**Algorithms — CS 102 — Spring 2014**
**Instructor: Achlioptas**
Homework 3, Network Flows

# 1 Dining

Cows are such finicky eaters. Each cow has a preference for certain foods and drinks and will consume no others. Farmer Mike has prepared fabulous foods and drinks for his cows, but he forgot to check them against their preferences. Now he needs to know if it's possible to give each cow a complete meal with what he has prepared. Specifically, he has cooked $f$ dishes and prepared $d$ drinks. For each of his $n$ cows he has two lists, one of the acceptable foods $F_i$, and one of acceptable drinks $D_i$. Farmer Mike must assign a dish and a drink to each cow. Each dish and each drink can only be consumed by one cow.

Propose an efficient algorithm that determines whether it is possible to give every cow a complete meal.

# 2 Outsourcing

Farmer Mike has decided to get into the social games market. He's trained his cows as programmers and is ready to start churning out low-quality generic games!

Farmer Mike has accepted $n$ projects from companies that want games and needs to assign his $m$ bovine programmers to the projects. Fortunately, cows are great multitaskers, and can work on up to 3 projects at once. On the other hand, not every cow has the right skillset for every project. Each cow $i$ has a subset $P_i$ of the $n$ projects that it is able to work on. Additionally, cows are not very reliable, and so Farmer Mike wants to assign $k$ cows to each project to have some redundancy. Finally, each cow $i$ has a color $c_i$. All cows of the same color have a tendency to miss the same bugs[1], so Farmer Mike wants to guarantee that for each project, not all assigned cows have the same color.

Can Farmer Mike fulfill all of his contracts under all these constraints? That is, give an algorithm that determines is there is an assignment of cows to projects such that each cow $i$ only works on projects in its set $P_i$, at least $k$ cows are working on each project, no cow is assigned to more than 3 projects, and no project has only one color of cow working on it.

# 3 Problem 7.22 from textbook

# 4 Problem 7.27 from textbook

# 5 Anything for a good bath

The ancient city of Rome had a complex network of aqueducts delivering water to public buildings and to those residents important enough to have plumbing in their homes. Suppose some new baths are being built, so the Roman engineers need to increase the maximum water flow that their network of aqueducts can carry - however, they've pillaged too few barbaric villages recently, so the engineers must keep costs down by replacing only one aqueduct segment. That is, if $G$ is the directed graph representing the network of aqueducts, they are only allowed to increase the capacity of a single arc in the entire graph.

[1] Now, you didn't see that coming, did you?

Let $f$ be a max flow for $G$ of value $v(f)$, let $G_r(f)$ be the corresponding residual graph, and let $(A, B)$ be a min cut where $A$ is the set of vertices reachable from the source $s$ in $G_r(f)$.

Design a procedure to determine if there exists a single segment (arc) that the engineers could replace with a new higher-capacity segment, so that in the resulting graph the value of the maximum flow would be strictly greater than $v(f)$. If it is possible, your procedure should identify the segment, otherwise it should report that no such segment exists.

Given the max flow $f$ your procedure should run in time $O(|E|)$, where $E$ is the set of arcs of $G$. Moreover, you should prove your procedure's correctness.