CIRCLE

FOR SSC CGL

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- CONCEPTUAL
- TRICKY
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GEOMETRY – CIRCLES AND MIXED FIGURES

In this final unit of Geometry we will be dealing with the **Circles and Mixed figures**. This is the **continuation of the Previous Chapter i.e. Triangles**. This chapter is also crucial for **SSC CGL Tier 1** Exam as circle is the most important topic of Geometry.

In the SSC CGL Tier 1 exam, the section on circles within the quantitative aptitude portion is pivotal for aspirants. It encompasses questions on various properties of circles, including chord lengths, tangents, secants, and their properties, area and circumference, and arc lengths. Additionally, candidates encounter problems related to angles subtended by chords at the center or on the circumference, cyclic quadrilaterals, and sectors and segments of circles.

Mastery of theorems and formulas related to circles is essential for tackling these questions efficiently. With usually 1-2 questions dedicated to circles, focused practice on this topic can aid candidates in securing crucial marks in the exam.

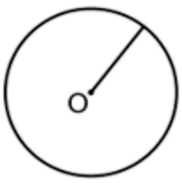
Circles

Let's start with the definition part.

A circle is a two – dimensional figure such that the distance from a point to the set of all points are equal.

OR,

A circle is a set of all points or locus of a point which are at the equal distance from the point.



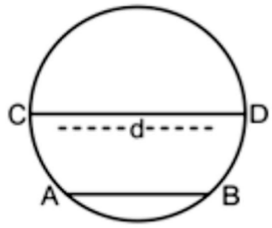
The distance from the centre (O) to any point on the circumference of the circle is equal and is called the radius of the circle (r).

[Note] Double the radius is called diameter (d) i.e. d = 2r.

Chord: A chord is a line segment joining the two points on the circumference of the circle.

[Diameter is the longest chord of a circle.]

In the diagram given below AB and CD are two chords of a circle. Also, CD = d (diameter) which is the longest chord.



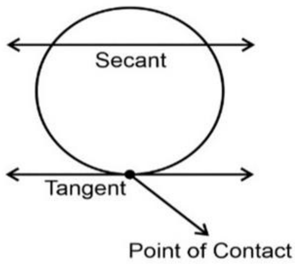
Tangent: A tangent is a line that touches a circle only at one point on the circumference of the circle.

Properties of Tangent:

1. Only **one tangent** can pass through one point of the circle.
2. The tangent at any point of a circle is **perpendicular** to the radius through the point of contact.

Secant: A line that touches the circle at two distinct points.

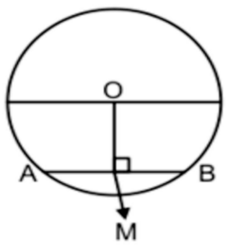
Look at the given following circle. We can easily distinguish between a tangent and a secant.



Important Theorems and Results

(1) Of the two chords of the circles, the one which is greater is nearer to the centre.

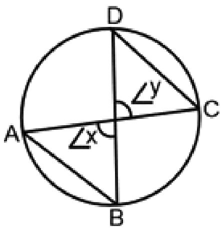
(2) The perpendicular from the centre of the circle bisects the chord i.e. radius always bisects the chord if perpendicular.



Also, if AM = MB, then OM is perpendicular to AB and vice versa.

(3) Equal chord of the circle always subtends the equal angles at the centre of the circle.

i.e. If **AB = CD** then **∠x = ∠y**.



Vice versa is also true.

(4) If the angles subtended by the two chords at the centre are equal then the chords are always equal.

(5) Equal chords of the circle are at equal distance from the centre.

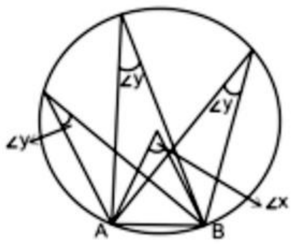
(6) Chords which are at equidistant from the centre of the circle are always equal.

(7) The length of the perpendicular from a point to a line is the distance of the line from the point.

(8) The angle subtended by an arc at the centre of the circle is double the angle subtended by it at any point on the circumference of the circle.

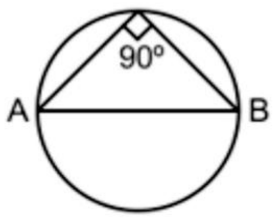
If you look at the given following diagram, ∠x is the angle subtended at the centre and ∠y is the angle subtended on the circumference of the circle i.e.

∠x = 2∠y



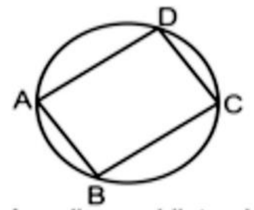
Also, the angle subtended by the circle on the same segment is equal. in the above diagram, we can see that the angle on the line segment AB is equal i.e. ∠y.

(9) The angle on the semicircle is the right angle or the angle made by the diameter on the circumference of the circle is 90˚.



(10) Cyclic Quadrilateral: A cyclic quadrilateral is a quadrilateral touching the boundaries/circumference of the circle.

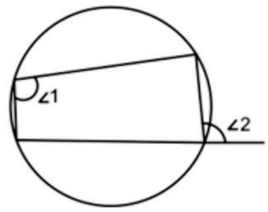
Note: The sum of the angles of a cyclic quadrilateral is 360˚. Also, note that the sum of the opposite angles of a cyclic quadrilateral is 180˚.



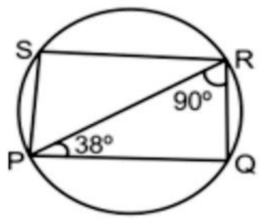
A cyclic quadrilateral

In the figure ABCD is a cyclic quadrilateral and ∠A + ∠B + ∠C + ∠D = 360˚.

Also, if a side of a quadrilateral is extended to produce an exterior angle, then that angle is equal to the opposite interior angle i.e. ∠1 = ∠2 (As shown in figure).



E.g. In the given figure where PQ is the diameter of the circle, find the sum of angles SPR and SRP.



Sol: At first glance you should notice that it is a cyclic quadrilateral.

In the triangle PQR, Sum of all angles

⇒ 38° + 90° + ∠PQR = 180 ⇒ ∠PQR = 52°

In the case of cyclic quadrilateral, the sum of angles of opposite faces are 180°.

⇒ ∠PQR + ∠PSR = 180° ⇒ 52° + ∠PSR = 180°

⇒ ∠PSR = 180° - 52° = 128°

Applying property of cyclic quadrilateral in angles SPQ and QRS.

First, let ∠SPR = x and ∠SRP = y.

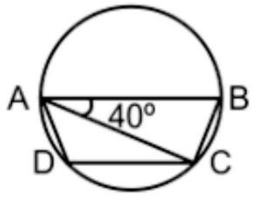
In ∠SPR,

⇒ 128° + x° + y° = 180° ⇒ x° + y° = 52° (**Ans.**)

E.g. A trapezium ABCD is such that all the four points lie on a circle. One of the parallel sides of the trapezium

(AB) is the diameter of a circle. If ÐCAB = 40°, find the value of CAD.

Sol:



If ABCD is cyclic, then ∠ABC + ∠CDA = 180°

Given, ABCD is a cyclic trapezium. Where AB ∥ CD Also, ∠CAB = 40°.

By angle sum property in triangle ABC,

⇒ ∠ABC = 180 - 90 - 40 = 50°

⇒ ∠CDA = 180 - 50 = 130°

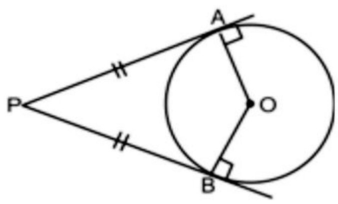
Also, ∠ACD = ∠CAB = 40° as AC is transversal to parallel sides AB and DC.

Now by angle sum property in triangle ADC, angle CAD = 180 - 130 - 40 = 10

Since angle in a semicircle is right angle. Then,

∠BCA = 90° (**Ans.**)

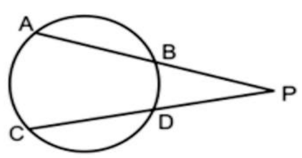
(11) The length of two tangents drawn from an external point is equal i.e. PA = PB.

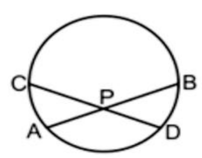


Also, tangent and radius make an angle of 90° at the point of contact of the two.

(12) Two chords which intersect each other internally or externally at any point, P then,

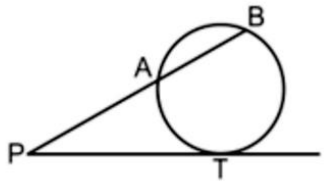
PA × PB = PC × PD.



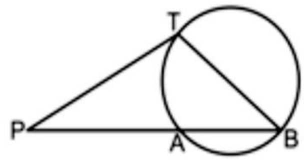


(13) If PAB is a secant and PT is a tangent, then,

PT^2 = PA × PB



E.g. In the given figure, TB passes through centre O, PT = 6 cm, PA = 4 cm and AB = x cm. What is the radius of the circle?



Sol: Since PT is the tangent to the circle and we know that in any circle,

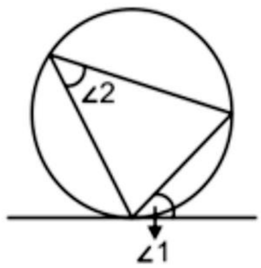
⇒ $PT^2 = PA \times PB$ ⇒ $(6)^2 = 4 \times (4 + x)$

⇒ $36 = 4 \times (4 + x)$ ⇒ $9 = 4 + x$ ⇒ $x = 5$ cm

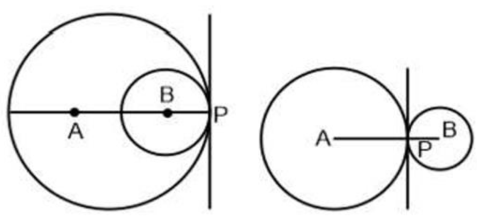
In triangle PTB,

⇒ $PB^2 = PT^2 + BT^2$ ⇒ $9^2 = 6^2 + BT^2$ ⇒ $BT = \frac{3\sqrt{5}}{2}$ cm (**Ans.**)

(14) Alternate segment theorem: Angle made by the chord and the tangent at the point of contact of circle in one segment is equal to the angle in the alternate segment i.e. ∠1 = ∠2 (below).



(15) What happens when circles touch each other internally or externally?



The first figure shows two circles touching each other internally while the other shows how they touch each other externally.

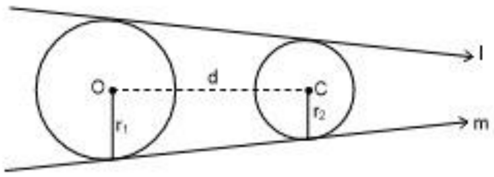
As you can see in the diagram, whenever centres intersect each other internally or externally then the point of contact lies on the line joining the centres of the two circles.

When touches internally, **distance = AP – BP**

When touches externally, **distance = AP + BP**

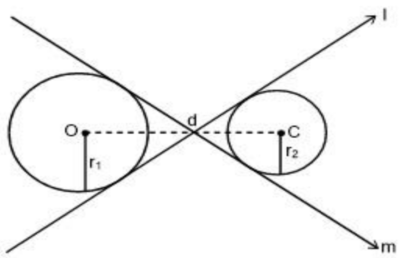
(16) Direct and Transverse Common Tangents:

(i) The **direct common tangents** to two circles meet on the line of centres and divide it externally in the ratio of the radii.



Length of DCT = $\sqrt{d^2 - (r_1 - r_2)^2}$

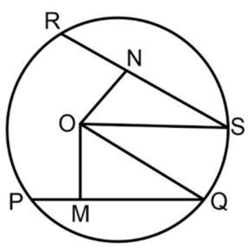
(ii) The **transverse common tangents** also meet on the line of centres and divide it internally in the ratio of the radii.



Length of TCT = $\sqrt{d^2 - (r_1 + r_2)^2}$

When you go through them once more, try solving the following question first by yourself then you can look at the solution.

E.g. In the given figure, PQ = 30, RS = 24, and OM = 12 cm then what is the value of ON?



Sol: We know that if we draw a perpendicular from the centre to any chord then the perpendicular divides the chord into two equal parts.

Therefore, OM divides PQ into PM and MQ

⇒ PM = MQ = 15 cm

Also, ON divides RS into two parts RN and NS

⇒ RN = NS = 12 cm

Since, OQ is the radius of the circle Therefore, using Pythagoras theorem So, in ΔOQM,

⇒ OQ = $\sqrt{(MO)^2 + (MQ)^2}$ = $\sqrt{(12)^2 + (15)^2}$

= $\sqrt{369}$ cm

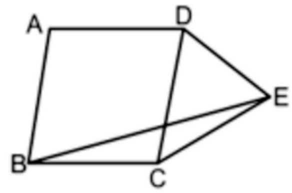
Also, OQ = OS = radius

⇒ ON = $\sqrt{(OS)^2 - (NS)^2}$ = $\sqrt{369 - (12)^2}$ = 15 cm (**Ans.**)

Mixed Figures

Let's discuss few questions about mixed figures. In these figures, 2 concepts like circle and square, rhombus and triangle, etc., or more than 2 concepts can be mixed. Here is just an introduction of these questions. We will discuss more of these in the exercise part.

E.g. In the given figure, ABCD is a rhombus and BCE is an isosceles triangle, with BC = CE, ∠CBE = 20˚ and ∠ADC = 78p, then what is the value (in degrees) of ∠DEC?



Sol: Let us look at the triangle BCE. In triangle BCE,

BC = CE (given)

and ∠CBE = 20˚ ⇒ ∠CEB = 20˚

So, ∠BCE = 180˚ – 2 × 20˚ = 140˚

As ABCD is a rhombus, AB || CD, therefore,

⇒ ∠ADC + ∠BCD = 180˚

⇒ ∠BCD = 180 – ∠ADC = 180 – 78 (∠ADC = 78, given)

⇒ ∠BCD = 102˚

Also, ∠DCE = ∠BCE – ∠BCD = 140˚ + 102˚ = 38˚

Also, in triangle CDE, CD = CB (Side of the rhombus) and we have been given that CB = CE

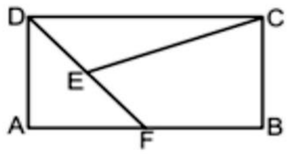
Therefore, CE = CD, So, ∠CDE = ∠CED

therefore, we have, ∠CDE + ∠CED + ∠DCE = 180

⇒ 2∠CDE + ∠DCE = 180

⇒ ∠CDE = $\frac{180 - 38}{2}$ = 71˚ (**Ans.**)

E.g. In the given figure, ABCD is a rectangle. F is a point on AB and CE is drawn perpendicular to DF. If CE = 60 cm and DF = 40 cm, then what is the area (in cm²) of the rectangle ABCD.



Sol: Construction: Join C to F

As we have to calculate the area of ABCD, we will calculate the area of triangle DFC first.

⇒ Area of DFC = $\frac{1}{2}$ DF × CE = $\frac{1}{2}$ 40 × 60 = 1200 cm^2

Also, Area of DFC = $\frac{1}{2}$ DC × BC = 1200

⇒ DC × BC = 2400

And we know that area of rectangle = DC × BC = 2400 cm^2 (**Ans.**)

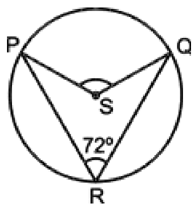
Let's look at some questions asked in SSC CGL Tier 1 previous year papers.

E.g. During a practice session in a stadium an athlete runs along a circular track and her performance is observed by her coach standing at point on the circle and also by her physiotherapist standing at the centre of the circle. The coach finds that she covers an angle of 72° in 1 min. what will be the angle covered by her in 1 second according to the measurement made by her physiotherapist? **[SSC CGL 2019]**

Sol: The angle subtended by the arc at the centre is double of the angle subtended by same arc at circumference of the circle.

Suppose the athlete runs from point P to Q in 1 minute and the coach and the physiotherapist observe him from point R and S respectively.

Given, ∠PSQ = 72°



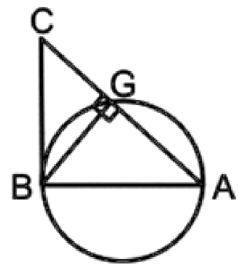
Then, ∠PSQ = 2 × 72 = 144°

Hence, according to physiotherapist the athlete covers an angle of 144° per minute

∴ In 1 second, the angle covered by the athlete according to physiotherapist = $\frac{144}{60}$ = 2.4° (**Ans.**)

E.g. AB is a diameter of a circle with centre O. CB is tangent to the circle at B. AC intersects the circle at G. If the radius of the circle is 6 cm and AG = 8 cm, then the length of BC is: **[SSC CGL 2019]**

Sol:



Angle made by diameter at any point on the circle is 90°.

Radius = 6 cm and AG = 8 cm

Suppose ∠BAG = x°

As in the figure: Triangle AGB is right angled

Hence, applying Pythagoras theorem:

BG = $\sqrt{(AB^2 - AG^2)} = \sqrt{(12^2 - 8^2)} = \sqrt{80} = 4\sqrt{5}$

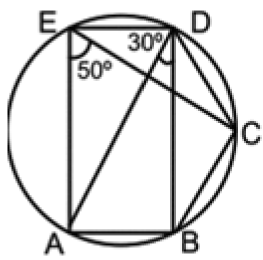
Now, In triangle AGB and ABC:

tan x = $\frac{BG}{AG} = \frac{BC}{AB}$

⇒ BC = $\frac{BG}{AG}$ × AB = $\frac{4\sqrt{5}}{8}$ × 12 = $6\sqrt{5}$ cm (**Ans.**)

E.g. Points A, D, C, B and E are concyclic. If ∠AEC = 50° and ∠ABD = 30°, then what is the measure (in degrees) of ∠CBD? **[SSC CGL 2020]**

Sol:

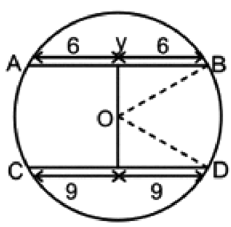


∠AEC = ∠ABC = 50°

∠ABC = ∠ABD + ∠DBC

∠DBC = 50 - 30 = 20° (**Ans.**)

E.g. In a circle with centre O, AB and CD are parallel chords on the opposite sides of a diameter. If AB = 12 cm, CD = 18 cm and the distance between the chords AB and CD is 15 cm, then find the radius of the circle (in cm). **[SSC CGL 2020]**



OA is perpendicular to AB and OX is perpendicular to CD.

OB = OD = radius.

In triangle OBY,

$OB^2 = OY^2 + 6^2 = OY^2 + 36$......(1)

In triangle ODX,

$\Rightarrow OD^2 - OX^2 + 9^2 = OX^2 + 81$...(2)

From Equation (1) and (2) we get,

$\Rightarrow OX^2 + 81 = OY^2 + 36$

$\Rightarrow 81 - 36 = OY^2 - OX^2$

$\Rightarrow 45 = (OY + OX)(OY - OX)$

Here OY + OX = 15......(3)

So one can say that OY - OX = 3.......(4)

From equation 3 and 4 we get,

OX = 6 cm and OY = 9.

$\Rightarrow OB^2 = OY^2 + 36 = 81 + 36 = 117$

$\Rightarrow OB = 3\sqrt{13}$. (**Ans.**)

E.g. A circle is inscribed in $\triangle ABC$, touching AB, BC, and AC at the points P, Q, and R, respectively. IF AB - BC = 4cm, AB - AC = 2cm and the perimeter of $\triangle ABC$ = 32 cm, then $\frac{BC}{2}$ (in cm) = ? **[SSC CGL 2021]**

Sol: AB - BC = 4 .....(1

AB - AC = 2 .....(2

Perimeter of ΔABC = 32

$\Rightarrow$ AB + BC + AC + = 32 .....(3

Adding equations (1, (2, and (3 -

$\Rightarrow 3AB = 32 + 4 + 2 = 38$

$\Rightarrow AB = \frac{38}{3}$

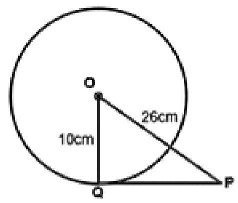
$\Rightarrow AB - BC = 4 \Rightarrow \frac{38}{3} - BC = 4$

$\Rightarrow BC = \frac{38-12}{3} = \frac{26}{3}$

$\Rightarrow \frac{BC}{2} = \frac{26}{6} = \frac{13}{3}$ (**Ans.**)

E.g. O is the centre of a circle of radius 10 cm. P is a point outside the circle and PQ is a tangent to the circle. What is the length (in cm) of PQ if the length OP is 26 cm? **[SSC CGL 2021]**

Sol:



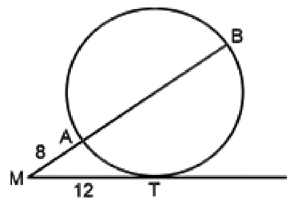
Using Pythagoras theorem:
$(\text{Hypotenuse})^2 = \text{Base}^2 + \text{perpendicular}^2$

$PQ^2 = OP^2 - OQ^2 = 26^2 - 10^2 = 676 - 100 = 576$

$\Rightarrow$ PQ = 24 cm (**Ans.**)

E.g. A tangent is drawn from point M to a circle, which meets the circle at T such that MT= 12 cm. A secant MAB intersects the circle at points A and B. If MA = 8 cm, what is the length (in cm) of the chord AB? **[SSC CGL 2022]**

Sol:



We know that, $MT^2 = MA \times MB$

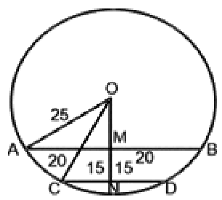
$\Rightarrow 12^2 = 8 \times MB$

$\Rightarrow MB = 18$

$\Rightarrow AB = MB - MA = 18 - 8 = 10$ cm (**Ans.**)

E.g. A chord of length 40 cm is drawn in a circle having a diameter of 50 cm. What is the minimum distance of the other parallel chord of length 30 cm in the same circle from the 40 cm long chord? **[SSC CGL 2022]**

Sol:



In triangle AOM,

$AO^2 = AM^2 + OM^2$

$\Rightarrow 25^2 = 20^2 + OM^2$

$\Rightarrow 625 - 400 = OM^2$

$\Rightarrow$ OM = 15 cm

In triangle CON,

$CO^2 = CN^2 + ON^2$

$\Rightarrow 25^2 = 15^2 + OM^2$

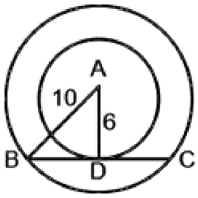
$\Rightarrow 625 - 225 = OM^2$

$\Rightarrow OM = 20$ cm

So, the distance between the chords MN = 20 – 15 = 5 cm (**Ans.**)

E.g. Two concentric circles are of radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle. **[SSC CGL 2023]**

Sol:



In triangle ABD

$\Rightarrow AB^2 = AD^2 + BD^2 \Rightarrow 100 = 36 + BD^2$

$\Rightarrow BD^2 = 64 \Rightarrow BD = 8$

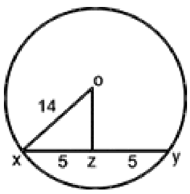
We know that,

$\Rightarrow$ BC = BD + DC = 8 + 8 = 16 cm (**Ans.**)

E.g. If the area of the circle is 616 cm^2 and a chord XY = 10 then find the perpendicular distance from the centre of the circle to the chord XY. **[SSC CGL 2023]**

Sol: Area of the circle = 616 cm^2

$\Rightarrow \pi r^2 = 616 \Rightarrow \frac{22}{7} \times r^2 = 616 \Rightarrow r^2 = 196 \Rightarrow r = 14$ cm



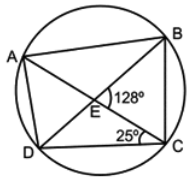
In triangle OZX by using Pythagoras theorem.

$OX^2 = OZ^2 + XZ^2 \Rightarrow 196 = OZ^2 + 25$

$\Rightarrow OZ = \sqrt{(196 - 25)} = \sqrt{171}$ cm (**Ans.**)

E.g. ABCD is a cyclic quadrilateral. Diagonals BD and AC intersect each other at E. If ∠BEC = 128° and ∠ECD = 25°, then what is the measure of ∠BAC? **[SSC CGL 2021]**

Sol: Angle made by the same arc on the perimeter is same.



ABCD is a cyclic quadrilateral.

∠ECD = 25° (Given)

Therefore, ∠DBA = 25° (Angle made by same arc on perimeter is same)

∠BEC + ∠AEB = 180° (Straight line)

128° + ∠AEB = 180° ⇒ ∠AEB = 52°

In triangle AEB,

∠DBA + ∠AEB + ∠BAC = 180°

Putting the values:

25° + 52° + ∠BAC = 180° ⇒ ∠BAC = 103° (**Ans.**)

So, you see, with the right approach it becomes easy to reach the concept. These questions might seem like a nightmare at first but when you apply concepts you could easily reach your answer. We should try to solve more questions of these types to command your brain. Try to use your pen less so that you save your time in the examination.

We will discuss different type of questions to prepare you better for the examination. This exercise will help you to deal with the main exam as we will discuss questions difficulty wise, as well as we will also discuss those question, which you can skip to save your time in the examination as all the questions have same marks but some of them are really time consuming. We will discuss over it, but for now, let's get to the exercise part.

Practice Questions

Easy Level

These are easy level questions based on what we have learned above. Try to solve all of them without using pen and paper. Try to calculate it in mind.

Time limit: Maximum 15 seconds per question.

Q:1 The radii of two circles which do not touch each other, are 10 cm and 7 cm while the length of direct common tangent is 4 cm. Find the distance between the centres of the given two circles.

1.3 cm 2.9 cm

3.5 cm 4.8 cm

Q:2 ABCD is a cyclic quadrilateral. The length of the AB = 15.2 cm, BC = x cm, CD = 22.8 cm, DA = 15 cm. If AC bisects BD then, find the length of the BC.

1.15.2 cm 2.22.5 cm
3.15 cm 4.18.5 cm

Q:3 If two circles with radius 25 cm and 16 cm respectively touch each other externally, then find the length of the common tangent to both the circles.

1.20 cm 2.30 cm
3.40 cm 4.50 cm

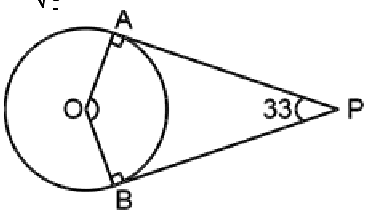
Q:4 Find the area of triangle formed by lines x = 0, y = 0 and 3x - 4y = 12.

1.6 2.16
3.24 4.12

Q:5 What is the length of the tangent (in cm) drawn to a circle of radius 28 cm from a point 53 cm from the center?

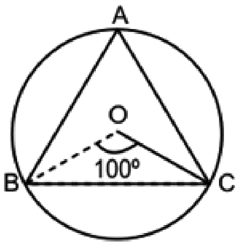
1.42 2.45
3.48 4.50

Q:6 In the given figure ∠APB = 33°, find the value of $\sqrt{\frac{1}{3}} \times \angle AOB$.



1.2 2.7
3.6 4.5

Q:7 In the following figure, find the angle BAC.

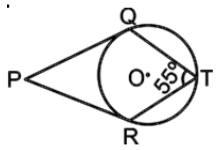


1.80° 2.50°
3.200° 4.25°

Q:8 If the radii of the two circles, not touching each other, are 8 cm and 7 cm respectively and the distance between the centres of the circles is 25 cm then find the length of the common transverse tangent.

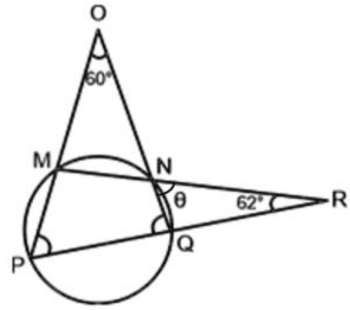
1.15 cm 2.20 cm
3.25 cm 4.30 cm

Q:9 PQ and PR are tangents to circle, QTR = 55°, find measure of QPR.



1.55° 2.70°
3.65° 4.45°

Q:10 In the given diagram ∠MON = 60° and ∠QRN = 62° then find the value of ∠QNR?



1.58° 2.84°
3.29° 4.61°

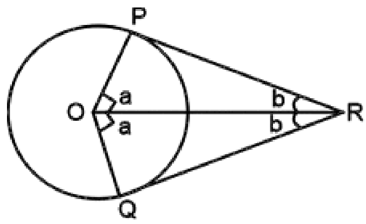
Q:11 Two circles having centers O_1 and O_2 having radii 16 and 9 respectively. A common tangent passes from outside of both the circles touching at points A and B respectively, the distance between the centers of the circle is 25. Find the length of AB.

1.24 cm 2.14 cm
3.10 cm 4.20 cm

Q:12 If the radii of the three circum-circles of a triangle are 4 cm, 4 cm and 2 cm respectively, what is the inradius of the triangle? (in cm)

1.0.5 2.1
3.1.5 4.2

Q:13 In the given figure, if a = 70° then what is the value of b?



1.20° 2.30°
3.40° 4.50°

Q:14 PQ and MN are the two chords that intersect on point O in the circle. If the length of MN is 11cm, PO is 4 cm, and QO is 7.5 cm then, find the difference between the ON and OM.

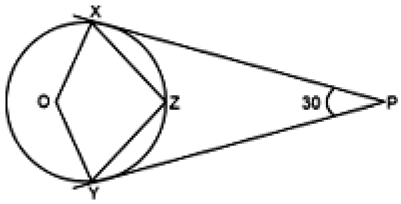
1.1 2.2
3.3 4.4

Q:15 In a cyclic quadrilateral ABCD, the ratio of angle A and B is 1 : 2 and the ratio of angle C and D is 5 : 4. What is the value of ∠ A?

1.15° 2.30°

3.40°　　4.60°

Q:16 In the given figure O is the center of the circle and ∠XPY = 30°. Find the average of the ∠XOY and ∠XZY.

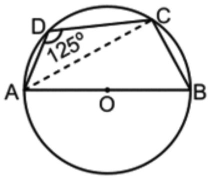


1.150°　　2.255°
3.127.5°　　4.105°

Q:17 In a circle with centre O, PQR is a tangent at the point Q on it. AB is a chord in the circle parallel to the tangent such that ∠BQR = 70º. What is the measure of ∠AQB?
1.60º　　2.35º
3.55º　　4.40º

Q:18 A circle is drawn outside a triangle and the sides of the triangle are 12 cm, 35 cm, and 37 cm then, find the radius of the circle.
1.17.5　　2.18.5
3.18.7　　4.16.5

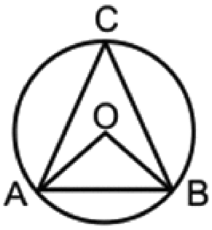
Q:19 In the given figure, AB is the diameter of the circle with centre O. Find the value of ∠CAB.



1.45º　　2.35º
3.55º　　4.90º

Q:20 The diameter of a circle is 30 cm. AB is a chord in the circle and AB subtends an angle of 60° at the centre of the circle. Find the length of chord AB.
1.15 cm　　2.19 cm
3.12 cm　　4.16 cm

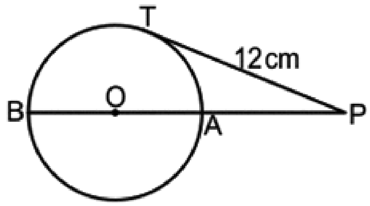
Q:21 In the following figure, O is the center of the circle. AB is a chord of length √3 times the radius of the circle. What is the value of ∠ ACB?



1.30°　　2.45°
3.60°　　4.75°

Q:22 An equilateral triangle PQR is inscribed in a circle and another circle with the same center 'O' is inscribed in the equilateral triangle whose radius is 11.4 cm. Find the radius of the larger circle.
1. 20　　2. 22.8
3. 14　　4. 23.2

Q:23 In the following figure, A tangent PT is drawn to the circle from a point P outside the circle. If the length of PT is 12 cm and the ratio of PA : PB = 4 : 9. Find the radius of the circle.



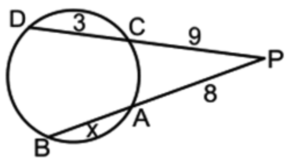
1.10 cm　　2.4 cm
3.9 cm　　4.5 cm

Q:24 If the radii of two circles are 5√3 and 11√3 respectively and the distance between the centers of these circles is $\frac{48}{\sqrt{3}}$. Find the number of common tangents that can be drawn to these circles.
1.None　　2.One
3.Three　　4.Four

Q:25 There are two circles with radius 9 cm and 5 cm. The distance between the centre of those two circles is 15 cm. Find the length of the transverse common tangent.
1.√29 cm　　2.5 cm
3.√23　　4.√31 cm

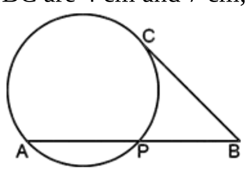
Q:26 A circle with centre 'O' has a radius of 20 cm. If the length of chord PQ is 32 cm, then find OA:OP given that PA : PQ = 1 : 2, given that 'A' is a point on PQ.
1. 3 : 5　　2. 3 : 9
3. 2 : 5　　4. 4 : 7

Q:27 In the given figure if PA = 8 cm , PC = 9 cm , CD = 3 cm find the value of AB.



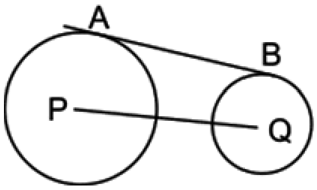
1.5.5 cm　　2.6.5 cm
3.7.5 cm　　4.11 cm

Q:28 Find the value of AP in the given figure, if BP and BC are 4 cm and 7 cm, respectively.



1.8.67 cm　　2.6.25 cm
3.7.55 cm　　4.8.25 cm

Q:29 What is the length of the common tangent AB of two circles with centers P and Q separated by 12.5 cm, if the radius of the circles are 6.5 cm and 3 cm respectively?



1.10 cm 2.11 cm
3.12 cm 4.13 cm

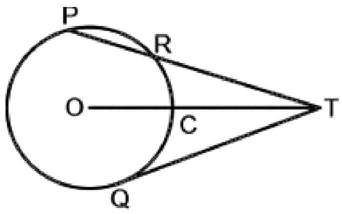
Q:30 The length of the largest chord of a circle is 42 cm whereas a smaller chord of length 16 cm is drawn perpendicular to it. Find the area (in cm^2) of the circle.
1.1386 2.1346
3.628 4.1196

Moderate Level

The level of questions is now increasing a bit. Try to avoid actual lengthy calculation as much as you can to get the answer.

Time limit: Maximum 25 seconds per question.

Q:1 In the given figure, QT is the tangent to the circle with radius 5 cm. If T is at a distance of 11 cm from the centre O and TR is 6 cm, then what is the length of chord PR?



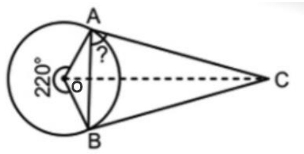
1.5 cm 2.10 cm
3.15 cm 4.20 cm

Q:2 In a circle with centre O, PQ is a diameter and RS a chord such that PQRS is a trapezium. If ∠RPQ = 25º, then find the value of ∠RPS.
1.45º 2.40º
3.54º 4.60º

Q:3 A tangent drawn from an external point ‘P’ meets a circle at point ‘Q’. If ‘O’ is the centre of the circle with a radius of 8 cm, and PQ = x cm and PO = (x + 2) cm, then find the value of ‘x’.
1.12 2.18
3.15 4.21

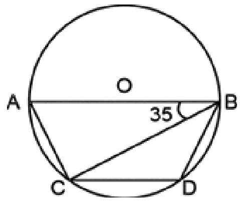
Q:4 If the area of the circle is 625π cm^2 and this circle have two equal chords opposite to each other of length 48 cm. Find the distance between the chords.
1.14 cm 2.7 cm
3.28 cm 4.15 cm

Q:5 In the given figure, from an external point C, tangents CA and CB are drawn to a circle with centerO. If ∠AOB = 220° (bigger arc of the circle), then find ∠CAB.



1. 50° 2. 60°
3. 70° 4. 80°

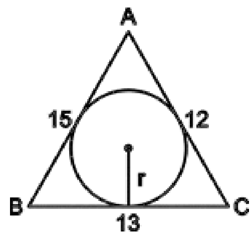
Q:6 In the given figure CD || AB, ∠CBA = 35°. Find the value of ∠CDB - ∠DBC.



1.150° 2.130°
3.105° 4.165°

Q:7 A tangent PQ is drawn to a circle that touches the circle at R. RS is a chord of the circle. If O is the center, and angle ROS = 130°, find measure of angle PRS.
1.50° 2.55°
3.60° 4.65°

Q:8 A scalene triangle ABC is given with sides 15 cm, 13 cm, and 12 cm, and a circle is drawn in the triangle having radius r. Find r.



1.$\sqrt{47}$ 2.$\sqrt{14}$
3.$\sqrt{15}$ 4.$\sqrt{29}$

Q:9 Chords AB and CD of a circle, when produced, meet at a point P outside the circle, If AB = 8 cm, BP = 4 cm and CD = 2 cm then find the value of DP.
1.8 m 2.5 m
3.6 m 4.2 m

Q:10 A chord is drawn inside the bigger circle of two concentric circles, with radii 25 and 15 cm, respectively. If the chord to the outer circle is such that, it touches the smaller circle at exactly 1 point, find the length of the chord.
1.32 cm 2.30 cm
3.40 cm 4.38 cm

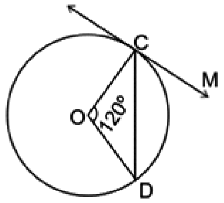
Q:11 A triangle is drawn inside a circle such that the sides of a triangle are given as 9 cm, 40 cm, and 41 cm then, find the radius (r) inside the triangle

1.5 | 2.5.85
3.4 | 4.7.6

Q:12 The distance of point P outside the circle from its centre is 13 cm. A chord RS is made such that when it produced, meets at P. If radius of circle is 5 cm, find PS × PR.

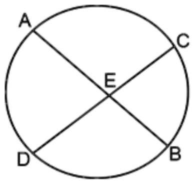
1.90 cm | 2.121 cm
3.65 cm | 4.144 cm

Q:13 In the given figure, O is the centre of a circle, CD is a chord and CM is the tangent at C. If ∠COD = 120°, then calculate ∠DCM.



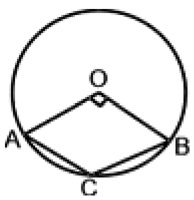
1.60° | 2.45°
3.30° | 4.15°

Q:14 In the given figure, the ratio of AE : CE = 5 : 3, EB = 30 cm and AB = 75 cm. What is the value of CD?



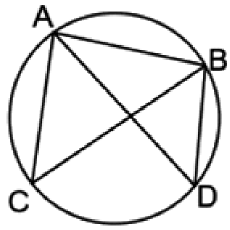
1.66 cm | 2.77 cm
3.88 cm | 4.99 cm

Q:15 In the given figure, O is the centre of the circle. If ∠AOB = 88° then what is the value of ∠ACB?



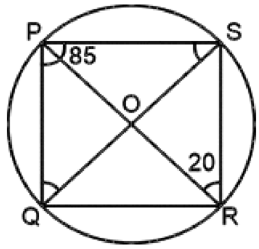
1.130° | 2.132°
3.134° | 4.136°

Q:16 In the given figure, BC is the diameter of the circle and ∠ABC : ∠ADB = 1 : 2, then what is the value of ∠ACB?



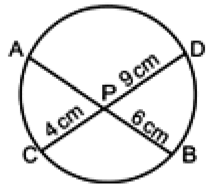
1.15° | 2.30°
3.45° | 4.60°

Q:17 In the given figure, ∠SPQ = 85° and ∠PRS = 20° then what is the value of ∠PSQ?



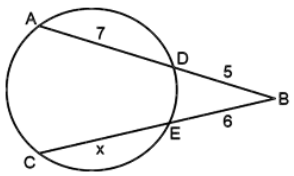
1.45° | 2.50°
3.75° | 4.60°

Q:18 Find the length of AP, if AB and CD are the chords of a circle intersect each other at P.



1.12 cm | 2.8 cm
3.6 cm | 4.7 cm

Q:19 In the given figure, what will be the value of x (in cm)?

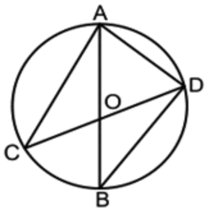


1.1 | 2.2
3.3 | 4.4

Q:20 In a circle with centre O, the diameter of the circle is 58 cm and the length of the chord is 42 cm. Find the distance between the chord and centre of the circle.

1.20 cm | 2.21 cm
3.25 cm | 4.15 cm

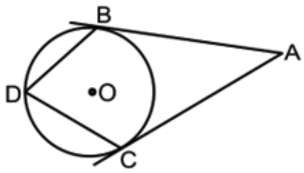
Q:21 In the following figure, AB and CD are diameters of the circle. If ∠AOD = 80°, then what is the value of ∠ADC?



1.30°　　2.40°
3.50°　　4.60°

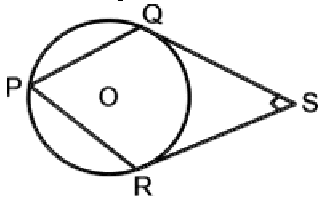
Q:22 MN and QR are two chords in a circle in different segments with center O and MR is the circle's diameter. When MN and QR are further extended, it meets at a point P outside the Circle. if the ∠RMN = 38°, ∠MPR = 32° then find the angle ∠QNR.
1.35°　　2.25°
3.30°　　4.20°

Q:23 In the following figure, AB and AC are two tangents drawn to the circle with center O. If ∠BDC : ∠BAC = 13 : 10, then what is the value of the angle ∠BAC?



1.40°　　2.50°
3.60°　　4.75°

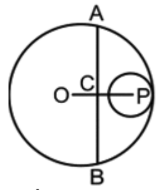
Q:24 In the given figure, QS and RS are two tangents to the circle with center O. If ∠QSR = 80° then what is the value of ∠QPR?



1.40°　　2.45°
3.50°　　4.55°

Q:25 A circle is drawn inside and outside a square having radius r and R respectively having a side of 38 cm. Find the value of $\frac{\sqrt{2}R + r}{R + r}$.
1.$(\sqrt{2} - 1)$　　2.$3(\sqrt{3} - \sqrt{2})$
3.$3(\sqrt{2} - 1)$　　4.$2(\sqrt{3} - 1)$

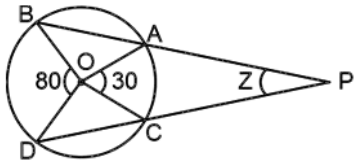
Q:26 In the figure given below, two circles with centres O and P and radii 5 cm and 1 cm respectively meet internally. A chord AB of the bigger circle perpendicularly bisects OP, where OP is twice of OC. What is the length of AB? (in cm)



1.$\sqrt{21}$　　2.$\sqrt{29}$
3.$2\sqrt{21}$　　4.$2\sqrt{29}$

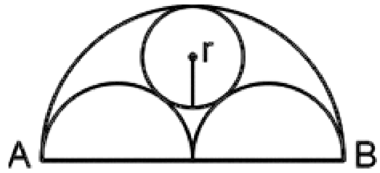
Q:27 PQRS is a cyclic quadrilateral. The tangents to the circle at the point P and R on it, intersect at X. If ∠PXR is equal to 22º, then find ∠PQR where ∠PSR is less than 90º:
1.101º　　2.121º
3.69º　　4.159º

Q:28 From the given figure find the value of ∠z.



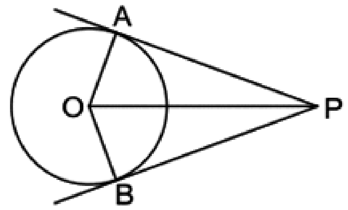
1.55°　　2.25°
3.50°　　4.110°

Q:29 From the given figure find the radius(r) of the smaller circle if AB = 12 cm.



1.2 cm　　2.3 cm
3.4 cm　　4.1 cm

Q:30 In the figure given below, O is the center of the circle of radius 9 cm. From a point P (OP = 15 cm), two tangents PA and PB are drawn to the circle. What is the value of $(PA + PB)^2$?



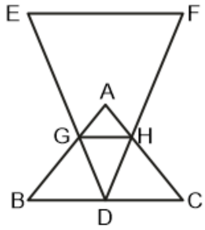
1.225　　2.441
3.576　　4.625

Hard Level

These 30 questions are based on core concepts of the chapter with lengthy but avoidable calculations if you read the question thoroughly. It will actually test your concept understanding skills. Try to solve as much as you can in the given time limit.

Time limit: 30 to 35 seconds per question.

Q:1 In the given figure, GH || BC || EF and area of the triangles ABC and DEF are 36 cm^2 and 81 cm^2 respectively. GH bisects AB whereas divides DE (DG : GE) in the ratio 1 : 2. What is the area of the quadrilateral AHDG? (in cm^2)

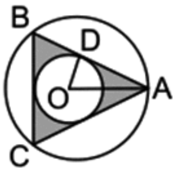


1.18 2.21
3.27 4.36

Q:2 AB is a chord in a circle having a radius of 29 cm. PQ is another chord in the same circle. If the length of AB is 42 cm and the length of PQ is 40 cm, then find the maximum possible perpendicular distance between AB and PQ.

1.43 cm 2.41 cm
3.48 cm 4.45 cm

Q:3 In the figure below, the two circles are concentric (O as centre) with a radius of 4 cm and 8 cm. Two tangents to the smaller are drawn from point A on the bigger circle, which meets the bigger circle again at B and C. What is the area of the shaded region? (in cm^2)

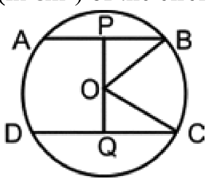


1.48 – 16π 2.48√3 – 16π
3.48√3 – 8π 4.48 – 8π

Q:4 ΔPQR is drawn to circumscribe a circle of radius √5 cm such that the segment QS and SR into which QR is divided by point of contact S are of length 3 cm and 5 cm respectively. Find the length of PR.

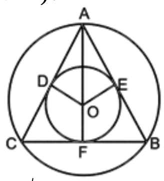
1.5 cm 2.1 cm
3.9 cm 4.3 cm
5.7 cm

Q:5 In the figure given below, AB (Length = 6 cm) and CD (Length = 8 cm) are two parallel chord 7 cm apart on opposite side of the center. PQ is perpendicular to both the chords and passes through center O. What is the area (in cm^2) of the circle? (Take π = 3.14.)



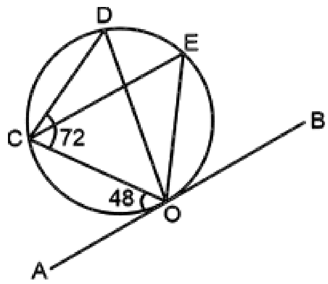
1.78.5 2.176.625
3.200.96 4.314

Q:6 In the figure given below, there are two concentric circles with center O, and radii 2 cm and 4 cm respectively. Three chords AB, BC and AC of the larger circle touches the smaller circle at E, F and D respectively. What is the area of the triangle ABC (in cm^2)?



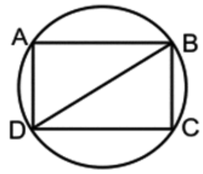
1.√3 2.3√3
3.6√3 4.12√3

Q:7 In the given figure AB is tangent with point O on it, ∠DCO = 72° and ∠AOC = 48° then, find the measure of ∠DOB + ∠CEO.



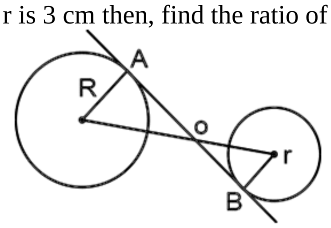
1.45° 2.150°
3.120° 4.75°

Q:8 In the following figure, ABCD is a cyclic quadrilateral such that ∠BAD : ∠BCD = 13 : 5, ∠ADB : ∠CDB = 1 : 3 and ∠ABD : ∠CBD = 1 : 2, then what is the value of ∠BDC?



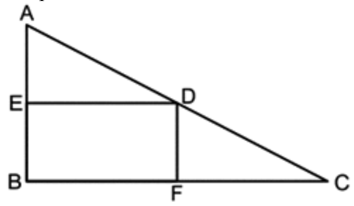
1.30° 2.60°
3.90° 4.120°

Q:9 In the given figure, O is the midpoint of AB, and the distance between the centers is $10(\sqrt{3})$. The radius of the bigger circle R is 7 cm and the radius of the smaller circle r is 3 cm then, find the ratio of r to AO.



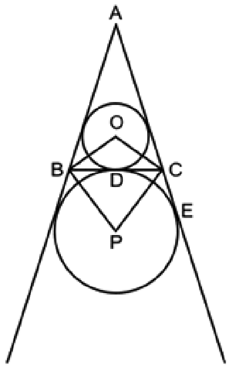
1. 3 : $5\sqrt{2}$ 2.$5\sqrt{2}$: 3
3.1 : $5\sqrt{2}$ 4. $5\sqrt{2}$: 1

Q:10 In the following figure, DE and DF are the perpendicular drawn from some point D on the hypotenuse of a right angled triangle ABC to the side AB and BC respectively. The ratio of area of triangles AED and DFC is 9: 16 and the area of the rectangle DEBF is 48 sq cm, then what is the area of triangle ABC? (in cm^2)



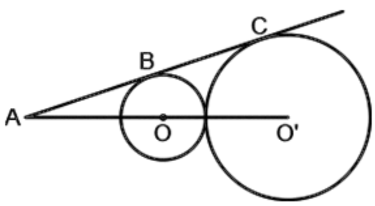
1.98 2.120
3.160 4.196

Q:11 In the following figure, the circle with center O is the incircle of the triangle whereas the other one with center P is the excircle of the triangle. What is the value of ∠OCP?



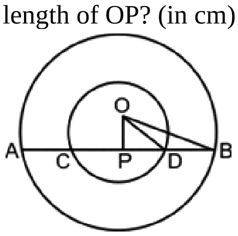
1.60° 2.90°
3.120°
4.Depends on the angles of the ABC

Q:12 In the given figure the centres of two circles are O and O'. ABC is the common tangent to the two circles. If the ratio of the radius of both the circles is 4 : 7 and AO' = 28, then find BC?



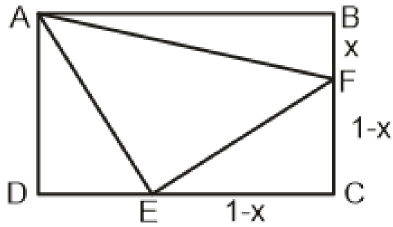
1.(48√7)11 2.$\frac{36}{11}$
3.$\frac{48}{11}$ 4.$\frac{32\sqrt{9}}{11}$

Q:13 In the following figure, O is the common center of two concentric circles of radii 5 cm and 8 cm respectively. A perpendicular is drawn from the center to the common chord. CD: AB = √3 : 4, then what is the length of OP? (in cm)



1.2 2.3
3.4 4.5

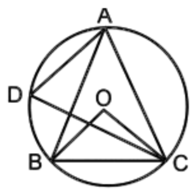
Q:14 In the figure given below, ABCD is a rectangle with two points E and F on the sides CD (= a cm) and BC (= b cm) respectively. F and E divides BC and DC in the ratio x : (1 – x). If AEF is a right angled triangle, then what is the value of x?



1.$\frac{1}{2}$ 2.$\frac{b}{a}$
3.$\frac{a}{b}$ 4.$\frac{b^2}{a^2}$

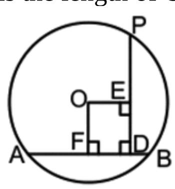
Q:15 A circle touches the side BC of triangle ABC at D & AB and AC is produced to E and F respectively to touch the circle. If AB = 10 cm, AC = 8.6 cm & BC = 6.4 cm then BE = ?
1.3.5 cm 2.2.2 cm
3.3.2 cm 4.2.5 cm

Q:16 In the figure given below, a circle with center O has two chords AB and AC of equal length. If ∠OBA = 15°, then what is the measure of ∠ADC?



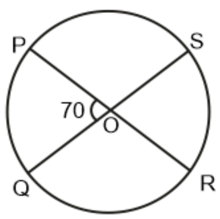
1.60° 2.75°
3.90° 4.45°

Q:17 In the following figure, the radius of the circle with center O is 5 cm. From a point P inside the circle, a perpendicular is drawn to a chord AB (3 cm from the center) which divides the chord (AD: DB) in the ratio 3 : 1. If the length of the perpendicular is 5 cm as well, what is the length of OP? (in cm)



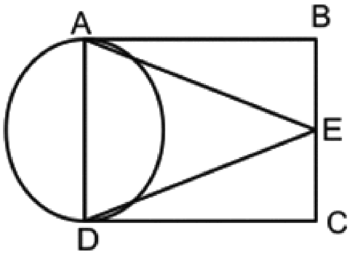
1.1 2.√3
3.√5 4.2√2

Q:18 In the given figure, O is the center of the circle. If ∠POQ = 70°, then what is the value of ∠QSR?



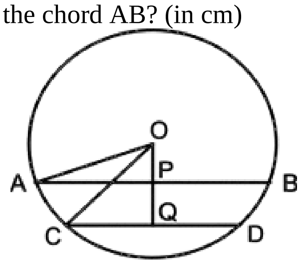
1.55°　　2.60°
3.65°　　4.70°

Q:19 ABCD is a rectangle such that AE and DE are two tangents from same point E. What is the value of $\frac{8\ BE - 2\ CE}{2\ BE + CE}$?



1.1　　2.2
3.3　　4.4

Q:20 In the figure given below, AB and CD are two parallel chords which are 1 cm apart. OQ is perpendicular to CD. If the radius of the circle is 5 cm and the lengths of AB and CD are in the ratio 4: 3, what is the length of the chord AB? (in cm)

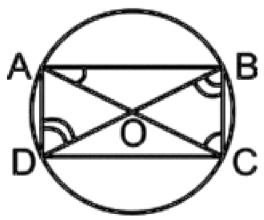


1.4　　2.6
3.8　　4.12

Q:21 Two circles of equal radii of 6 cm intersect each other such that each circle passes through the centre of the other circle. Find the length of the chord that is common to both the circles.

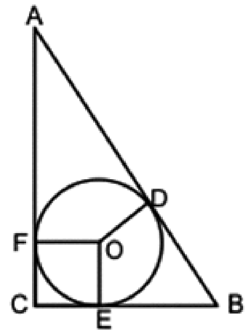
1.6√3 cm　　2.2√5 cm
3.6√2 cm　　4.4√3 cm

Q:22 In the figure given below, ABCD is a cyclic quadrilateral such that ∠BAC = ∠BCA and ∠DBC = ∠BDA. If ∠ABC = 80°, then what is the measure of ∠ABD?



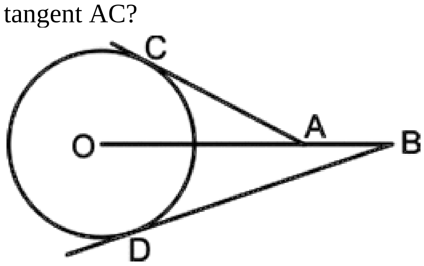
1.30°　　2.40°
3.50°　　4.60°

Q:23 In the following figure, ABC is a right-angled triangle with ∠ACB = 90°. The radius of the circle inscribed in the triangle is 3 cm and AC = 7 cm. What is the value of BC?



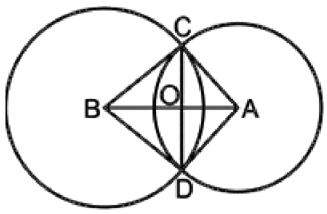
1.20 cm　　2.21 cm
3.24 cm　　4.25 cm

Q:24 In the figure given below tangents are drawn from two points A and B to a circle of radius 5 cm and center O which meets the circle at C and D respectively. Given that the $\frac{AC}{BD} = \frac{1}{2}$ and $\frac{OA}{OB} = \frac{\sqrt{61}}{13}$, then what is the length of the tangent AC?



1.4　　2.5
3.6　　4.8

Q:25 Two circles of radii 4 cm and 4√3 cm and centers A and B touches each other at points C and D. Distance between A and B is 8 cm. What is the area of the segment formed by the chord CD in the shorter segment bigger circle? (in cm^2)



1.12π – 8√3　　2.8π – 12√3
3.4π – 8√3　　4.12π – 4√3

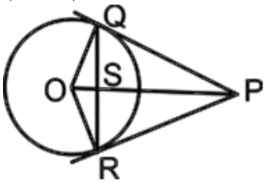
Q:26 A tangent is drawn from a point P outside of the circle at A. The distance between the centre and point P is 26 cm and the area of the circle is 324π cm^2. Find the length of the tangent.

1.18 cm　　2.14 cm
3.16 cm　　4.12 cm

Q:27 A circle is circumscribed around an equilateral triangle of side 3 cm. What is the area of the circle (in cm^2)?

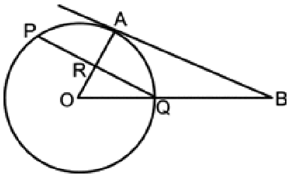
1.3π 2.8π
3.9π 4.16π

Q:28 In the figure below, a circle of radius 3 cm and center at O is given with a pair of tangents drawn from point P which touches the circle at Q and R respectively. The length of the tangents is found to be 3 cm. If the line OP cuts the chord QR at S, then what is the length of PS (in cm)?



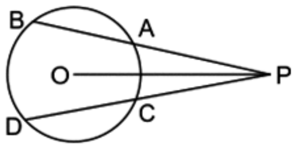
1.$\frac{1}{\sqrt{3}}$ 2.$\frac{3}{\sqrt{2}}$
3.$\frac{3}{2}$ 4.$\frac{\sqrt{3}}{2}$

Q:29 In the figure given below, AB is a tangent to the circle with center O. PQ is a chord parallel to AB and Q is the middle point of OB. If the radius of the circle is $3\sqrt{3}$ cm, what is the length of the chord PQ?



1.4.5 2.6
3.7.5 4.9

Q:30 In the figure given below, O is the center of the circle of radius 5 cm. From an external point P, (OP = 13 cm) two secants are drawn to the circle. What is the value of $\frac{PA.PB + PC.PD}{72}$?



1.1 2.2
3.3 4.4

Calculative Only

These questions are totally calculation based. You can get the answer without doing much brainstorming by just stick to the calculation part.

Tip: Wherever possible, try to implement each of the concepts we discussed in this chapter. Try to eliminate options to avoid lengthy calculations.

Q:1 Two chords of 65.25 cm and 84.36 cm intersect each other and if the length of their common chord is 96.25 then find the distance between their centers.

1.142.13 cm 2.128.13 cm
3.124.13 cm 4.132.13 cm

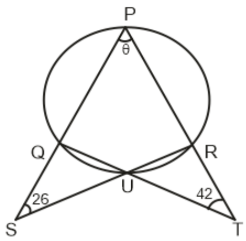
Q:2 PQ and RS are 2 chords of a circle with center O and PS is the diameter of that chord. When PQ and RS are extended further they meet at point K. If ∠PKR = 36 and ∠SPQ = 28 then find the value of ∠RQS.

1.36° 2.74°
3.26° 4.64°

Q:3 O is the center of the circle. PQ is the diameter of the circle, PQ is extended forward outside the circle where point A is situated, a point B is situated on the circle AB is tangent to the circle if the length of arc PB is 277.55 cm and the length of arch BQ is 237.9 then find the value of ∠PAB.

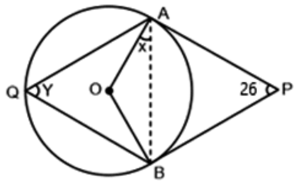
1.18.92° 2.6.92°
3.12.8° 4.173.08°

Q:4 In the given figure ∠PSR = 26° and ∠PTR = 42°, then find the value of ∠QPR.



1.76° 2.56°
3.68° 4.112°

Q:5 In the given figure O is the center of the circle, and OA and OB are the radius of the circle. If ∠APB= 26 then what will be the difference between x and y?



1.34° 2.64°
3.74° 4.84°

Conceptual Only

These questions are only concept based. They have calculation part but that can be easily avoided if you brainstrom enough on th questions and their options.

Tip: Don't try to SOLVE every question to get the answer. Just find the right concept and apply it there.

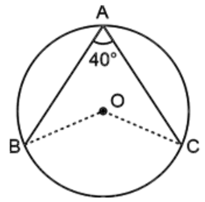
Q:1 If the angle made by an arc at the center of the circle is 270º. Find the angle made by the same arc at any point on the circumference.

1.90º 2.135º
3.180º 4.120º

Q:2 In a triangle, each side equal to 12cm, two circles are drawn inside and outside the triangle having radii r and R respectively. Find the ratio of r : R.
1.3 : 4 2.1 : 4
3.1 : 2 4.2 : 1

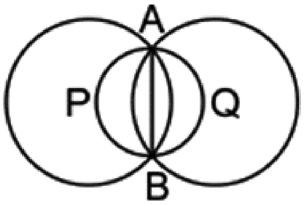
Q:3 PQRS is a cyclic quadrilateral and Its diagonal PR and SQ intersects at K at the right angle. If $KP^2 + KP^2 + KR^2 + KS^2 = 196$ cm then, find the area of the circle.
1.343 cm^2 2.154 cm^2
3.616 cm^2 4.84 cm^2

Q:4 In the given figure, find ∠OBC.



1.50° 2.80°
3.90° 4.45°

Q:5 Two circles of equal radius and different centers P and Q intersect at two points A and B. If the length of PQ is equal to the radius of the circles and a new circle with AB as the diameter is drawn, what is the ratio of areas of the circle with AB as the diameter and the area of the circle with P as the center?



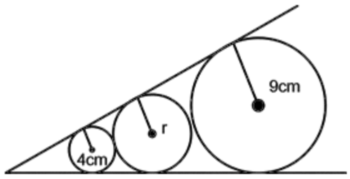
1.$\frac{3}{4}$ 2.$\frac{1}{\sqrt{2}}$
3.$\frac{\sqrt{3}}{2}$ 4.1

Tricky Questions

These are tricky questions. These questions will look very hard at one instance but when you'll find the actual concept applied there, you will solve them immediately.

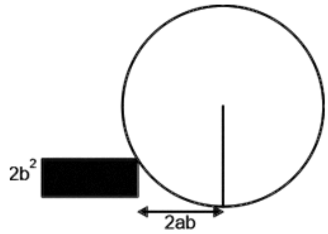
Tip: Eliminating the options is the best approach to do these questions.

Q:1 In the given figure, if the radius of the largest circle is 9 cm and radius of the smallest circle is 4 cm. Find the radius of the circle with radius 'r'.



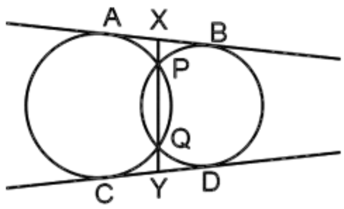
1.6 cm 2.36 cm
3.13 cm 4.5 cm

Q:2 A brick of height $(2b^2)$ meter is placed on the ground such that it is touching a circle 2ab meter away from the point where the circle touches the ground. Find the radius of the circle.



1.$(a + b)^3$ 2.$(a + b)^2$
3.$a^2 - b^2$ 4.$a^2 + b^2$

Q:3 Two circles intersect at P and Q. PQ when extended to both sides meet two direct common tangents AB and CD at X and Y respectively. If AB = 14 cm and XY = 25 cm then, find the length of PQ.



1.$\sqrt{29}$ 2.$\sqrt{329}$
3.$\sqrt{429}$ 4.$\sqrt{119}$

Q:4 AB and AC are two tangents to a circle. AB = $3x^2 + 2x + 10$ and AC = 11. Find the value of x.
1.-3 2.-1
3.1 4.3

Q:5 AB and CD are two parallel chords of a circle such that AB = 6 cm and CD = 2AB. Both chords are on the same side of the center of the circle. If the distance between them is equal to one-fourth of the length of the CD, then the diameter of the circle is:
1.5$\sqrt{3}$ cm 2.4$\sqrt{3}$ cm
3.6$\sqrt{5}$ cm 4.4$\sqrt{5}$ cm

Some Innovative Questions

These kind of questions have not asked yet in the exam, but as the pattern of every exam is evolving, we should be prepared for all type of questions. These questions will also prepare you to think beyond the traditional questions and their solving pattern.

Tip: Try to eliminate options to avoid all lengthy calculation.

Q:1 A circle with center O and a diameter AB = 50 cm is drawn. A chord PQ is drawn and the distance between the chord and the center of the circle is $\frac{7}{25}$th of the radius of the circle. Another chord MN is drawn which intersects chord PQ at Z such that ZQ = $\frac{1}{5}$th of PZ. Find the minimum value of MN.

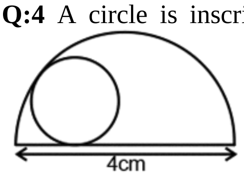
1.$16\sqrt{5}$ cm	2.16 cm
3.$8\sqrt{5}$ cm	4.$12\sqrt{5}$ cm

Q:2 In a circle of diameter DF with center O. Find the value of ∠DBC.
Statement I : ∠BAD = 65° and DF is parallel to BC.
Statement II : ∠ADB = 40° in triangle AED.
1.Either statement I or II is sufficient
2.Both statement I and II are sufficient
3.Only statement I is sufficient
4.Only statement II is sufficient

Q:3 Two circles with centres O and K touch each other externally at a point R. AB is a common tangent to both the circles passing through R. PQ is another tangent to the circles touching them at T and U respectively and also cutting AB at S. TU measures 12 cm and the point S is at distance of 10 cms and 8 cms from the centres of the circles. What is the area of the triangle SOK if ∠OSK = 90°?

1.$3(4 + \sqrt{7})$ cm^2	2.$6(4 + \sqrt{7})$ cm^2
3.$6(2 + 2\sqrt{7})$ cm^2	4.$3(2 + 2\sqrt{7})$ cm^2

Q:4 A circle is inscribed in a semi-circle as shown:-



The radius of the circle possibly is:

1.$\sqrt{2} + 1$	2.$\sqrt{2} - \frac{1}{2}$
3.$(4 - \sqrt{2})$	4.$2\sqrt{2} + 1$

Q:5 PQRS is a rectangle inscribed in a circle of radius 61 cm. Find the possible length and breadth of the rectangle such that length is less than its breadth.

1.26 and 140 cm	2.22 and 120 cm
3.11 and 60 cm	4.24 and 122 cm

Questions to be Skipped

If you don't have enough time in exam to solve all the questions, try to skip these questions in the first attempt. We should know what question can be skipped in the exam so that we can maximise our efficiency. Here, are some questions that can be skipped in the first attempt.

Tip: Try to understand the core of the problem, if it is an easy question with too many lengthy calculations, skip it in first attempt and try it again after solving other questions.

Q:1 A Circle with center O and radius r has a chord XY which is extended to a point P outside the circle and PZ is a tangent to the Circle. If the length of the chord XY is 18.8 cm, the length of PY is 23.4 cm and the length from P from the center O is 44.6 cm. find the area of the circle.

1.2452.27 cm^2	2.3963.27 cm^2
3.4258.27 cm^2	4.3148.27 cm^2

Q:2 Triangle PQR is a right-angled triangle at Q. A semicircle is drawn with diameter QR such that it cuts PR at M. If PQ = 46.25 cm, and PM = 14.75 cm, then find the perimeter of the semicircle.

1.285.65 cm	2.353.17 cm
3.485.25 cm	4.312.25 cm

Q:3 Two circles touch each other. Two common tangents to a circle meet at point P and no tangent passes through the point where the circles touch each other. These tangents touch the larger circle at points B and C. If the radius of the larger circle is 476 cm and CP = 765 cm, then find the area of the smaller circle.

1.25699.20 cm^2	2.256989.20 cm^2
3.67769.20 cm^2	4.56456.20 cm^2

Q:4 The diameter of a circle is PQ and RS is the chord of the circle, PQ and RS intersect each other at M, and O is the center of the circle. If PM = 22.17 cm, RS = 206.45 cm and radius = 136.20 then find the length of MS (Nearest Approximate).

1. 205.5, or 316.75	2. 176.95, or 152.75
3. 176.95, or 31.75	4. 145.95, or 31.75

Q:5 A circle with center O and two chords PQ and RS. Both the chords are parallel to each other on opposite sides of the center and the distance between the chords is 86 cm. The length of chord PQ is 122 cm and the length of chord RS is 118 cm. Then find the radius of the circle.

1.44.45 cm	2.86.63 cm
3.73.84 cm	4.61.84 cm

Answers and Solutions

Easy Level

Q:1 (3) Let the required distance be 'y' cm.
Length of direct common tangent = √(distance between centres)^2 - $(r_1 - r_2)^2$
So, 4 = √y^2 - $(10 - 7)^2$
Or, 16 = $y^2 - 9$
Or, $y^2 = 25$
So, y = ±5 (Since length cannot be negative, therefore, we will take the positive root only)
Hence, y = 5

Q:2 (2)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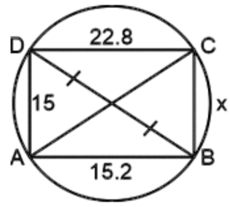


In the cyclic quadrilateral ABCD, AC bisects BD

So, $\frac{AD}{BC} = \frac{AB}{DC} \Rightarrow \frac{15}{x} = \frac{15.2}{22.8} \Rightarrow x = \frac{45}{2} = 22.5$ cm

Q:3 (3) $R_1 = 25$ cm and $R_2 = 16$ cm

(Length of common tangent)^2 = (Distance between the centres of the circle)^2 - $(R_1 - R_2)^2$

In the given question, the two circles touch each other externally, then the distance between their centres is equal to the sum of their radii.

Now, according to the formula

(Common Tangent)$^2 = (25 + 16)^2 - (25 - 16)^2 = (41)^2 - (9)^2 = (41 + 9)(41 - 9) = 50 \times 32 = 1600$

$\Rightarrow$ Common tangent = $\sqrt{1600} = 40$

Hence the value of common tangent is 40 cm.

Q:4 (1) Given: $3x - 4y = 12$

Writing the line in intercept form,

$\Rightarrow \left(\frac{3x}{12}\right) - \left(\frac{4y}{12}\right) = 1 \Rightarrow \frac{x}{4} - \frac{y}{3} = 1$

Hence the length of intercept on x and y axis is 4 and 3 respectively.

Area = $\frac{1}{2} \times$ Base $\times$ Height = $\frac{1}{2} \times 4 \times 3 = 6$

Q:5 (2) Tangent $\perp$ Radius

$\therefore$ [Distance of external point from center]^2 = [Radius]^2 + [Length of tangent]^2 [$\because$ Pythagoras theorem]

$\Rightarrow 53^2 = 28^2$ + [Length of tangent]^2

$\Rightarrow 2025$ = [Length of tangent]^2

Length of tangent = 45 cm

Q:6 (2) AOBP is a quadrilateral

So, $\angle PAO + \angle PBO = 180°$

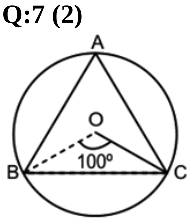
$\Rightarrow \angle AOB + \angle BPA = 180°$

$\Rightarrow \angle AOB + 33° = 180°$

$\Rightarrow \angle AOB = 147°$

Value of $\sqrt{\frac{1}{3} \times \angle AOB} = \sqrt{\frac{1}{3} \times 147} = \sqrt{49} = 7$

Q:7 (2)

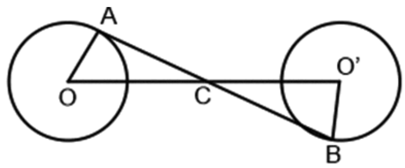


The angle made by an arc at the center is double the angle made by the same arc at any point on the circumference.

So, $\angle BOC = 2 \times \angle BAC$

$\angle BAC = \frac{100}{2} = 50°$

Q:8 (2) $AB^2 = (OO')^2 - (OA + O'B)^2$



From the formula, we get

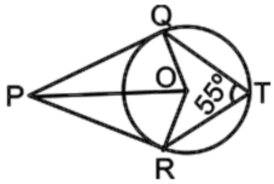
$AB^2 = (25)^2 - (8 + 7)^2$

$\Rightarrow AB^2 = 625 - 225 = 400$

$\Rightarrow AB = \sqrt{400} = 20$

Hence, the value of a common transverse tangent is 20 cm

Q:9 (2) Join points Q, R and P to the centre of circle O.



QOR = 2 × QTR (angle at the centre is twice the angle at other part of circle)

QOR = 2 × 55 = 110°

In triangle POQ,

PQO = 90° (Angle made by tangent)

Also, QOP = $\frac{1}{2}$ × QOR

QOP = $\frac{1}{2} \times 110 = 55°$

QOP + PQO + QPO = 180°

90° + 55° + QPO = 180°

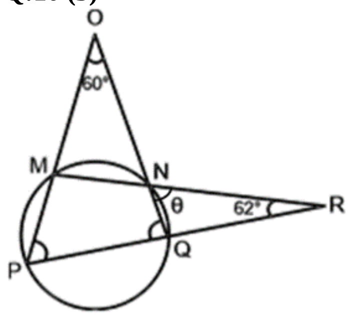
QPO = 35°

Similarly, OPR = 35°

QPR = OPR + QPO

QPR = 35 + 35 = 70°

Q:10 (3)



In the above diagram,

We have given that $\angle MON = 60°$ and $\angle QRN = 62°$

$\angle QNR = \theta$

The value of exterior angle is equal to the opposite interior angle.

$\angle RNQ = \angle MNO = \theta$

$\angle PQN = 62° + \theta$ (Exterior angle of triangle QNR)

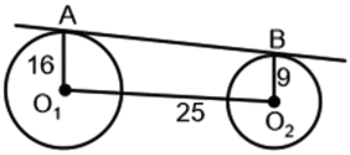
$\angle PMN = 60° + \theta$ (Exterior angle of triangle MNO)

In cyclic quadrilateral sum of the opposite angle is 180°.

So, $62° + \theta + 60° + \theta = 180°$

$\angle QNR = \theta = \frac{180° - 122°}{2} = 29°$

Q:11 (1)



We can find the given value by the following formulae directly

Length of direct common tangent = $\sqrt{D^2 - (R - r)^2}$

D is the distance between centers, R and r are the radius of two circles

$\Rightarrow$ AB $= \sqrt{D^2 - (R - r)^2} = \sqrt{25^2 - (16 - 9)^2}$

$= \sqrt{625 - (49)} = \sqrt{576} = 24$ cm

Q:12 (2) $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

where r_1, r_2 and r_3 are radii of ex-circles and r = inradius

$\Rightarrow \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = \frac{1}{r} \Rightarrow 1 = \frac{1}{r} \Rightarrow r = 1$ cm

Q:13 (1) a = 70°

The tangent to the circle makes a right angle with the radius of the circle.

Sum of all angles of a triangle is 180°.

$\angle P = \angle Q = 90°$ {PR and QR are tangents to the circle}

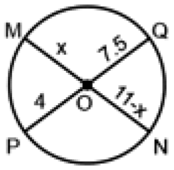
Now, in triangle POR,

$\angle P + \angle O + \angle R = 180° \Rightarrow 90° + 70° + \angle R = 180°$

$\Rightarrow \angle R = 180° - 160° = 20°$

Hence the value of b is 20°.

Q:14 (1)



We know that if the two chords in a circle intersect each other on point O then,

$\Rightarrow OP \times OQ = OM \times ON \Rightarrow 4 \times 7.5 = x(11 - x)$

$\Rightarrow 30 = 11x - x^2 \Rightarrow x^2 - 11x + 30 = 0 \Rightarrow x = 6, 5$

If x = 6,

Then MO = 6, NO = (11 - 6) = 5

So, the difference = (6 - 5) = 1

Q:15 (2) Let the $\angle$ A and $\angle$ B be x and 2x

Let $\angle$ C and $\angle$ D be 5y and 4y

$\Rightarrow \angle A + \angle C = 180°$ [$\because$ In cyclic quadrilateral opposite angles are supplementary]

$\Rightarrow x + 5y = 180$ ---- (1)

$\Rightarrow \angle B + \angle D = 180° \Rightarrow 2x + 4y = 180$ ---- (2)

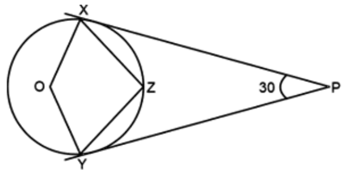
2 × Equation (1) – Equation (2)

$\Rightarrow 6y = 180 \Rightarrow y = 30°$

Substituting in equation (1)

$\Rightarrow x = 30° \Rightarrow \angle A = 30°$

Q:16 (3)



$\Rightarrow \angle XPY = 30°$

$\Rightarrow \angle OXP = \angle OYZ = 90°$ (Tangent on a circle)

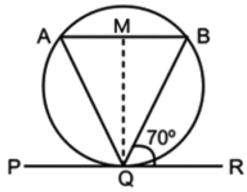
$\Rightarrow \angle YOX + \angle OXP + \angle OYZ + \angle XPY = 360°$

$\Rightarrow \angle YOX = 360° - (90° + 90° + 30) = 150°$

$\Rightarrow \angle XZY = 90° + \frac{\angle XPY}{2} = 90 + 15 = 105°$

Required average = $\frac{150° + 105°}{2} = 127.5°$

Q:17 (4)



In the given figure, the chord AB gets bisected by the perpendicular

The middle of AB be M(say)

$\Rightarrow$ AM = MB

$\Rightarrow \angle QMB = \angle QMA$

$\Rightarrow$ MQ = MQ

$\Rightarrow \Delta QMA \cong \Delta QMB$

$\Rightarrow \angle AQM = \angle BQM = 20°$ …. (90° - 70°)

$\Rightarrow \angle AQB = 2 \times 20° = 40°$

Q:18 (2) The sides of a triangle are 12,35 and 37 and its forms a triplet. Hence, it is right angle triangle.

$\Rightarrow$ H = 37, B = 35 and P = 12

The radius of the circle outside the triangle

$\Rightarrow R = \frac{H}{2} = \frac{37}{2} = 18.5$

Q:19 (2) The angle made by diameter in the circle is a right angle.

Since AB is a diameter. So, angle ACB is 90º.

And The sum of the opposite angle of cyclic a quadrilateral is 180º.

Since ABCD is a cyclic quadrilateral.

So, 125º + $\angle$ABC = 180º

$\angle$ABC = 55º

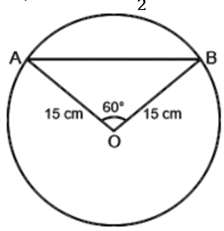
In right-angled triangle ACB, The sum of three angles is 180º

So, 90º + 55º + $\angle$CAB = 180º

$\Rightarrow \angle$CAB = 180º - 145º = 35º

Q:20 (1) Given, diameter = 30 cm

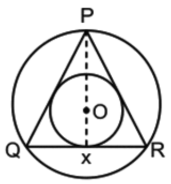
So, radius = $\frac{30}{2}$ = 15 cm



In the given figure, let ‘O’ represent the centre of the circle.
We have, OB = OA = 15 cm (radii of the circle)
Since, $\angle AOB = 60^o$
So, $\angle OAB = \angle ABO = (180 – 60) \div 2 = 60^o$
So, $\angle OAB$ is an equilateral triangle.
Therefore, AB = AO = OB = 15 cm

Q:21 (3) Let the radius be r. So, $AB = r\sqrt{3}$
In triangle AOB
$AB^2 = AO^2 + BO^2 - 2\ AO.BO.\cos\angle AOB$ [$\because$ Cosine rule]
$\Rightarrow 3r^2 = r^2 + r^2 - 2r^2 \cos\angle AOB$
$\Rightarrow \cos\angle AOB = -\frac{1}{2}$
$\therefore \angle AOB = 120°$
$\angle ACB = \frac{\angle AOB}{2}$ [$\because$ Angle subtended by chord on any point of circle is half the central angle]
$\Rightarrow \angle ACB = 60°$

Q:22 (2) Equilateral triangle PQR having two circles, one is inner and another is outer.
Inner circle radius = 11.4 cm (given)
We know that in an equilateral triangle PQR, Centroid, and Circumcentre coincide.
'O' being the centroid divided the median POX in ratio of 2 : 1



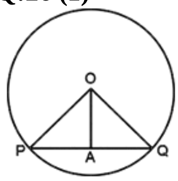
OX = inner circle radius = 11.4 cm
PO = outer circle radius = 2 × 11.4 = 22.8 cm
Hence, the radius of the bigger circle = 22.8 cm.

Q:23 (4) According to the **tangent-secant theorem**: "When a **tangent** and a **secant** are drawn from one single external point to a circle, the square of the length of the **tangent** segment must be equal to the product of lengths of whole **secant** segment and the exterior portion of **secant** segment."
Let, PA = 4x and PB = 9x
According to question,
$PA \times PB = PT^2 \Rightarrow 4x \times 9x = 144 \Rightarrow x = 2$
So, PA = 8 and PB = 18
AB = PB - PA = 18 - 8 = 10
And $OA = \frac{AB}{2} = \frac{10}{2} = 5$ cm

Q:24 (3) Given, Centre to centre distance = $\frac{48}{\sqrt{3}} = 16\sqrt{3}$
Sum of radii = $5\sqrt{3} + 11\sqrt{3} = 16\sqrt{3}$
Since sum of radii is equal to the centre to centre distance, circles touch each other at one point. Hence two direct common tangents can be drawn, and one transverse common tangent can be drawn.
Total 3 tangents can be drawn.

Q:25 (1) Length of transverse common tangent = $\sqrt{D^2 - (R + r)^2}$ ---- (1)
Where D = distance between the centre of two circles
R and r = Radius of the two circles
Calculations:
Using equation (1), we get
$\Rightarrow$ Length = $\sqrt{15^2 - (9+5)^2} = [\sqrt{225 - 196}] = \sqrt{29}$ cm

Q:26 (1)



In the right triangle AOP, OP = 20 cm
Also, $\frac{PA}{PQ} = \frac{1}{2}$
Or, $\frac{PA}{32} = \frac{1}{2}$
So, PA = 16 cm
So, $OA^2 = OP^2 – PA^2$
Or, $OA = \sqrt{(400 – 256)} = 12$
So, required ratio = 12 : 20 = 3 : 5

Q:27 (1) As we know that,
If P is an exterior point to the circle, then,
$PA \times PB = PC \times PD \Rightarrow 8 \times PB = 9 \times 12$
$\Rightarrow$ PB = 13.5 cm
Therefore, AB = 13.5 - 8 = 5.5 cm

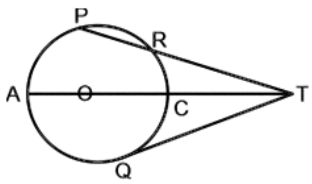
Q:28(4) We know that $BC^2 = BA \times BP$
So, $7^2 = BA \times 4$
Or, BA = 12.25 cm
So, AP = 12.25 – 4 = 8.25 cm

Q:29 (3) Length of direct common tangent = $\sqrt{PQ^2 - (R - r)^2} = \sqrt{12.5^2 - (6.5 - 3)^2} = \sqrt{\{156.25 – 12.25\}} = \sqrt{144} = 12$ cm

Q:30 (1) Area of the circle = πr^2 ---- (1)
Let the radius of the circle be r.
The largest chord of the circle is the diameter of the circle
Hence, $r = \frac{\text{Diameter}}{2} = \frac{42}{2} = 21$ cm
Area of the circle = $\frac{22}{7} \times 21^2 = 22 \times 63 = 1386$

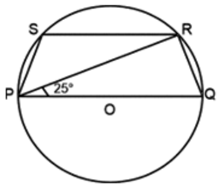
Moderate Level

Q:1 (2) OT = 11 cm
Radius, OC = 5 cm
TR = 6 cm
If any two chords, say AB and CD, of a circle, intersect each other at a point E outside a circle, then $AE \times EB = EC \times ED$



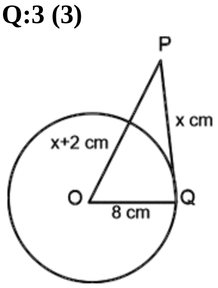
In the above figure, TA = OT + OA = 11 + 5 = 16 cm
TC = TA - AC = 16 - 10 = 6 cm
Now, TR × TP = TC × TA
⇒ 6 × TP = 6 × 16 ⇒ TP = 16 cm
So, PR = TP - TR
⇒ PR = 16 - 6 = 10 cm
Hence, the length of PR is 10 cm

Q:2 (2)



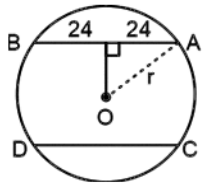
In ΔRPQ,
⇒ ∠PRQ = 90° (Angle subtended by diameter on the circle)
⇒ ∠RPQ + ∠PRQ + ∠RQP = 180°
⇒ 25° + 90° + ∠RQP = 180°
⇒ ∠RQP = 65°
Also,
⇒ ∠RQP + ∠PSR = 180° (opposite angles of a cyclic quadrilateral)
⇒ ∠PSR = 180° – 65° = 115°
⇒ ∠SRP = 25° (alternate angles)
Therefore, ∠RPS = 180° – (115° + 25°) = 40°

Q:3 (3)



In the given figure, ∠OQP = 90° (since, the radius is perpendicular to the tangent at the point of contact)
So, in ∠OPQ, $PO^2 = OQ^2 + PQ^2$
Or, $(x + 2)^2 = 8^2 + x^2$
Or, $x^2 + 4x + 4 = 64 + x^2$
So, 4x + 4 = 64
So, x = (64 – 4) ÷ 4 = 15

Q:4 (1) Area of the circle = 625π cm^2
length of the chord = 48 cm

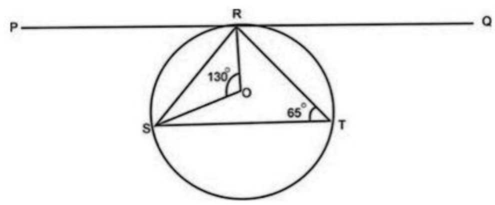


Let the radius of the circle be r.
Area of the circle = πr^2
⇒ $625\pi = \pi r^2 \Rightarrow r^2 = 625 \Rightarrow r = 25$ cm
Let, The length between the center and chord be x
Now, Using the figure above;
⇒ $x^2 = 25^2 - 24^2 \Rightarrow x^2 = 49 \Rightarrow x = 7$ cm
Hence, the distance between the chords = 2x = 14 cm

Q:5 (3) The angle between radius and tangent is 90°
∠AOB = 220° (bigger arc of the circle)
so Smaller arc of the circle = 360° - bigger arc = 360° - 220° = 140°
In triangle AOB
∠OAB = ∠OBA = $\frac{180° - 140°}{2} = 20°$
(AO = OB = Radius)
∠CAO = 90°
So, ∠CAB = ∠CAO - ∠OAB
∠CAB = (90° - 20°) = 70°

Q:6 (3) ∠ACB = 90° [angle on semicircle]
In triangle ABC
⇒∠ABC + ∠BCA + ∠CAB = 180°
⇒ 35° + 90° + ∠CAB = 180°
⇒∠CAB = 55°
ABCD is a cyclic quadrilateral
⇒∠CAB + ∠CDB = 180°
⇒ 55° + ∠CDB = 180°
⇒∠CDB = 125°
∠DCB = ∠CBA [AB || CD]
In triangle BCD
⇒ ∠BCD + ∠CDB + ∠DBC = 180°
⇒ 35° + 125° + ∠DBC = 180°
⇒ ∠DBC = 20°
Value of ∠CDB - ∠DBC = 125° - 20° = 105°

Q:7 (4)

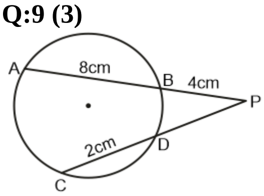


Let T be point on major arc RS.
∠RTS = $\frac{∠ROS}{2}$(angle made by chord at centre is double the angle made by it at some other part of circle)
∠RTS = $\frac{130}{2}$ = 65°
Since angle between chord and tangent is equal to angle made by the chord on circle.
∠PRS = ∠RTS = 65°

Q:8 (2) We know that,
r = $\frac{\text{area}}{s}$ (s = semi-perimeter)
s = $\frac{a+b+c}{2}$
Here, a, b, and c are sides of the triangle
s = $\frac{15+13+12}{2} = \frac{40}{2} = 20$
Area = $\sqrt{s(s-a)(s-b)(s-c)}$

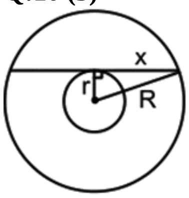
$= \sqrt{20(20-15)(20-13)(20-12)}$
$= \sqrt{20 \times 5 \times 7 \times 8} = 20\sqrt{14}$
Hence, r = $\frac{20\sqrt{14}}{20} = \sqrt{14}$

Q:9 (3)



Let DP = x cm
So, PB × PA = PD × PC
⇒ 4 × 12 = x(x + 2) ⇒ $x^2 + 2x - 48 = 0$
⇒ $x^2 + 8x - 6x - 48 = 0$ ⇒ x(x + 8) – 6(x + 8) = 0
⇒ (x + 8)(x – 6) = 0
So, x = 6 {Since length of DP cannot be negative}

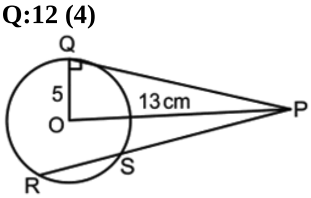
Q:10 (3)



Let radius of larger circle, R = 25 cm
And radius of smaller circle, r = 15 cm
Since chord touches smaller circle at just one point, it is a tangent to small circle.
Let length of chord = 2x
We know that the perpendicular from centre bisects the chord. Therefore,
$R^2 = x^2 + r^2 \Rightarrow 25^2 = x^2 + 15^2 \Rightarrow x^2 = 625 - 225$
⇒ $x^2 = 400$ ⇒ x = 20
Length of chord = 2x = 40

Q:11 (3) The sides of a triangle are given as 9, 40, and 41.
So, it is a right-angle triangle,
H = 41, B = 9, and P = 40
Formulae for the radius of the circle inside the triangle
⇒ $r = \frac{P+B-H}{2} = \frac{40+9-41}{2} = \frac{49-41}{2} = \frac{8}{2} = 4$

Q:12 (4)

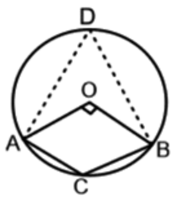


Pythagoras theorem,
$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$
According to the **tangent-secant theorem**: "When a **tangent** and a **secant** are drawn from one single external point to a circle, square of the length of **tangent** segment must be equal to the product of lengths of whole **secant** segment and the exterior portion of **secant** segment."
In a right angled triangle,
$PQ^2 = OP^2 + OQ^2 = 169 - 25 = 144$
According to theorem, PQ^2 = PS × PR = 144 cm

Q:13 (1) OC and OD are the radii of the circle.
OC = OD
COD forms an isosceles triangle.
∠OCD = ∠ODC
In triangle ODC:
∠OCD + ∠ODC + ∠COD = 180° (Angle sum property)
⇒ 2∠OCD + 120 = 180° ⇒ 2∠OCD = 60°
⇒ ∠OCD = 30°
We know that the tangent is perpendicular to the radius of a circle.
⇒ ∠OCM = 90°
⇒ ∠OCD + ∠DCM = 90°
⇒ ∠DCM = 60°

Q:14 (2) EB = 30 cm
AB = 75 cm
If two chords PQ and RS intersect each other at point T inside the circle then,
PT × QT = RT × ST
In the given question,
AE = AB - EB = 75 - 30 = 45 cm ...(i)
Now, AE : CE = 5 : 3
Let the value of AE and CE be 5x and 3x respectively.
From equation (i), we get,
⇒ 5x = 45 ⇒ x = 9
So, CE = 3x = 27 cm
Now, AE × EB = CE × ED
⇒ 45 × 30 = 27 × ED ⇒ ED = 50 cm
Now, CD = CE + ED = 27 + 50 = 77 cm
Hence the value of CD is 77 cm.

Q:15 (4) ∠AOB = 88°
Central angle is double the angle of the major arc.
The sum of the opposite angles of a cyclic quadrilateral is 180°.
In the given figure,



The lines from A and B touches the circumference of the circle at point D, such that ∠ADB = $\frac{1}{2}$ × ∠AOB
∠AOB = 88°
Then, ∠ADB = $\frac{1}{2}$ × 88°
⇒ ∠ADB = 44°
Now, In quadrilateral ABCD, ∠C + ∠D = 180°
⇒ ∠C + 44° = 180° ⇒ ∠C = 180° - 44° = 136°
Hence the value of ∠ACB is 136°.

Q:16 (4) Let ∠ ACB = θ
∠ CAB = 90° [∵ BC is diameter and angle in a semicircle is 90°]

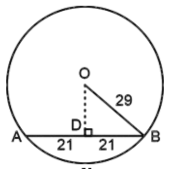
$\angle$ ACB + $\angle$ CAB + $\angle$ ABC = 180° [∵ Angle sum property of triangle ABC]
∴$\angle$ ABC = 90 - θ
$\angle$ ADB = $\angle$ ACB = θ [∵ Angle subtended by a chord on the same side of the segment is equal]
∴ $\angle$ ABC : $\angle$ ADB = $\frac{90-\theta}{\theta}$
⇒ $\frac{90-\theta}{\theta} = \frac{1}{2}$ ⇒ 180 – 2θ = θ ⇒ 180 = 3θ
∴$\angle$ ACB = θ = 60°

Q:17 (3) ∠SPQ = 85°
∠PRS = 20°
1. Angles in the same segment of a circle are equal.
2. Sum of all the angles of a triangle is 180°.
In the given question, ∠PRS = ∠PQS [Angles in the same segment]
⇒ ∠PQS = 20°
Now, in triangle PQS
⇒ ∠P + ∠Q + ∠S = 180°
⇒ 85° + 20° + ∠S = 180°
⇒ ∠S = 180° - 105°
⇒ ∠S = 75°
Hence the value of ∠PSQ is 75°.

Q:18 (3) Theorem: If two chords intersect each other inside a circle the products of their segments are equal.
AB & CD are two chords, intersecting at P
PD = 9 cm, PC = 4 cm, BP = 6 cm
By theorem,
AP × PB = PD × PC
⇒ AP × 6 = 9 × 4
⇒ AP = $\frac{36}{6}$ = 6 cm

Q:19 (4) AD = 7 cm, BD = 5 cm and, BE = 6 cm
If the two chords say AB and CD, of a circle, intersect each other at point E outside the circle, then,
EA × EB = EC × ED
In the given question,
BD × BA = BE × BC
⇒ 5 × (7 + 5) = 6 × (6 + x) ⇒ 5 × 12 = 6 × (6 + x)
⇒ 6 + x = 10 ⇒ x = 4
Hence, the value of x is 4 cm

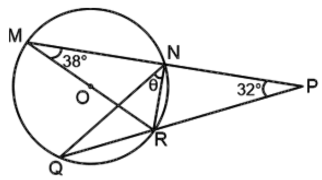
Q:20 (1) Pythagoras theorem, $H^2 = B^2 + P^2$
H = Hypotenuse, B = base and P = Perpendicular
We have the following figure,



OB = radius = 29 cm
DB = $\frac{AB}{2} = \frac{42}{2}$ = 21 cm
OD = distance between the centre and the chord
Using equation (1), we get
⇒ $OB^2 = DB^2 + OD^2$ ⇒ $29^2 = 21^2 + OD^2$
⇒ OD^2 = 841 - 441 = 400 ⇒ OD = 20 cm

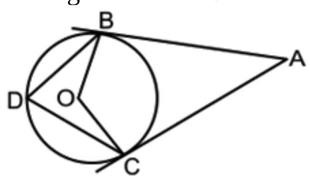
Q:21 (3) If AB and CD are diameters, then O is the centre of the circle.
For chord AD, angle subtended at centre = ∠AOD = 80°
∴∠ACD = ∠$\frac{AOD}{2}$ = 40° [∵ Angle subtended by the chord on any point of the circle is half the central angle]
In triangle CAD
⇒ ∠CAD = 90° [∵ Diameter subtends right angle on the circle]
⇒ ∠ACD + ∠CAD + ∠ADC = 180° [∵ Angle sum property of triangle]
40° + 90° + ∠ADC = 180°
∴∠ADC = 50°

Q:22 (4)



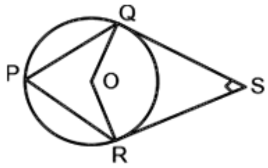
∠RMN = 38°, ∠MPR = 32°
∠MNR = 90° (The angle subtended in a semicircle is a right angle)
∠MRQ = 32° + 38° = 70° (The exterior angle of the given triangle equals to the sum of the two opposite interior angles present in that triangle)
∠MRQ = ∠QNM = 70°
∠QNR = 90° - 70° = 20°

Q:23 (2) Suppose ∠BDC and ∠BAC are 13x and 10x respectively.
Joining OB and OC:



In quadrilateral OBAC:
$\angle$ BOC + $\angle$ BAC + $\angle$ OBA + $\angle$ OCA = 360°
$\angle$ OBA = $\angle$ OCA = 90° [∵ Tangent ⊥ Radius]
So, $\angle$ BOC = 180° - 10x
Now, $\angle$ BDC = $\frac{1}{2}$ × ($\angle$ BOC) [∵ For chord BC, central angle $\angle$ BOC is twice the angle subtended on any point of the circle at the same side of the segment]
⇒ 13x = $\frac{1}{2}$ (180° - 10x) = 90 – 5x
⇒ 18x = 90 ⇒ x = 5
Hence, $\angle$ BAC = 10x = 50°

Q:24 (3) ∠QSR = 80°
The angle between the radius of the circle and the tangent to the circle is 90°.
The sum of all angles of a quadrilateral is 360°.
The angle on the circumference of the circle is half the angle at the centre of the circle when subtended from the same arc.
In the figure given,



$\angle Q = \angle R = 90°$

Now, in quadrilateral QORS, $\angle Q + \angle O + \angle R + \angle S = 360° \Rightarrow 90° + \angle O + 90° + 80° = 360°$

$\Rightarrow \angle O + 260° = 360° = 100°$

Now, $\angle QPR = \frac{1}{2} \times \angle O = \frac{1}{2} \times 100° = 50°$

Hence, the value of $\angle QPR$ is 50°

Q:25 (3) We know that,

$r = \frac{a}{2} = \frac{38}{2} = 19$

$\Rightarrow R = \frac{a}{\sqrt{2}} = \frac{38}{\sqrt{2}}$

Rationalize $\frac{38}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \frac{38}{\sqrt{2}} = \frac{38}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 19\sqrt{2}$

Value of $\frac{(\sqrt{2}R + r)}{(R + r)} = \frac{\sqrt{2}\times19\sqrt{2}+19}{19\sqrt{2}+19} = \frac{\sqrt{2}\times19\sqrt{2}+19}{19(\sqrt{2}+1)} = \frac{38+19}{19(\sqrt{2}+1)}$

$= \frac{57}{19(\sqrt{2}+1)} = \frac{3}{(\sqrt{2}+1)} = \frac{3}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{3(\sqrt{2}-1)}{(2-1)}$

$= 3(\sqrt{2} - 1)$

Q:26 (3) Length of OP = Radius of big circle – Radius of small circle = 5 – 1 = 4 cm

$\Rightarrow OC = \frac{OP}{2} = 2$ cm

Join OA,

In triangle OAC,

$\Rightarrow$ OA = 5 cm

$\Rightarrow$ OC = 2 cm

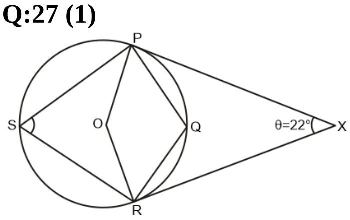
$\Rightarrow AC = \sqrt{(OA^2 - OC^2)}$ [∵ Pythagoras theorem]

$\Rightarrow \sqrt{(5^2 - 2^2)} = \sqrt{21}$ cm

$\Rightarrow AB = 2 \times AC$ [∵ Perpendicular from center bisects the chord]

$\Rightarrow AB = 2\sqrt{21}$ cm

Q:27 (1)



$\angle ORX + \angle OPX = 180°$ (each is an angle made by tangent and is equal to 90°)

Also, as sum of angles of quadrilateral is 360°

Thus, $\angle POR + \angle PXR = 180°$

$\Rightarrow \angle POR + 22° = 180°$

$\Rightarrow \angle POR = 158°$

Hence, $\angle PSR = 158°/2 = 79°$ (central angle $\angle POR$ is twice the angle subtended on any point of the circle at the same side of the segment)

Therefore, $\angle PQR = 180° - 79° = 101°$

Q:28 (2) If we have $\angle BOD = 80°$ and $\angle AOC = 30°$

From the given figure, we know that

$\Rightarrow \angle z = \frac{1}{2}(\angle BOD - \angle AOC) = \frac{1}{2} \times (80° - 30°) = \frac{1}{2} \times 50° = 25°$

Q:29 (1) AB = 12 cm

Radius of the smaller circle with radius $r = \frac{AB}{6}$ ---- (1)

Where AB = radius of the bigger semi-circle.

Using equation (1), we get

$\Rightarrow r = \frac{12}{6} = 2$ cm.

Q:30 (3) OA ⊥ PA [∵ Tangent ⊥ Radius]

$PO^2 = PA^2 + OA^2$ [∵ Pythagoras theorem]

$\Rightarrow 15^2 = PA^2 + 9^2 \Rightarrow 225 = PA^2 + 81$

∴ PA = 12 cm

PB = PA = 12 cm [∵ Lengths of pair of tangents drawn from same external points are equal]

∴ $(PA + PB)^2 = 24^2 = 576$

Hard Level

Q:1 (1) In triangles AGH and ABC: GH || BC

$\frac{AG}{AB} = \frac{AH}{BC} = \frac{1}{2}$ [∵ BPT theorem]

∠ A is common.

So, AGH ~ ABC

So, $\frac{\text{Area (AGH)}}{\text{Area (ABC)}} = \left(\frac{AG}{AB}\right)^2 \Rightarrow \frac{\text{Area(AGH)}}{36} = \frac{1}{4}$

$\Rightarrow$ Area (AGH) = 9 cm^2

In triangles DGH and DEF: GH || EF

$\frac{DG}{DE} = \frac{DH}{EF} = \frac{1}{3}$ [∵ BPT theorem]

∠ D is common

So, DGH ~ DEF

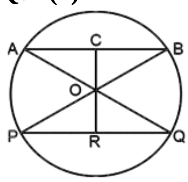
So, $\frac{\text{Area (DGH)}}{\text{Area (DEF)}} = \left(\frac{DG}{DE}\right)^2 \Rightarrow \frac{\text{Area (DGH)}}{81} = \frac{1}{9}$

$\Rightarrow$ Area (DGH) = 9 cm^2

Hence,

Area of AGDH = Area (AGH) + Area (DGH) = 18 cm^2

Q:2 (2)



In the given figure, let 'O' represent the centre of the circle.

So, OB = OQ = 29 cm (radii of the circle)

Let OC be perpendicular to AB,

Then, $AC = CB = (\frac{42}{2}) = 21$ cm (Since the perpendicular from the centre to a chord bisects the chord)

In the right △OBC,

$\Rightarrow OC^2 + CB^2 = OB^2 \Rightarrow OC^2 = 29^2 - 21^2$

$\Rightarrow OC = \sqrt{400} = 20$ cm {Since, length cannot be negative}

Similarly, let OR be perpendicular to PQ, then $RQ = (\frac{PQ}{2})$ = 20 cm

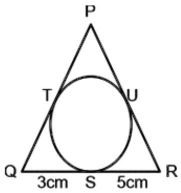
In the right △ORQ,

$\Rightarrow OR^2 + RQ^2 = OQ^2 \Rightarrow OR^2 = 29^2 - 20^2$

$\Rightarrow$ OR = $\sqrt{441}$ = 21 cm {Since, length cannot be negative}
Since we want the maximum possible distance between AB and PQ, so, we assume AB and PQ are on opposite sides of the centre of the circle
So, maximum possible perpendicular distance = OC + OR = (21 + 20) = 41 cm

Q:3 (2) In triangle ADO, $\angle$ D = 90° [$\because$ Tangent $\perp$ radius]
$\Rightarrow OA^2 = OD^2 + AD^2 \Rightarrow 8^2 = 4^2 + AD^2$
$\therefore$ AD = $4\sqrt{3}$ cm
AB = 2 × AD [$\because$ Perpendicular from centre bisects the chord] = $8\sqrt{3}$ cm
Also sin $\angle$ DAO = $\frac{OD}{OA} = \frac{1}{2}$
$\therefore$ sin $\angle$ DAO = 30°
$\angle$ DAC = 60° [$\because\angle$ DAC = 2 × $\angle$ DAO]
$\therefore$ Triangle ABC is an equilateral triangle
Area of the shaded region = Area of triangle ABC – Area of smaller circle = $\frac{\sqrt{3}}{4}$ × side^2 – πr^2 = $\frac{\sqrt{3}}{4}$× $(8\sqrt{3})^2$ - $\pi \times 4^2$
= $(48\sqrt{3} - 16\pi)$ cm^2

Q:4 (3)

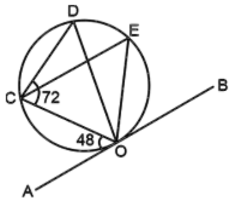


We know that: The length of two tangents from a common external point to a circle are equal.
QS = QT = 3 cm
RS = RU = 5cm
Let the length of PT = TU = x cm
Now, length of QR = 3 + 5 = 8 cm
Length of PQ = (x + 3) cm
Length of PR = (x + 5) cm
Semi perimeter of a triangle (s) = $\frac{8+(x+3)+(x+5)}{2}$ = (x + 8) cm
Area of a triangle = $\sqrt{(s(s-a)(s-b)(s-c))}$ [here a, b and c are sides and s is the perimeter]
= $\sqrt{(x+8)(x+8-8)(x+8-(x+3))(x+8-(x+5))}$
= $(\sqrt{(x+8)(x)(5)(3)})$
We know that:
Inradius of a triangle r = $\frac{\text{area of a triangle}}{\text{semi perimeter of a triangle}}$
$\Rightarrow \sqrt{5} = \frac{\sqrt{(x+8)(x)(5)(3)}}{(x+8)}$
By squaring both sides
$\Rightarrow 5 = \frac{(x+8)(x)(5)(3)}{(x+8)^2}$ $\Rightarrow$ 5x + 40 = 15x $\Rightarrow$ x = 4 cm
Required, length of PR = (x + 5) = (4 + 5) = 9 cm

Q:5 (1) Let OP be x and OQ be 7 - x.
OB = OC = r
According to the question;
$\Rightarrow x^2 + 3^2 = 4^2 + (7 - x)^2 \Rightarrow 9 = 16 + 49 - 14x$
$\Rightarrow$ 14x = 56 $\Rightarrow$ x = 4
Hence the radius, $r^2 = 3^2 + 4^2 \Rightarrow r = 5$
Area of the circle = πr^2 = 3.14 × 25 = 78.5

Q:6 (4) In triangle ADO
$\angle$ ADO = 90° [$\because$ Tangent $\perp$ Radius]
$AO^2 = AD^2 + OD^2$
$\Rightarrow 4^2 = AD^2 + 2^2$
$\Rightarrow AD^2 = 16 - 4 = 12$
$\therefore$ AD = $2\sqrt{3}$ cm
Perpendicular from the center bisects the chord
$\therefore$ AD = DC = $2\sqrt{3}$ cm
AC = $4\sqrt{3}$ cm
sin $\angle$DAO = $\frac{DO}{AO} = \frac{2}{4} = \frac{1}{2}$
$\therefore\angle$DAO = 30°
Similarly $\angle$OCD = 30°
$\angle$DAE = 2 × $\angle$DAO = 60°
[$\because$ The line joining the external point and the center bisects the angle between the tangents]
Similarly,
$\angle$DCF = 60°
$\therefore$ Triangle ABC is an equilateral triangle
Area of ABC = $\sqrt{3} \times \frac{\text{side}^2}{4} = \sqrt{3} \times \frac{(4\sqrt{3})^2}{4} = 12\sqrt{3}$ cm^2

Q:7 (3)



Given,
$\Rightarrow\angle$DCO = 72° and, $\angle$AOC = 48°
$\Rightarrow\angle$DCO = $\angle$DOB = 72° [alternate segment]
$\Rightarrow\angle$AOC = 48°
AB is a straight line
$\Rightarrow\angle$AOC + $\angle$COD + $\angle$DOB = 180°
$\Rightarrow$ 48° + $\angle$COD + 72° = 180°
$\Rightarrow\angle$COD = 60°
Now, In triangle COD
$\Rightarrow\angle$DCO + $\angle$COD + $\angle$ODC = 180°
$\Rightarrow$ 72° + 60° + $\angle$ODC = 180°
$\Rightarrow\angle$ODC = 48°
$\Rightarrow\angle$ODC = $\angle$CEO = 48°
Value of $\angle$DOB + $\angle$CEO = 72° + 48° = 120°

Q:8 (3) $\angle$BAD : $\angle$BCD = $\frac{13}{5}$
$\Rightarrow \frac{\angle BAD}{180 - \angle BAD} = \frac{13}{5}$ [$\because$ In a cyclic quadrilateral, opposite angles are supplementary]
$\Rightarrow$ 5$\angle$BAD = 180 × 13 – 13 × $\angle$BAD
$\Rightarrow$ 18 $\angle$BAD = 180 × 13
$\therefore$ $\angle$BAD = 130° and $\angle$BCD = 50
Let $\angle$ADB and $\angle$CDB be x and 3x respectively
$\therefore$ $\angle$ADC = x + 3x = 4x
$\Rightarrow$ $\angle$ABC = 180° - $\angle$ADC [$\because$ In a cyclic quadrilateral, opposite angles are supplementary]
$\therefore$ $\angle$ABC = 180 – 4x

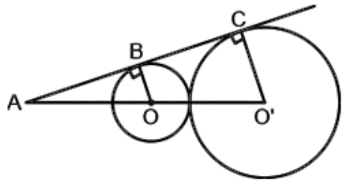
$\Rightarrow \angle ADB = \frac{1}{1+2} \times \angle ABC = 60 - \frac{4x}{3}$
$\Rightarrow \angle CBD = \frac{2}{1+2} \times \angle ABC = 120 - \frac{8x}{3}$
In triangle ABD
$\Rightarrow \angle BAD = 180° - \angle ADB - \angle ABD$ [∵ Angle sum property of triangle ABD]
$\Rightarrow \angle BAD = 180 - x - 60 + \frac{4x}{3}$
$\Rightarrow 130 = 120 + \frac{x}{3} \Rightarrow x = 30°$
$\therefore \angle BDC = 3x = 90°$

Q:9 (1) Given, R = 7 cm, r = 3 cm and D = $10\sqrt{3}$
We know that,
$\Rightarrow AB = \sqrt{D^2 - (R+r)^2} = \sqrt{(10\sqrt{3})^2 - (7+3)^2}$
$= \sqrt{(100 \times 3) - 100} = \sqrt{300 - 100} = \sqrt{200} = 10\sqrt{2}$
Now, $AO = OB = \frac{10}{2\sqrt{2}} = 5\sqrt{2}$
Ratio = r : AO = 3 : $5\sqrt{2}$

Q:10 (1) In triangle AED and DFC
$\angle E = \angle F = 90°$ [Given]
$\angle A = \angle D$ [∵ DF ∥ AB, Corresponding angles are equal]
∴ AED ~ DFC [∵ AA similarity]
$\Rightarrow \frac{\text{Area (AED)}}{\text{Area (DFC)}} = \left(\frac{AE}{DF}\right)^2 = \left(\frac{ED}{FC}\right)^2$
$\Rightarrow \frac{9}{16} = \left(\frac{AE}{DF}\right)^2 = \left(\frac{ED}{FC}\right)^2 \Rightarrow \frac{3}{4} = \frac{AE}{DF} = \frac{ED}{FC}$
Let AE and DF be 3x and 4x respectively
Let ED and FC be 3y and 4y respectively
Area of rectangle EDFB = Length × Breadth = DE × DF
$\Rightarrow 48 = 3y \times 4x$
$\therefore xy = 4$
AB = AE + EB
⇒ AE + DF [∵ In rectangle Opposite sides are equal]
$\Rightarrow 3x + 4x = 7x$
BC = BD + DC
$\Rightarrow ED + DC = 3y + 4y = 7y$
Area of triangle ABC $= \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 7x \times 7y = \frac{49xy}{2}$
$= 98\ cm^2$

Q:11 (2) Let $\angle ACD = \theta$
$\therefore \angle OCD = \frac{\theta}{2}$ [∵ Line joining incentre and vertex will bisect the angle]
$\angle DCE = 180° - \angle ACD$ [∵ Angle in a straight line is 180°]
$\angle DCE = 180° - \theta$
$\angle DCP = \frac{\angle DCE}{2}$ [∵ Line joining excentre and vertex will bisect the exterior angle]
$\angle DCP = 90° - \frac{\theta}{2}$
$\angle OCP = \angle OCD + \angle DCP$
$\Rightarrow \frac{\theta}{2} + 90° - \frac{\theta}{2}$
$\therefore \angle OCP = 90°$

Q:12 (1)



In Δ ABO and Δ ACO'
$\angle B = \angle C = 90°$
$\angle A = \angle A$ = common
Then, Δ ABO ~ Δ ACO'
$\frac{OB}{O'C} = \frac{AO}{AO'} = \frac{4}{7}$
Let AO = 4x and AO' = 7x
AO' = 28
$\Rightarrow 7x = 28$ or $x = 4$
⇒ AO = 16 cm
⇒ OO' = 7x - 4x = 3x = 12 cm
Let OB = 4y and O'C = 7y
OB + O'C = OO'(sum of radiuses of 2 circles)
$\Rightarrow 4y + 7y = 12 \Rightarrow y = \frac{12}{11}$
$BC = 2\sqrt{(r_1r_2)} = 2\sqrt{4y.7y} = 2\sqrt{28y^2} = 2y \times 2\sqrt{7} = \frac{48\sqrt{7}}{11}$
$\Rightarrow y = \frac{12}{11}$

Q:13 (3) Let OP be x cm
If CD : AB = $\frac{\sqrt{3}}{4}$
PD : PB = $\sqrt{\frac{\sqrt{3}}{4}}$
∵ $PD = \frac{CD}{2}$ and $PB = \frac{AB}{2}$ [∵ Perpendicular from the centre bisects the chord]
In triangle OPD
$PD^2 = OD^2 - OP^2$ [∵ Pythagoras theorem] $= 5^2 - x^2$
$PD = \sqrt{(25 - x^2)}$
In triangle OPB
$PB^2 = OB^2 - OP^2$ [∵ Pythagoras theorem]
$\Rightarrow 8^2 - x^2$
$PB = \sqrt{(64 - x^2)}$
$\Rightarrow \frac{PD}{PB} = \frac{\sqrt{25 - x^2}}{\sqrt{64 - x^2}} \Rightarrow \frac{\sqrt{3}}{4} = \frac{\sqrt{25 - x^2}}{\sqrt{64 - x^2}}$
Squaring both the side
$\Rightarrow \frac{3}{16} = \frac{25 - x^2}{64 - x^2} \Rightarrow 192 - 3x^2 = 400 - 16x^2$
$\Rightarrow 13x^2 = 208 \Rightarrow x^2 = 16 \Rightarrow x = 4$ cm

Q:14 (4) In triangle ABF
$AF^2 = AB^2 + BF^2$ [∵ Pythagoras theorem]
In triangle CEF
$EF^2 = CE^2 + CF^2$ [∵ Pythagoras theorem]
In triangle ADE
$AE^2 = AD^2 + DE^2$ [∵ Pythagoras theorem]
In triangle AEF
$AF^2 = AE^2 + EF^2$
$AB^2 + BF^2 = AD^2 + DE^2 + CE^2 + CF^2$
AB = CD = a
BC = AD = b
BF = xb
CF = (1 – x) b
DE = ax
EC = (1 – x) a

Substituting

$\Rightarrow a^2 + x^2b^2 = b^2 + a^2x^2 + (1 - x)^2a^2 + (1 - x)^2b^2$

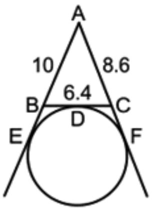
$\Rightarrow a^2 \{1 - x^2 - (1 - x)^2\} = b^2 \{1 - x^2 + (1 - x)^2\}$

$\Rightarrow a^2 \{2x - 2x^2\} = b^2 \times \{2 - 2x\}$

$\Rightarrow a^2 \times 2x \times (1 - x) = b^2 \times 2 \times (1 - x)$

$\Rightarrow a^2 \times x = b^2 \Rightarrow x = \frac{b^2}{a^2}$

Q:15 (4)



Given: AB = 10 cm, BC = 6.4 cm, AC = 8.6 cm

As the circle touches side BC at D, let BD = x, DC = y

BD + DC = 6.4 (Given in the question)

$\Rightarrow x + y = 6.4$ --------------- eqn (1)

Then, BE = x, CF = y (Lengths of tangents from the same point outside the circle are equal)

As AE touches the circle at point E and AF touches the circle at point F only:

AE = AF

$\Rightarrow$ AB + BE = AC + CE

$\Rightarrow 10 + x = 8.6 + y$

$\Rightarrow y - x = 1.4$ -------------- eqn (2)

From eqn 1 and eqn 2, we get,

x = 2.5 cm = BE

Q:16 (2) Let ∠BAC be θ

∴ ∠BOC = 2θ [∵ Angle subtended by a chord in centre is twice the angle subtended it by at any point on the segment]

In triangle ABC,

AB = AC [Given]

∠ABC = ∠ACB [∵ Angle opposite to equal sides are equal]

∠BAC + ∠ABC + ∠ACB = 180° [∵ Angle sum property of triangle]

$\Rightarrow \theta + 2 \times \angle ABC = 180° \Rightarrow \angle ABC = 90 - (\frac{\theta}{2})$

In triangle OBC

OB = OC [Radii]

∠OBC = ∠OCB [∵ Angle opposite to equal sides are equal]

∠BOC + ∠OBC + ∠OCB = 180° [∵ Angle sum property of triangle]

2θ + 2 × ∠OBC = 180°

∠OBC = 90 – θ

∴ ∠OBA = ∠ABC - ∠OBC

so, $15° = \frac{\theta}{2}$

∴ θ = 30°

$\angle ABC = 90 - \frac{\theta}{2} = 75°$

∠ADC = ∠ABC = 75° [∵ Angle subtended by the chord on same side of the segment is equal]

Q:17 (4) Construct OE ⊥ PD and OF ⊥ AB

In triangle OAF

$OA^2 = OF^2 + AF^2 \Rightarrow 5^2 = AF^2 + 3^2$

∴ AF = 4 cm

AB = 2 × AF = 8 cm [∵ Perpendicular from center bisects the chord]

$FD = AD - AF = \frac{3}{3+1} \times AB - AF = 6 - 4 = 2$ cm

In triangle POE

OE = FD = 2 cm

PE = PD – ED = 5 – 3 = 2 cm [∵ ED = OF]

$OP^2 = OE^2 + PE^2 = 8$

∴ $OP = 2\sqrt{2}$ cm

Q:18 (1) ∠POQ = 70°

Angles in the same segment of a circle are equal.

The sum of all angles of a triangle is 180°.

In the given figure, O is the center of the circle,

So, OP = OQ ⇒∠OPQ = ∠OQP

and, OS = OR ⇒∠ORS = ∠OSR

Now in triangle POQ,

∠P + ∠Q + ∠O = 180°

⇒ ∠P + ∠P + 70° = 180° ⇒ 2∠P = 110° ⇒ ∠P = 55°

Now, ∠RPQ = ∠QSR [Angle in the same segment]

⇒ ∠QSR = 55°

Hence the value of ∠QSR is 55°.

Q:19 (2) In triangle ABE and DCE

AB = CD [∵ Opposite sides of rectangle are same]

∠ B = ∠ C = 90° [∵ ABCD is a rectangle]

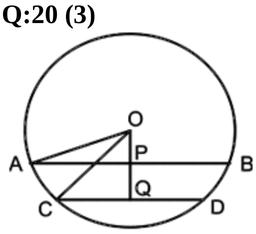
AE = AD [∵ Lengths of tangents drawn from an external point are equal]

ABE ≅ DCE [∵ RHS congruency]

∴ BE = CE

$\Rightarrow \frac{8\,BE - 2\,CE}{2\,BE + CE} = \frac{6BE}{3BE} = 2$

Q:20 (3)



Perpendicular from centre bisects the chord

∴ P and Q are midpoints of the chord

If AB : CD = 4 : 3

Then AP : CQ = 4 : 3

Let AP and CQ be 4x and 3x respectively

$OP = \sqrt{(OA^2 - AP^2)}$ [∵ Pythagoras theorem]

$OP = \sqrt{(25 - 16x^2)}$

$OQ = \sqrt{(OC^2 - CQ^2)}$ [∵ Pythagoras theorem]

$OQ = \sqrt{(25 - 9x^2)}$

PQ = OQ – OP

$1 = \sqrt{(25 - 9x^2)} - \sqrt{(25 - 16x^2)}$

Let $x^2 = t$

$1 = \sqrt{(25 - 9t)} - \sqrt{(25 - 16t)}$ ---- 1)

Equation 1 × $\sqrt{(25 - 9t)} + \sqrt{(25 - 16t)}$

$\sqrt{(25 - 9t)} + \sqrt{(25 - 16t)} = (25 - 9t) - (25 - 16t)$

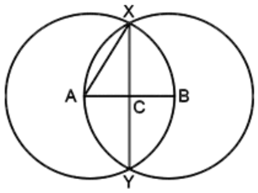
[∵ $(a - b)(a + b) = a^2 - b^2$]

$\sqrt{(25 - 9t)} + \sqrt{(25 - 16t)} = 7t$ ---- 2)

Equation 1 + 2

$2 \times \sqrt{(25 - 9t)} = 1 + 7t$
Squaring on both the sides
$\Rightarrow 100 - 36t = 1 + 49t^2 + 14t$
$\Rightarrow 49t^2 + 50t - 99 = 0$
$\Rightarrow 49t^2 + 99t - 49t - 99 = 0$
$\Rightarrow t(49t + 99) - 1(49t + 99) = 0$
$\Rightarrow (t - 1)(49t + 99) = 0$
$\Rightarrow t = 1$ [∵ It has to be positive]
$\Rightarrow x = 1$
$\Rightarrow AP = 4x = 4$ cm
$\Rightarrow AB = 2 \times AP = 8$ cm

Q:21 (1)



In the given figure, let 'A' and 'B' represent the centres of the two circles and let XY represent the common chord.
Since the two circles have equal radii and pass through the centre of each other, so, XY bisects AB at 'C'.
We have AB = radii = 6 cm
$\Rightarrow AC = \frac{6}{2} = 3$ cm
$\Rightarrow AX$ = radii = 6 cm
We have, $\angle ACX = 90^\circ$ {Since a line drawn from the centre of the circle is the perpendicular bisector of the chord}
$\Rightarrow CX = CY = \left(\frac{XY}{2}\right)$
$\Rightarrow AX^2 = AC^2 + CX^2$
$\Rightarrow CX^2 = 6^2 - 3^2 = 27$
$\Rightarrow CX = 3\sqrt{3}$ cm {Since length cannot be negative}
So, $XY = 2 \times 3\sqrt{3} = 6\sqrt{3}$ cm

Q:22 (1) Let, $\angle BAC = \angle BCA = a$
$\angle ADB = \angle DBC = b$
For chord AB of the circle, the angle subtended by it at the same side of the sector should be same
$\therefore \angle ADB = \angle BCA$
$b = a$
For chord BC of the circle, the angle subtended by it at the same side of the sector should be same
$\therefore \angle CAB = \angle CDB$
$\therefore \angle CDB = a$
$\angle ADC = \angle ADB + \angle CDB = a + a = 2a$
Let $\angle ABD = x$
$\angle ABC = \angle ABD + \angle DBC = x + b = x + a$
Given $\angle ABC = 80^\circ$
$x + a = 80^\circ$
$\angle ADC = 180^\circ - \angle ABC$ [∵ In cyclic quadrilateral, opposite angles are supplementary]
$\angle ADC = 180^\circ - 80^\circ$
$\Rightarrow 2a = 100^\circ$
$\Rightarrow a = 50^\circ$
$\Rightarrow x + 50^\circ = 80^\circ$
$\therefore x = 30^\circ$
$\angle ABD = 30^\circ$

Q:23 (3) In CEOF,
$\Rightarrow \angle C = 90^\circ$ [∵ Given]
$\Rightarrow \angle E = \angle F = 90^\circ$ [∵ Tangent ⊥ Radius]
OE = OF [∵ Both are radius]
∴ CEOF is a square
$\Rightarrow CE = CF = r = 3$ cm
$\Rightarrow AF = AC - CF = 7 - 3 = 4$ cm
AD = AF = 4 cm [∵ Lengths of tangents from the same external point is same]
Let BD = BE = x
$\therefore AB = 4 + x$
$BC = x + 3$
$AB^2 = AC^2 + BC^2$ [∵ Pythagoras theorem]
$\Rightarrow (4 + x)^2 = 7^2 + (3 + x)^2$
$\Rightarrow 16 + x^2 + 8x = 49 + 9 + 6x + x^2 \Rightarrow 2x = 42$
$\Rightarrow x = 21$ cm $\Rightarrow BC = x + 3 = 24$ cm

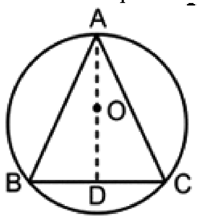
Q:24 (3) Let the length of the line segment AC = a
∴ Length of line segment BD = 2a
In triangle ODB
$\angle D = 90^\circ$ [∵ Tangent ⊥ Radius]
$\Rightarrow OB^2 = OD^2 + BD^2 = 5^2 + (2a)^2 = 25 + 4a^2$
In triangle OCA
$\Rightarrow \angle C = 90^\circ$
$\Rightarrow OA^2 = OC^2 + AC^2$
$\Rightarrow OA^2 = 5^2 + a^2$
$\Rightarrow OA^2 = 25 + a^2$
$\Rightarrow \frac{OA^2}{OB^2} = \left(\frac{\sqrt{61}}{13}\right)^2 \Rightarrow \frac{25 + a^2}{25 + 4a^2} = \frac{61}{169}$
$\Rightarrow 4225 + 169a^2 = 1525 + 244a^2 \Rightarrow 2700 = 75a^2$
$\Rightarrow a^2 = 36 \Rightarrow a = 6$ cm

Q:25 (2) In triangle BCA,
$BC^2 + AC^2 = AB^2$
∴ BCA is a right angled triangle [∵ Inverse Pythagoras Theorem]
Due to symmetry $\angle COB = \angle DOB = 90^\circ$
∴ Area of triangle ABC $= \frac{1}{2} \times AC \times BC = \frac{1}{2} \times AB \times CO$
$\Rightarrow \frac{1}{2} \times 4\sqrt{3} \times 4 = \frac{1}{2} \times 8 \times CO \Rightarrow CO = 2\sqrt{3}$ cm
Due to symmetry, $CD = 2 \times CO = 4\sqrt{3}$ cm
In triangle BCD, all the sides are $4\sqrt{3}$ cm
∴ It is an equilateral triangle and sector angle = 60°
Area of segment = Area of sector - Area of equilateral triangle BCD $= \frac{60}{360} \times \pi r^2 - \frac{\sqrt{3}}{4} \times side^2 = \frac{1}{6} \times \pi \times (4\sqrt{3})^2 - \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2 = 8\pi - 12\sqrt{3}$ cm²

Q:26 (4) Area of the circle = 324π cm²
Distance from centre of the circle to point P = 26 cm.
Area of circle $= \pi r^2$ ---- (1)
$PA^2 = OB \times BP$ ---- (2)
Where PA = length of the tangent, OB = radius and BP = OP - OB
Let the radius be x.
Using equation (1)
$\Rightarrow \pi x^2 = 324\pi \Rightarrow x = 18$ cm
According to the question;

OB = 18 cm and BP = 26 - 18 = 8 cm
Using equation (2), we get
⇒ $PA^2 = 18 \times 8 = 144$
⇒ PA = 12 cm

Q:27 (1) Let a circle (o, r)
ABC is an equilateral triangle.

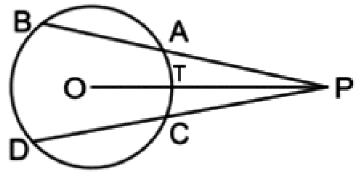


Here, AB = BC = CA = 3 cm
AD is a perpendicular bisector
BD = DC = $\frac{3}{2}$ cm
In triangle ABD, we have
AD = $\sqrt{(AB^2 - BD^2)} = \sqrt{[3^2 - (\frac{3}{2})^2]} = \sqrt{[9 - (\frac{9}{4})]} = \frac{3\sqrt{3}}{2}$
As the altitude of the equilateral triangle is divided in the ratio 2 : 1 by the center of the circle
Since, OA = $\frac{2}{3}$ of AD = $\frac{2}{3} \times \frac{3\sqrt{3}}{2} = \sqrt{3} = r$
Hence, Area of the circle = $\pi r^2 = \pi \times \sqrt{3} \times \sqrt{3} = 3\pi$

Q:28 (2) In triangle OQP
∠ OQP = 90° [∵ Tangent ⊥ Radius]
PR = PQ = 3 cm [Given]
∴ ∠POQ = ∠QPO
∠ OQP + ∠ POQ + ∠ QPO = 180° [Angle sum property of a triangle]
90° + 2 × ∠ QPO = 180°
∴ ∠ QPO = 45°
∠ QPR = 2 × ∠ QPO = 90° [∵ Line joining external point and centre bisects the angle between the tangents]
In triangle PQR
PQ = PR [∵ Tangents drawn from the same point are same in length]
∴∠ Q = ∠ R = 45° [∵∠ P = 90°]
In triangle PQS
⇒ sin ∠Q = $\frac{PS}{PQ} \Rightarrow \frac{1}{\sqrt{2}} = \frac{PS}{3} \Rightarrow PS = \frac{3}{\sqrt{2}}$ cm

Q:29 (4) OB = 2 × OQ = $6\sqrt{3}$ cm
In triangle OAB
sin ∠ABO = $\frac{OA}{OB} = \frac{1}{2}$
[∵ OA = radius = $3\sqrt{3}$]
⇒ ∠ABO = 30°
⇒ ∠RQO = 30° [∵ PQ ∥ AB]
⇒ RQ = OQ cos ∠RQO
⇒ $3\sqrt{3} \times \cos 30° = \frac{9}{2}$ cm
⇒ PQ = 2 × RQ = 9 cm [∵ Perpendicular from centre bisects chord]

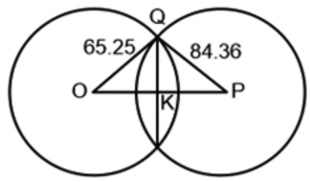
Q:30 (4)



Let a tangent be drawn from point P to the circle which meets the circle at T
$PO^2 = r^2 + PT^2$ [∵ Tangent ⊥ Radius and Pythagoras theorem]
⇒ $169 = 25 + PT^2 \Rightarrow PT^2 = 144$
Secant law: PA. PB = PC. PD = PT^2 = 144
∴ $\frac{PA.PB + PC.PD}{72} = \frac{144 + 144}{72} = 4$

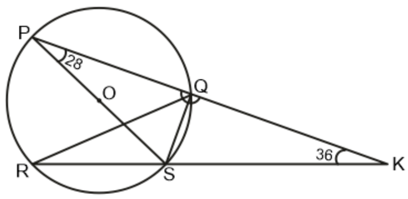
Calculative Only

Q:1 (2)



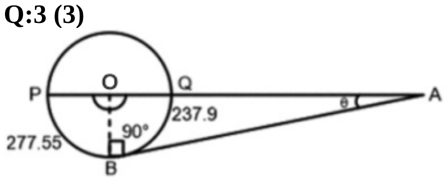
Triangle OPK and QPK are right-angle triangles.
QK = $\frac{96.25}{2}$ = 48.125
$OQ^2 = QK^2 + OK^2 \Rightarrow 65.25^2 = 48.125^2 + OK^2$
⇒ $1941.55 = OK^2 \Rightarrow OK = \sqrt{1941.55} = 44.06$
$PQ^2 = QK^2 + PK^2 \Rightarrow 84.36^2 = 48.125^2 + PK^2$
⇒ $4800.59 = PK^2 \Rightarrow PK = \sqrt{4800.59} = 69.28$
So, the distance between the centres OP = OK + PK
= 44.06 + 69.28 = 113.34 cm

Q:2 (3)



∠PKR = 36°, ∠SPQ = 28°, ∠PQS = 90° and ∠RQS = x
∠PQS = 90° (The angle on the semicircle by its diameter is always 90°)
∠RSP = ∠SPQ + ∠PKR = 36 + 28 = 64°
∠RQP = ∠RSP = 64°
∠RQS = ∠PQS - ∠RQP = 90° - 64° = 26°

Q:3 (3)



Length of Arc PB = 277.55 cm
Arc QB = 237.9 cm
∠OBA = 90°
The ratio of Arc PB : Arc QB = $\frac{277.55}{237.9} = \frac{7}{6}$

The ratio of Arc PB and Arc QB is also the same as the ∠POB and ∠QOB

So, ∠POB : ∠QOB = 7x : 6x

$\Rightarrow 7x + 6x = 180° \Rightarrow x = \frac{180°}{13}$

$\angle QOB = 6x = 6 \times \frac{180°}{13} = 83.08°$

In triangle OAB:

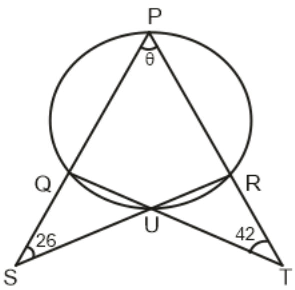
∠QOB + ∠OBA + ∠OAB = 180°

⇒ 83.08° + 90 + ∠OAB = 180°

∠OAB = 180° - 173.08°

⇒ ∠OAB = ∠PAB = 6.92°

Q:4 (2)



∠PSR = 26°

∠PTQ = 42°

Let's assume that ∠QPR = θ

In the cyclic quadrilateral exterior angle is equal to the opposite interior angle.

∠QUS = ∠TUR = ∠QPR = θ

The value of the exterior angle of a triangle is equal to the sum of the values of its two interior opposite angles.

In triangle QUS and RUT:

∠UQP = θ + 26°

∠URP = θ + 42°

in a cyclic quadrilateral, the sum of its opposite angle is 180°

⇒ θ + 26° + θ + 42° = 180°

⇒ 2θ = 112°

∠QPR = θ = 56°

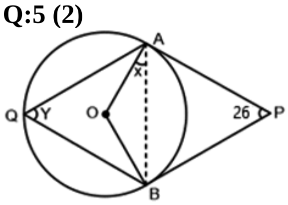
Shortcut method:

∠QPR = 90° - (∠PSR + ∠PTQ)/2

∠QPR = 90° - (26° + 42°)/2

∠QPR = 90° - 34° = 56°

Q:5 (2)



∠APB= 26°

We know that

∠AOB + ∠APB = 180°

⇒ ∠AOB + 26° = 180° ⇒ ∠AOB = 154°

OA = OB = radius of the circle

Triangle OAB is an isosceles triangle

∠OAB = ∠OBA = x

⇒ ∠OAB + ∠OBA + ∠AOB = 180

⇒ x + x + 154° = 180° ⇒ x = 13°

$\angle AQB = \frac{1}{2}(\angle AOB)$ (The angle at the center is twice the angle at the circumference)

$\angle AQB = \frac{1}{2}(154°) = 77° = y$

Required difference = y - x = 77° - 13° = 64°

Conceptual Only

Q:1 (2) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

So, The angle at the centre = 2 × the angle at any point on the circumference.

Therefore, the angle made by an arc at the circumference $= \frac{270°}{2} = 135°$

Q:2 (3) We know that,

$r = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = \frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}$

$R = \frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12}{3\sqrt{3}} = 4\sqrt{3}$

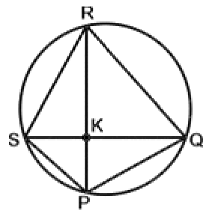
Ratio = r : R = 1 : 2

Alternate answer:

The ratio between r and R in an equilateral triangle is always 1 : 2 or,

The ratio between R and r in an equilateral triangle is always 2 : 1 (points to be remembered)

Q:3 (3)



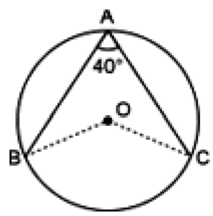
We have given,

$KP^2 + KP^2 + KR^2 + KS^2 = 196$ cm

Radius $r = \sqrt{(KP^2 + KP^2 + KR^2 + KS^2)} = \sqrt{196} = 14$ cm

So, the area of the circle $= \frac{22}{7} \times 14 \times 14 = 616\ cm^2$.

Q:4 (1) The angle subtended by an arc at the centre is twice the angle subtended at the circumference.



Join BC

Let the angle subtended by an arc at the centre be x.

Using the above concept, we get

⇒ ∠BOC = x = 2 × 40 = 80

We know that sum of angles of triangle = 180°

Thus in ΔBOC,

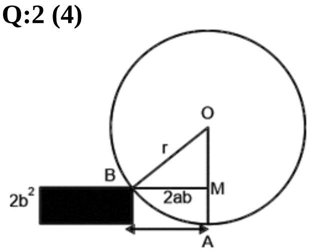
$\Rightarrow \angle BOC + \angle OBC + \angle OCB = 180°$
Since, OB = OC (radius)
$\Rightarrow \angle OBC = \angle OCB \Rightarrow 80 + 2\angle OBC = 180$
$\Rightarrow \angle OBC = 50°$

Q:5 (1) Length of common chord of two equal circles = $\sqrt{(4r^2 - d^2)}$, where d = distance between the centres of the circles.
d = r [Given]
$\therefore AB = \sqrt{(4r^2 - r^2)} = \sqrt{3}\,r$
The radius of the circle with AB as diameter = $\frac{\sqrt{3}r}{2}$
Area of circle with AB as diameter = $\pi \times (\sqrt{\frac{3r}{2}})^2 = \frac{3\pi r^2}{4} = \frac{3}{4} \times \pi r^2 = \frac{3}{4} \times$ Area of the circle with P as center
$\therefore$ Required ratio = $\frac{3}{4}$

Tricky Questions

Q:1 (1) If such figure is given then the radius of the middle circle will be,
$\Rightarrow$ radius = $(\sqrt{R_1R_2})$ ---- (1)
Where R_1 = radius of smaller circle and R_2 = radius of larger circle
$\Rightarrow$ Radius of the circle with radius 'r' = $\sqrt{9 \times 4}$
$\Rightarrow r = \sqrt{36} = 6$ cm

Q:2 (4)

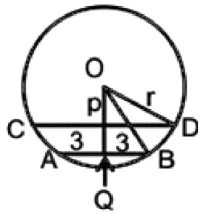


Let r be the radius of the circle. OA = OB = r
AM = height of the brick = $2b^2$
so, OM = OA - AM = $r - 2b^2$.
In ΔOMB using Pythagoras theorem,
$OB^2 = OM^2 + BM^2$
$\Rightarrow r^2 = (r - 2b^2)^2 + (2ab)^2$
$\Rightarrow r^2 = r^2 + 4b^4 - 4b^2r^2 + 4a^2b^2$
$\Rightarrow r = \frac{4b^2(a^2 + b^2)}{4b^2} = a^2 + b^2$

Q:3 (3) To solve such types of questions we have a direct formula to be remembered i.e.
$\Rightarrow XY^2 = AB^2 + PQ^2$
$\Rightarrow PQ^2 = XY^2 - AB^2 = 25^2 - 14^2 = 625 - 196 = 429$
$\Rightarrow PQ = \sqrt{429}$

Q:4 (2) Tangents to a circle from a point are equal in length.
So, AB = AC
$\Rightarrow 3x^2 + 2x + 10 = 11 \Rightarrow 3x^2 + 3x - x - 1 = 0$
$\Rightarrow 3x(x + 1) - 1(x + 1) = 0 \Rightarrow (x + 1)(3x - 1) = 0$
$\Rightarrow x = -1, \frac{1}{3}$

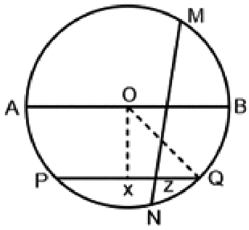
Q:5 (3) The perpendicular drawn from the center to the chord of a circle bisects the chord.
Given AB = 6,
CD = 2AB = 12,
PQ = $\frac{CD}{4} = 3$.
OP ⊥ CD and OQ ⊥ AB
Suppose OP = x



According to the concept:
PD = $\frac{CD}{2} = 6$ cm
QB = $\frac{AB}{2} = 3$ cm
From Pythagoras theorem in triangle OPD:
$r^2 = x^2 + 6^2$ ………………..(1)
In triangle BOQ:
$r^2 = (x + 3)^2 + 3^2 = x^2 + 18 + 6x$ ………..(2)
Equating both equations:
$x^2 + 6^2 = x^2 + 18 + 6x$
$\Rightarrow 6x = 18 \Rightarrow x = 3$
From equation 1:
$r^2 = 3^2 + 6^2 = 9 + 36 = 45 \Rightarrow r = 3\sqrt{5}$ cm
Diameter = $2r = 6\sqrt{5}$ cm

Some Innovative Questions

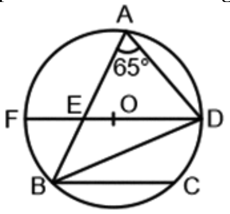
Q:1 (1) Diameter = 50 cm
Distance between the chord and center of the circle = $\frac{7}{25}$th of the radius of the circle
Intersecting Chords Theorem: The Intersecting Chords Theorem states that when two chords of a circle intersects within the circle, the product of the segments of one chord is equal to the product of the segments of the other chord



radius of the circle = $\frac{50}{2} = 25$ cm
Distance between the chord and center of the circle = $\frac{7}{25} \times 25 = 7$ cm
Using Pythagoras theorem;
$\Rightarrow OQ^2 = OX^2 + XQ^2 \Rightarrow XQ^2 = 25^2 - 7^2$
$\Rightarrow$ XQ = 24 cm and PQ = 2 × XQ
$\Rightarrow$ PQ = 2 × 24 = 48 cm
We know that,
$\Rightarrow$ ZQ = $\frac{1}{5}$th of PZ such that = $\frac{1}{6}$th of PQ
Let the value of ZQ and PQ be x and 6x

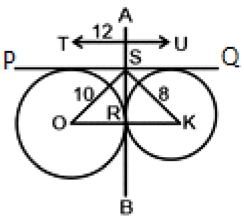
According to the question;
⇒ $6x = 48 \Rightarrow x = 8$
Using the intersecting Chords Theorem;
⇒ $PZ \times ZQ = MZ \times ZN \Rightarrow MZ \times ZN = 40 \times 8$
Using AM - GM inequality,
⇒ $\frac{MZ + ZN}{2} \geq \sqrt{(MZ \times ZN)} \Rightarrow MZ + ZN \geq (2 \times 8\sqrt{5})$
⇒ $MZ + ZN \geq 16\sqrt{5} \Rightarrow MN \geq 16\sqrt{5}$

Q:2 (3) The angle subtended by an arc at the center of the circle is twice the angle subtended by it at any other point in the remaining part of the circle.



From the figure drawn above;
In triangle ABD, we join OB
Where, OB = OD
Now using the concept described above, we get
⇒ $\angle BOD = 2 \times \angle BAD = 2 \times 65 = 130°$
Now, in triangle OBD,
⇒ $\angle OBD + \angle ODB + \angle BOD = 180$
⇒ $2 \times \angle ODB + 130 = 180$ ($\angle OBD = \angle ODB$, angles opposite to equal sides are equal)
⇒ $\angle ODB = \frac{180 - 130}{2} = \frac{50}{2} = 25°$
We also know chord BC is parallel to DF,
⇒ $\angle ODB = \angle DBC = 25°$
Hence, only statement I is sufficient and statement II is not sufficient.

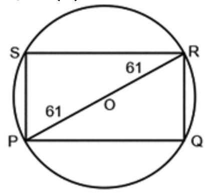
Q:3 (2) Given that
Two circles touch each other externally, AB tangent to common point R and PQ is the tangent touch at T and U.



TU = 12 cm, OS = 10 cm, KS = 8 cm
According to the question,
SR = SU and SR = ST it means $ST = SU = SR = \frac{12}{2} = 6$ cm
SR is the height of triangle SOK.
In triangle ORS, By Pythagoras theorem
$OR = \sqrt{(OS^2 - SR^2)} = \sqrt{(10^2 - 6^2)} = 8$ cm
Similarly, In triangle KRS is a right angled triangle
$KR = \sqrt{(SK^2 - SR^2)} = \sqrt{(8^2 - 6^2)} = 2\sqrt{7}$cm
$OK = OR + RK = 8 + 2\sqrt{7}$
Hence, Area of the triangle $SOK = \frac{1}{2} \times SR \times OK = \frac{1}{2} \times 6 \times \left(8 + 2\sqrt{7}\right) = 6(4 + \sqrt{7})$ cm^2

Q:4 (2) A circle is inscribed in a semi-circle
The radius of the semicircle = 2 cm
The area of a semicircle = $\frac{\pi r^2}{2} = \frac{\pi \times 2^2}{2} = 2\pi$
As we know, The area of a circle should be less than the area of a semicircle.
Option 1: Area $= \pi \times (\sqrt{2} + 1)^2 = \frac{\pi}{4} \times \left(2 + 1 + 2\sqrt{2}\right) = (3 + \sqrt{2}) \times \pi > 2\pi$
Option 2: Area $= \pi \times (\sqrt{2} - \frac{1}{2})^2 = \pi \times (2 + \frac{1}{4} - \sqrt{2}) = 0.85\pi < 2\pi$
Option 3: Area $= \pi \times (4 - \sqrt{2})^2 = \pi \times (16 + 2 - 8\sqrt{2}) = (18 - 11.31)\pi = 6.68\pi > 2\pi$
Option 4: Area $= \pi \times (2\sqrt{2} + 1)^2 = \pi \times (8 + 1 + 4\sqrt{2}) = 14.65\pi > 2\pi$
Hence, the required possible radius of circle is $\sqrt{2} - \frac{1}{2}$.

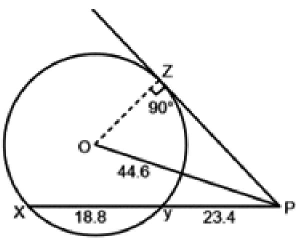
Q:5 (2)



The radius of the circle = 61 cm
So, the Diameter of the circle PR = 61 × 2 = 122 cm
PQRS is a rectangle in which PR is the diagonal of the rectangle.
So, PR is the Hypotenuse.
(11, 60, 61) is a Pythagorean triplet
So, (22, 120, 122) are also Pythagoras triplets
In triangle PQR
$(PR)^2 = (PQ)^2 + (QR)^2 \Rightarrow (122)^2 = (120)^2 + (22)^2$
⇒ $14884 = 14884 \Rightarrow$ LHS = RHS
So, the possible length and breadth are 22 and 120 cm.

Questions to be Skipped

Q:1 (4)



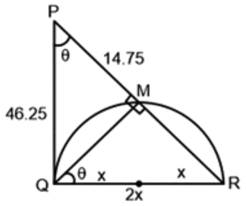
XY = 18.8 cm, PY = 23.4 cm and OP = 44.6 cm
$\angle OZP = 90°$ (the angle between the radius and the tangent will always is 90°)
We know that,
$PZ^2 = PY.PX$
⇒ $PZ^2 = 23.4 \times (23.4 + 18.8)$
⇒ $PZ = \sqrt{987.48} = 31.42$ cm
Triangle OZP is a right-angled triangle
$OP^2 = PZ^2 + OZ^2$
⇒ $44.6^2 = 31.42^2 + OZ^2$
⇒ $OZ = \sqrt{1001.94}$

Radius = OZ = 31.65 cm

So, the area of the circle = $\pi R^2 = \left(\frac{22}{7}\right) \times 31.65^2$

$= 3148.27\ cm^2$

Q:2 (2)



PQ = 46.25 and PM = 14.75

Let's assume QR = 2x

The radius of semicircle r = x

Triangle PMQ is a right-angled triangle

$QM^2 = PQ^2 - PM^2 = 46.25^2 - 14.75^2 = 46.25^2 - 14.75^2$

= 1921.5

⇒ QM = 43.83 cm

ΔPMQ is similar to the ΔQMR

So, by using the similarity

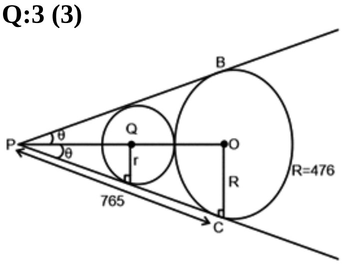
$\Rightarrow \frac{QR}{PQ} = \frac{QM}{PM} \Rightarrow \frac{2x}{46.25} = \frac{43.83}{14.75}$

Radius x = 68.71 cm

Perimeter of semicircle = πR + 2R

= 3.14 × 68.71 + 2 × 68.71 = 353.17 cm

Q:3 (3)



OC = R = 476 cm

PC = 765 cm

ΔOCP is a right-angled-triangle

$OP^2 = PC^2 + OC^2 = 765^2 + 476^2 = 585225 + 226576$

$\Rightarrow OP = \sqrt{811801} = 901$ cm

$\sin\theta = \frac{OC}{OP} = \frac{476}{901}$

We know that

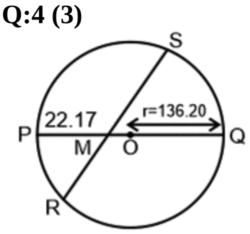
$\Rightarrow \frac{r}{R} = \frac{1-\sin\theta}{1+\sin\theta} \Rightarrow \frac{r}{476} = \frac{1-\frac{476}{901}}{1+\frac{476}{901}} \Rightarrow \frac{r}{476} = \frac{\frac{901-476}{901}}{\frac{901+476}{901}}$

$\Rightarrow \frac{r}{476} = \frac{425}{1377} \Rightarrow r = \frac{202300}{1377} = 146.91$ cm

The area of the smaller circle = πr^2

$= 3.14 \times 146.91^2 = 67769.20\ cm^2$

Q:4 (3)



PM = 22.17 cm and RS = 206.45 cm

Let's assume MS = x

RS = RM + MS

RM = 206.45 - x

Radius = 136.20

So, PQ = 136.20 × 2 = 272.4

MQ = PQ - PM = 272.4 - 22.17 = 250.23 cm

We know that

RM × MS = PM × MQ

⇒ (206.45 - x) × MS = 22.17 × 250.23

$\Rightarrow 206.45x - x^2 = 5547.60$

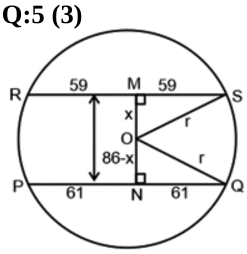
$\Rightarrow x^2 - 206.45x + 5547.60 = 0$

$\Rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{206.45\pm\sqrt{206.45^2-4\times1\times5547.60}}{2\times1}$

$\Rightarrow x = \frac{206.45+\sqrt{20431.2025}}{2}$ or $\frac{206.45-\sqrt{20431.2025}}{2}$

$\Rightarrow x = \frac{206.45 + 142.94}{2}$ or $\frac{(206.45 - 142.94)}{2} = 176.95$ or 31.75

Q:5 (3)



Distance between the chords MN = 86 cm

Let's assume the length of OM = x

ON = 86 - x

Radius = r

PQ = 122, So, NQ = $\frac{122}{2}$ = 61 cm

RS = 118, So, SM = $\frac{118}{2}$ = 59 cm

Triangle OMS and ONQ are the right angle triangle

So, $OS^2 = OM^2 + SM^2$

$\Rightarrow r^2 = x^2 + 59^2 = x^2 + 3481$........1

And $OQ^2 = ON^2 + NQ^2$

$\Rightarrow r^2 = (86 - x)^2 + 61^2 = 11117 + x^2 - 172x$...........2

Now equating equations 1 and 2

$\Rightarrow x^2 + 3481 = 11117 + x^2 - 172x$

$\Rightarrow 11117 - 3481 = 172x \Rightarrow x = \frac{7636}{172} = 44.40$ cm

$\Rightarrow r^2 = x^2 + 3481 = 44.40^2 + 3481 = 5452.36$

⇒ r = 73.84 cm

Previous Year Questions

Questions asked in SSC CGL Tier 1 Exam

These are the questions asked in SSC CGL Tier 1 Exam in past few years. Try to solve all these question by the tips and tricks you've learned in this chapter. It will help you to connect the concepts we've learned, and the questions asked in exam.

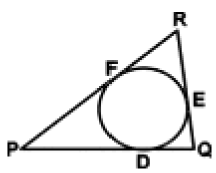
Q:1 What is the radius of the circle that circumscribes the triangle ABC whose sides are 16, 30, and 34 units, respectively? **[SSC CGL 2023]**
1.16 units 2.17 units
3.28 units 4.34 units

Q:2 Two circles of radii 10 cm and 5 cm touch each other externally at a point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. The length of PQ is equal to: **[SSC CGL 2023]**
1.$10\sqrt{2}$ cm 2.$8\sqrt{2}$ cm
3.$9\sqrt{2}$ cm 4.$6\sqrt{2}$ cm

Q:3 A 15-cm-long perpendicular is drawn from the centre of a circle to its 40-cm-long chord. Find the radius of the circle. **[SSC CGL 2023]**
1.25 cm 2.27 cm
3.22 cm 4.20 cm

Q:4 Select the correct option with respect to the given statement. Two tangents are drawn at the end of the diameter of a circle. **[SSC CGL 2022]**
1.They intersect each other.
2.They pass through origin.
3.They are parallel to each other
4.They are perpendicular to each other.

Q:5 In the given figure, a circle is inscribed in $\triangle PQR$, such that it touches the sides PQ, QR and RP at points D, E, F respectively. If the lengths of the sides PQ = 15 cm, QR = 11 cm and RP = 13 cm, then find the length of PD. **[SSC CGL 2022]**



1.9 cm 2.8 cm
3.7.5 cm 4.8.5 cm

Q:6 The side of an equilateral triangle is 12 cm. What is the radius of the circle circumscribing this equilateral triangle? **[SSC CGL 2022]**
1.$6\sqrt{3}$ cm 2.$4\sqrt{3}$ cm
3.$9\sqrt{3}$ cm 4.$5\sqrt{3}$ cm

Q:7 How many circles can be drawn that pass through two fixed points? **[SSC CGL 2022]**
1.Infinite 2.Only two
3.One or two 4.Only one

Q:8 Let O be the centre of a circle and P be the point outside the circle. If PAB is a secant to the circle intersecting the circle at A and B and PT is the tangent segment drawn from P, find the length of PT, if PA = 3 cm and AB = 9 cm. **[SSC CGL 2022]**
1.$3\sqrt{3}$ cm 2.$4\sqrt{3}$ cm
3.6 cm 4.8 cm

Q:9 Find the number of common tangents, if $r_1 + r_2 = C_1C_2$. (With usual notations, r_1 & r_2 and C_1 & C_2 are the radii and centres of the two circles.) **[SSC CGL 2022]**
1.1 2.0
3.3 4.4

Q:10 Two circles touch each other externally at P. AB is a direct common tangent to the two circles, A and B are points of contact, and $\angle PAB = 40°$. The measure of $\angle ABP$ is: **[SSC CGL 2022]**
1.45° 2.55°
3.50° 4.40°

Q:11 The length of the tangent to a circle from a point P is 15 cm. Point P is 17 cm away from the centre. What is the radius of the circle? **[SSC CGL 2022]**
1.7 cm 2.9 cm
3.8 cm 4.4 cm

Q:12 O is the center of this circle. Tangent drawn from a point P, touches the circle at Q. If PQ = 24 cm and OQ = 10 cm, then what is the value of OP? **[SSC CGL 2022]**
1.26 cm 2.52 cm
3.13 cm 4.15 cm

Q:13 Two equal circles of radius 8 cm intersect each other in such a way that each passes through the centre of the other. The length of the common chord is: **[SSC CGL 2022]**
1.$8\sqrt{3}$ 2.$\sqrt{3}$
3.$2\sqrt{3}$ 4.$4\sqrt{3}$

Q:14 A chord of length 42 cm is drawn in a circle having diameter 58 cm. What is the minimum distance of other parallel chord of length 40 cm in the same circle from 42cm long chord? **[SSC CGL 2022]**
1.4 cm 2.1 cm
3.3 cm 4.2 cm

Q:15 The number of parallel tangents of a circle with a given tangent is: **[SSC CGL 2022]**
1.1 2.2
3.3 4.4

Q:16 PQRS is a cyclic quadrilateral and PQ is the diameter of the circle. If $\angle RPQ = 23°$, then what is the measure of $\angle PSR$? **[SSC CGL 2021]**
1.157° 2.113°
3.123° 4.147°

Q:17 A circle is circumscribed on a quadrilateral ABCD. If $\angle$ DAB = 100°, $\angle$ ADB = 35° and CDB = 40°, then find the measure of $\angle$DBC. **[SSC CGL 2021]**
1.45° 2.40°
3.60° 4.35

Q:18 AB and CD are two chords in a circle with centre O and AD is the diameter. When produced, AB and CD meet at the point P. If $\angle$DAP = 27°, $\angle$APD = 35°, then what is the measure (in degrees) of $\angle$DBC?
[SSC CGL 2021]
1.32 2.30
3.26 4.28

Q:19 Point P, Q, R, S and T tie in this order on a circle with centre O. If chord TS is parallel to diameter PR and $\angle$RQT = 58°, then find the measure (in degrees) of $\angle$RTS. **[SSC CGL 2020]**
1.29 2.32
3.45 4.58

Q:20 A ΔABC has sides 5 cm, 6 cm and 7 cm. AB extended touches a circle at P and AC extended touches the same circle at Q. Find the length (in cm) of AQ.
[SSC CGL 2020]
1.13 2.9
3.11 4.12

Questions asked in different exams

Apart from SSC CGL Exam, we should be aware of the questions asked in other exams as well. It will help you to understand the variety as well as difficulty level of other exams, so that if a question of some different pattern will in exam, you will be aware of it.

Q:1 A tangent at point A of a circle of radius 6 cm meets a line through the centre O at point B. If OB = 10 cm, then the length of AB (in cm) is equal to:
[SSC CHSL 2023]
1.5 2.6
3.4 4.8

Q:2 Two circles of radii 12 cm and 13 cm are concentric. The length of the chord of the larger circle that touches the smaller circle is: **[SSC CHSL 2023]**
1.8 cm 2.18 cm
3.10 cm 4.12 cm

Q:3 A direct common tangent is drawn to two circles of radius 25 cm and 20 cm. The centres of the two circles are 35 cm apart. What is the length (in cm) of the tangent?
[SSC CHSL 2023]
1.$25\sqrt{2}$ 2.$25\sqrt{3}$
3.$20\sqrt{3}$ 4.$20\sqrt{2}$

Q:4 PQR is an equilateral triangle inscribed in a circle. S is any point on the arc QR. Measure of $\frac{1}{2}\angle PSQ$ is:
[SSC CPO 2023]
1.20° 2.15°
3.30° 4.60°

Q:5 Two circles of radius 13 cm and 15 cm intersect each other at points A and B. If the length of the common chord is 12 cm, then what is the distance between their centres? **[SSC CPO 2023]**
1.$\sqrt{145}+\sqrt{184}$ 2.$\sqrt{131}+\sqrt{181}$
3.$\sqrt{145}+\sqrt{169}$ 4.$\sqrt{108}+\sqrt{189}$

Q:6 E, F, G, and H are four points lying on the circumference of a circle to make a cyclic quadrilateral. If $\angle$FGH = 57°, then what will be the measure of the $\angle$HEF? **[SSC CPO 2023]**
1.33° 2.123°
3.93° 4.143°

Q:7 Let C be a circle with centre O and radius 5 cm. Let PQ be a tangent to the circle and A be the point of tangency. Let B be a point on PQ such that the length of AB is 12 cm. If the line joining O and B intersects the circle at R, find the length of BR (in cm).
[SSC CPO 2023]
1.2 2.13
3.6 4.8

Q:8 In a circle centred at O, PQ is a tangent at P. Furthermore, AB is the chord of the circle and is extended to Q. If PQ = 12 cm and QB = 8 cm, then the length of AB is equal to: **[SSC CPO 2023]**
1.20 cm 2.4 cm
3.10 cm 4.8 cm

Q:9 What is the length (in cm) of the transverse common tangent between two circles with radii 6 cm and 4 cm, given that the distance between their centres is 14 cm?
[SSC CPO 2023]
1.$2\sqrt{6}$ 2.$4\sqrt{6}$
3.$5\sqrt{6}$ 4.$3\sqrt{6}$

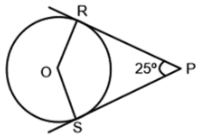
Q:10 If the distance between the centres of two circles is 12 cm and the radii are 5 cm and 4 cm, then the length (in cm) of the transverse common tangent is:
[SSC CPO 2023]
1.9 2.$\sqrt{143}$
3.$\sqrt{63}$ 4.7

Q:11 In a circle of radius 14 cm. an arc subtends an angle of 90° at the centre. The length of the arc (in cm) is equal to: Take ($\pi = \frac{22}{7}$) **[SSC CPO 2023]**
1.22 2.18
3.20 4.24

Q:12 RA and RB are two tangents from a common point 'R' outside the circle with centre 'O', having a radius of 6 cm. If $\angle$ARB = 70°, find the measure of the angle AOB.
[SSC CHSL 2022]
1.130° 2.110°

3.120°　　　　4.80°

Q:13 In the given figure, if ∠RPS = 25°, the value of ∠ROS is _________.



[SSC CHSL 2022]

1.155°　　　　2.145°
3.135°　　　　4.165°

Q:14 The radii of two circles are 4 cm and 7 cm and the distance between their centres is 12 cm. Find the length of the direct common tangent to the circles.
[SSC CHSL 2022]

1.3√6 cm　　　　2.2√6 cm
3.2√15 cm　　　　4.3√15 cm

Q:15 AB is a diameter of a circle with centre O. If C is any point on the circle such that ∠BAC = 42°, then find the measure of ∠BOC. **[SSC CHSL 2021]**

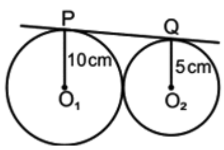
1.84°　　　　2.42°
3.60°　　　　4.63°

Answers and Solutions

Questions asked in SSC CGL Tier 1 Exam

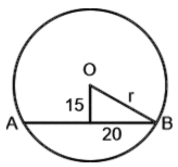
Q:1 (2) Dimensions of the triangle = 16, 30, and 34
In △ABC
By Pythagoras theorem
⇒ $AB^2 = AC^2 + BC^2$
⇒ $34^2 - 30^2 = AC^2$
⇒ $AC^2 = 1156 - 900 = 256 = 16^2$
Hence, it is a right-angled triangle
We know that circumradius = $\frac{H}{2} = \frac{34}{2}$ = 17 units

Q:2 (1)



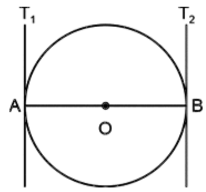
We two circles touch externally then, the length of PQ = $\sqrt{r_1 \times r_2} = 2\sqrt{10 \times 5} = 2\sqrt{5 \times 2 \times 5} = 10\sqrt{2}$ cm

Q:3 (1) The chord AB length of a circle is 40 cm.
A 15-cm-long perpendicular is drawn from the centre of a circle O to the chord AB.
Let the radius of the circle be r cm.



By using Pythagoras theorem,
$r^2 = 15^2 + 20^2 = 225 + 400 = 625$
r = 25 cm

Q:4 (3)

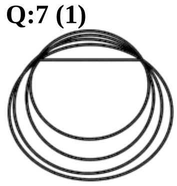


In the figure given above,
AB = diameter and T_1 and T_2 are the tangents
Hence, we can see that both tangents are parallel to each other.

Q:5 (4) The lengths of tangents drawn from an external point to a circle are equal
Let the value of ER = x
Hence using the concept given;
⇒ ER = FR = x
⇒ PF = 13 - x and PD = 13 - x (PF = PD)
⇒ QE = 11 - x and QD = 11 - x (QE = QD)
Now, QP = PD + QD
⇒ 15 = 13 - x + 11 – x ⇒ 2x = 24 – 15 ⇒ x = 4.5
We know that, PD = 13 - x = 13 - 4.5 = 8.5 cm

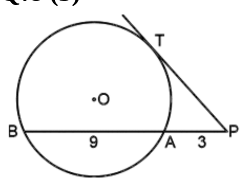
Q:6 (2) The side of an equilateral triangle is 12 cm.
The radius of a circle circumscribing an equilateral triangle = $\frac{\text{side}}{\sqrt{3}}$
According to the concept,
Radius of the circle = $\frac{12}{\sqrt{3}} = \frac{4 \times 3}{\sqrt{3}} = 4 \times \frac{(\sqrt{3})^2}{\sqrt{3}} = 4\sqrt{3}$
The radius of the circle circumscribing this equilateral triangle is 4√3 cm.

Q:7 (1)



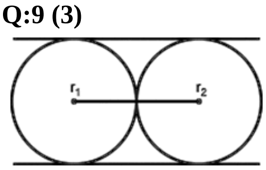
We can see in the above figure there infinite circles can be drawn that pass through two fixed points.

Q:8 (3)



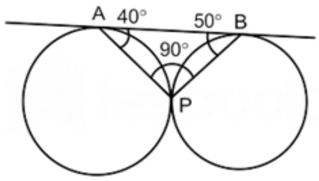
we know that, $PT^2 = PA \times PB$
⇒ $PT^2 = 3 \times 12 = 36$
⇒ PT = 6 cm

Q:9 (3)



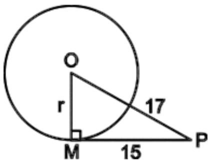
As given in the figure, if $r_1 + r_2 = C_1C_2$
the two circles touches each other only at one point.
Hence, we can see from the figure that there are 3 common tangents.

Q:10 (3)



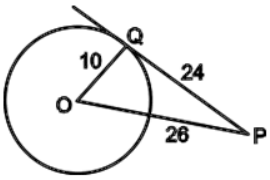
According to the concept, ∠APB = 90°
Considering ΔAPB,
∠ABP = 90° - ∠PAB = 90° - 40° = 50°
∴ The measure of ∠ABP is 50°.

Q:11 (3) Let the radius of the circle be r



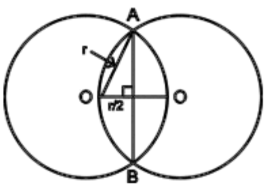
Using Pythagoras theorem in triangle OMP, we get
$\Rightarrow r^2 = 17^2 - 15^2 = 289 - 225 = 64 \Rightarrow r = 8$ cm

Q:12 (1) O is the centre of this circle. Tangent drawn from a point P, touches the circle at Q.
PQ = 24 cm and OQ = 10 cm
1. If a tangent is drawn on a circle from an external point, it is perpendicular to the radius at the point of tangency.
2. In a right-angled triangle, $\text{Hypotenuse}^2 = \text{Base}^2 + \text{Height}^2$



According to the concept, ∠OOP 90°.
Hence, OP is the hypotenuse of AOQP which is a right-angled triangle.
Now, $OP = \sqrt{(24^2 + 10^2)} = 26$ cm
The value of OP is 26 cm.

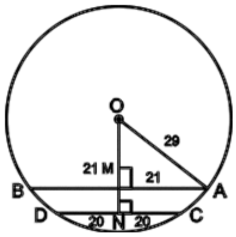
Q:13 (1)



In the following figure, we have
OA = r, OO' = r
AM is perpendicular to OO'
Hence, $OM = O'M = \frac{r}{2}$
Common tangent = AB = 2AM
Using Pythagoras theorem, we get

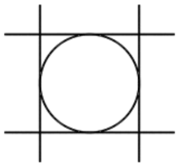
$\Rightarrow AM^2 = OA^2 - OM^2 = r^2 - (\frac{r}{2})^2 = r^2 - \frac{r^2}{4} = \frac{3r^2}{4}$
$\Rightarrow AM = \sqrt{\frac{3r}{2}}$
Now, $AB = 2 \times \sqrt{\frac{3r}{2}} = \sqrt{3}r$
We have the value of radius i.e. r = 8 cm
Therefore, the length of the common tangent = $8\sqrt{3}$ cm

Q:14 (2)



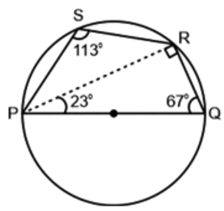
We have the following figure,
Diameter = 58 cm, radius = 29 cm
If we have another chord and we have to find the minimum distance between them, then the chord will be on the same side.
MN is the minimum distance between the chords
OM is the perpendicular on chord AB which divides the chord in two equal halves
Hence, BM = AM = 29 cm
Using Pythagoras theorem in triangle OMA, we get
$\Rightarrow OA^2 = OM^2 + AM^2 \Rightarrow 29^2 = OM^2 + 21^2$
$\Rightarrow OM^2 = 841 - 441 = 400 \Rightarrow OM = 20$ cm
Similarly using Pythagoras theorem in triangle ONC, we get
$\Rightarrow ON^2 = 29^2 - 20^2 \Rightarrow ON = 21$
Hence, the distance between parallel chord = OM - ON = MN = 21 - 20 = 1 cm

Q:15 (1)



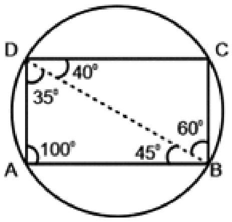
We can see in the above figure, the number of parallel tangents of a circle with a given tangent is only one.

Q:16(2)



If PQ is the diameter, then ∠PRQ will be 90º (Angle made by a diameter in a segment is 90º)
∠PQR = 180 - 90 - 23 = 67º
In a cyclic quadrilateral, Opposite angles are supplementary.
Thus, ∠PQR + ∠PSR = 180
⇒ ∠PSR = 180 - 67 = 113º

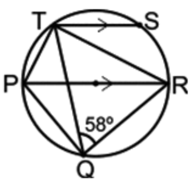
Q:17 (3)



In triangle DAB,
∠DBA + ∠DAB + ∠ADB = 180°
⇒ 100° + 35° + ∠DBA = 180°
⇒ ∠DBA = 45°
Since the circle is circumscribed on a quadrilateral ABCD, ABCD is a cyclic quadrilateral.
So, ∠ADC + ∠ABC = 180°
⇒ (∠ADB + ∠CDB) + (∠ABD + ∠DBC) = 180°
⇒ ∠DBC = 60°
∴ The measure of ∠DBC is 60°.

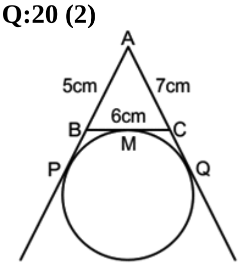
Q:18 (3) AB and CD are two chords in a circle with centre O and AD is a diameter.
AB and CD produced meet at a point P outside the circle.
∠APD = 25^0 and ∠DAP = 39^0
Exterior angle theorem: An exterior angle of a triangle is equal to the sum of its two opposite non-adjacent interior angles. The angle subtended from a diameter is 90^0. The opposite angles of a triangle are equal.
According to the question -
In ΔADP -
∠ADC = ∠DAP + ∠DPA = $25^0 + 39^0 = 64^0$
Angles in the same segment are equal.
∠ADC = ∠ABC = 64^0 ....(I
Since AD is a diameter.
So, ∠ABD = 90^0
⇒ ∠ABC + ∠CBD = 90^0
⇒ 64^0 + ∠CBD = 90^0 [From (i)]
⇒ ∠CBD = 26^0

Q:19 (2)



∠RQT = ∠RPT = 58°(Angle on the segment)
∠PTR = 90° (angle in a semi-circle is equal to 90°)
∠PRT = 180 - 90 - 58 = 32°
∠RTS = ∠PRT = 32° (Alternate interior angle).

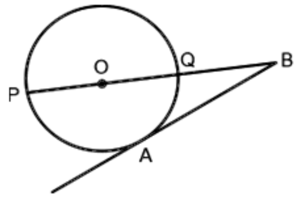
Q:20 (2)



Let BM = x and CM = (6 - x)
As one knows that the length of tangents drawn from the same external point is equal.
So one can say that BM = BP = x and CM = CQ = 6 - x
Similarly, AP = AQ
⇒ 5 + x = 7 + 6 – x ⇒ x = 4
AQ = 7 + 6 - 4 = 9 cm.

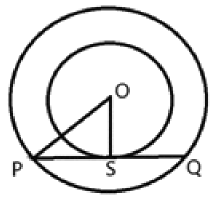
Questions asked in different exams

Q:1(4) A circle of radius (r) = 6 cm
OB = 10 cm
A line PQ through the centre O meets at point B



PQ = 2 × 6 = 12 cm, PB = OP + OB = 6 + 10 = 16 cm, QB = OB - OQ = 10 - 6 = 4 cm
According to the tangent theorem,
AB^2 = PB × QB = 16 × 4 = 64
AB = 8 cm

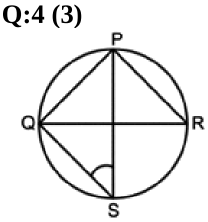
Q:2 (3) Two circles of radii 12 cm and 13 cm are concentric.
The chord PQ touches the small circle at S



OP = 13 cm, OS = 12 cm
In triangle OPS,
$OP^2 = OS^2 + PS^2$
$PS^2 = 13^2 - 12^2 = 169 - 144 = 25$
PS = 5 cm
Hence, the chord length PQ = 2 × 5 = 10 cm

Q:3 (3) Distance between the centres of two circles is 35 cm. Radii are 25 cm and 20 cm.
Direct common tangent = $\sqrt{D^2 - (R - r)^2}$ ---- (1)
Where D = distance between the centre of two circles
R and r are the radius of two circles
Using equation (1), we get
⇒ $\sqrt{35^2 - (25 - 20)^2} = \sqrt{1200} = 20\sqrt{3}$

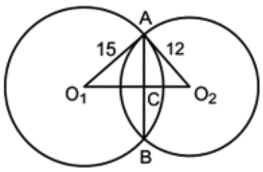
Q:4 (3)



Given, that PQR is an equilateral triangle

So, all the angles of triangle PQR are 60°
∠PRQ and ∠PSQ are on the same side of arc PQ
∠PRQ = ∠PSQ = 60°
The value of $\frac{1}{2}$∠PSQ = $\frac{1}{2} \times 60 = 30°$

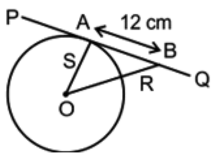
Q:5 (4)



Given AB = 12 cm
We know that the line joining both centres cut the common chord into two equal parts then,
AC = $\frac{12}{2}$ = 6 cm
Now, applying Pythagoras theorem in triangle O_1AC
⇒ $O_1A^2 = O_1C^2 + AC^2$
⇒ $O_1C^2 = O_1A^2 - AC^2 = 15^2 - 6^2 = 189$
⇒ $O_1C = \sqrt{189}$
Similarly, In triangle O_2CA
⇒ $O_2A^2 = O_2C^2 + AC^2$
⇒ $O_2C^2 = O_2A^2 - AC^2 = 12^2 - 6^2 = 144 – 36 = 108$
⇒ $O_2C = \sqrt{108}$
The distance between their centres = $\sqrt{189} + \sqrt{108}$

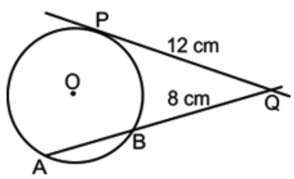
Q:6 (2) E, F, G and H are four points lying on the circumference of a circle to make a cyclic quadrilateral.
If ∠FGH = 57°
As we know,
The sum of opposite angles in a quadrilateral = 180°
According to the question
⇒ ∠FGH + ∠HEF = 180°
⇒ 57° + ∠HEF = 180°
⇒ ∠HEF = 180° - 57° = 123°

Q:7 (4) The radius of a circle (C) = 5 cm



AB = 12 cm
In a triangle OAB, $OB^2 = OA^2 + AB^2 = 25 + 144 = 169$
OB = $\sqrt{169}$ = 13 cm
Radius OA = OR = 5 cm
BR = OB - OR = 13 - 5 = 8 cm

Q:8 (3) In a circle centred at O, PQ is a tangent at P
If PQ = 12 cm and QB = 8 cm



According to the circle tangent theorem
⇒ $PQ^2 = AQ \times BQ$
⇒ $12^2 = (AB + 8) \times 8$
⇒ AB + 8 = $\frac{144}{8}$ = 18 ⇒ AB = 18 - 8 = 10 cm

Q:9 (2) Two circles with radii 6 cm and 4 cm.
Distance between their centres is 14 cm.
Transverse common tangent = $\sqrt{D^2 - (R + r)^2}$
We have, D = 14 cm, r_1 = 6 cm and r_2 = 4 cm
Now, using the above formula we get,
⇒ Transverse common tangent = $\sqrt{14^2 - (6+4)^2}$
= $\sqrt{96} = 4\sqrt{6}$

Q:10 (3) Distance between the centres of two circles is 12 cm. Radii are 5 cm and 4 cm.
Transverse common tangent = $\sqrt{D^2 - (R+r)^2}$ ---- (1)
Where D = distance between the centre of two circles
R and r are the radius of two circles
Using equation (1), we get,
⇒ $\sqrt{12^2 - (5+4)^2} = \sqrt{63}$

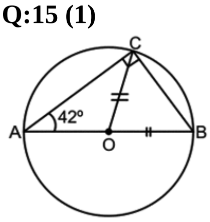
Q:11(1) A circle of radius (r) = 14 cm
An arc subtends an angle of (θ) = 90° at the centre
Arc length (l) = $r\theta \times \frac{\pi}{180} = 14 \times 90 \times \frac{\frac{22}{7}}{180} = 22$ cm

Q:12 (2) If ∠ARB = 70º , then ∠ARO = 35º
In Triangle(OAR) , ∠OAR = 90º and ∠ARB = 35º
∠ROA = 180 - 90 - 35 = 180 - 125 = 55º
Similarly, ∠ROB = 55º
Thus, ∠AOB = 110º

Q:13 (1) According to the question,
in quadrilateral OSPR,
⇒ ∠ P + ∠ S + ∠ O + ∠ R = 360°
⇒ 25° + 90° + ∠ O + 90° = 360°
⇒ ∠ ROS = 360° - 205° = 155°

Q:14 (4) Length of the direct common tangent = $\sqrt{D^2 - (R-r)^2}$ ---- (1)
Where D = distance between the centre of two circles
R and r are the radius of two circles
Using equation (1), we get
⇒ $\sqrt{12^2 - (7-4)^2} = \sqrt{135} = 3\sqrt{15}$ cm

Q:15 (1)



In △ABC, ∠A + ∠B + ∠C = 180°
[∵Angle sum property]
⇒ 42° + ∠B + 90° = 180°
[∵Angle in a semi-circle is 90°]
⇒ ∠B = 180° - 90°- 42° = 48°
In △BOC, ∠OBC + ∠BOC + ∠BCO = 180°
⇒ 48° + 48° + ∠BOC = 180°
[∵Angles opposite to equal sides are equal]
⇒ ∠BOC = 180° - 48° - 48° = 84° [BO = CO = r]