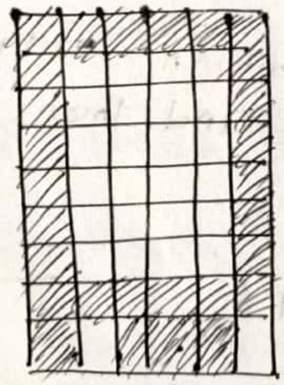


fig e : $F \oplus B \cap I^c$

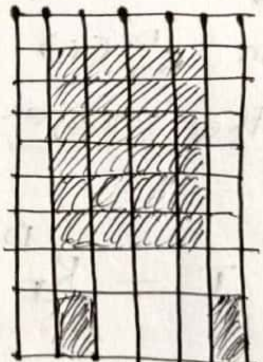


fig f : H

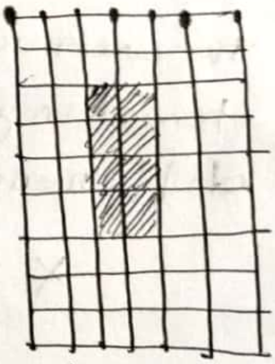


fig g :- $H \cap I^c$

* Fig (a) shows the simple image containing one hole
* Fig (b) shows the complement of I
* Fig (c) shows array formed according to equation (1)

* Fig d is F dilated with SE (3x3) where elements are all 1's i.e

* Fig (e) shows the geodesic dilation of F using I^c as a mask

* Fig (f) shows that the hole is filled and most of the image I was unchanged

* Fig (g) shows the final result $H \cap I^c$

(C) Border Clearing

In order to clean the border, we use a procedure based on Morphological reconstruction

* In this application, we use original image as the mask and the following marker image

$$F(x,y) = \begin{cases} I(x,y) & \text{if } (x,y) \text{ is on border of } I \\ 0 & \text{otherwise} \end{cases} \quad \text{--- (1)}$$

This algorithm first computes the Morphological reconstruction $R_I^D(F)$ (which extracts the objects touching the ~~object~~) and then computes the

border difference

$$X = I - R_I^D(F) \quad \text{--- (2)}$$

i.e it will obtain an image x, with no objects touching the border.

Questions

1. Define (i) Radiance (ii) Luminance (iii) Brightness
2. Write a note on Primary and secondary colours
3. Define Hue, Saturation, intensity, chromaticity & colour
4. Explain RGB, CMY, CMYK and HSI colour model
5. Explain the procedure to convert RGB to HSI & Vice versa
6. Problems (Refer page 8)
7. Explain Intensity slicing and Gray level to colour transformation w.r.t pseudocolour image processing.
8. Explain Image Pyramids
9. What is subband coding? Explain 2 channel filter bank and 2D, four band filter bank for subband coding
10. Find 2×2 Haar transform (H_2)
11. Find 4×4 Haar transform (H_4)
13. What are the properties of Haar Transform
13. Write a note on Reflection & Translation in morphological image processing
14. Explain Dilation process in Image processing
15. Explain Erosion process in Image processing
16. With diagram explain opening & closing operation.

17) Explain Hit or Miss Transformation Operation

18) Explain boundary extraction process with diagram

19) —||— Region filling, Convex hull —||—

20) —||— Thinning, Thickening —||—