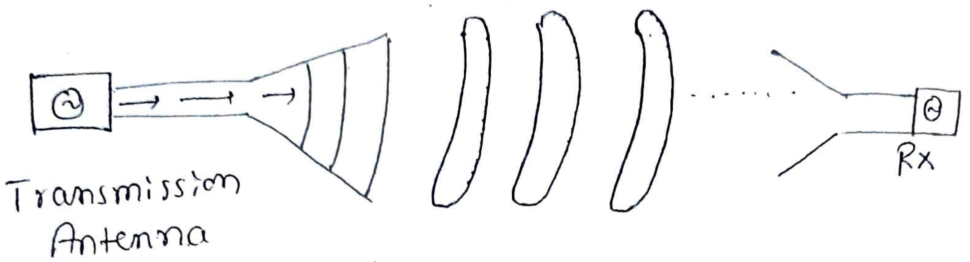
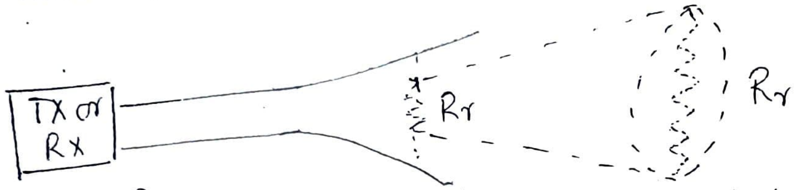
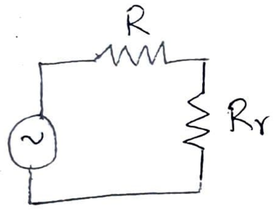
Antenna Basics

Antenna is a transition device which converts a guided wave into free space wave. Antennas are extensively used in satellite, radar, mobile communication.



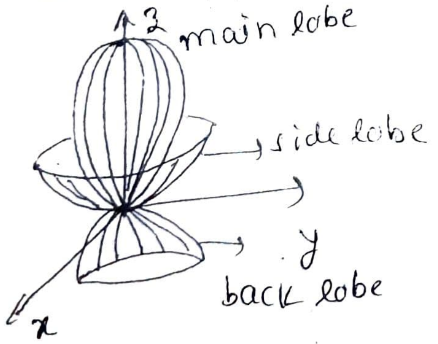
Antenna appear to the transmission lines as a resistance known as radiation resistance.





The radiation resistance of an antenna is that equivalent resistance which would dissipate the same amount of power as the antenna radiates, when current in that resistance equals the output current at the antenna terminals.

<u>patterns of antenna</u>: All antennas are directive. They radiate more power in one direction when compared with other.

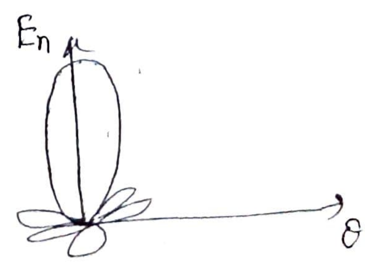


Radiation pattern is defined as a graphical representation of the radiation properties of the antenna as a function of directional coordinates.

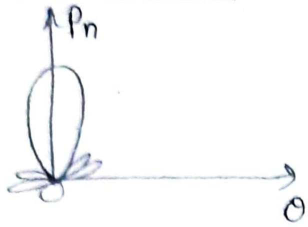
<u>Normalized pattern</u>

1. Electric field pattern

$$E_n = \frac{F_\theta(\theta, \phi)}{F_\theta(\theta, \phi)_{max}}$$



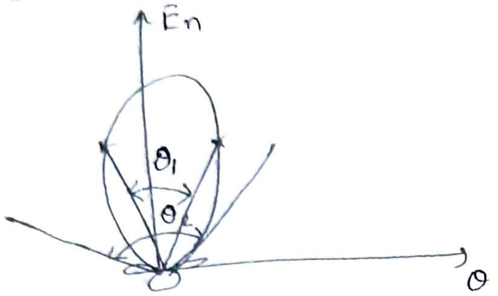
Power pattern



$$P_n(\theta,\phi) = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

Half power Beam width (HPBW):

It is the angle between two points in the antenna field pattern at which field becomes $\frac{1}{\sqrt{2}}$ times the maximum value.

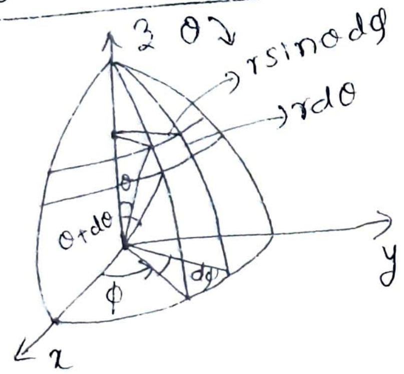


θ_1 represents HPBW.

Beam width between first nulls (BWFN): It is the angle between two points in the field pattern where the field becomes null for the first time.

Generally BWFN $\approx$ 2 HPBW, θ_2 represents BWFN.

Beam Area:



$$\text{surface area} = \iint_{\theta\,\phi} ds$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} (r\,d\theta)(r\sin\theta)\,d\phi$$

$$= r^2\int_{\theta=0}^{\pi}\sin\theta\,d\theta\,[\phi]_0^{2\pi}$$

$$= 2\pi r^2\,[-\cos\theta]_0^{\pi} = 4\pi r^2$$

$$\therefore \boxed{A = 4\pi r^2}$$

$dA = r^2\sin\theta\,d\theta\,d\phi$, Beam solid angle is the product of two angles.

$$\frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi = d\Omega$$

Beam area is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta,\phi)$ maintained its maximum value over Ω_A and was zero elsewhere.

The total surface area of the sphere $A = 4\pi r^2$

$\therefore$ The total solid angle subtended $= \frac{A}{r^2} = 4\pi$ steradians

$1 sr = (1 rad)^2 = \left(\frac{180}{\pi}\right)^2 = 3282.8064^\circ$

Total solid angle $= 4\pi\ sr = 4\pi(3282.8064) = 41253^\circ$

The beam area is also defined by

$$\Omega_A = \int_\theta \int_\phi P_n(\theta,\phi) \sin\theta\, d\theta\, d\phi = \int_\theta \int_\phi P_n(\theta,\phi)\, d\Omega$$

Beam area is approximately given by $\Omega_A \approx \theta_{HP} \phi_{HP}$

Radiation Intensity: Radiation intensity is the power radiated per unit solid angle. The power radiated in terms of radiation intensity is given by,

$$W = \int_\theta \int_\phi U\, d\Omega$$

Radiation intensity is measured in watts/sr.

Beam Efficiency: The antenna field pattern consists of a major lobe and some minor lobes.

$$\Omega_A = \Omega_m + \Omega_m$$

Beam efficiency is the ratio of area of major lobe to that of total area.

$$\epsilon_m = \frac{\Omega_m}{\Omega_A}$$

Stray factor is the ratio of total area of the minor lobes to that of total beam area of the antenna.

$$\epsilon_m = \frac{\Omega_m}{\Omega_A}$$

Directivity: It is the ratio of maximum power density to that of average power density over a sphere as observed in the far field of antenna.

$$D = \frac{P_{max}(\theta,\phi)}{P_{avg}(\theta,\phi)}$$

Directivity of antenna is always more than 1.

$$D = \frac{P_{max}(\theta,\phi)}{\frac{1}{4\pi}\iint_{\theta\,\phi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi} = \frac{P_{max}(\theta,\phi)}{\frac{1}{4\pi}\iint_{\theta\,\phi} P(\theta,\phi)\, d\Omega}$$

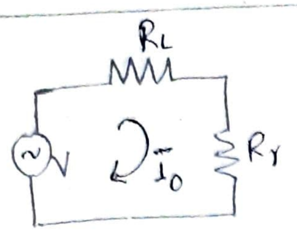
$$= \frac{4\pi}{\iint_{\theta\,\phi} \frac{P(\theta,\phi)}{P_{max}(\theta,\phi)}\, d\Omega} = \frac{4\pi}{\iint_{\theta\,\phi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A}$$

Directivity is inversely proportional to the beam area.
Directivity is dimensionless quantity.

Isotropic Antenna: Isotropic antenna is the one which radiates equally in all the directions. Directivity of isotropic antenna is unity.


← field pattern of isotropic antenna

Gain of Antenna: Gain of an antenna is practically observed value of directivity. Gain and Directivity are related by, $G = KD$, where K is antenna efficiency factor and $0 \le K \le 1$



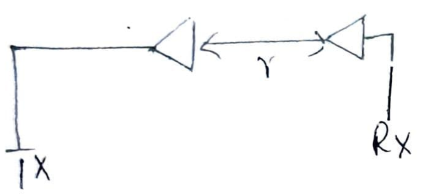
Consider a communication system with internal resistance R_L and antenna having a radiation resistance of R_r is connected as the load.

Power i/p to the circuit $= I_0^2 (R_L + R_r)$

Power delivered to antenna $= I_0^2 R_r$

efficiency $K = \dfrac{R_r}{R_L + R_r}$

Field strength at a particular point:



Consider an antenna which is transmitting a power of W_t watts in the free space. Field strength at any distance 'r' from the antenna is required.

The power density at any point distant 'r' from the antenna is given by,

$$S_r = \frac{W_t}{4\pi r^2}$$

If the antenna is having a gain of G_t, then the above formula is rewritten as,

$$S_r = \frac{W_t G_t}{4\pi r^2} \quad - (1)$$

By using Poynting theorem, the power density at any point can be calculated as,

$$S_r = E^2 / Z_0 \quad - (2)$$

where Z_0 is the free space impedance $= 120\pi$

from (1) and (2)

$$\frac{W_t G_t}{4\pi r^2} = \frac{E^2}{Z_0} = \frac{E^2}{120\pi}$$

$$\therefore E = \frac{\sqrt{30 W_t G_t}}{r} \quad v/m$$

1. A radio station radiates a total power of 10kW and has a power gain of 30. Find the field intensity at a distance of 100km from the antenna for free space propagation.

$G_t = 30, \; r = 100\,km, \quad W_t = 10\,kW$

$$E = \frac{\sqrt{30 W_t G_t}}{r} = \frac{\sqrt{30 \times 10 \times 10^3 \times 30}}{100 \times 10^3} = 30\,mv/m$$

2. Find the half power beam width and hence directivity of an antenna with $E(\theta) = \cos^2\theta, \; 0 \le \theta \le \pi/2$

HPBW represents $\frac{1}{\sqrt{2}} = \cos^2\theta$

$\therefore \theta = 33^\circ \qquad \therefore HPBW = 2\theta = 66^\circ$

Assume $\theta_{HP} = \phi_{HP}$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\theta_{HP}\phi_{HP}} = \frac{41253}{(66)(66)} = 9.47$$

3 Find the beam area if $E_n = \sin\theta, \; 0 \le \theta \le \pi$

$$\Omega_A = \int_\theta \int_\phi P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

$$P_n = E_n^2 \quad \therefore \Omega_A = \iint_{\theta\,\phi} \sin^2\theta \sin\theta \, d\theta \, d\phi = \iint_{\theta\,\phi} \sin^3\theta \, d\theta \, d\phi$$

$$\Omega_A = \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi = \left(\frac{4}{3}\right) \cdot 2\pi = \frac{8\pi}{3} \; sr$$

4 Calculate the directivity, $E_n = \sin\theta\cos\phi, \; 0 \le \theta, \phi \le \pi$

$$D = \frac{4\pi}{\Omega_A}$$

$$\Omega_A = \iint_{\theta\ \phi} P_n(\theta,\phi)\sin\theta\, d\theta\, d\phi = \iint_{\theta\ \phi} \sin^3\theta \cos^2\phi\, d\theta\, d\phi$$

$$= \int_{\theta=0}^{\pi} \sin^3\theta\, d\theta \int_{\phi=0}^{\pi} \cos^2\phi\, d\phi = \frac{2\pi}{3}\ sr$$

$$\therefore D = \frac{4\pi}{\Omega_A} = 6$$

5. An isotropic radiator has a field strength given by $E = \frac{10I}{r}$ V/m in the free space. Find the radiation resistance of the antenna.

$$E = \frac{10I}{r}, \quad R_r = ?$$

power density; $S_r = \frac{E^2}{Z_0} = \frac{100I^2}{r^2 Z_0}$

$$P = \iint \vec{S_r}\cdot\vec{ds} = \frac{100I^2}{Z_0}\iint \frac{1}{\cancel{r^2}}\vec{a_r}\cdot \cancel{r^2}\sin\theta\, d\theta\, d\phi\, \vec{a_r}$$

$$= \frac{100I^2}{Z_0}\int_{\theta=0}^{\pi}\sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi = \frac{100I^2}{Z_0}[-\cos\theta]_0^{\pi}\cdot 2\pi$$

$$P = \frac{400\pi I^2}{Z_0} \quad -(1)$$

This power is equated to the power radiated by the isotropic point source.

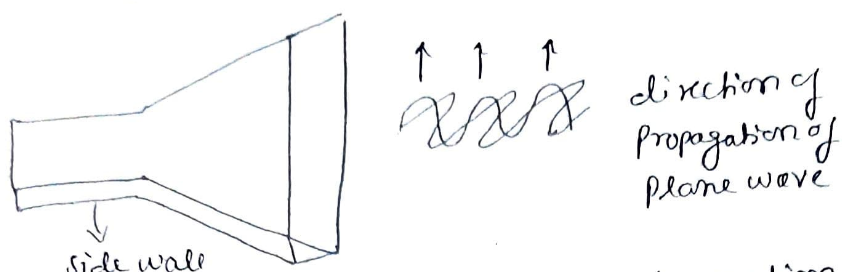
$$P = I^2 R_r \quad -(2)$$

from (1) and (2) we get

$$I^2 R_r = \frac{400\pi I^2}{120\pi} \qquad \therefore R_r = 3.33\,\Omega$$

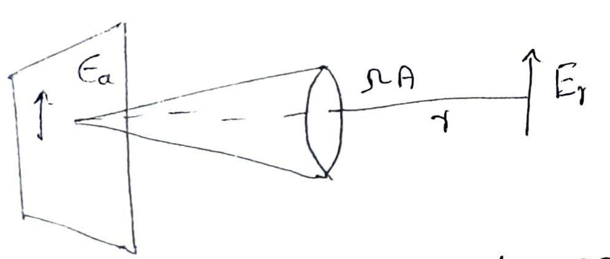
Resolution of Antenna: It is defined as half of the beam width between first nulls.

Antenna Aperture : consider a horn antenna which is immersed in e/m wave in the free space. The physical connecting area of the antenna is known as physical aperture of the antenna. It is denoted by Ap.



The antenna cannot collect the information over all of its physical aperture because the voltage at the side walls will be zero.
we define effective aperture as the actual collecting area of an antenna and is denoted by Ae. The ratio of Ae to Ap is known as aperture efficiency, and its value lies between 0.5 to 0.85.

Relation between effective aperture and directivity



consider a conical antenna as shown in the diagram
Let 'r' be the distance of measurement at which the electric field intensity is Er.
The transmitted power from the antenna is given by

$$P = \frac{E^2 Ap}{Z_0}$$

since antenna has an effective aperture Ae, the above expression is modified as,

$$P = \frac{E^2 Ae}{Z_0} \quad -①$$

The same power P can also be written in terms of received electric field Intensity as,

$$P = \frac{E_r^2 r^2 \Omega_A}{Z_0} \quad -(2)$$

where Ω_A is the beam area of the receiving antenna.

The received electric field intensity E_r is related to transmitted electric field intensity E by the relation,

$$E_r = \frac{E A_e}{r\lambda} \quad -(3)$$

using (3) in (2) we get,

$$P = \left(\frac{E A_e}{r\lambda}\right)^2 \frac{r^2 \Omega_A}{Z_0} = \frac{E^2 A_e^2 \Omega_A}{\lambda^2 Z_0} \quad -(4)$$

eqn (1) and (4) represents same quantity

$$\therefore \frac{\cancel{E^2} \cancel{A_e}}{\cancel{Z_0}} = \frac{\cancel{E^2} A_e^{\cancel{2}} \Omega_A}{\lambda^2 \cancel{Z_0}}$$

$$\lambda^2 = A_e \Omega_A \qquad \therefore A_e = \lambda^2 / \Omega_A$$

$$\therefore \text{Directivity } D = \frac{4\pi}{\Omega_A} = \frac{4\pi A_e}{\lambda^2}$$

The directivity of the antenna is directly proportional to the effective aperture and frequency of transmission.

consider an isotropic antenna, The effective aperture of this antenna is :

$$A_e = \frac{D\lambda^2}{4\pi}, \quad D = 1 \qquad \therefore A_e = \frac{\lambda^2}{4\pi} = 0.0796\lambda^2$$

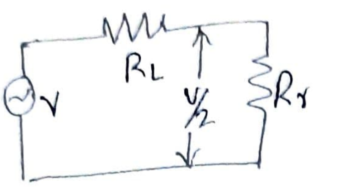
<u>Effective height:</u> In free space the voltage induced in the antenna is related to the height by the relation

$$V = Eh,$$

As the dimension of antenna increases, its ability to collect the information also increases.

because of constructional difficulties, the height of the antenna Cannot be increased beyond a point.

Relation between effective height and effective aperture



consider a circuit with internal resistance R_L and antenna which has a radiation resistance of R_r.

Maximum power is transferred to the antenna when $R_L = R_r$.

The power dissipated by the antenna is:

$$P = \frac{(V/2)^2}{R_r} = \frac{V^2}{4R_r} = \frac{E^2 h^2}{4R_r} \quad - (1)$$

According to the poynting theorem, the power at any point is given by,

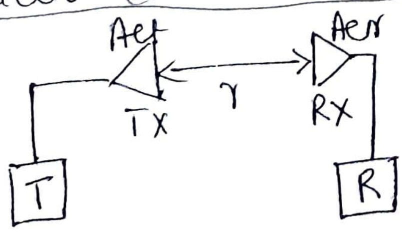
$$P = S_r A_e = \frac{E^2 A_e}{Z_0} \quad - (2)$$

from (1) and (2), we get

$$\frac{\cancel{E^2} h^2}{4R_r} = \frac{\cancel{E^2}}{Z_0} A_e$$

$$\therefore h = \sqrt{\frac{4 R_r A_e}{Z_0}}$$

Radio communication link (Friis Transmission Formula)



consider two antennas, one transmitting and another receiving. They are separated by a distance 'r' in between them in the free space. Let the transmitter antenna has an effective aperture of Aet and receiver antenna has an effective aperture of Aer.

The Power density at any point distant 'r' from the transmitter antenna is given by

$$S_r = \frac{P_t}{4\pi r^2}$$

Let the gain of transmitter antenna be G_t

$$\therefore S_r = \frac{P_t G_t}{4\pi r^2}$$

The received power by receiving antenna is:

$$P_r = S_r A_{er}$$

$$= \frac{P_t G_t A_{er}}{4\pi r^2}$$

but $G_t = 4\pi A_{et}/\lambda^2$

$$\therefore P_r = P_t \cdot \frac{\cancel{4\pi} A_{et} A_{er}}{\cancel{4\pi} r^2 \lambda^2}$$

$$\boxed{\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2}}$$

1. Calculate the transmitter power where aperture areas of transmitter and receiver antennas are 15 sqm and 5 sqm. The transmission distance being 25km and frequency of transmission is 10GHz. Assume that the power received is 10KW. Compute the gains of transmitted and receiver antennas.

$$\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2}$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03\,m$$

$$\frac{10\times10^3}{P_t} = \frac{5\times15}{(25\times10^3)^2 (0.03)^2}$$

$$G_t = \frac{4\pi A_{et}}{\lambda^2} = \frac{4\pi * 15}{(0.03)^2} = 209.43 * 10^3$$

$$\therefore P_t = 75MW$$

$$G_r = \frac{4\pi A_{er}}{\lambda^2} = \frac{4\pi * 5}{(0.03)^2} = 70 * 10^3$$

2) compute the path loss over a distance of 10km occurring in the case of transmitter at a frequency of 5MHz when the gain of antennas used are 100.

$\frac{P_r}{P_t} = ?$ $r = 10km$, $f = 5MHz$, $\lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^6} = 60m$

$G_t = G_r = 100$

$$\frac{P_r}{P_t} = \frac{A_{er}A_{et}}{r^2\lambda^2} = \frac{\frac{G_t\lambda^2}{4\pi}\cdot\frac{G_r\lambda^2}{4\pi}}{r^2\lambda^2} = \frac{G_rG_t\lambda^2}{(4\pi)^2r^2}$$

$$= \frac{(100)(100)(60)^2}{(4\pi*10*10^3)^2} = 2.27*10^{-3}$$

$$\left(\frac{P_r}{P_t}\right)_{dB} = 10\log_{10}\left(\frac{P_r}{P_t}\right) = -26.42dB$$

3) The power received by the receiver antenna at a distance of 0.5km over a free space at a frequency of 1GHz is 10.8mW. Calculate the input to the transmitter antenna if the gain of transmitter antenna & receiver antenna is 25dB, and 20dB respectively. The gain is w.r.t. isotropic antenna.

$r = 0.5km$, $f = 1GHz$, $\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3m$

$G_{t(dB)} = 25dB = 10\log_{10}G_t$ $\therefore G_t = 316.22$

$G_{r(dB)} = 20dB = 10\log_{10}G_r$ $\therefore G_r = 100$

$$\frac{P_r}{P_t} = \frac{G_tG_r\lambda^2}{(4\pi r)^2} \Rightarrow \frac{10.8*10^{-3}}{P_t} = \frac{(100)(316.22)(0.3)^2}{(4\pi*0.5*10^3)^2}$$

$\therefore P_t = 150W$

4) Two space crafts are separated by 100Mm. Each has an antenna with D=1000, operating at 2.5GHz. If aircraft A receiver requires 20dB over 1PW, what transmitter power is required on aircraft B to achieve this signal level?

$r = 100 \times 10^6 m$, $f = 2.5 \times 10^9$, $c = f\lambda$ $\therefore \lambda = \frac{3 \times 10^8}{2.5 \times 10^9} = 0.12m$

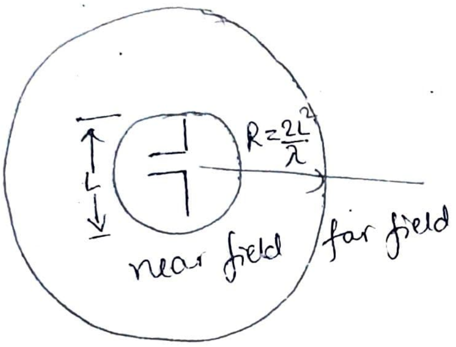
P_r = 20dB over 1pW

$20dB = 10\log_{10} P_{rt}$ $\therefore P_{rt} = 100$

$\therefore P_r = P_{rt} \times 1pW = 100pW$

$$P_t = \frac{P_r r^2 (4\pi)^2}{G_r G_t \lambda^2} = \frac{(10^{-10})(10^8)(10^8)(4\pi)^2}{(1000)(1000)(0.12)^2} = 10.96kW$$

Antenna field zones



Consider an antenna whose maximum dimension is 'L'. Draw a circle with radius $R = 2L^2/\lambda$. At a fixed frequency if the field is measured at a distance which is less than R, then that field is known as near field, or Fresnel zone. If the field measurement is made at a distance which is greater than R, then it is known as far field or Fraunhofer zone.

In the near field, the field pattern is dependent on the distance of measurement and in the far field, the pattern is independent of the distance.

The antenna characteristics such as side lobes, back lobes, are more clearly observed in the far field, pattern. Therefore generally field measurement is done at the far of distance from the antenna.