CS 374 Fall 2015
Homework 0
Problem 1

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1. Prove that for any positive integer n and any set $X \subseteq \{1, 2, ..., 2n\}$ such that $|X| = n + 1$, there exist two distinct elements a, b in X such that a is a multiple of b.

We know from the original problem statement that any integer can be written as the product of an odd number and a power of 2.

For the set X, let us create at most n bins that correspond to the odd numbers in X. There are n bins at most because there are $2n$ elements in the set X and odd numbers are half of them. $2n/2 = n$.

Each member of X can be represented as the product of an odd number and a power of 2. This means each member of X can be assigned to a bin based on the product of the *largest odd number corresponding to the bins* that fits into that member and a power of 2. The odd number will be its bin.

Note that selecting $n + 1$ members from n bins means that we will have to draw from one bin more than one time, by the Pigeonhole Principle. Drawing from a bin more than once means that we will choose a multiple of a number already chosen from that bin.

Thus, there exist two distinct elements a, b in X such that a is a multiple of b, for any positive integer n and any set $X \subseteq \{1, 2, ..., 2n\}$ such that $|X| = n+1$.

Citations:

- Nitesh Nath
- Alek Festekjian
- Margaret Fleck's book