Midterm Microeconomics 2024

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This version: November 2024

Problem 1. (30 points) Consumer Theory with the Weak Axiom of Revealed Preference. WARP.

Let $X = \mathbb{R}^L_+$ be the standard commodity space with $L \geq 2$.

Define a preference function $r : X \times X \to \mathbb{R}$ associated with a preference relation $\succeq \subseteq X \times X$, such that $r(x, y) \geq 0$ if and only if $x \succeq y$ for any $x, y \in X$ (and $r(x, y) > 0$ if $x \succ y$). Notice that $\succeq$ may not be rational.

Consider the following particular model of consumer behavior. For each $j \in \{1, \cdots, J\}$, let U_j be a a finite collection of utilities such that $u \in U_j$, is $u : X \to \mathbb{R}$ it's a utility function that is continuous, strictly monotone and concave. Now, consider a consumer that has the following preferences (multiple-utility–MU–model):

$$r(x, y) = \max_{j \in \{1, \cdots, J\}} \min_{u \in U_j} (u(x) - u(y)).$$

Intuitively, this MU model says that the consumer does not have a single utility to compare between any two consumption bundles, it may have many such utilities. In order to decide if she likes x better than y, this consumer first picks the worst case over a particular criterion U_j $\min_{u \in U_j}(u(x) - u(y))$, and then if there is at least one criterion (some j) for which $\min_{u \in U_j}(u(x) - u(y)) \geq 0$ the comparison is favorable to x with respect to y, then it will declare that $r(x, y) \geq 0$ (i.e., the max over j is equivalent to the previous statement).

Let $\mathcal{A} \subseteq 2^X \setminus \emptyset$ be a collection of menus/choice sets

We define a choice correspondence $c : \mathcal{A} \to 2^X \setminus \emptyset$ by $c(A) \subseteq A$. We say that a choice correspondence admits a MU preference function rationalization if

$$c(A) = \{a \in A : r(a, b) \geq 0 \forall b \in A\},$$

where r is a MU such that $r(x, y) = \max_{j \in \{1, \cdots, J\}} \min_{u \in U_j}(u(x) - u(y))$ for any $x, y \in X$.

Let $\succeq^D$ be defined as the direct revealed preference such that $x \succeq^D y \iff x \in c(A)$ and $y \in A$, and $x \succ^D y \iff x \in c(A)$ and $y \in A$ but $y \notin c(A)$. We say WARP holds if $x \succeq^D y$ implies not $y \succ^D x$.

1. (10 points) Show that associated preferences $\succeq$ to the MU preference function above may fail to be rational. Hint: Build a particular model that fails transitivity or completeness, that it's a MU preference function. You can simplify the problem assuming for this numeral only that X is finite and that $\mathcal{A}$ consists of tuples and triples $\{x, y\}$, $\{x, y, z\}$.

2. (10 points) Show that when the multiple criteria are coherent, namely $U_j \cap U_i \neq \emptyset$, then a choice correspondence that admits a MU preference function rationalization satisfies WARP. Namely, if $x \succeq^D y$ then it cannot be that $y \succ^D x$.

3. (10 points) Partial converse. Assume now that $\mathcal{A}$ is such that it contains every tuple and triple choice set such that $\{x, y\}, \{x, y, z\} \in \mathcal{A}$ for every $x, y, z \in X$. Show that if a choice correspondence c defined on this collection of choice sets satisfies WARP then there is a MU preference function that rationalizes it. For this exercise you can assume that the preference relation that rationalizes this data set is a continuous preference relation and that $X \subseteq \mathbb{R}^L$. (Hint: note that we are assuming that the choice correspondence it's nonempty).

Problem 2. (50 points) Quasilinear Welfare Analysis.

Consider a finite dataset of observations $O^T = \{p^t, x^t\}_{t=1}^T$.

Consider a consumer that maximizes a quasilinear utility function, such that:

$$x^t \in arg \max_{x \in X} u(x) - p^t \cdot x,$$

for a given price vector $p^t \in \mathbb{R}^L_{++}$, where u is a continuous, concave, strictly increasing utility function, and $X \subseteq \mathbb{R}^L_+$.

1) (10 points) Show that the dataset O^T is such that there exists numbers $v^t \in \mathbb{R}$ such that:

QuasilinearAfriat Inequalities

$$v^t - v^s \geq p^t \cdot (x^t - x^s) = p^t(x^t - x^s)$$

for all $t, s \in \{1, \cdots, T\}$ (Note: the dot product notation is omitted as indicated above).

2) (10 points) Show that we can bound below the utility difference between any two bundles x^t, x^s, $u(x^t) - u(x^s)$, by:

$$u(x^t) - u(x^s) \geq p^t(x^t - x^1) + p^1(x^1 - x^2) + \cdots + p^n(x^n - x^s),$$

for any arbitrary subsequence of data points $(p^i, x^i)_{i=1}^n \subseteq O^T$. (This is useful for welfare analysis).

3) (10 points) The indirect utility of a price p^t is given by:

$$v(p^t) = \max_x (u(x) - p^t x).$$

Show that if $p^s x^t \geq (>) p^t x^t$ then $v(p^t) - v(p^s) \geq (>) 0$.

4) (10 points) Show that if O^T satisfies the Quasilinear Afriat Inequalities, there is a concave, strictly increasing continuous utility function u, such that:

$$x^t \in arg \max_{x \in X} (u(x) - p^t \cdot x),$$

for a given $t \in \{1, \cdots, T\}$.

5) (10 points) Say that inflation rate is such that, and out-of-sample (not in O^T) price p^{T+1} is such that $x^1(p^2 - p^1) \geq 0, x^2(p^3 - p^2) \geq 0, \cdots, x^T(p^{T+1} - p^T) \geq 0$. What can I conclude in terms of welfare for this individual: Is she better-off or worse-off in $T+1$, with respect to every $t \in \{1, \cdots, T\}$?

Problem 3. (Attraction Effect and Choice Overload)

(20 points) (Too much choice/Choice overload and the Attraction Effect) Propose a model for consumer behavior that captures the following stylized facts.

- Choice Overload: Consumers are more likely to choose an outside option (that we equate in this problem to not choosing) when the size of the menu is too big. Hint: The choice set is $X \cup \{o\}$ where o is an outside option (not in X). Let X be finite. Menus are of the form $A \cup \{o\}$ where $A \subseteq X$, i.e., the outside option is always available to the consumer.

- Attraction Effect:The **attraction effect** is the observation that if we add an alternative to a menu, then some existing item becomes more attractive to the consumer and is picked by her. This introduced object is usually called the decoy.

In formal terms, $b^- \in X$ be the decoy, so we are going to study its impact in the choice of the object of interest $b \in X$. The observation is the following:

$$b \in c(A \cup \{b^-\})$$

$$b \notin c(A), b \in A.$$

In probabilities, it means that

$$p(b, A \cup \{b^-\}) > p(b, A).$$

1. You should describe with words what is the consumer choice algorithm. The model can be deterministic or stochastic, your choice.
2. Then you should describe what is "new" in your model and how that relates to some behavioral or bounded rationality feature we learned in class, and how it differs from utility maximization.
3. Finally you should write the behavioral maximization problem and show how your new model predicts the two stylized facts. Also show, that the rational benchmark cannot deal with this models (neither random utility or deterministic utility maximization can predict the choice overload or attraction effect).
4. You should try to explain the model to me in the best way you can, by using words, diagrams and examples. However, you should have also formality, make the effort to write down the model using math, this is the only way to get full credit.
5. Describe what other testable implications your proposed model has.

Grading Criterion

1. Work group is not allowed, if I find out that several of you have essentially the same model, you will receive the total grade of this question divided by the number of students that share the same response.
2. The last question will be graded on the following 5 items: Logical consistency of the decision algorithm. Clarity of the explanation of the decision algorithm. Degree of success in explaining the behavioral effects. Clarity in the explanation of the behavioral effect using the decision algorithm. Ability of your model to be able to have testable implications (not everything goes).