Meeg342 - Heat Exchanger Design Project

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1 Problem Definition

In this design scenario we look to understand the characteristics of the different properties of a heat exchanger. The team is given key constraints that limit the design. The affect of mass flowrate, number of tubes, and length of tubes will be varied and their affects will be studied. This counter-flow concentric-tube heat exchanger needs to heat 290K with supply waste water which is at 360K. The main constrains/specifications of the system are the following:

1. The length of each tube is not to exceed 10 m
2. The outlet temperature of the hot water must be at least 10K higher than the inlet temperature of the cold water, namely, $T_{h,o} \geq 300K$
3. The maximum number of tubes allowed is 8
4. The waste hot water flow rate is 100 liters/minute
5. Both the inner and outer tube diameters are held constant at 22mm and 45mm respectively

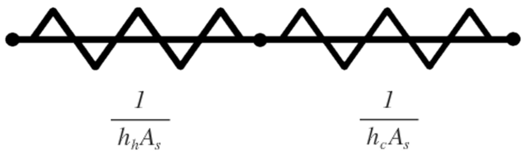
2 Single Tube Varying Mass Flowrate

To first understand more about the design problem the team first studied a single pipe at constant length with the cold water flowing in the inner tube and the hot water in the outer tube annulus. The only variable varied in this experiment was the mass flowrate of the cold temperature stream. This provides insight into how the cold water flow has an effect on the outlet temperatures. The problem is formulated as the following:

$$C_h = \dot{m}_h C_{ph} \quad C_h = \dot{m}_c C_{pc} \quad \rightarrow \quad C_r = \frac{C_{min}}{C_{max}} \tag{1}$$

$$NTU = \frac{1}{R_{total} C_{min}} \tag{2}$$

To calculate this NTU value first the total resistance needs to be calculated for the heat exchanger. The thermal resistance circuit diagram can be constructed and drawn as the following:



The internal pipe of the heat exchanger is taken to be "very thin", so the conduction though this pipe surface can be modeled as zero. The first step to finding the total resistance requires finding the convection coefficient for both the inner and outer surface of the heat exchanger. This requires the Reynolds number along with the Nusslet number for each surface. To calculate the Reynolds number we can use the equations below. It is important to note that the cross sectional area A_c is different for the inner pipe and outer pipe (i.e. the outer pipe uses the hydraulic diameter, D_h, which is the difference between the outer diameter and the inner diameter.)

$$Re_D = \frac{\rho D_i V_i}{\mu} = \frac{D_i \dot{m}_i}{\mu A_c} \tag{3a}$$

$$Re_{Dh} = \frac{\rho D_h V_i}{\mu} = \frac{D_h \dot{m}_o}{\mu A_c} \tag{3b}$$

It is important to note that the diameter used for the outer tube, D_h, used the hydraulic diameter which is the difference between the outer diameter and the inner diameter. Once the Reynolds number is known for both parts, the Nusslet number can then be calculated using the equations below.

$$Re_D < 3000 : \quad Nu_D = 4.36 \tag{4a}$$

$$Re_D > 3000 : \quad Nu_D = 0.023 Re_D^{0.8} Pr^n \tag{4b}$$

$$Re_{Dh} < 3000: \quad Nu_{Dh} = 5.74 \tag{5a}$$

$$Re_{Dh} > 3000: \quad Nu_{Dh} = 0.023 Re_{Dh}^{0.8} Pr^{n} \tag{5b}$$

Where the following is true for both n values:

$$n = \begin{Bmatrix} 0.3 & if & T_s < T_m \\ 0.4 & if & T_s > T_m \end{Bmatrix} \tag{6}$$

Equations (7) was then used to find the convection coefficient for each tube. Once both h's were found, the total resistance of the system could be calculated using (8) below, where the surface area is the surface area of the inner pipe.

$$\bar{h}_i = \frac{Nu_D k_{fluid}}{D_i} \tag{7a}$$

$$\bar{h}_o = \frac{Nu_{Dh} k_{fluid}}{D_h} \tag{7b}$$

$$R_{total} = \frac{1}{\bar{h}_i A_s} + \frac{1}{\bar{h}_o A_s} \tag{8}$$

Following the calculation of total resistance, the minimum and maximum capacity rate needed to be calculated, as seen in equation (1), which is used to determine the NTU value as stated above. Along with the C_{min} and C_{max} value, the C_r is needed to find the heat exchanger effectiveness which can be found using (1) from above. To find the heat exchanger effectiveness equation (9) was used. Then with the application of equation (10) the rate of heat transfer of the system can be determined.

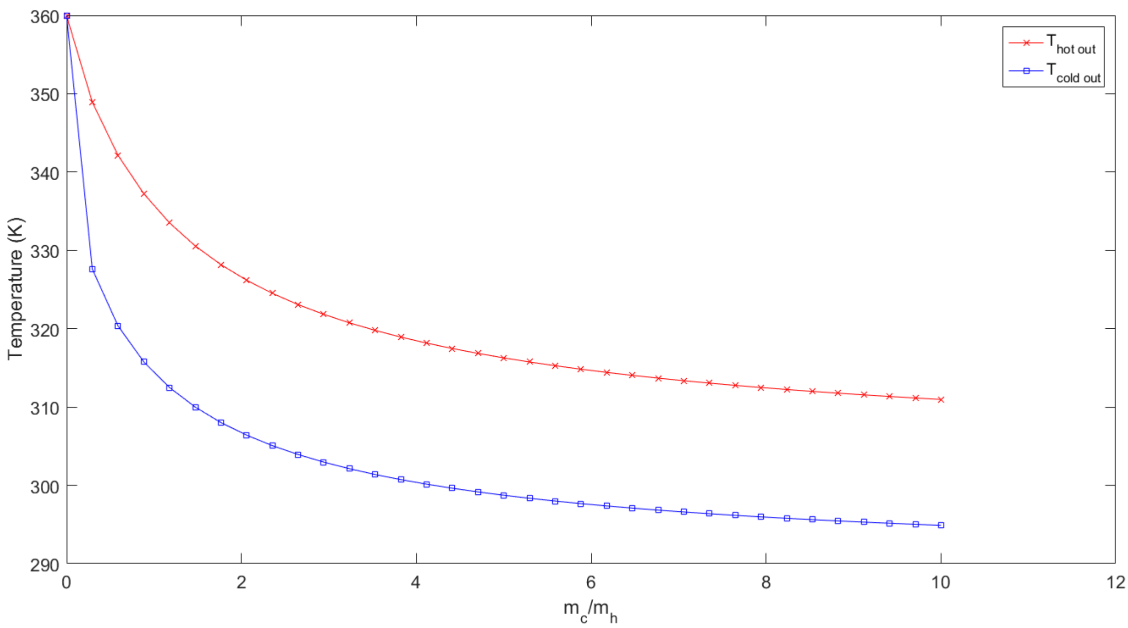
$$\varepsilon = \frac{1 - exp\left(-NTU\left(1 - C_r\right)\right)}{1 - C_r exp\left(-NTU\left(1 - C_r\right)\right)} \tag{9}$$

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \tag{10}$$

This can then be applied to energy balance where the outlet temperatures vary based on the different mass flowrates. We can then find both the hold and cold outlet temperatures of the system. This entire process can be repeated for a different cold mass flowrate and the results can be compared. The results can be seen below.

$$q = C_c\left(T_{c,o} - T_{c,i}\right) \tag{11a}$$

$$q = C_h\left(T_{h,i} - T_{h,o}\right) \tag{11b}$$



3 Multiple Tubes Constant Mass Flowrate

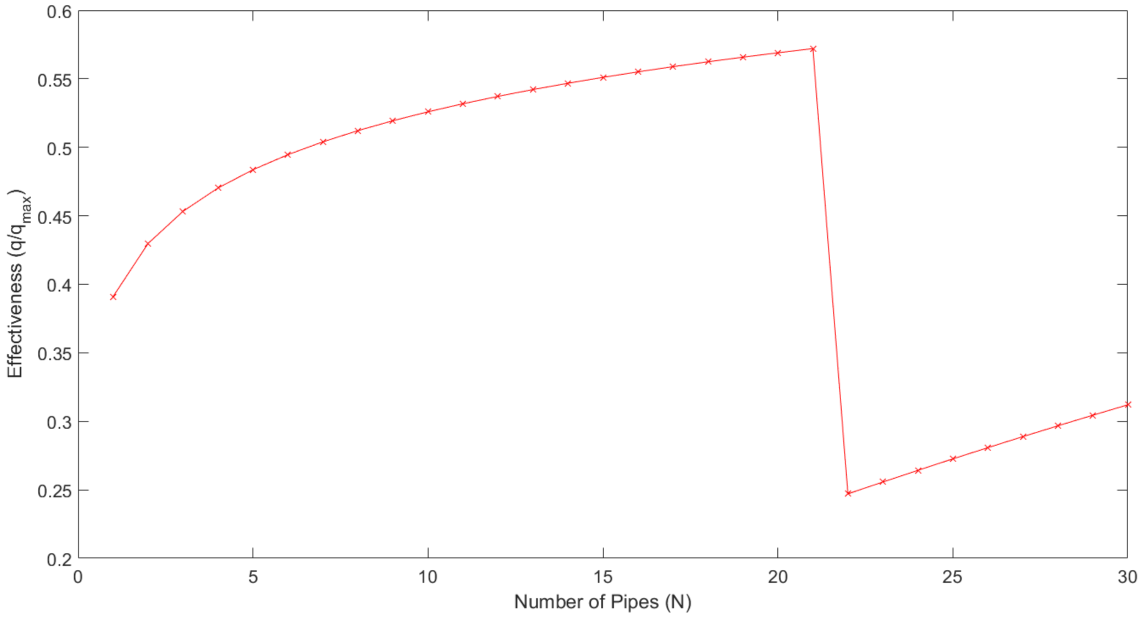
Next the system is formulated for multiple tubes. This is done by finding the individual velocity/mass flowrate that will be flowing through each pipe. The team assumes that the waste water and internal flowrate coming into the heat exchanger will be split evenly to each 10 meter long heat exchanger. All other constants besides number of pipes will be held constant. As seen in part 1 the Nusslet number for each will be calculated using the Reynolds number. This Reynolds number uses a mass flowrate that is a "fraction" of the total that comes into the system.

$$Re_D = \frac{\rho D_i V_{i,partial}}{\mu} = \frac{D_i \dot{m}_i}{N \mu A_c} \tag{12a}$$

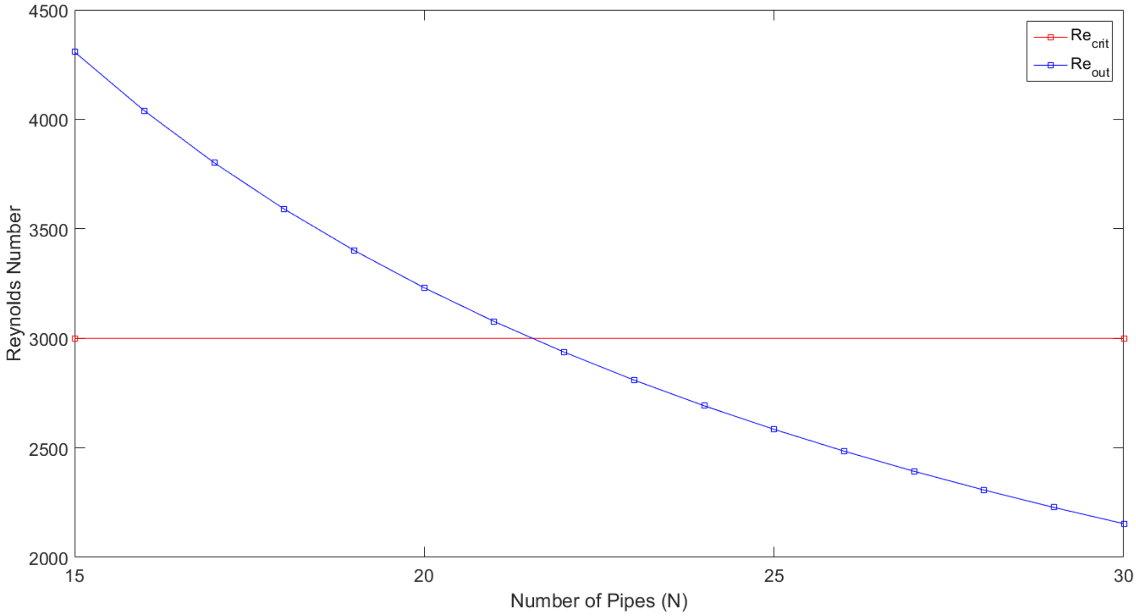
$$Re_{Dh} = \frac{\rho D_h V_{i,partial}}{\mu} = \frac{D_h \dot{m}_o}{N \mu A_c} \tag{12b}$$

Using the above equations and equations (4) and (5) the Nusslet number can be calculated and thus the convection coefficients can be finally calculated. When the total resistance is calculated the number of tubes becomes important again. The convection coefficient calculated is for a single pipe, thus we want to apply this to the entire surface area of the system. This calculation of the total resistance can be seen below.

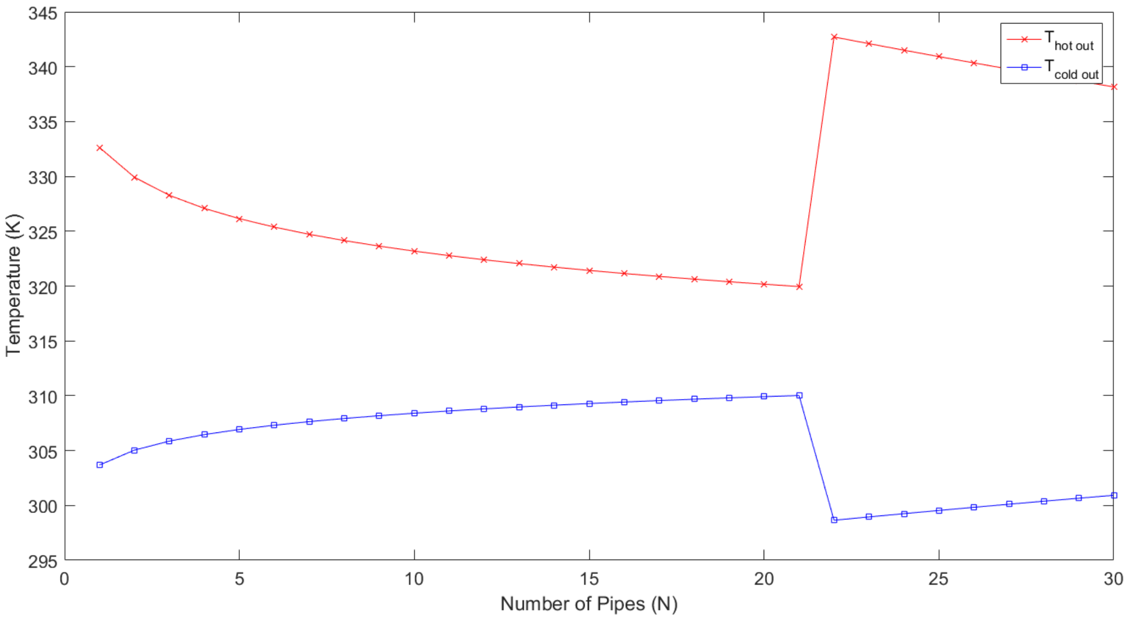
$$R_{total} = \frac{1}{\bar{h}_i N A_s} + \frac{1}{\bar{h}_o N A_s} \tag{13}$$



From the results, it is observed that the effectiveness of the system drops significantly when using approximately 22 tubes. This is because the outside Reynolds number, see figure below, transitions from turbulent to laminar which causes the Nusslet number to change in the process. Note that this effectiveness was calculated with a constant cold water mass flowrate of 2 kg/s.



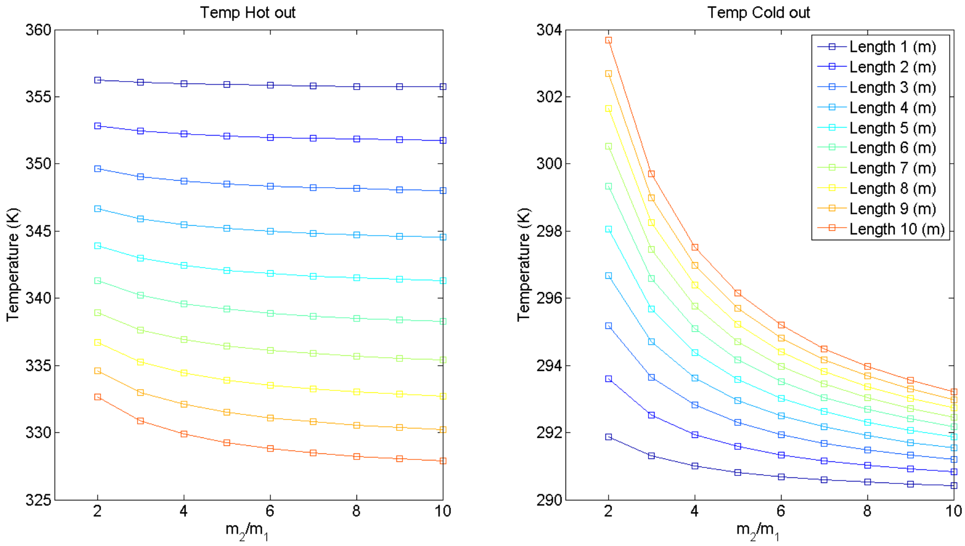
This is an interesting numerical problem with the equations that are used to simulate the flow. Because we do not have an equation representing the transition state from turbulent to laminar we instead have this piecewise function that causes jumps like theses. We can gain insight to how the system would preform with these number of tubes by looking at how the outlet temperatures change with the number of tubes.



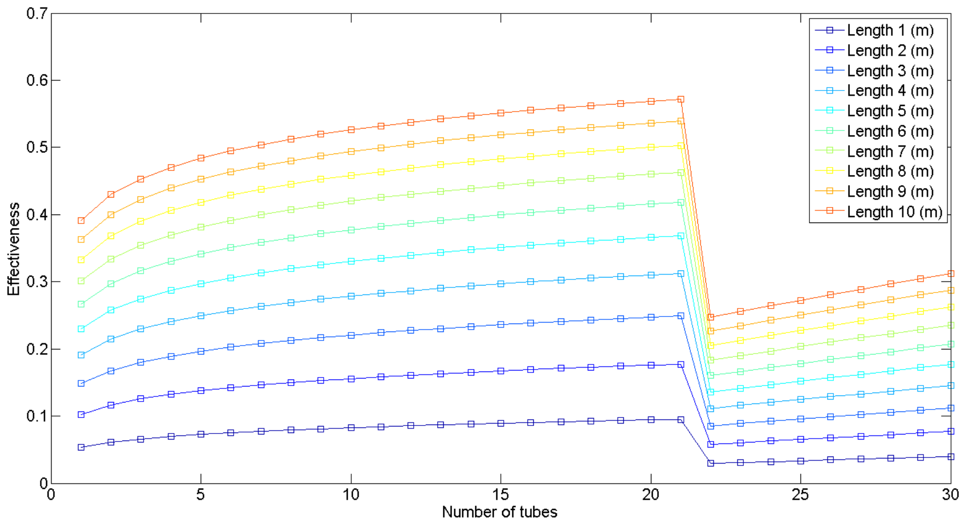
As seen above the outlet temperatures provide an interesting insight into the behavior of the system. We can see that there is the expected jump at 22 tubes, but it is interesting to see that the outlet temperatures actually "reset" to close to their inlet temperatures. This is tied to the Reynolds number, and that the outside flow has now transitioned from turbulent to laminar, and thus it has a decrease in its ability to transfer heat.

4 Varying Lengths and Mass Flowrate

When the design team varied the lengths of the heat exchanger for one tube, the results were interesting. From the results, seen in the figure below, as the length of the tube got longer the hot outlet temperature decreased. However for the cold water leaving the system, the temperature coming out had the opposite effect. As the length of the tube increased the temperature increased and as the mass flow rate ratio increased the temperatures all converged to relatively the same range of temperatures, thus the mass flow rate has little impact after a mass flow ratio of 7.



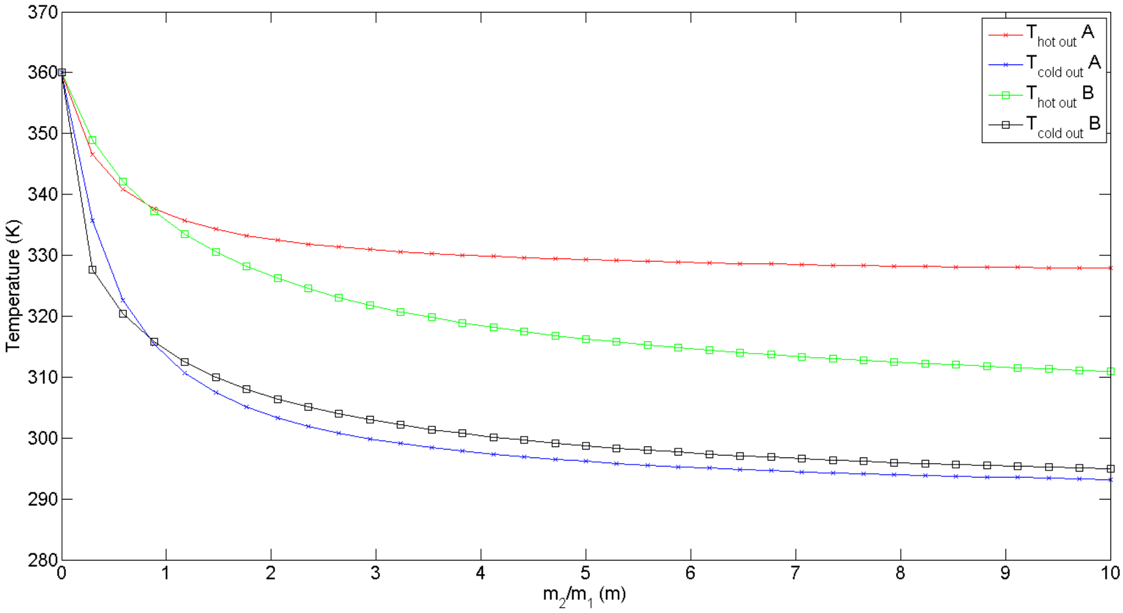
To see how effectiveness changes in a parallel system, the number of tubes of varying length in the system will be compared to the overall effectiveness. As seen in the figure below, as the length of the tube increased from one to ten meters, so did the effectiveness. Like before, the effectiveness drops significantly due to the fact that the Reynolds number changes due to the transition to laminar flow in the outer tube. This shows that for an efficient design a heat exchanger with approximately 22 tubes and a length of 10 meters will result in the most effective heat exchanger. This configuration is optimal because each fluid stream is still in turbulance and has the lowest mass flowrates.



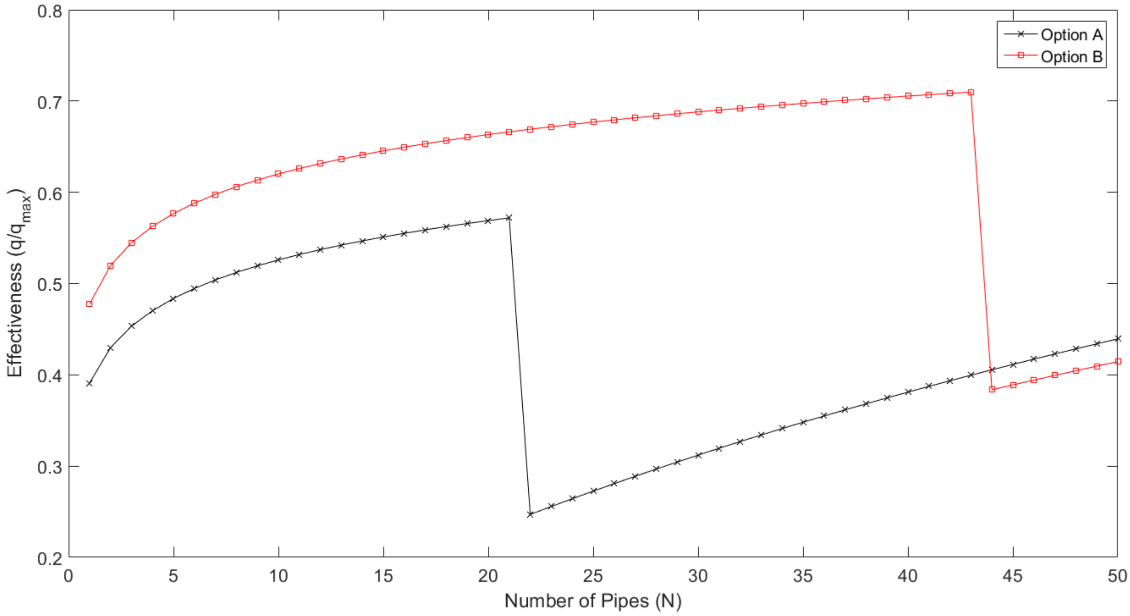
5 Configuration Flow for Option B

Option A consisted of having the fluids travel in counter-flow configuration with the waste water fluid traveling in the outer tube and the cold water traveling through the inner tube. For Option B however, the fluids remain in the counter-flow setup but the cold water now travels through the outer tube and the waste water is traveling through the inner tube.

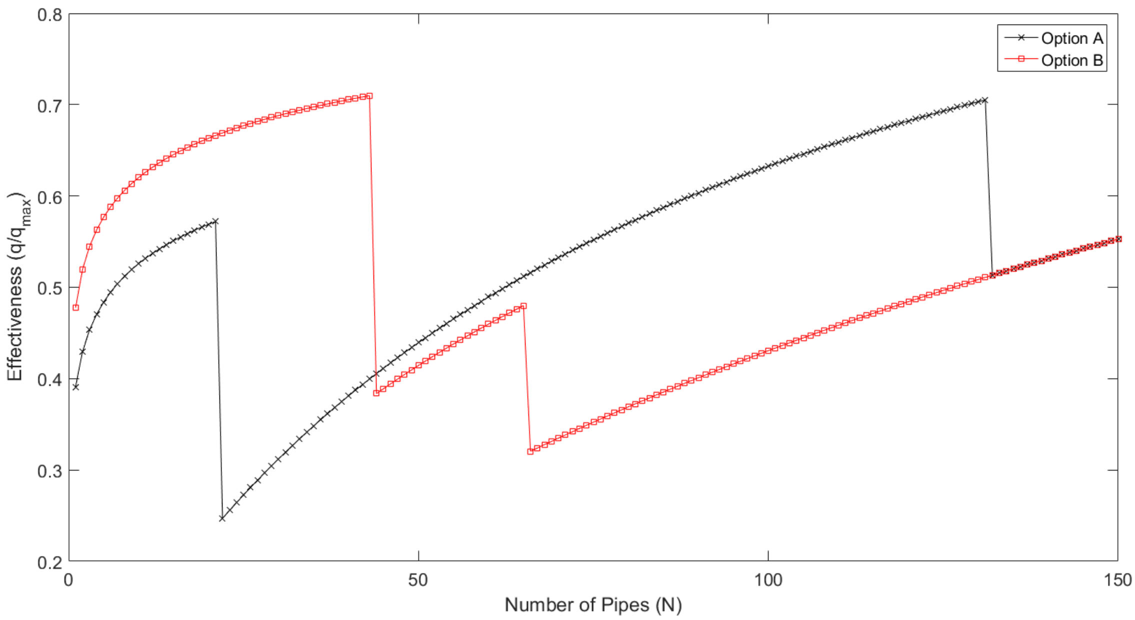
When calculating the outlet temperatures of the hot and cold fluids as the mass flow ratio increased, the plot looked relatively similar. Option A needed to plotted against Option B and a gap between the two data sets can be seen. This can be seen in the figure below and the results show that when the mass flow ratio is below 1, option A is better because the final temperature at the cold water outlet is hotter. However if the mass flow ratio is above 1, option B is greater because the outlet temperature is greater than that of option A. Both options meet the constraint of the outlet of the waste water being 10K higher. Note that the cold mass flowrate is held constant for all cases at 2 kg/s.



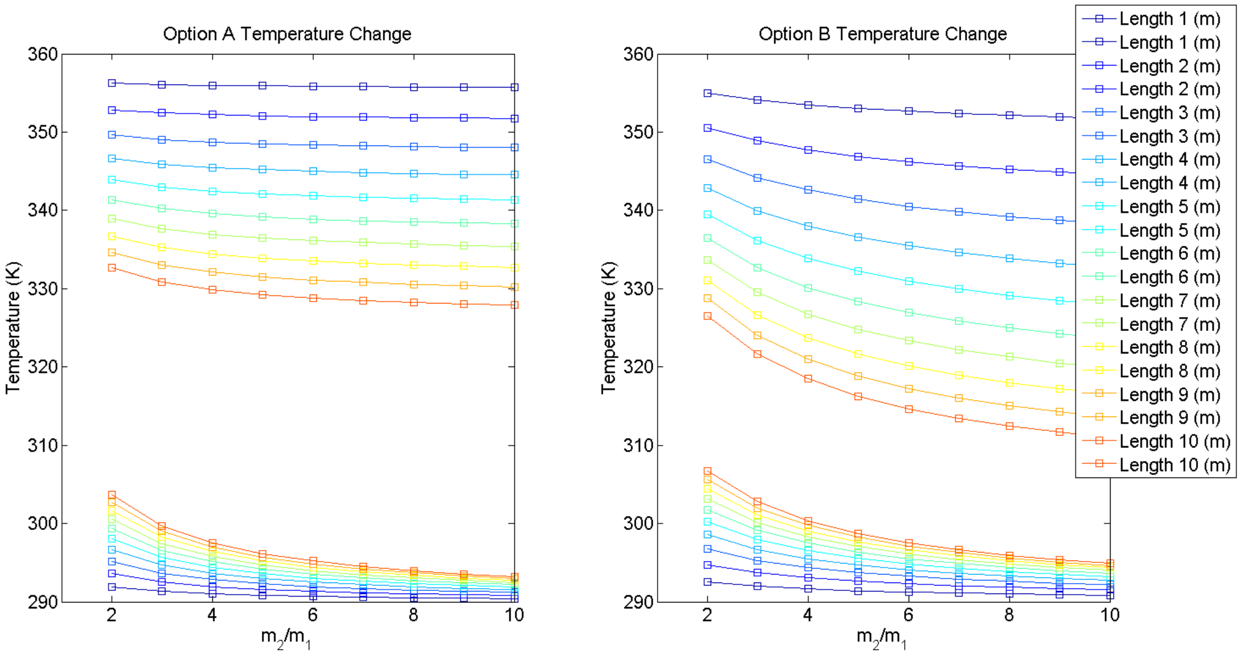
To judge the performance of the two systems, one can look at each of their effectiveness. From the figure below, it can be seen that the overall effectiveness of Option B is far greater than that of Option A. The reason for the effectiveness being greater in Option B is that the cold water flow Reynolds number for the outer flow is greater compared to the cold water flow Reynolds number in option A which increases the overall total resistance which in turn reduces the value of the NTU value and increasing the effectiveness.



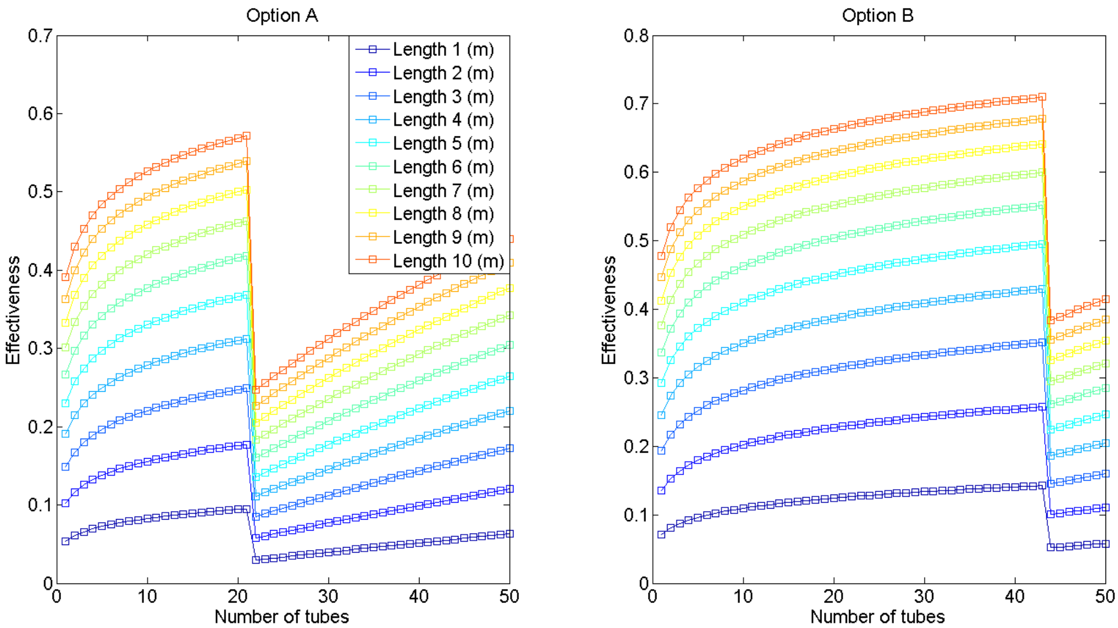
It is interesting to note that their is still the transition to laminar flow of the varying cold mass flowrate. When the cold water is on the outside of the exchanger this happens at a much higher mass flow rate because of the Reynolds number effect described above. It is important to note that when the number of pipes becomes very large we have the hot water transition to a laminar flow. As seen in the figure below this brings the two different configurations to the same performance level when the number of pipes become very large.



For the final part of analyzing option B, the outlet temperatures needed to be observed with different lengths. To do this the value of the mass flow ratio was a constant value for one iteration and the length varied. This produced the figure seen below just with at different ratios of mass flow rate. It can be deduced that the heat exchanger with larger length tubes and small mass flow ratio produces a hotter cold outlet temperature. Refer to Appendix A for individual graphs of hot water outlet and cold water outlet temperatures.



When analyzing the effectiveness of the heat exchanger as the number of tubes increase at different length values, it can be seen in the figure below that a heat exchanger with the largest length also performs the best with approximately 43 tubes. Although the heat exchanger cannot use more than 8 tubes, option B still outperforms Option A at the greatest length of 10 meters.

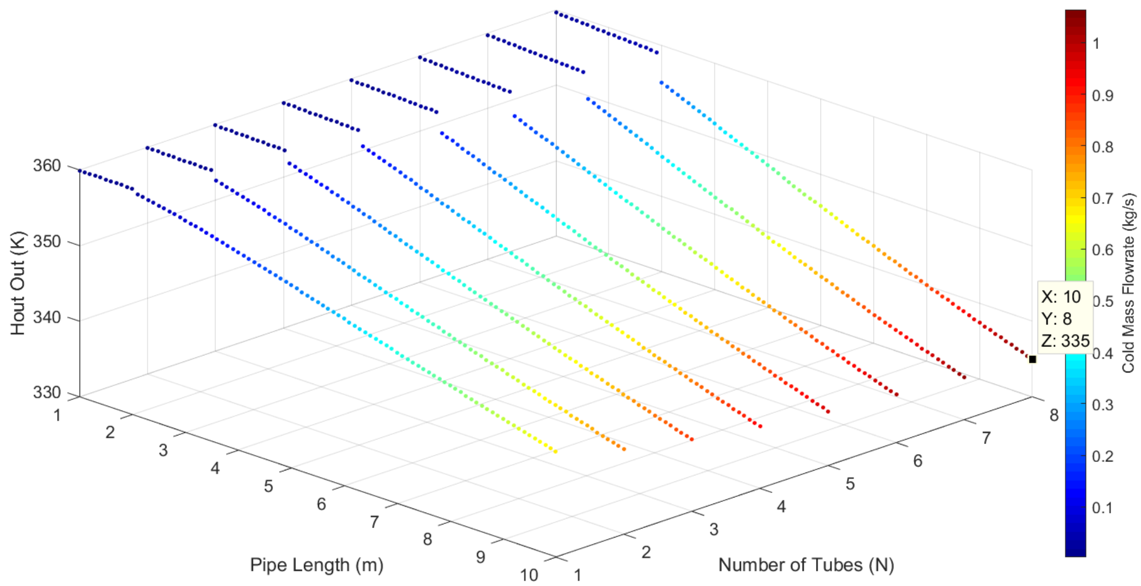


6 The Final Design

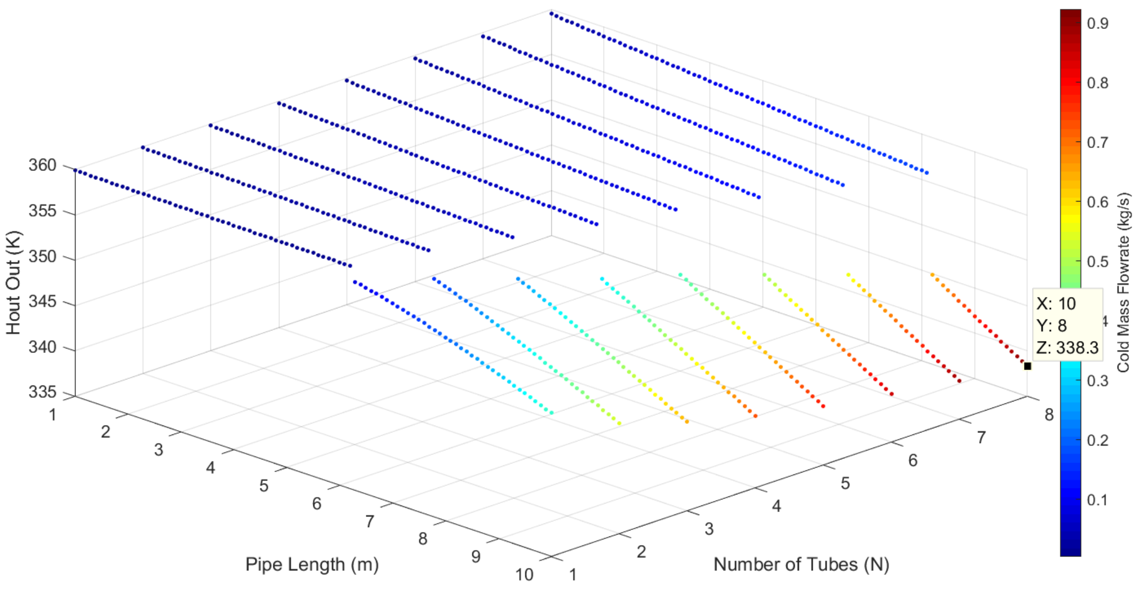
First the team looked at the overall effectiveness of different options. It can be seen that when the mass flowrate is increased the outlet temperatures approach their original inlet temperatures. When the number of tubes is studied it can be seen that the effectiveness increases with the number of tubes till around 22 tubes when there is a transition to laminar flow. When the outlet temperatures are studied as the number of tubes increase it can be seen that the outlet temperatures converge towards each other (towards an equilibrium state of equal outlet temperatures). Finally when the length of the individual heat exchanger pipe is studied the longer the pipe, the higher effectiveness the overall system has. Finally Option B provides the highest effectiveness as compared to Option A because of the affect of the Reynolds number on the varying cold water rate.

To find the best combination of all these variables the design team wants to maximize the mass flowrate through the system while also maintaining the correct output temperatures. Due to the constraints of the design having to have a cold outlet temperature of at least 330K and a hot outlet temperature of at least 290K, the mass flow needed to be relatively low as seen in the temperature figures below. Also, when comparing the effectiveness above between Option A and Option B, Option B overall is the better option because it resulted in a higher effectiveness using the same number of tubes. Due to the constraints of the problem, the heat exchanger can only have a max of 8 pipes can the pipes cannot exceed more than 10 meters in length.

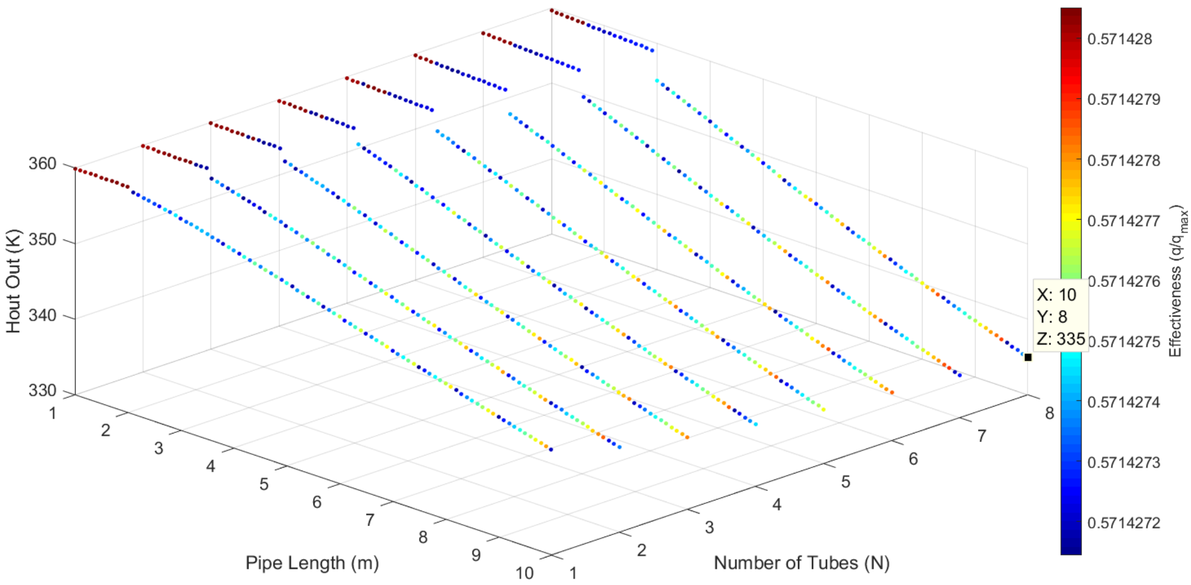
However, after compiling all the data and inputting the problem constraints, the results found are the complete opposite. The configuration with the greatest mass flowrate while maintaining the constraints is actually Option A, as seen in the figures below.



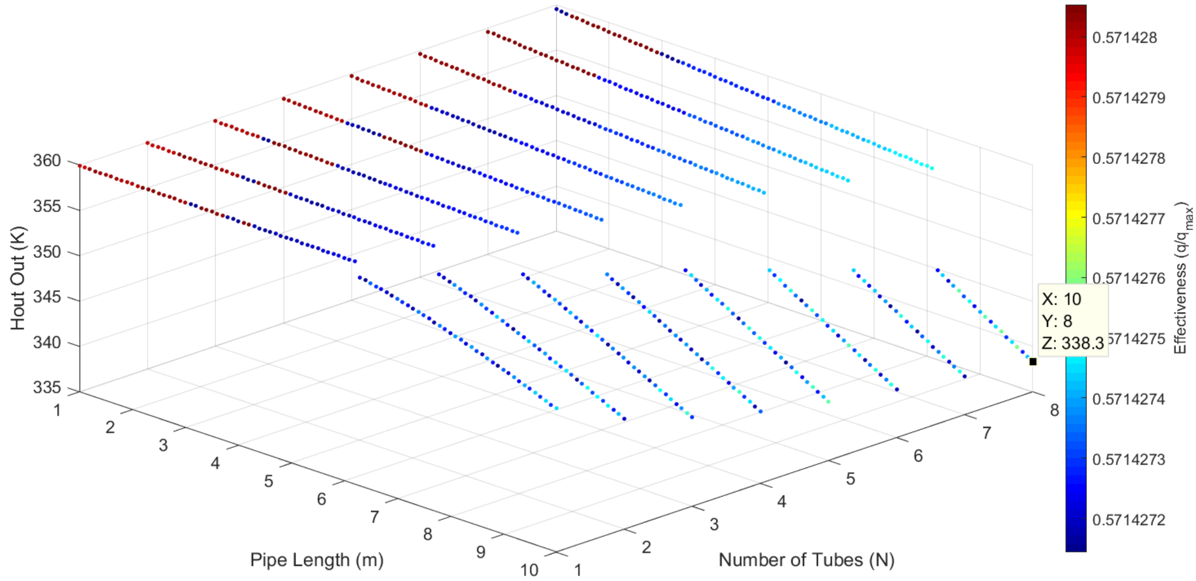
(a) Option A - Mass Flowrates



(b) Option B - Mass Flowrates

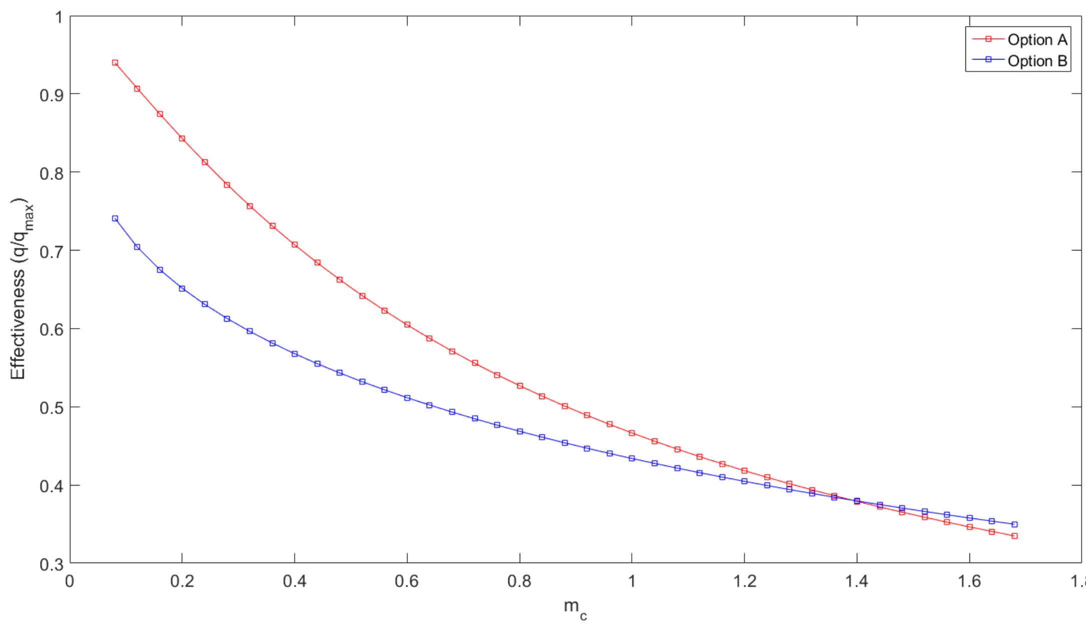


(c) Option A - Total Effectiveness



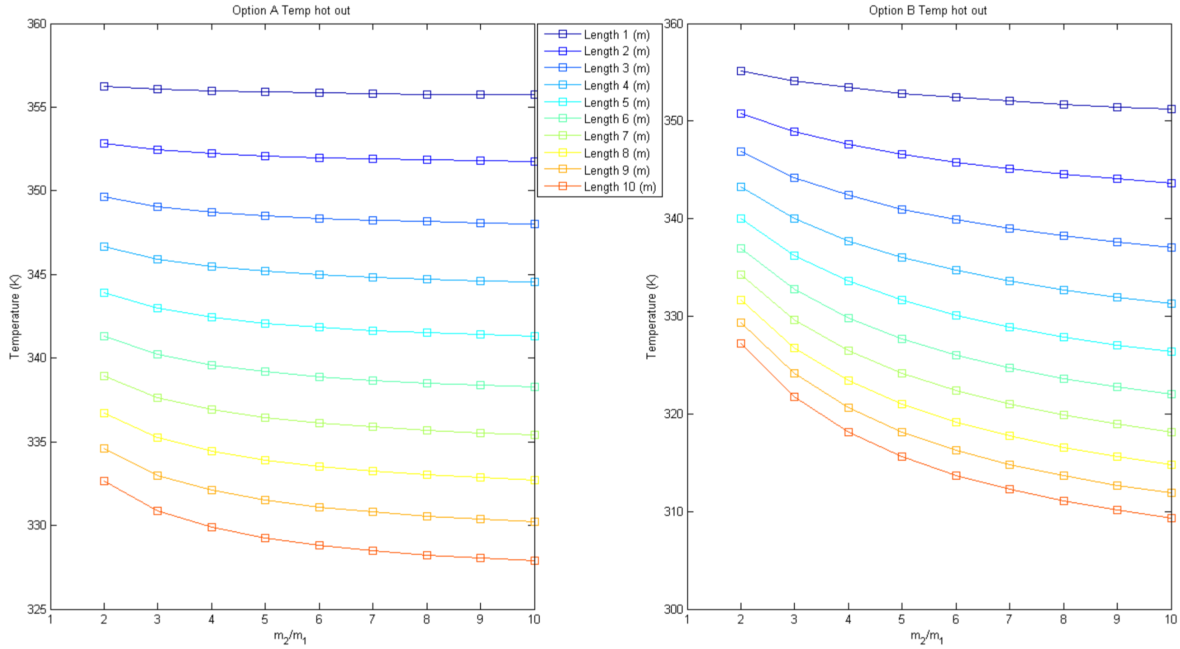
(d) Option B - Total Effectiveness

Also, the previous results showed that Option B would be the better configuration yielding a greater effectiveness, but that figure was produced with a mass flow ratio of 2:1. When the mass flow ratio became less than 1 however, the effectiveness in Option A is actually better than the effectiveness in Option B. Below is a figure that plots how the effectiveness changes with the cold mass flowrate. With a maximum flowrate of 1.064 for Option A and 0.923 for Option B it can be see at these values Option A has a higher efficiency.



In summary, the design team recommends the construction of a heat exchanger with 8 pipes and with a length of 10. The team recommends that it is configured as Option A with waste water traveling on the outer tube and the cold water traveling in the inner tube. This is backed by the 4D plot comparing the pipe length, number of tubes, Hout, and the cold mass flowrate.

7 Appendix



Option A and Option B hot outlet temperatures plotted with varying tube length.

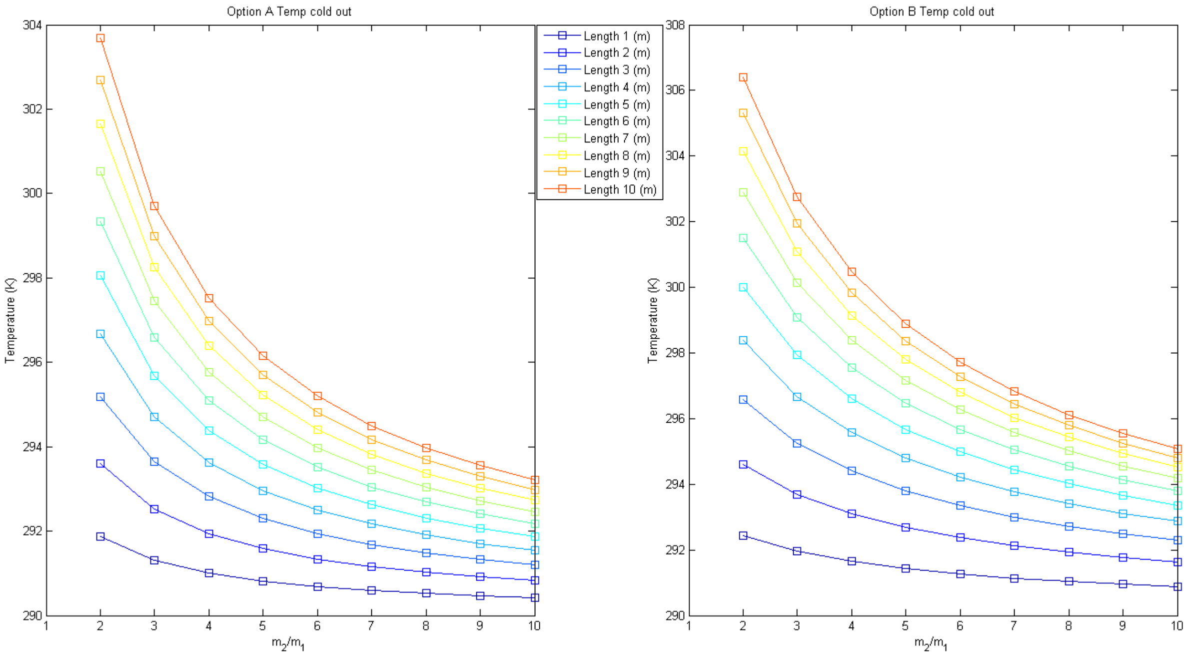


Figure 1

Option A and Option B cold outlet temperatures plotted with varying tube length.