# Sharpness-Aware Minimization For Efficiently Improving Generalization

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ICLR 2021 (Spotlight)

# Contents

# Motivation

# Problem setting

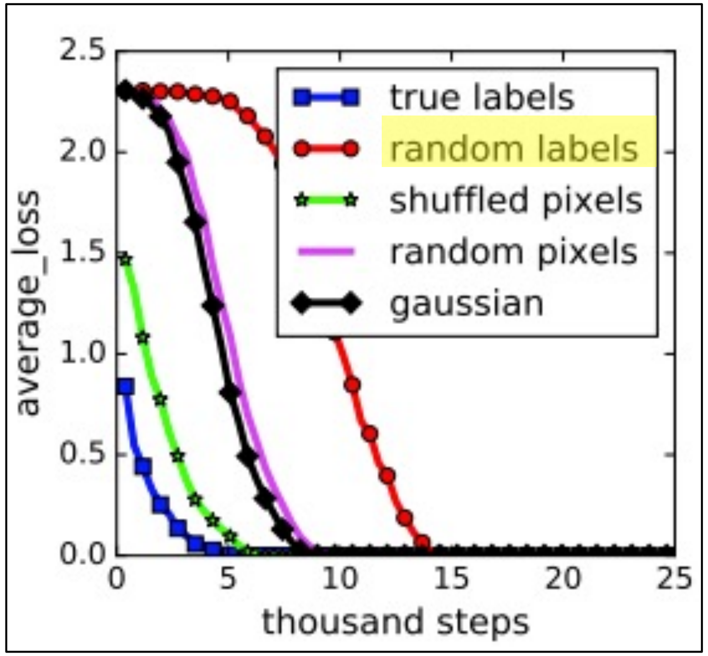
- Notation
  - Training dataset : $S = \cup_{i=1}^{n} \{(x_i, y_i)\} \sim \mathcal{D}$
  - Model parameter : $w \in \mathcal{W} \subseteq \mathbb{R}^k$
  - Empirical loss function : $L_S(w) = \frac{1}{n}\sum_{i=1}^{n} l(w, x_i, y_i)$
  - Population loss function : $L_{\mathcal{D}}(w) = \mathbb{E}_{(x,y)\sim\mathcal{D}}[l(w, x, y)]$

- Objective
  - Select model parameter $w$ having low population loss $L_{\mathcal{D}}(w)$

* Conventional approach : $\min_{\mathrm{w}} L_S(w) + \lambda\|w\|_2^2$

# Minimizing training loss is not sufficient

- Deep neural networks easily fit random labels
- Explicit regularization is not sufficient for controlling generalization error



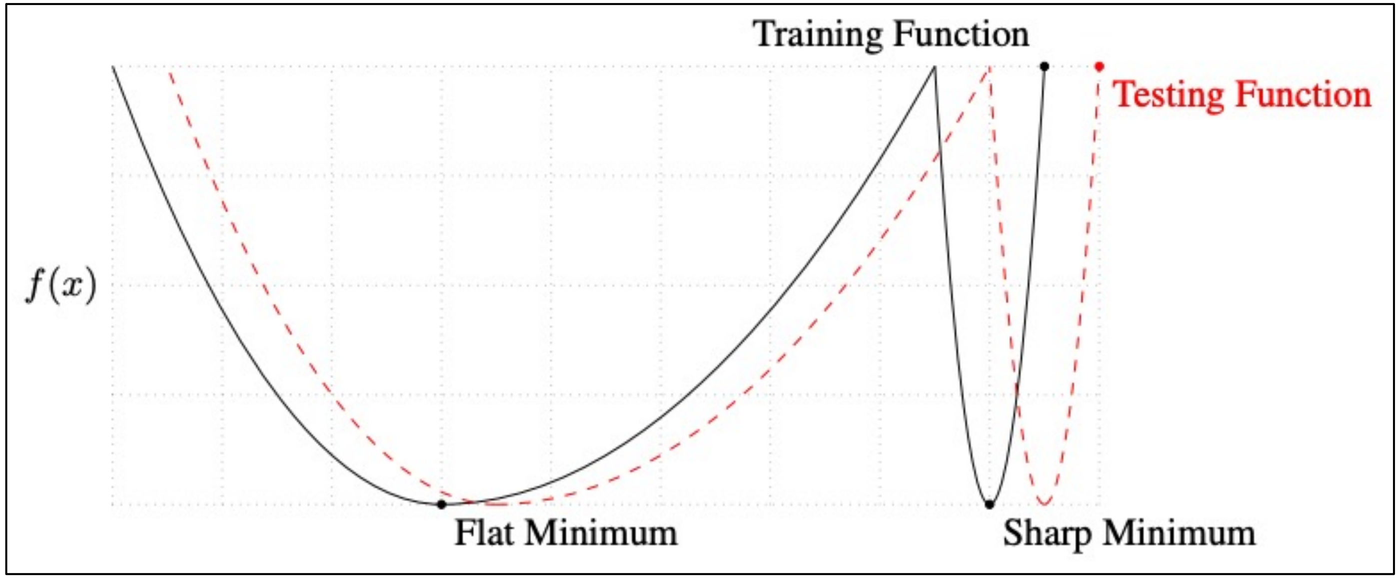

Training loss decaying with steps

| data aug | dropout | weight decay | top-1 train | top-5 train | top-1 test | top-5 test |
|---|---|---|---|---|---|---|
| ImageNet 1000 classes with the original labels | | | | | | |
| yes | yes | yes | 92.18 | 99.21 | 77.84 | 93.92 |
| yes | no | no | 92.33 | 99.17 | 72.95 | 90.43 |
| no | no | yes | 90.60 | 100.0 | 67.18 (72.57) | 86.44 (91.31) |
| no | no | no | 99.53 | 100.0 | 59.80 (63.16) | 80.38 (84.49) |
| Alexnet (Krizhevsky et al., 2012) | | | - | - | - | 83.6 |
| ImageNet 1000 classes with random labels | | | | | | |
| no | yes | yes | 91.18 | 97.95 | 0.09 | 0.49 |
| no | no | yes | 87.81 | 96.15 | 0.12 | 0.50 |
| no | no | no | 95.20 | 99.14 | 0.11 | 0.56 |

Performance on ImageNet with true labels and random labels

Zhang, Chiyuan, et al. "Understanding deep learning (still) requires rethinking generalization." *ICLR 2017.*

# Loss Landscape v.s. Generalization



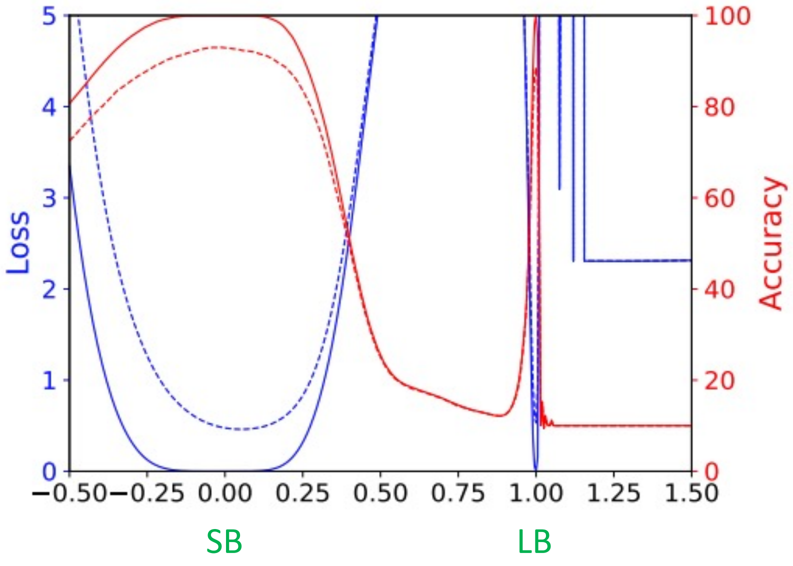

If there exists a shift from empirical loss to population loss, flat minimum is more robust than sharp minimum.

- Smoothening loss landscape : Skip connection, Batch normalization
- Escaping basins of sharp minima : Small-batch training

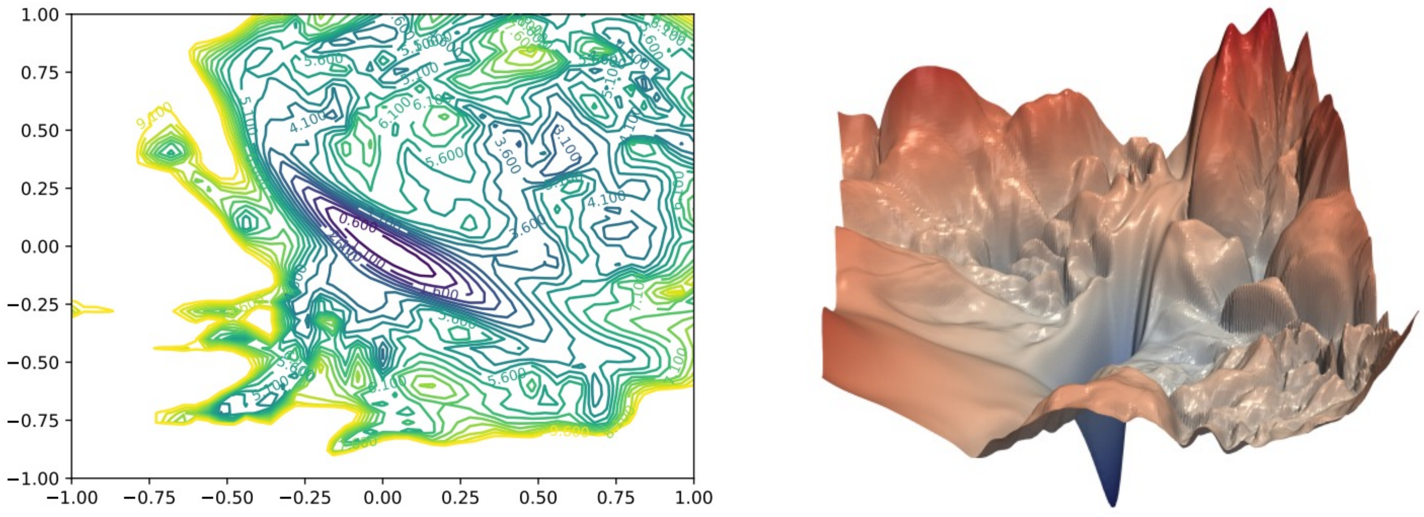

*Keskar, Nitish Shirish, et al. "On large-batch training for deep learning: Generalization gap and sharp minima." ICLR 2017.*
*Santurkar, Shibani, et al. "How does batch normalization help optimization?." NeurIPS 2018.*

# How to analyze sharpness?

1D linear interpolation




2D contour plot and its 3D visualization



$L(\alpha w_l^* + (1-\alpha) w_s^*)$ with $-1 \leq \alpha \leq 1$

($w_s^*$ : converged w/ Small Batch (SB) training)

($w_l^*$ : converged w/ Large Batch (LB) training)

$L(w + \alpha\delta + \beta\eta)$ with $-1 \leq \alpha, \beta \leq 1$

($\delta$ and $\eta$ are random weights of the same size of w)

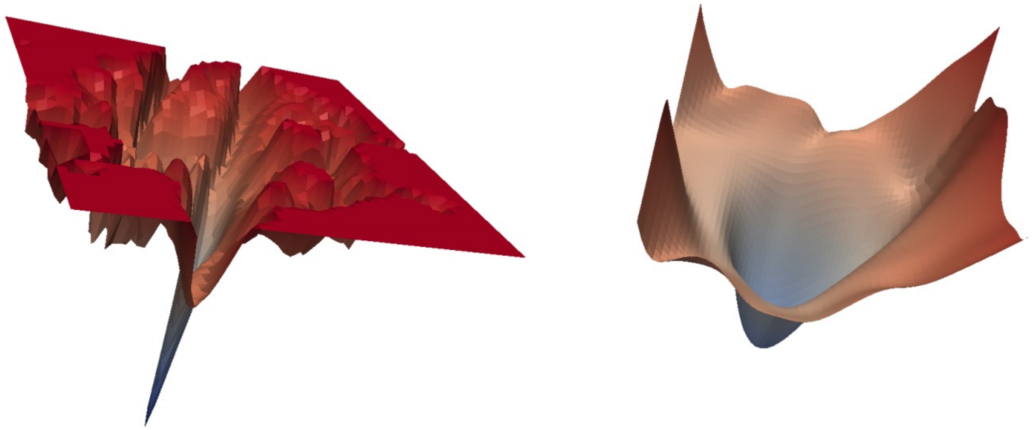
(Filter-wise normalization in aware of *scale invariance* )

Previous studies have shown better generalization, however, yet struggle to beat a range of state-of-the-art models

Li, Hao, et al. "Visualizing the loss landscape of neural nets.", *NeurIPS 2018.*

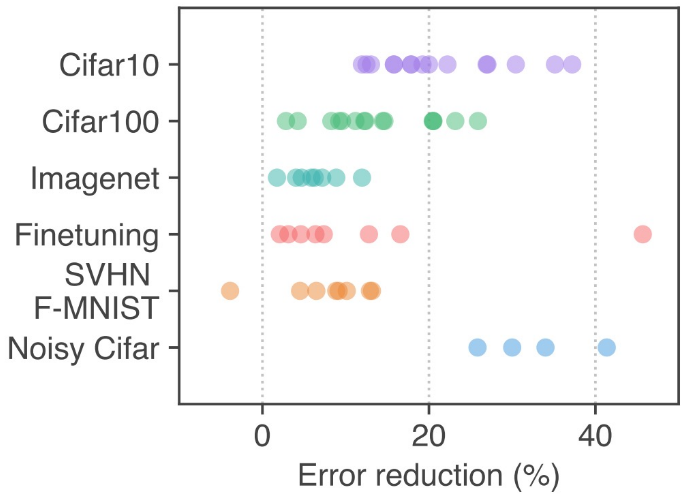
# Sharpness-Aware Minimization (SAM)

- Find a wide and flat minima by simultaneously minimizing loss and sharpness
  1. Seeks parameters that lie in neighborhoods having uniformly low loss
  2. Conduct a rigorous empirical study across a range of widely studied computer vision tasks



(a) Without SAM (b) With SAM

3D visualization of the loss landscape
around the converged solutions




Error rate reduction by switching to SAM
(dataset / model / augmentation)

# Preliminary

# PAC versus PAC-bayes

- PAC (Probably Approximately Correct)

With probability greater than $1-\delta$,

$$L_{\mathcal{D}}(w) \le L_S(w) + \sqrt{\frac{1}{2n}\left(\ln|\mathcal{H}| + \ln\left(\frac{2}{\delta}\right)\right)}$$

- PAC-bayes

With probability greater than $1-\delta$,

$$\mathbb{E}_{w\sim Q}[L_{\mathcal{D}}(w)] \le \mathbb{E}_{w\sim Q}[L_S(w)] + \sqrt{\frac{1}{2(n-1)}\left(KL(Q|P) + ln\left(\frac{n}{\delta}\right)\right)}$$

Dziugaite, Gintare Karolina, and Daniel M. Roy. "Computing nonvacuous generalization bounds for deep (stochastic) neural networks with many more parameters than training data." *UAI 2016*

# Component-wise level bound

$$\left[\mathbb{E}_{w\sim Q}L_{\mathcal{D}}(w)\right] \leq \mathbb{E}_{w\sim Q}[L_S(w)] + \sqrt{\frac{1}{2(n-1)}\left(KL(Q|P) + ln\left(\frac{n}{\delta_j}\right)\right)}$$

Let $P = \mathcal{N}\left(0, c\exp\left(\frac{1-j}{k}\right)I\right)$ for $j \in \mathbb{N}$ and $Q = \mathcal{N}(w, \sigma^2 I)$

- $KL(Q|P) \leq \frac{1}{2}\left[1 + k\log\left(1 + \frac{\|w\|_2^2}{k\sigma^2}\right)\right]$
- $ln\left(\frac{n}{\delta_j}\right) \leq \log\left(\frac{n}{\delta}\right) + 2\log(6n+3k)$

Please refer to Appendix in paper

$$\mathbb{E}_{\epsilon\sim\mathcal{N}(0,\sigma^2 I)}[L_{\mathcal{D}}(w+\epsilon)] \leq \mathbb{E}_{\epsilon\sim\mathcal{N}(0,\sigma^2 I)}[L_S(w+\epsilon)] + \sqrt{\frac{1}{2(n-1)}\left(\frac{1}{2}\left[1 + k\log\left(1+\frac{\|w\|_2^2}{k\sigma^2}\right)\right] + \log\left(\frac{n}{\delta}\right) + 2\log(6n+3k)\right)}$$

Kingma, Diederik P., and Max Welling. "Auto-encoding variational bayes.", *ICLR 2014*

# How can we constrain the neighborhood?

- Directly constraining $\epsilon$ is not intuitive due to high-dimensional nature
- Rather propose to use *norm*

If $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$, then $\|\epsilon\|_2^2 \sim \chi^2(k\sigma^2)$

$$\Pr\left(\|\epsilon\|_2^2 - k\sigma^2 \geq 2\sigma^2\sqrt{kt} + 2t\sigma^2\right) \leq \exp(-t)$$

With probability greater than $1 - 1/\sqrt{n}$,

$$\|\epsilon\|_2^2 \leq k\sigma^2 + 2\sigma^2\sqrt{kt} + 2t\sigma^2 = k\sigma^2\left(1 + 2\sqrt{t/k} + 2t/k\right)$$
$$= k\sigma^2\left(1 + 2\sqrt{\ln\sqrt{n}/k} + 2\ln\sqrt{n}/k\right) (:= \rho^2)$$

Laurent, Beatrice, and Pascal Massart. "Adaptive estimation of a quadratic functional by model selection." *Annals of Statistics* (2000)

# PAC-bayes Generalization Bound for SAM

$$\mathbb{E}_{\epsilon\sim\mathcal{N}(0,\sigma^2 I)}[L_{\mathcal{D}}(w+\epsilon)]$$

$$\leq \mathbb{E}_{\epsilon\sim\mathcal{N}(0,\sigma^2 I)}[L_S(w+\epsilon)] + \sqrt{\frac{1}{2(n-1)}\left(\frac{1}{2}\left[1+k\log\left(1+\frac{\|w\|_2^2}{k\sigma^2}\right)\right]+\log\left(\frac{n}{\delta}\right)+2\log(6n+3k)\right)}$$

$$\leq \left(1-\cancel{\frac{1}{\sqrt{n}}}\right)\max_{\|\epsilon\|_2\leq\rho} L_S(w+\epsilon)+\cancel{\frac{1}{\sqrt{n}}}+\sqrt{\frac{1}{4(n-1)}\left(1+k\log\left(1+\frac{\|w\|_2^2}{k\sigma^2}\right)+2\log\left(\frac{n}{\delta}\right)+4\log(6n+3k)\right)}$$

$$\leq \max_{\|\epsilon\|_2\leq\rho} L_S(w+\epsilon)+\sqrt{\frac{1}{4(n-1)}\left(1+k\log\left(1+\frac{\|w\|_2^2}{\rho^2}\left(1+\sqrt{\frac{2\ln(n)}{k}}\right)^2\right)+2\log\left(\frac{n}{\delta}\right)+4\log(6n+3k)\right)}$$

# SAM optimizer

# Framing into min-max optimization problem

$$\min_{w} \max_{\|\epsilon\|_2 \le \rho} L_S(w+\epsilon) + \sqrt{\frac{1 + k\log\left(1 + \frac{\|w\|_2^2}{\rho^2}\left(1 + \sqrt{\frac{2\ln(n)}{k}}\right)^2\right) + 2\log\left(\frac{n}{\delta}\right) + 4\log(6n+3k)}{4(n-1)}}$$

$$= \min_{w} \max_{\|\epsilon\|_2 \le \rho} L_S(w+\epsilon) + \mathrm{h}\left(\frac{\|w\|_2^2}{\rho^2}\right) \approx \min_{w} \max_{\|\epsilon\|_p \le \rho} L_S(w+\epsilon) + \lambda\|w\|_2^2$$

$$= \min_{w} \max_{\|\epsilon\|_p \le \rho} L_S(w+\epsilon) - L_S(w) + L_S(w) + \lambda\|w\|_2^2$$

Sharpness Training Loss Regularizer

How quickly the training loss can be increased by moving from $w$ to a nearby parameter value

# Solving min-max optimization problem

**1.** $\boldsymbol{\max_{\|\epsilon\|_p \le \rho} L_S(w+\epsilon)}$

$$\epsilon^*(w) = \underset{\|\epsilon\|_p \le \rho}{\arg\max}\, L_S(w+\epsilon) \approx \underset{\|\epsilon\|_p \le \rho}{\arg\max}\, L_S(w) + \epsilon^T \nabla_w L_S(w) = \underset{\|\epsilon\|_p \le \rho}{\arg\max}\, \epsilon^T\, \nabla_w L_S(w)$$

$$\approx \rho \frac{sign\big(\nabla_w L_S(w)\big) |\nabla_w L_S(w)|^{\frac{q}{p}}}{\big(\|\nabla_w L_S(w)\|_q\big)^{\frac{q}{p}}} \;(:= \hat{\epsilon}(w)) \;\; where \quad \frac{1}{p} + \frac{1}{q} = 1$$

If $p = 2,$ then $\|\hat{\epsilon}(w)\|_2 = \rho \quad and \quad \hat{\epsilon}(w) = \rho \nabla_w L_s(w) / \|\nabla_w L_S(w)\|_2$

# Solving min-max optimization problem

**2.** $\boldsymbol{\min_{w} L_S(w + \hat{\epsilon}(w))}$

$$\nabla_w L_S(w_t + \hat{\epsilon}(w_t)) = \frac{d\big(w + \hat{\epsilon}(w)\big)}{dw} \nabla_w L_S(w)\Big|_{w_t + \hat{\epsilon}(w_t)}$$

$$= \nabla_w L_S(w)|_{w_t + \hat{\epsilon}(w_t)} + \cancel{\frac{d\hat{\epsilon}(w)}{dw} \nabla_w L_S(w)|_{w_t + \hat{\epsilon}(w_t)}}$$

Hessian computation

$$\approx \nabla_w L_S(w)|_{w_t + \hat{\epsilon}(w_t)}$$

By Stochastic Gradient Descent (SGD), $w_{t+1} = w_t - \eta \nabla_w L_S(w)|_{w_t + \hat{\epsilon}(w_t)}$

# Experiment

# Image classification

| Model | Augmentation | CIFAR-10 SAM | CIFAR-10 SGD | CIFAR-100 SAM | CIFAR-100 SGD |
|---|---|---|---|---|---|
| WRN-28-10 (200 epochs) | Basic | **2.7**$_{\pm 0.1}$ | 3.5$_{\pm 0.1}$ | **16.5**$_{\pm 0.2}$ | 18.8$_{\pm 0.2}$ |
| WRN-28-10 (200 epochs) | Cutout | **2.3**$_{\pm 0.1}$ | 2.6$_{\pm 0.1}$ | **14.9**$_{\pm 0.2}$ | 16.9$_{\pm 0.1}$ |
| WRN-28-10 (200 epochs) | AA | **2.1**$_{\pm <0.1}$ | 2.3$_{\pm 0.1}$ | **13.6**$_{\pm 0.2}$ | 15.8$_{\pm 0.2}$ |
| WRN-28-10 (1800 epochs) | Basic | **2.4**$_{\pm 0.1}$ | 3.5$_{\pm 0.1}$ | **16.3**$_{\pm 0.2}$ | 19.1$_{\pm 0.1}$ |
| WRN-28-10 (1800 epochs) | Cutout | **2.1**$_{\pm 0.1}$ | 2.7$_{\pm 0.1}$ | **14.0**$_{\pm 0.1}$ | 17.4$_{\pm 0.1}$ |
| WRN-28-10 (1800 epochs) | AA | **1.6**$_{\pm 0.1}$ | 2.2$_{\pm <0.1}$ | **12.8**$_{\pm 0.2}$ | 16.1$_{\pm 0.2}$ |
| Shake-Shake (26 2x96d) | Basic | **2.3**$_{\pm <0.1}$ | 2.7$_{\pm 0.1}$ | **15.1**$_{\pm 0.1}$ | 17.0$_{\pm 0.1}$ |
| Shake-Shake (26 2x96d) | Cutout | **2.0**$_{\pm <0.1}$ | 2.3$_{\pm 0.1}$ | **14.2**$_{\pm 0.2}$ | 15.7$_{\pm 0.2}$ |
| Shake-Shake (26 2x96d) | AA | **1.6**$_{\pm <0.1}$ | 1.9$_{\pm 0.1}$ | **12.8**$_{\pm 0.1}$ | 14.1$_{\pm 0.2}$ |
| PyramidNet | Basic | **2.7**$_{\pm 0.1}$ | 4.0$_{\pm 0.1}$ | **14.6**$_{\pm 0.4}$ | 19.7$_{\pm 0.3}$ |
| PyramidNet | Cutout | **1.9**$_{\pm 0.1}$ | 2.5$_{\pm 0.1}$ | **12.6**$_{\pm 0.2}$ | 16.4$_{\pm 0.1}$ |
| PyramidNet | AA | **1.6**$_{\pm 0.1}$ | 1.9$_{\pm 0.1}$ | **11.6**$_{\pm 0.1}$ | 14.6$_{\pm 0.1}$ |
| PyramidNet+ShakeDrop | Basic | **2.1**$_{\pm 0.1}$ | 2.5$_{\pm 0.1}$ | **13.3**$_{\pm 0.2}$ | 14.5$_{\pm 0.1}$ |
| PyramidNet+ShakeDrop | Cutout | **1.6**$_{\pm <0.1}$ | 1.9$_{\pm 0.1}$ | **11.3**$_{\pm 0.1}$ | 11.8$_{\pm 0.2}$ |
| PyramidNet+ShakeDrop | AA | **1.4**$_{\pm <0.1}$ | 1.6$_{\pm <0.1}$ | **10.3**$_{\pm 0.1}$ | 10.6$_{\pm 0.1}$ |

Comparison of SAM and SGD for the test error rate on CIFAR-{10, 100}
(Similar trend is observed on SVHN and Fashion-MNIST)

# Image classification

| Model | Epoch | SAM | | Standard Training (No SAM) | |
|---|---|---|---|---|---|
| | | Top-1 | Top-5 | Top-1 | Top-5 |
| ResNet-50 | 100 | **22.5**$_{\pm 0.1}$ | 6.28$_{\pm 0.08}$ | 22.9$_{\pm 0.1}$ | 6.62$_{\pm 0.11}$ |
| | 200 | **21.4**$_{\pm 0.1}$ | 5.82$_{\pm 0.03}$ | 22.3$_{\pm 0.1}$ | 6.37$_{\pm 0.04}$ |
| | 400 | **20.9**$_{\pm 0.1}$ | 5.51$_{\pm 0.03}$ | 22.3$_{\pm 0.1}$ | 6.40$_{\pm 0.06}$ |
| ResNet-101 | 100 | **20.2**$_{\pm 0.1}$ | 5.12$_{\pm 0.03}$ | 21.2$_{\pm 0.1}$ | 5.66$_{\pm 0.05}$ |
| | 200 | **19.4**$_{\pm 0.1}$ | 4.76$_{\pm 0.03}$ | 20.9$_{\pm 0.1}$ | 5.66$_{\pm 0.04}$ |
| | 400 | **19.0**$_{\pm <0.01}$ | 4.65$_{\pm 0.05}$ | 22.3$_{\pm 0.1}$ | 6.41$_{\pm 0.06}$ |
| ResNet-152 | 100 | **19.2**$_{\pm <0.01}$ | 4.69$_{\pm 0.04}$ | 20.4$_{\pm <0.0}$ | 5.39$_{\pm 0.06}$ |
| | 200 | **18.5**$_{\pm 0.1}$ | 4.37$_{\pm 0.03}$ | 20.3$_{\pm 0.2}$ | 5.39$_{\pm 0.07}$ |
| | 400 | **18.4**$_{\pm <0.01}$ | 4.35$_{\pm 0.04}$ | 20.9$_{\pm <0.0}$ | 5.84$_{\pm 0.07}$ |

Comparison of SAM and SGD for the test error rate on ImageNet

# Finetuning the pretrained model

Pretrained on ImageNet

Pretrained on ImageNet and unlabeled JFT

| Dataset | EffNet-b7 + SAM | EffNet-b7 | Prev. SOTA (ImageNet only) | EffNet-L2 + SAM | EffNet-L2 | Prev. SOTA |
|---|---|---|---|---|---|---|
| FGVC_Aircraft | $6.80_{\pm 0.06}$ | $8.15_{\pm 0.08}$ | **5.3** (TBMSL-Net) | $\mathbf{4.82}_{\pm 0.08}$ | $5.80_{\pm 0.1}$ | 5.3 (TBMSL-Net) |
| Flowers | $\mathbf{0.63}_{\pm 0.02}$ | $1.16_{\pm 0.05}$ | 0.7 (BiT-M) | $\mathbf{0.35}_{\pm 0.01}$ | $0.40_{\pm 0.02}$ | 0.37 (EffNet) |
| Oxford_IIIT_Pets | $\mathbf{3.97}_{\pm 0.04}$ | $4.24_{\pm 0.09}$ | 4.1 (Gpipe) | $\mathbf{2.90}_{\pm 0.04}$ | $3.08_{\pm 0.04}$ | 4.1 (Gpipe) |
| Stanford_Cars | $5.18_{\pm 0.02}$ | $5.94_{\pm 0.06}$ | **5.0** (TBMSL-Net) | $4.04_{\pm 0.03}$ | $4.93_{\pm 0.04}$ | **3.8** (DAT) |
| CIFAR-10 | $\mathbf{0.88}_{\pm 0.02}$ | $0.95_{\pm 0.03}$ | 1 (Gpipe) | $\mathbf{0.30}_{\pm 0.01}$ | $0.34_{\pm 0.02}$ | 0.63 (BiT-L) |
| CIFAR-100 | $\mathbf{7.44}_{\pm 0.06}$ | $7.68_{\pm 0.06}$ | 7.83 (BiT-M) | $\mathbf{3.92}_{\pm 0.06}$ | $4.07_{\pm 0.08}$ | 6.49 (BiT-L) |
| Birdsnap | $\mathbf{13.64}_{\pm 0.15}$ | $14.30_{\pm 0.18}$ | 15.7 (EffNet) | $\mathbf{9.93}_{\pm 0.15}$ | $10.31_{\pm 0.15}$ | 14.5 (DAT) |
| Food101 | $7.02_{\pm 0.02}$ | $7.17_{\pm 0.03}$ | 7.0 (Gpipe) | $\mathbf{3.82}_{\pm 0.01}$ | $3.97_{\pm 0.03}$ | 4.7 (DAT) |
| ImageNet | $15.14_{\pm 0.03}$ | 15.3 | **14.2** (KDforAA) | $\mathbf{11.39}_{\pm 0.02}$ | 11.8 | 11.45 (ViT) |

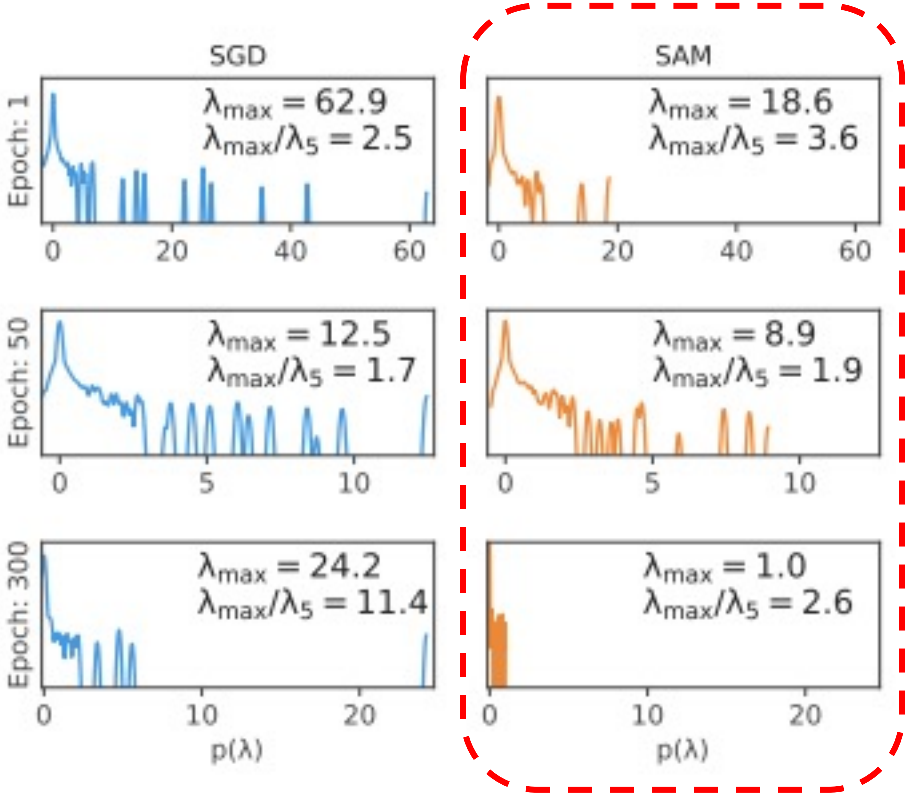
Top-1 error rates for finetuning EffNet-b7 or EffNet-L2 to various target datasets

# Robustness to label noise

| Method | Noise rate (%) | | | |
|---|---|---|---|---|
| | 20 | 40 | 60 | 80 |
| Sanchez et al. (2019) | 94.0 | 92.8 | 90.3 | 74.1 |
| Zhang & Sabuncu (2018) | 89.7 | 87.6 | 82.7 | 67.9 |
| Lee et al. (2019) | 87.1 | 81.8 | 75.4 | - |
| Chen et al. (2019) | 89.7 | - | - | 52.3 |
| Huang et al. (2019) | 92.6 | 90.3 | 43.4 | - |
| MentorNet (2017) | 92.0 | 91.2 | 74.2 | 60.0 |
| Mixup (2017) | 94.0 | 91.5 | 86.8 | 76.9 |
| MentorMix (2019) | **95.6** | **94.2** | 91.3 | **81.0** |
| SGD | 84.8 | 68.8 | 48.2 | 26.2 |
| Mixup | 93.0 | 90.0 | 83.8 | 70.2 |
| Bootstrap + Mixup | 93.3 | 92.0 | 87.6 | 72.0 |
| SAM | 95.1 | 93.4 | 90.5 | 77.9 |
| Bootstrap + SAM | 95.4 | 94.2 | 91.8 | 79.9 |

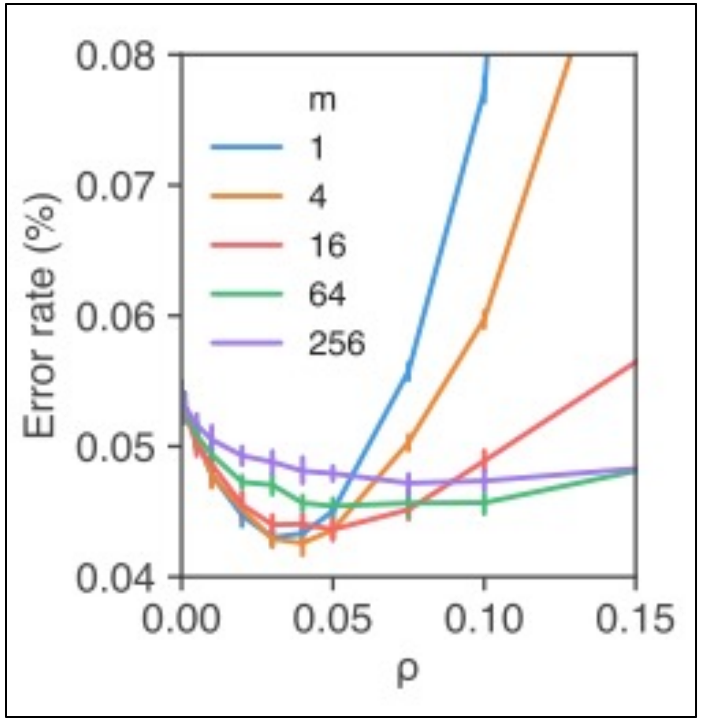
Test accuracy on clean test dataset while trained with noisy labels

# Sharpness and Generalization




Evolution of hessian spectrum along epoch

- SGD : to sharp minima
- SAM : to flat minima




Effect of batch size($m$) to test error rate

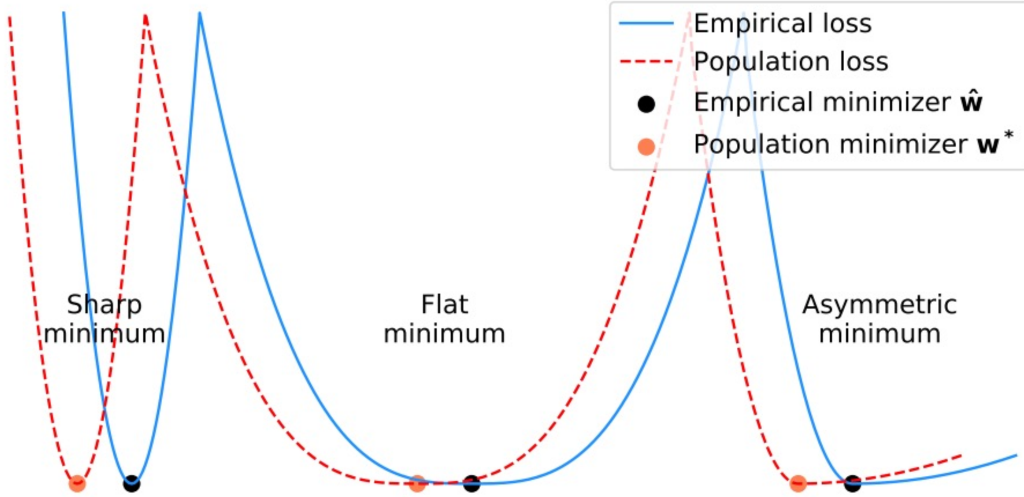
- Small batch training leads to flat minima
- Model gets less robustness to hyperparameter $\rho$

# Critique

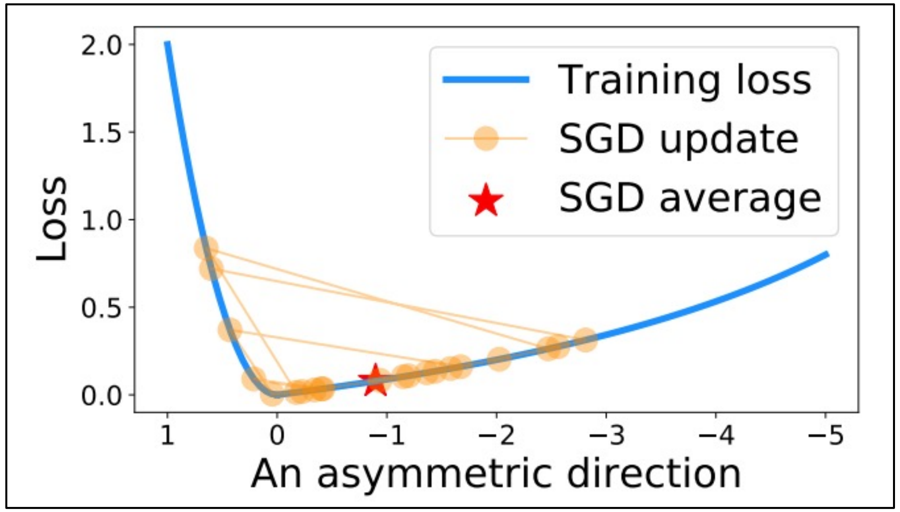
# In-between sharpness and flatness?

Known to be in sharp minima

By sampling the loss function in a neighborhood of LB solutions, we observe that it rises steeply only along a small dimensional subspace (e.g. 5% of the whole space); <u>on most other directions, the function is relatively flat</u>




Notion of sharpness and flatness may be oversimplification

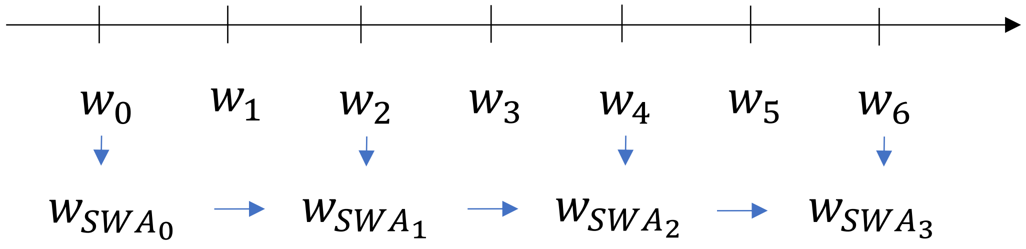


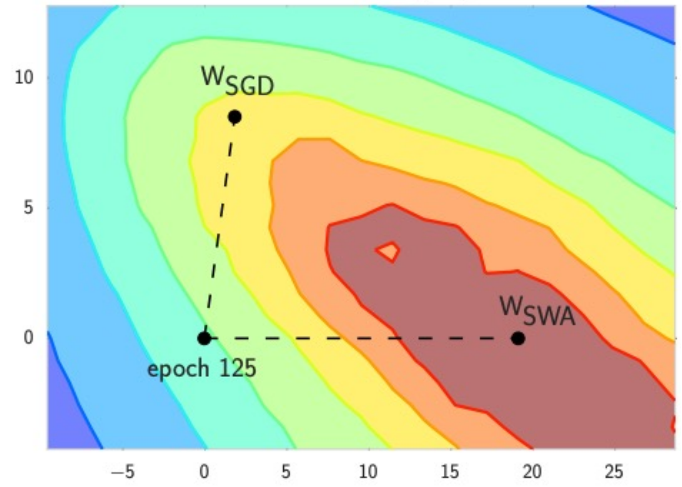

Averaging SGD trajectory leads to flat side

*Keskar, Nitish Shirish, et al. "On large-batch training for deep learning: Generalization gap and sharp minima." ICLR 2017.*
*He, Haowei, Gao Huang, and Yang Yuan. "Asymmetric valleys: Beyond sharp and flat local minima." NeurIPS 2019.*

# How to free the choice of batch size?

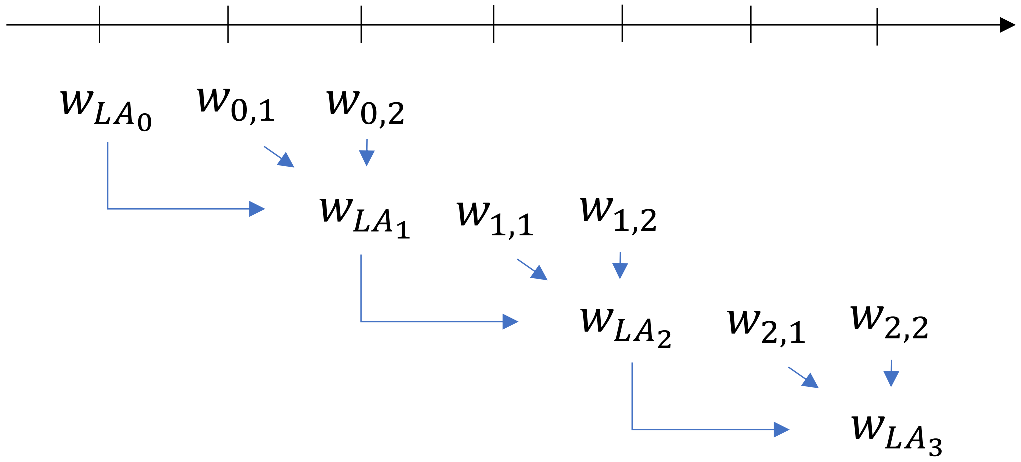
**Stochastic Weight Averaging (SWA), *UAI 2018***

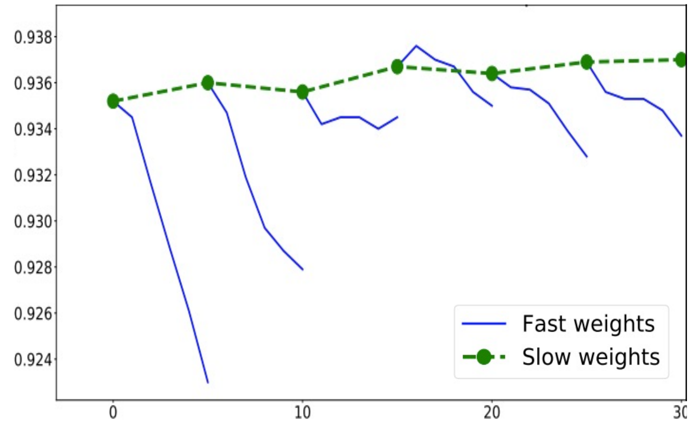



Contour plot of test loss surface

**Look ahead(LA), *NeurIPS 2019***




Test accuracy along epoch


*Izmailov, Pavel, et al. "Averaging weights leads to wider optima and better generalization.", UAI 2018.*
Zhang, Michael R., et al. "Lookahead optimizer: k steps forward, 1 step back.", NeurIPS, 2019.

Thank you for your attention

# Appendix

# Learning algorithm w/ PAC-bayes bound

1. Fix a probability $\delta > 0$ and a distribution $P = \mathcal{N}(\mu_P, \sigma_P^2 I)$
2. Collect an i.i.d. dataset $S$ of size $n$
3. Compute the optimal distribution $Q = \mathcal{N}(\mu_Q, \sigma_Q^2 I)$ that minimizes

$$\mathbb{E}_{w\sim Q}[L_S(w)] + \sqrt{\frac{1}{2(n-1)}\left(KL(Q|P) + ln\left(\frac{n}{\delta}\right)\right)}$$

4. Return the randomized classifier given by $Q$

* *Closed form expression of KL divergence :* $KL(Q|P) = \frac{1}{2}\left[\frac{k\sigma_Q^2 + \|\mu_P - \mu_Q\|_2^2}{\sigma_P^2} - k + k\ln\left(\frac{\sigma_P^2}{\sigma_Q^2}\right)\right]$

Dziugaite, Gintare Karolina, and Daniel M. Roy. "Computing nonvacuous generalization bounds for deep (stochastic) neural networks with many more parameters than training data." *UAI 2016*

# How can we set the prior $P$ in advance?

- Predefine values for $\sigma_P^2$ and pick the best one in that set
  - Set $\delta_j = \frac{6\delta}{\pi^2 j^2}$ and $\sigma_P^2 \in T \coloneqq \left\{ c \exp\left(\frac{1-j}{k}\right) \middle| j \in \mathbb{N} \right\}$

Let $A_j$ is an event s.t. $\mathbb{E}[Z] > \bar{Z} + g(\delta_j)$ where $\Pr\left(A_j\right) < \delta_j$

Then, $\Pr\left(\cup_{j \in \mathbb{N}} A_j\right) < \sum_{j \in \mathbb{N}} \delta_j = \frac{6\delta}{\pi^2} \sum_{j \in \mathbb{N}} \frac{1}{j^2} = \delta$

$\therefore$ With probability $1 - \delta$, PAC-bayes bound holds for all $\sigma_P^2 \in T$

Langford, John, and Rich Caruana. "(Not) bounding the true error." *NIPS 2001*