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Optimization Project - Part B

In Part B of the Deep Space Nine power optimization project, we at the Starfleet Corps of Engineers were tasked with adjusting our solution from the initial stage of the project to ensure that our solution was presented in a more fair and balanced manner. This was done after learning that the distribution of the nodes was split into seven separate subgroups, numbered 1 through 7. This then raised the question of how the demand could be satisfied while still ensuring that one subgroup would not be significantly lacking in terms of the proportion of its demand met.

To address the issue with regards to fairness in the distribution of power among the groups of residents, we first created a metric to measure the fairness of a solution. Our fairness metric we determined is as follows:

$$min \ \frac{Demand\ Satisfied\ for\ group\ i}{Total\ Demand\ for\ group\ i}\ for\ i\ = 1, ..., 7$$

This metric will return the value of the lowest proportion of demand satisfied for a group, thus attempting to showcase the minimum standard of fairness achieved. The solution given by the existing LP model returns:

Group 1 had proportion 1 of its demand satisfied
Group 2 had proportion 0.958333 of its demand satisfied
Group 3 had proportion 0.66 of its demand satisfied
Group 4 had proportion 0.583333 of its demand satisfied
Group 5 had proportion 0.636364 of its demand satisfied
Group 6 had proportion 0.35 of its demand satisfied
Group 7 had proportion 0.285714 of its demand satisfied

Therefore, the value that our fairness metric returns is 0.285714. One shortcoming of using a ratio of quantity satisfied versus quantity demanded is that not all groups will get the same amount of flow. Because some groups have smaller aggregate demand relative to other groups, it is possible there will be a large disparity of flow supplied to small versus larger groups. Also, adding a constraint will cause the calculated value of the model to decrease from the maximum total flow from part A. Based on the percentage we chose, some groups may receive close to 100% of flow demanded (which is more fair) but may also only supply closer to X% of quantity demanded (less fair) for other groups. There could be a situation where the metric scores are fair (percent of demand satisfied is close to 100%) but one solution may have energy flow more evenly distributed across the nodes in it’s group than the other (which is more fair in context).

Our next step in the process was to create an LP model to maximize our fairness metric such that the highest threshold of fairness could be achieved.

<u>LP Model:</u>

Parameters:

N = set of nodes in network given by data - indexed i (i = 1,2,...,n)

A = set of directed arcs in network, one for each conduit given by data

G = set of groups (of nodes) in network given by data - indexed g (g = 1,2,...,n)

c_a = maximum capacity of conduit a, one parameter for each a $\in$ A

d_i = maximum demand of node i, one parameter for each i $\in$ N

d_g = maximum demand of group g, one parameter for each g $\in$ G

Variables:

x_a = flow on arc a, one variable for each a $\in$ A

y_i = demand satisfied for each node i, one variable for each i $\in$ N

y_g = demand satisfied for each group g, one variable for each g $\in$ G

Z = ratio of group demand satisfied (y_g) over maximum group demand (d_g) → (y_g / d_g)

$$\max \quad Z$$

$$\text{s.t.} \quad \sum_{a \in \delta^-(i)} X_a - \sum_{a \in \delta^+(i)} X_a = Y_i \qquad \forall i \in N \setminus \{1\}$$

$$-C_a \leq X_a \leq C_a \qquad \forall a \in A$$

$$0 \leq Y_i \leq d_i \qquad \forall i \in N$$

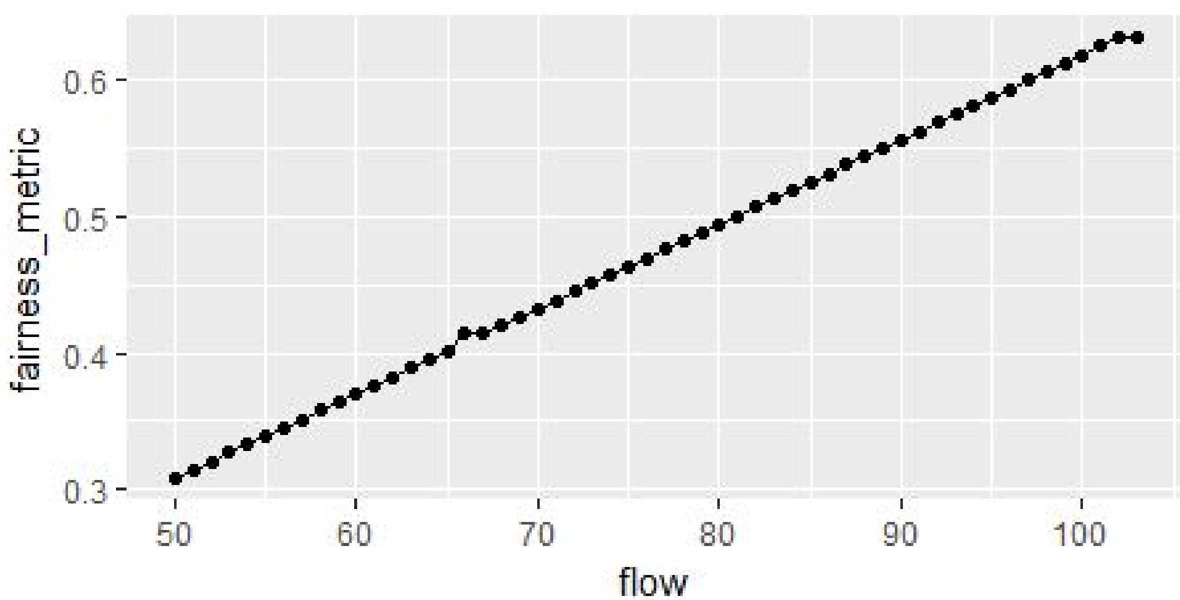
$$Z \leq \min\left(\frac{Y_g}{d_g}\right) \qquad \forall g \in G$$

Where $\delta^+(i)$ = Set of all arcs leaving i

$\delta^-(i)$ = Set of all arcs entering i

In the code for this metric and its ensuing LP model, we added another constraint (m.addConstr(flow.sum('*',sink)>=.95*103)), which ensures that 95% of the maximum possible power (103) is flowing into the network. It sums the flow of all demand satisfied from each node into the sink node 0 and creates a constraint for it to be at least .95 * 103. We implemented the constraint for Z to be less than or equal to the minimum of group demand satisfied over maximum group demand for each group. This created a “group is only as strongest as its weakest link” policy in which the minimum of Z was maximized. The capacity constraint, maximum demand constraint for nodes, and flow constraint were all the same as Part A of this project. The maximum demand constraint for groups was not needed because it was already implied and satisfied from the maximum demand constraint for nodes. For our fairness metric constraint, we aggregated the amount of flow for each group and appended it to an empty list. Next, we divided each value in the list by the total quantity of energy demanded for each group, which resulted in a list of ratios (representing our fairness metric percentage). Finally, we iterated through the objects in this list and created a constraint for our fairness metric variable, Z, to be less than or equal to each object in the list of fairness ratios.

In order to confirm the efficiency of our fairness metric, the Harmony Council requested to see a complete trade-off curve that demonstrated the greatest level of fairness achieved with varying levels of flow. To confirm our belief that this was indeed the strongest metric possible, we plotted the value of our fairness metric against a range total demand satisfied. The results are below:



The trade-off curve between the fairness metric and the flow shows that maximizing the fairness metric will yield an optimal solution no matter what percentage of maximum possible demand (103) is given as a constraint. This shows that the fairness essentially does not trade-off with the total power supplied. This is due to the fact that our metric is only concerned with a single minimum value out of all the ratios, while other metrics may utilize every single ratio of demand satisfied/demand required. This was another shortcoming of our metric. An alternative fairness metric that will address this issue is as follows:

$$\sum_{i=1}^{7} m_i$$

Where $m_i = \frac{\textit{Demand Satisfied for group } i}{\textit{Total Demand for group } i}$ for i = 1,...,7

Adding up these ratios, we will be considering the proportion of demand satisfied for every group rather than focusing just on the minimum of one. This should create a trade-off between this fairness metric and the total power supplied. However, because smaller groups require smaller amounts of power, their ratios will be easier to increase and therefore will be increased by the metric if applied. This will result in a complete trade-off between fairness and the total power supplied. For example, if 3 groups out of the 7 only require a quarter of the power of a 4th group, those groups will be filled with power with priority over the 4th one simply due to the fact that a ratio of 1 is far easier to achieve with these smaller demand requirements. An LP model to this alternative fairness metric is as follows:

$$\max \quad \sum_{i=1}^{7} m_i$$

$$\text{s.t.} \quad \sum_{a\in\delta^-} x_a - \sum_{a\in\delta^+} x_a = y_i \quad \forall i \in N \setminus \{1\}$$

$$-c_a \le x_a \le c_a \quad \forall a \in A$$

$$0 \le y_i \le d_i \quad \forall i \in N$$

$$m_i = \frac{y_g}{d_g} \quad \forall g \in G$$

$\delta^+(i)$ = set of all arcs leaving i

$\delta^-(i)$ = set of all arcs entering i

The next step in our work was to consult with an engineering team that could increase the power flow within the station. After realizing that a more optimal power solution may stem from a physical upgrade of the conduits themselves, Deep Space Nine decided to negotiate a purchase of bolts, each of which increased the capacity of a conduit by 1 unit. It would take three hours for an engineer to access each conduit upon which installation would occur and an additional one hour for each bolt installed. The task given to our group was to determine which improvements should be made to maximize the total demand fulfilled. We created a parameter, expressed as H in the ILP below, to determine the set number of hours required by the engineers to make improvements.

ILP Model:

Parameters:

N = set of nodes in network given by data - indexed i (i = 1,2,...,n)

A = set of directed arcs in network, one for each conduit given by data

c_a = maximum capacity of conduit a, one parameter for each a $\in$ A

d_i = maximum demand of node i, one parameter for each i $\in$ N

H = the total number of hours the engineers worked

Variables:

x_a = flow on arc a, one variable for each a $\in$ A

y_i = demand satisfied for each node i, one variable for each i $\in$ N

b_a = binary variable (0 or 1), equals 1 if engineers work on conduit a, 0 if not

m_a = number of bolts installed on conduit a

$$\max \sum_{i \in N} y_i$$

s.t.

$$\sum_{a \in \delta^-} x_a - \sum_{a \in \delta^+} x_a = y_i \qquad \forall i \in N \setminus \{1\}$$

$$-c_a \le x_a \le c_a \qquad \forall a \in A$$

$$0 \le y_i \le d_i \qquad \forall i \in N$$

$$H = \sum_{a \in A} (3b_a + m_a)$$

$$b = [0,1] \qquad m_a \ge 0$$

$\delta^+(i)$ = set of all arcs leaving i

$\delta^-(i)$ = set of all arcs entering i

In the code created we simulated the ILP to find the theoretical maximum of total demand that could be satisfied if the engineering team was given unlimited hours to work. Firstly, we added the new hypothetical capacity to each conduit and made it bidirectional to account for the two-way energy flow. Then, we updated the arc dictionary so that this increase in capacity would be accounted for. Next, we inserted the new variables required to simulate this process which included one for the total engineering hours, a Gurobi variable to hold a maximum ceiling for the increase in capacity, and a binary variable that would indicate whether a particular conduit was worked on or not. Lastly, we implemented constraints that updated the flow capacity to account for a maximum increase, set a max improvement to be greater than 0 on the occasion that the binary variable set earlier was 1, and finally set the amount of time an engineer works to 3 times the binary variable plus the improvement capacity. Following the implementation and execution of this code, the theoretical maximum result that we achieved came out to 154 units of power given the unlimited access to engineering hours.

While 154 units of power is the maximum possible generation of power given an unlimited access to engineering hours, the engineering team wanted to understand how the time it spares for power grid improvements impacts the amount of power that the network will be able to deliver, within the range of 40 to 80 hours. Thus, we created another trade-off curve which displayed the amount of flow and power delivered based upon hours worked by the engineering team. The resulting graph is shown below, exhibiting a consistent, piecewise increase in the maximum flow possible, culminating in the 154 units of power to be generated given 80 hours of work by the engineering team. The engineering team can utilize this data to determine what energy they would like to theoretically maximize at based upon their allotted work hours.